QRKT-GAN: Neural ODE-Inspired Generative Adversarial Network with Numerical Runge-Kutta Methods for Quantum Visual Transformer-Based Generator and Discriminator

Cătălin-Alexandru Rîpanu

Supervisor: Șl. dr. ing. Dumitru-Clementin Cercel

Faculty of Automatic Control and Computers,
National University of Science and Technology POLITEHNICA Bucharest

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Context

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for such performance . . . millions to billions of neurons are required.

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Numerous solutions have been developed to mitigate this problem:

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1. Grid Search and Random Search [2, 3]

2. Group Sparsity Regularizers [4]

3. Dimensionality Reduction and Kernel-Sharing [5, 6]

4. Network Weights Splitting [7]

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unfortunately...

these **classical** methods have inherent drawbacks in their logic.

Generative Adversarial Networks (GANs)
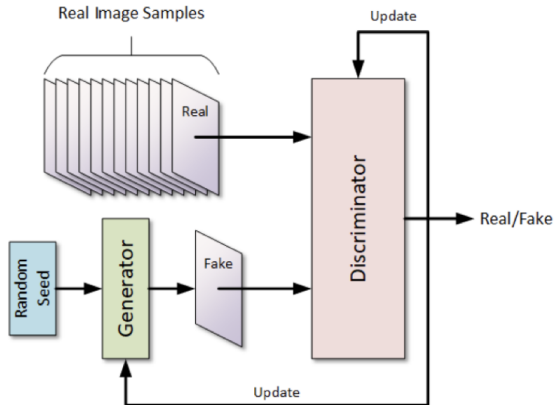
Generative Adversarial Networks (GANs)

Figure: The Generative Adversarial Network Architecture [9]
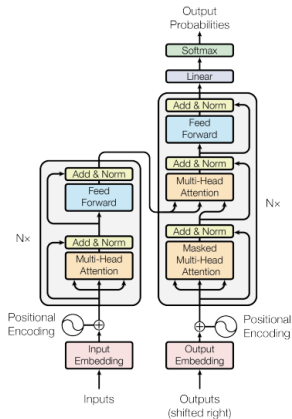
The Transformer

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Figure: The Transformer Architecture [10]

The Visual Transformer
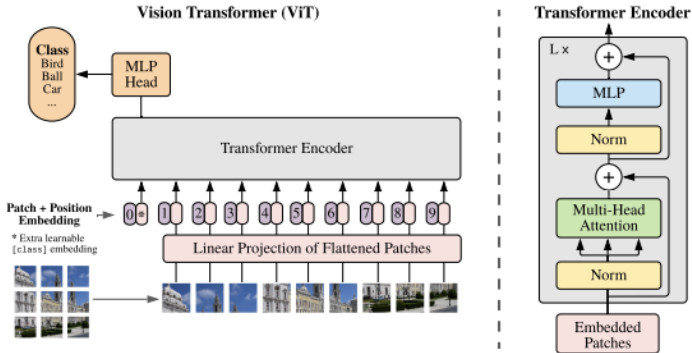
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Figure: The Visual Transformer Architecture [10, 11]

The Visual Transformer (2)

Thus, the transformation at each layer is defined as:

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$$W = X + \text{MHA}(\text{Norm}(X), \text{Norm}(X), \text{Norm}(X)) \qquad (1)$$

$$X' = W + \text{MLP}(\text{Norm}(W)) \qquad (2)$$

where $X, X' \in \mathbb{R}^{N \times D}$.

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And. . . the Multi-Head Attention Mechanism can be expressed as:

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$$\text{Attention}(V, K, Q) = \text{softmax}\left(\frac{QK^T}{\sqrt{D_k}}\right) V \tag{3}$$

$$\text{MHA}(V, K, Q) = \text{Concat}(\text{single\_head}_i) W^O, i = 1 : h \tag{4}$$

$$\text{single\_head}_i = \text{Attention}(VW_i^V, KW_i^K, QW_i^Q) \tag{5}$$

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where:

- $W_i^K \in \mathbb{R}^{D_x \times D_k},\ W_i^V \in \mathbb{R}^{D_x \times D_v},\ W_i^Q \in \mathbb{R}^{D_x \times D_k}$
- $W^O \in \mathbb{R}^{hD_v \times D_x}$

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$$\hat{y}_i^m = y_i^m + G(y_i^m, Y^m), \quad 1 \le i \le L, \tag{6}$$

