Temă Laborator 5-6

Seminar

Pb 1 d) $X=\begin{pmatrix}-3 & 6\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$, $Y=\begin{pmatrix}e & e^3\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}$

$$\left.\begin{array}{l} 2-X \Leftrightarrow -X+2 \\ -X=(-1)\cdot X = X\cdot(-1)=\begin{pmatrix}3 & -6\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}\end{array}\right\} \Rightarrow -X+2=\begin{pmatrix}3+2 & -6+2\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix} \Rightarrow$$

$$\Rightarrow -X+2=\begin{pmatrix}5 & -4\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$$

$$X^3=\begin{pmatrix}(-3)^3 & 6^3\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}=\begin{pmatrix}-27 & 216\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$$

$$\cos\left(\frac{\pi}{6}\cdot X\right)=\cos\begin{pmatrix}\frac{\pi}{6}\cdot(-3) & \frac{\pi}{6}\cdot 6\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}=$$

$$=\cos\begin{pmatrix}-\frac{\pi}{2} & \pi\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}=\begin{pmatrix}\cos(-\frac{\pi}{2}) & \cos\pi\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$$

$$=\begin{pmatrix}\cos(\frac{\pi}{2}) & \cos\pi\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}=\begin{pmatrix}0 & -1\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}$$

$$Y^{-1}=\begin{pmatrix}\frac{1}{e} & \frac{1}{e^3}\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}$$

$$\ln Y=\begin{pmatrix}\ln(e) & \ln(e^3)\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}=\begin{pmatrix}1 & 3\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}$$

Pb 2 d) $X\cdot Y=\begin{pmatrix}-3 & 6\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}\cdot\begin{pmatrix}e & e^3\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}=\begin{pmatrix}-3e & -3e^3 & 6e & 6e^3\\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32}\end{pmatrix}$

$$\left.\begin{array}{l} \frac{X}{Y}=X\cdot Y^{-1} \\ Y^{-1}=\begin{pmatrix}\frac{1}{e} & \frac{1}{e^3}\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}\end{array}\right\} \Rightarrow \frac{X}{Y}=\begin{pmatrix}-3 & 6\\ \frac{1}{8} & \frac{7}{8}\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{e} & \frac{1}{e^3}\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}=$$

$$= \begin{pmatrix} -\frac{3}{e} & -\frac{3}{e^3} & \frac{6}{e} & \frac{6}{e^3} \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$|X-Y^2|$

$$Y^2 = \begin{pmatrix} e^2 & e^6 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

$$X-Y^2 = \begin{pmatrix} -3-e^2 & -3-e^6 & 6-e^2 & 6-e^6 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$|X-Y^2| = \begin{pmatrix} |-3-e^2| & |-3-e^6| & |6-e^2| & |6-e^6| \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$= \begin{pmatrix} -(-3-e^2) & -(-3-e^6) & -(6-e^2) & -(6-e^6) \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$= \begin{pmatrix} e^2+3 & e^6+3 & e^2-6 & e^6-6 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

Pb 3| d) $X: \begin{pmatrix} -1 & 1 \\ 2p & q \end{pmatrix}$ $Y: \begin{pmatrix} 0 & 1 \\ q & 7q \end{pmatrix}$

X v.a. bine definită $\Rightarrow \begin{cases} -1; 1 \in \mathbb{R} \\ 2p \geq 0 \Rightarrow p \geq 0 \\ q \geq 0 \\ 2p+q=1 \end{cases} \Rightarrow \begin{cases} p \geq 0 \\ q \geq 0 \end{cases}$

Y v.a. bine definită $\Rightarrow \begin{cases} 0; 1 \in \mathbb{R} \\ q \geq 0 \\ 7q \geq 0 \Rightarrow q \geq 0 \\ q+7q=1 \Rightarrow 8q=1 \Rightarrow q=\frac{1}{8} \geq 0 \end{cases} \Rightarrow q \geq 0$

$2p+q=1 \Rightarrow 2p = 1-\frac{1}{8} \Rightarrow 2p = \frac{7}{8} \Rightarrow p = \frac{7}{16} \geq 0$

$X: \begin{pmatrix} -1 & 1 \\ \frac{7}{8} & \frac{1}{8} \end{pmatrix}$ $Y: \begin{pmatrix} 0 & 1 \\ \frac{1}{8} & \frac{7}{8} \end{pmatrix}$

4) d) $X : \begin{pmatrix} -3 & 6 \\ \frac{1}{8} & \frac{7}{8} \end{pmatrix}$, $Y : \begin{pmatrix} e & e^3 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

$\mathbb{P}(X \cdot Y \le e^4) = ?$

$X \cdot Y = \begin{pmatrix} -3e & -3e^3 & 6e & 6e^3 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$ $X \cdot Y^{-1} = \begin{pmatrix} -\frac{3}{e} & -\frac{3}{e^3} & \frac{6}{e} & \frac{6}{e^3} \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$

$$\left.\begin{array}{l} e \simeq 2,72 \Rightarrow 6 > 2 \cdot e \\ 6e^3 > e^4 \Rightarrow 6 > e \end{array}\right\} \Rightarrow 6e^3 > e^4 \ (A)$$

$\mathbb{P}(X \cdot Y \le e^4) = \frac{1}{32} + \frac{3}{32} + \frac{7}{32} = \frac{11}{32}$

