

# Category Theory by Example

Ivan Murashko, Alexey Radkov, Marat Minshin

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# Introduction

You just looked at yet another introduction to Category Theory. The subject mostly consists of a lot of definitions that are related each others and we wrote the book to collect all of them in one place to be easy checked and updated in future when we decide to refresh our knowledge about the field of math. Therefore the book was written mostly for our category theory studying purposes but we will appreciate if somebody else find it useful.

The topics(chapters) cover the base definitions (??, ?? and ??), ??, ??, ?? and also include important results from the category theory such as Yoneda's lemma (see chapter 1) and Curry-Howard-Lambek correspondence (see ??).

There are a lot of examples in each chapter. The examples cover different category theory application areas. We assume that the reader is familiar with the corresponding area and the example(s) can be passed if not. I.e. anyone can choose the suitable example(s) for (s)he.

The most important examples are related to the set theory. The set theory and category theory are very close related. Each one can be considered as an alternative view to another one.

There are a lot of examples from programming languages which include Haskell, Scala, C++. The source files for programming languages examples (Haskell, C++, Scala) can be found on github repository [8].

The examples from physics are related to quantum mechanics that is the most known for me. For the examples We were inspired by the Bob Coecke article [1].

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# Chapter 1

## Yoneda's lemma

TBD

### 1.1 Examples

#### 1.1.1 Quantum mechanics

Flori interpretation of quantum mechanics

TBD





# Bibliography

- [1] Coecke, B. Introducing categories to the practicing physicist / Bob Coecke. — 2008. — <https://arxiv.org/abs/0808.1032>.
- [2] Murashko, I. Category theory. — <https://github.com/ivanmurashko/articles/tree/master/cattheory/src>. — 2018.



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