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The output $\hat{Y}^m = [\hat{y}_1^m, \hat{y}_2^m, \ldots, \hat{y}_L^m]$ is then fed to the MLP:

$$y_i^{m+1} = \hat{y}_i^m + H(\hat{y}_i^m), \quad 1 \leq i \leq L, \tag{7}$$

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Over the time interval $[m, m+1]$, using Lie-Trotter decomposing method [13, 14]:

$$\frac{dy_i}{dt} = H(y_i) + G(y_i, Y) \tag{8}$$

Neural Runge-Kutta Method (RK4)

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$$y_i(t+1) = y_i(t) + \sum_{j=1}^{n} \gamma_j F_{ij} \tag{9}$$

$$F(y_i, Y) = F_i \tag{10}$$

$$F_{ij} = F_i(y_i + \sum_{p=1}^{j-1} \beta_{jp} F_{ip}, Y) \tag{11}$$

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Thus

$$y_i(t+1) = y_i(t) + \frac{1}{6}(F_{i1} + 2F_{i2} + 2F_{i3} + F_{i4}) \tag{12}$$

The Quantum Visual Transformer
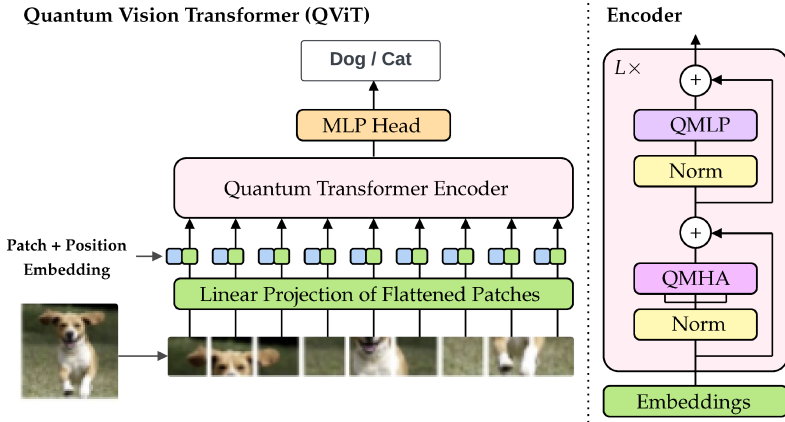
The Quantum Visual Transformer

Figure: Visual Quantum Transformer [15]

Quantum Gates

Quantum Gates

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad\qquad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$R_X(\theta) = \begin{bmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{bmatrix} \quad R_Z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

$$R_Y(\theta) = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$
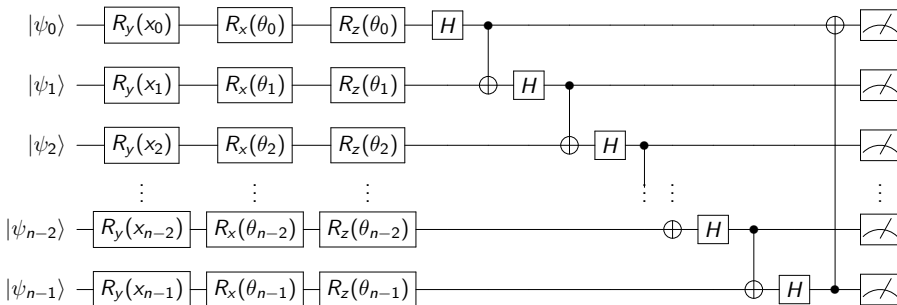
The Variational Quantum Circuit

One can use such techniques. . .

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One can use such techniques. . . only in another reality.

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Figure: The Variational Quantum Circuit used in QRKT-GAN

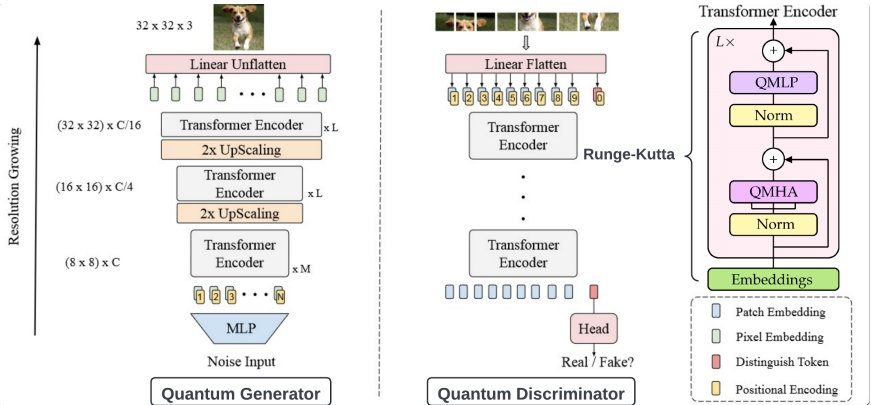# Proposed Solution (QRKT-GAN)

