

Cod Hamminguz detector in corector de 0 errore

$A = [1100]$ - bit de informatie

$m = 4$ nr. bitilor de info

$$2^k \geq m + k + 1 \Rightarrow k$$

k - nr. bitilor de control

$$k = 3$$

$2^0, 2^1, \dots, 2^{k-1}$ - pozitii bitilor de control

pt. $k = 3$

$2^0, 2^1, 2^2 = 1, 2, 4$ - pozitii bitilor de control

$$V = [c_1 c_2 a_3 c_4 a_5 a_6 a_7]$$

$$a_3 = 1 \quad a_5 = 1 \quad a_6 = 0 \quad a_7 = 0$$

c_1, c_2, c_4 - bit de control

$n = m + k$ - nr. bitilor de transmitere

$$H \cdot V^T = 0 \Rightarrow c_1, c_2, c_4 = ?$$

$$H = [3 \times 7] \quad V = [1 \times 7]$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cc|c} & & \oplus \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{array}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ 1 \\ c_4 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} 0 \cdot c_1 \oplus 0 \cdot c_2 \oplus 0 \cdot 1 \oplus 1 \cdot c_4 \oplus 1 \cdot 1 \oplus 1 \cdot 0 \oplus 1 \cdot 0 = 0 \\ 0 \cdot c_1 \oplus 1 \cdot c_2 \oplus 1 \cdot 1 \oplus 0 \cdot c_4 \oplus 0 \cdot 1 \oplus 1 \cdot 0 \oplus 1 \cdot 0 = 0 \\ 1 \cdot c_1 \oplus 0 \cdot c_2 \oplus 1 \cdot 1 \oplus 0 \cdot c_4 \oplus 1 \cdot 1 \oplus 0 \cdot 0 \oplus 1 \cdot 0 = 0 \end{array}$$

$$\rightarrow c_4 = 1 \rightarrow c_2 = 1 \rightarrow c_1 = 0$$

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$$V = [0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0]$$

$$T_x \rightarrow R_x$$

$$V \quad E$$

E - cuvânt recepționat

$$E = [1 \times u] \quad [1 \times 7]$$

$$H \cdot E^T = z \Rightarrow \text{par. err.}$$

$$\textcircled{1} \text{ Dacă } z = 0 \Rightarrow \text{fără err.}$$

$$\textcircled{2} \text{ Dacă } z \neq 0 \Rightarrow \text{ } \} \text{ o err.}$$

la poziția dată de z

Ex:

$$1. \ E = [0 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0]$$

$$H \cdot E^T = z$$

$$z = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Err. pe poz. 2

(conversie din binar în zecimal a vectorului z).

$$E_c = [0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0]$$

$$2. \ E = [1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0]$$

$$H \cdot E^T = z \Rightarrow z = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

par. 3.

$$E_c = [1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0]$$

nu rezultă eroarea dublă //

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Cod Hamming detector de erori pare in corctor de o erori

$$A = [1100]$$

$$m = 4$$

$$2^K > m + K + 1$$

$$K = 3$$

$$V_2 = [C_0 \ C_1 \ C_2 \ a_3 \ C_4 \ a_5 \ a_6 \ a_7]$$

$$H_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$a_3 = 1 \quad a_5 = 1 \quad a_6 = 0 \quad a_7 = 0$$

C_0 - bit de paritate

C_1, C_2, C_4 - biti de control

$$\underline{H_2} \cdot \underline{V_2}^T = \underline{0}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ 1 \\ C_4 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1 \cdot C_4 \oplus 1 \cdot 1 \oplus 1 \cdot 0 \oplus 1 \cdot 0 = 0$$

$$1 \cdot C_2 \oplus 1 \cdot 1 \oplus 1 \cdot 0 \oplus 1 \cdot 0 = 0$$

$$1 \cdot C_1 \oplus 1 \cdot 1 \oplus 1 \cdot 1 \oplus 1 \cdot 0 = 0$$

$$1 \cdot C_0 \oplus 1 \cdot C_1 \oplus 1 \cdot C_2 \oplus 1 \cdot 1 \oplus 1 \cdot C_4 \oplus 1 \cdot 1 \oplus 1 \cdot 0 \oplus 1 \cdot 0 = 0$$

$$C_4 = 1 \quad C_2 = 1 \quad C_1 = 0 \quad C_0 = 0$$

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$$V_2 = [0 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0]$$

$$T_x \rightarrow R_x$$

$$\frac{1}{2} \quad E_2$$

$$H_2 \cdot E_2^T = Z_2$$

$$Z_2 = \begin{bmatrix} Z_c \\ \hline Z_p \end{bmatrix} \quad \begin{array}{l} Z_c = K \times 1 \\ Z_p = 1 \times 1 \end{array}$$

$$\textcircled{1} \quad \begin{array}{l} Z_c = 0 \\ Z_p = 0 \end{array} \quad \Rightarrow \quad \text{f erori}$$

$$\textcircled{2} \quad \begin{array}{l} Z_c = 0 \\ Z_p \neq 0 \end{array} \quad \Rightarrow \quad \exists \text{ o eroare pe poziția } \\ \text{bitului de paritate} \\ (E_{\text{corectat}})$$

$$\textcircled{3} \quad \begin{array}{l} Z_c \neq 0 \\ Z_p = 0 \end{array} \quad \Rightarrow \quad \exists \text{ mai multe erori} \\ \text{(se retransmite packet)}$$

$$\textcircled{4} \quad \begin{array}{l} Z_c \neq 0 \\ Z_p \neq 0 \end{array} \quad \Rightarrow \quad \exists \text{ o eroare pe poziția } \\ \text{dată de } Z_c \\ (E_{\text{corectat}})$$

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