

I. Să se calculeze integralele iterate

1) $\int_2^4 dx \int_0^{x^2} x dy;$

6) $\int_1^{\frac{\pi}{2}} dx \int_0^x \cos(x+y) dy;$

2) $\int_2^6 dx \int_0^x \frac{y}{x} dy;$

7) $\int_{-1}^1 dy \int_{2y}^y (x-y)e^y dx;$

3) $\int_1^3 dy \int_0^{\sqrt{y}} \sqrt{y} dx;$

8) $\int_0^{\frac{\pi}{2}} d\varphi \int_1^{\cos \varphi} r \sin \varphi \ln r dr;$

4) $\int_1^2 dy \int_0^{y^3} \frac{4}{y^5} dx;$

9) $\int_0^1 dx \int_0^1 \frac{x^2 dy}{1+y^2};$

5) $\int_1^5 dy \int_0^{\sqrt{y}} xy dx;$

10) $\int_0^{2\pi} d\varphi \int_{a \sin \varphi}^a \rho d\rho.$

II. Să se schimbe ordinea de integrare în integralele iterate; să se reprezinte domeniul:

1) $\int_0^2 dx \int_0^x f(x,y) dy;$

6) $\int_0^{\frac{R\sqrt{2}}{2}} dx \int_0^x f dy + \int_{\frac{R\sqrt{2}}{2}}^R dx \int_0^{\sqrt{R^2-x^2}} f dy$ 7)

2) $\int_0^4 dx \int_{3x^2}^{12x} f(x,y) dy;$

$$\int_1^2 dx \int_{\ln x}^{3x} f(x,y) dy;$$

3) $\int_0^1 dy \int_0^{y^2+y} f(x,y) dx;$

8) $\int_{\frac{\pi}{4}}^{\pi} dx \int_{\cos x}^{\sin x} f(x,y) dy;$

4) $\int_0^1 dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y) dx;$

9) $\int_{-1}^1 dx \int_{x^2}^{2x^2-1} f(x,y) dy;$

5) $\int_3^6 dx \int_0^{\sqrt{12x-x^2}} f(x,y) dy;$

10) $\int_0^2 dx \int_{\sqrt{2x-x^2}}^{2\sqrt{x}} f(x,y) dy.$

III. Să se calculeze integralele duble pe domeniul D , mărginit de liniile indicate:

1) $\iint_D xy dx dy; \quad D: y = x^2, \quad y^2 = x$

2) $\iint_D x^2 y dx dy, \quad D: y = x^2, \quad y = 4$

$$3) \iint_D y^2 x dx dy, D: x^2 + y^2 = 4, x + y - 2 = 0$$

$$4) \iint_D (xy + y) dx dy, D: 0 \leq y \leq 1, y \leq x \leq 2 - y$$

$$5) \iint_D e^{x+y} dx dy, D: y = e^x, x = 0, y = 2$$

$$6) \iint_D \sqrt{xy - y^2} dx dy, D: 1 \leq y \leq 2, y \leq x \leq 10y$$

$$7) \iint_D (2x + y) dx dy, D \text{ este triunghiul cu vârfurile } A(-2, -2), B(-1, 2), C\left(-1, -\frac{3}{2}\right)$$

$$8) \iint_D (x^2 + y^2) dx dy, a) D: x = \sqrt{2}, y = x, x^2 + y^2 = 8,$$

$$b) D: y - x, y = \sqrt{3}x, x^2 + y^2 = 8 \ (x \geq 0, y \geq 0).$$

IV. Să se calculeze integralele duble, folosind schimbul de variabilă:

$$1) \iint_D \frac{x dx dy}{\sqrt{x^2 + y^2}}, D: x^2 + y^2 = 4, x^2 + y^2 = 16, x \geq 0, y \geq 0$$

$$2) \iint_D e^{x^2+y^2} dx dy, D: x^2 + y^2 \leq 1$$

$$3) \iint_D \sqrt{x^2 + y^2} dx dy, D: x^2 + y^2 \leq 6x, y \geq 0$$

$$4) \iint_D \cos \sqrt{x^2 + y^2} dx dy, D: \frac{\pi^2}{4} \leq x^2 + y^2 \leq 4\pi^2$$

$$5) \iint_D \sqrt{25 - x^2 - y^2} dx dy, D: x^2 + y^2 \leq 9$$

$$6) \iint_D (x^2 + y^2) dx dy, D: (x^2 + y^2)^2 = a^2 (x^2 - y^2), y = 0, (x > 0, y > 0)$$

$$7) \iint_D xy dx dy, D: (x^2 + y^2)^2 = 2a^2 xy$$

$$8) \iint_D \sqrt{25 - \frac{x^2}{16} - \frac{4^2}{25}} dx dy, D: \frac{x^2}{16} + \frac{4^2}{25} = 1$$

$$9) \iint_D dx dy, D: x^2 + y^2 - 4x = 0, x^2 + y^2 - 6x = 0, y = 0, y = x$$

$$10) \iint_D xy dx dy, D: y = x^3, y = 2x^3, y^2 = 2x, y^2 = 3x$$

$$11) \iint_D (x^2 + y^2) dx dy, D: xy = 1, xy = 2, y = 2x, y = 5x, x \geq 0, y \geq 0$$

$$12) \iint_D \sqrt{xy} dx dy, D: xy = 1, xy = 3, y^2 = x, y^2 = 2x$$

V. Să se calculeze ariile domeniilor, mărginite de liniile:

$$1) y = x, y = x^2$$

$$6) x^2 + y^2 + 4y = 0, x^2 + y^2 + 2y = 0$$

$$2) y = x^2 + 1, x - y + 3 = 0$$

$$7) x^2 + y^2 = 16, x^2 + y^2 - 8x = 0, y = 0$$

$$3) y = x, y = 3x, x = 2$$

$$8) x^2 + y^2 + 4x = 0, x^2 + y^2 + 4y = 0$$

$$4) y = 2x^2 - 2x, y = -x^2 + 1$$

$$9) x^4 + y^4 = 2a^2 xy$$

$$5) x = 4 - y^2, x + y - 4 = 0$$

$$10) xy = 2, xy = 3, y = 3x, y = 5x$$

VI. Să se calculeze volumul corpurilor mărginite de suprafețele:

$$1) x + 2y - z = 0, x - 2y + 5 = 0, 2x + 3y - 18 = 0, z = 0$$

$$2) z = 16 - x^2 - y^2, x = \pm 3, y = \pm 3, z = 0$$

$$3) y = x^2, x = y^2, z = 12 + y - x^2$$

$$4) z = \sqrt{x^2 + y^2}, x^2 + y^2 = 4, z = 0$$

$$5) z = x^2 + y^2, x^2 + y^2 = a^2, z = 0$$