Proposed Solution (QRKT-GAN)

Figure: The QRKT-GAN Architecture. Image inspired from [15, 16]

MNIST Classification

MNIST Classification
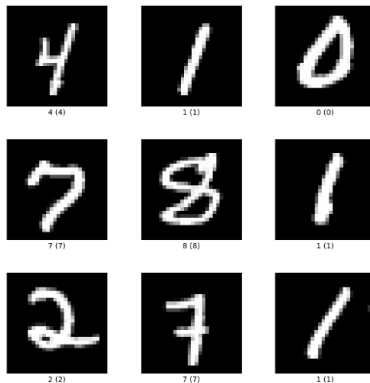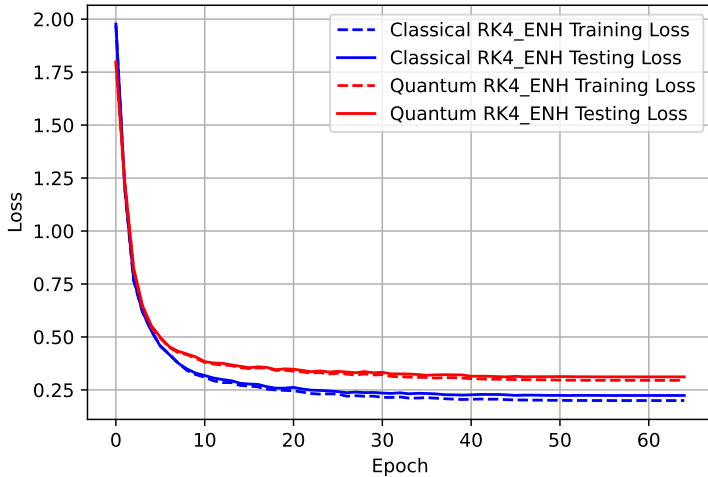

Figure: Examples from MNIST[1]

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[1]`https://www.tensorflow.org/datasets/catalog/mnist`

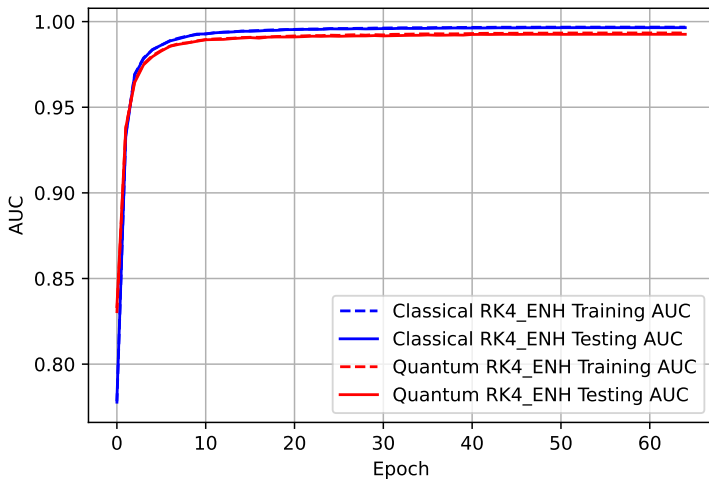Figure: Cross-entropy loss evolution during learning

Figure: AUC score evolution during learning

Configurations:

Configurations:

- **Patch Size**: 14
- **Hidden Size**: 6
- **Classical and Quantum ODE-Transformer Blocks**: 3
- **Classical and Quantum Attention Heads**: 2
- **Hidden QMLP Size**: 3

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ODE	Train Time (s)	Accuracy	F1 Score	Best AUC Epoch	# Parameters	# Qubits
RK4_ENH	1842.04	95%	95%	54	5971	-
QRK4_ENH	3539.44	91%	91%	48	3520	357

Table: MNIST metrics for the quantum and classical configurations

CIFAR-10 Classification

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[2]https://www.tensorflow.org/datasets/catalog/cifar10