$\mathbb{P}(X \cdot Y \ge 7 \mid X < 0) = \frac{\mathbb{P}((X \cdot Y \ge 7) \cap (X < 0))}{\mathbb{P}(X < 0)} = \frac{0}{\frac{1}{8}} = 0$

$\mathbb{P}(X \cdot Y < 9 \mid Y > 3) = \frac{\mathbb{P}((X \cdot Y < 9) \cap (Y > 3))}{\mathbb{P}(Y > 3)} = \frac{\frac{3}{32}}{\frac{3}{4}} = \frac{\cancel{3}}{\cancel{32}_{8}} \cdot \frac{4}{\cancel{3}} = \frac{1}{8}$

$\mathbb{P}\left(\frac{X}{Y} < 1\right) = \mathbb{P}(X \cdot Y^{-1} < 1) = \frac{1}{32} + \frac{3}{32} + \frac{21}{32} = \frac{25}{32}$

$\mathbb{P}(|X - Y^2| \ge 3) = \frac{1}{32} + \frac{3}{32} + \frac{21}{32} = \frac{25}{32}$

$|X - Y^2| = \begin{pmatrix} e^2+3 & e^6+3 & e^2-6 & e^6-6 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$

VI $\mathbb{P}\left(\frac{X}{Y} < |X - Y^2|\right) = \mathbb{P}(X \cdot Y^{-1} < |X - Y^2|) = \frac{1}{32^2} + \frac{3}{32^2} + \frac{7}{32^2} + \frac{21}{32^2} + \frac{3}{32^2} +$
$+ \frac{9}{32^2} + \frac{21}{32^2} + \frac{63}{32^2} + \frac{7}{32^2} + \frac{21}{32^2} + \frac{147}{32^2} + \frac{21}{32^2} + \frac{63}{32^2} + \frac{147}{32^2} + \frac{441}{32^2}$

$-\frac{3}{e} < e^2+3$; $-\frac{3}{e^3} < e^2+3$; $\frac{6}{e} < e^2+3$; $\frac{6}{e^3} < e^2+3$

(1) $-\frac{3}{e} < e^6+3$; $-\frac{3}{e^3} < e^6+3$; $\frac{6}{e} < e^6+3$; $\frac{6}{e^3} < e^6+3$

$-\frac{3}{e} < e^2-6$; $-\frac{3}{e^3} < e^2-6$; $\frac{6}{e} > e^2-6$; $\frac{6}{e^3} < e^2-6$

$-\frac{3}{e} < e^6-6$; $-\frac{3}{e^3} < e^6-6$; $\frac{6}{e} < e^6-6$; $\frac{6}{e^3} < e^6-6$

$\mathbb{P}(X \cdot Y^{-1} < |X - Y^2|) = \frac{1+3+7+21+3+9+21+63+7+21+147+21+63+147+441}{1024}$

$= \frac{975}{1024}$

VII $\mathbb{P}\left(\frac{X}{Y} < |X - Y^2|\right) = \mathbb{P}(X \cdot Y^{-1} < |X - Y^2|) = \mathbb{P}(X \cdot Y^{-1} - |X - Y^2| < 0)$

$$X\cdot Y^{-1}-|X-Y^2|=\begin{pmatrix} -\frac{3}{e} & -\frac{3}{e^3} & \frac{6}{e} & \frac{6}{e^3} \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}-\begin{pmatrix} e^2+3 & e^6+3 & e^2-6 & e^6-6 \\ \frac{1}{32} & \frac{3}{32} & \frac{7}{32} & \frac{21}{32} \end{pmatrix}$$

$$=\left(\begin{matrix} -\frac{3}{e}-e^2-3 & -\frac{3}{e}-e^6-3 & -\frac{3}{e}-e^2+6 & -\frac{3}{e}-e^6+6 & -\frac{3}{e^3}-e^2-3 \\ \frac{1}{32^2} & \frac{3}{32^2} & \frac{7}{32^2} & \frac{21}{32^2} & \frac{3}{32^2} \end{matrix}\right.$$

$$\begin{matrix} -\frac{3}{e^3}-e^6-3 & -\frac{3}{e^3}-e^2+6 & -\frac{3}{e^3}-e^6+6 & \frac{6}{e}-e^2-3 & \frac{6}{e}-e^6-3 \\ \frac{9}{32^2} & \frac{21}{32^2} & \frac{63}{32^2} & \frac{7}{32^2} & \frac{21}{32^2} \end{matrix}$$

$$\begin{matrix} \frac{6}{e}-e^2+6 & \frac{6}{e}-e^6+6 & \frac{6}{e^3}-e^2-3 & \frac{6}{e^3}-e^6+3 & \frac{6}{e^3}-e^2+6 \\ \frac{49}{32^2} & \frac{147}{32^2} & \frac{21}{32^2} & \frac{63}{32^2} & \frac{147}{32^2} \end{matrix}$$

$$\left.\begin{matrix} \frac{6}{e^3}-e^6+6 \\ \frac{441}{32^2} \end{matrix}\right)$$

Utilizând ① obținem:

$$\mathbb{P}(X\cdot Y^{-1}-|X-Y^2|<0)=\frac{1+3+7+21+3+9+21+63+7+21+147+21+63+147+441}{1024}=\frac{975}{1024}$$