CIFAR-10 Classification

Figure: Examples from CIFAR-10[2]

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[2]https://www.tensorflow.org/datasets/catalog/cifar10
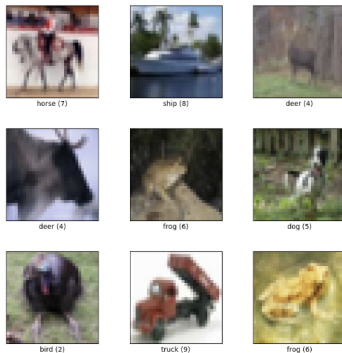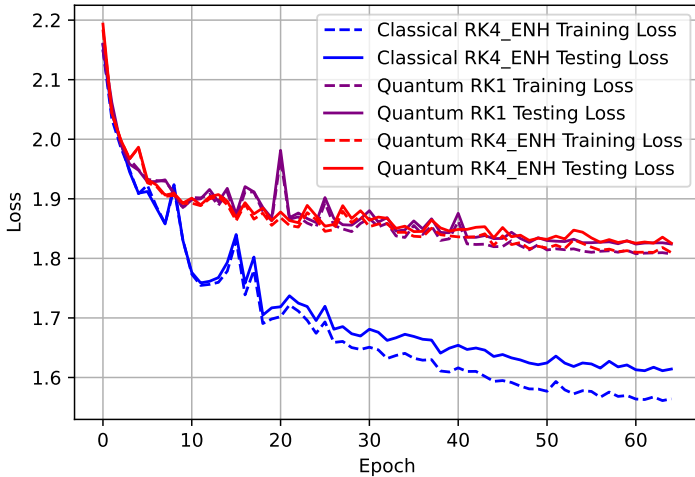
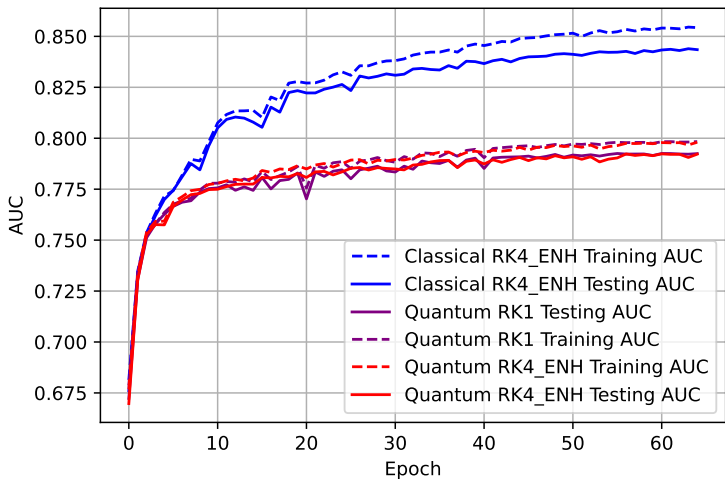Figure: Cross-entropy loss evolution during learning

Figure: AUC score evolution during learning

Configurations:

Configurations:

- **Patch Size**: 16
- **Hidden Size**: 12
- **Classical and Quantum ODE-Transformer Blocks**: 1
- **Classical and Quantum Attention Heads**: 6
- **Hidden QMLP Size**: 6

Configurations:

- **Patch Size**: 16
- **Hidden Size**: 12
- **Classical and Quantum ODE-Transformer Blocks**: 1
- **Classical and Quantum Attention Heads**: 6
- **Hidden QMLP Size**: 6

ODE	Train Time (s)	Accuracy	F1 Score	Best AUC Epoch	# Parameters	# Qubits
RK4_ENH	1685.04	42%	42%	65	33634	-
QRK4_ENH	16724.79	34%	33%	61	20590	390
QRK1	10909.40	33%	33%	56	20590	336

Table: CIFAR-10 metrics for the quantum and classical configurations

IMDb Classification

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[3]https://www.tensorflow.org/datasets/catalog/imdb_reviews

IMDb Classification

Label	Text
0 (neg)	"I have been known to fall asleep during films, but this is usually due to a combination of things including, really tired, being warm and comfortable on the settee and having just eaten a lot. However on this occasion I fell asleep because the film was rubbish [...]"
1 (pos)	"This is a film which should be seen by anybody interested in, effected by, or suffering from an eating disorder. It is an amazingly accurate and sensitive portrayal of bulimia in a teenage girl, its causes and its symptoms. The girl is played by one of the most brilliant young actresses working in cinema today, Alison Lohman, who was later so spectacular in 'Where the Truth Lies' [...]"

Table: Movie Reviews[3]

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[3]https://www.tensorflow.org/datasets/catalog/imdb_reviews
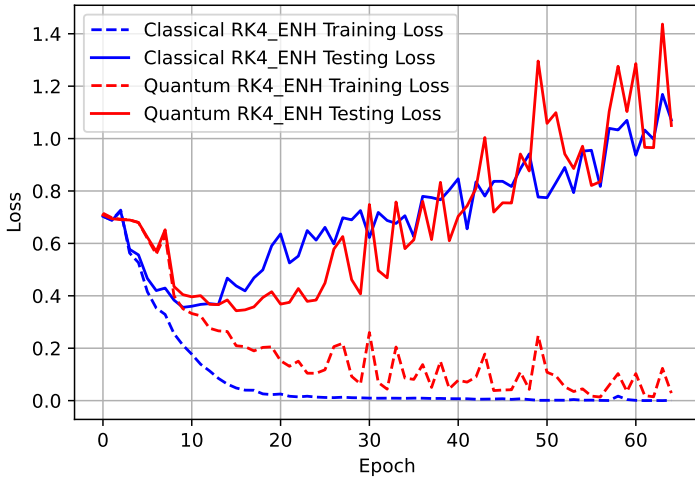
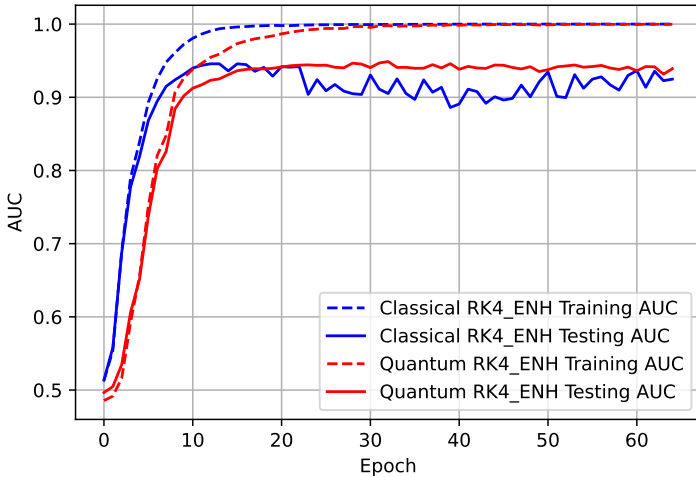Figure: Cross-entropy loss evolution during learning

Figure: AUC score evolution during learning

Configurations:

Configurations:

- **Max sequence length**: 512
- **Classical / Quantum Hidden Size**: 12 / 6
- **Classical / Quantum ODE-Transformer Blocks**: 1 / 1
- **Classical / Quantum Attention Heads**: 6 / 2
- **Classical / Quantum Hidden MLP Size**: 6 / 3

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- **Classical / Quantum ODE-Transformer Blocks**: 1 / 1
- **Classical / Quantum Attention Heads**: 6 / 2
- **Classical / Quantum Hidden MLP Size**: 6 / 3

ODE	Train Time (s)	Accuracy	F1 Score	Best AUC Epoch	# Parameters	# Qubits
RK4_ENH	3328.61	85%	85%	13	499316	-
QRK4_ENH	9033.13	**85**%	**85**%	33	**243896**	141

Table: IMDb metrics for the classical and quantum configurations

Synthetic Data Generation

Synthetic Data Generation

Metric	QRKT-GAN	TransGAN
Inception Score (IS)	74.89	52.31
Fréchet Inception Distance (FID)	66.78	46.97

Table: Performance Metrics for TransGAN and QRKT-GAN on CIFAR-10

Synthetic Data Generation

Metric	QRKT-GAN	TransGAN
Inception Score (IS)	74.89	52.31
Fréchet Inception Distance (FID)	66.78	46.97

Table: Performance Metrics for TransGAN and QRKT-GAN on CIFAR-10

Figure: Generated Images using QRKT-GAN

Technologies used for QRKT-GAN:

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- JAX [17] and Flax [18]
- Tensorflow Quantum [19]
- Qiskit [20]
- Pytorch [21]
- Tensor Circuit [22]

Conclusion

Keywords:

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- GANs
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$$|\text{Thank you!}\rangle$$

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