Plesu Cătălin TI-206. Testarea 4

V - 11

Ex. 1.

a) $y' + \frac{2y}{x} = \frac{1}{x}.$

① $y = uv \qquad y' = u'v + v'u$

$u'v + v'u + \frac{2uv}{x} = \frac{1}{x}$

② $v'u + v\left(\underbrace{u' + \frac{2u}{x}}\right) = \frac{1}{x}$

③

$u' + \frac{2u}{x} = 0 \Rightarrow u' = -\frac{2u}{x} \Rightarrow$

$\Rightarrow \frac{du}{dx} = -\frac{2u}{x} \Rightarrow \frac{du}{2u} = -\frac{dx}{x} \Rightarrow$

$\ln 2u = \ln -x \Rightarrow 2u = -x \Rightarrow$

$\Rightarrow \boxed{u = \frac{-x}{2}}$

④ $v'u = \frac{1}{x}$

$v' \cdot \left(-\frac{x}{2}\right) = \frac{1}{x} \Rightarrow v' = -\frac{2}{x^2}$

$v = \int -2x^{-2} dx = \frac{\cancel{+}2x^{-1}}{\cancel{-1}}$

$\boxed{v = \frac{2}{x}}$

$f(z) = -\frac{x}{2} \cdot \frac{2}{x} \Rightarrow \boxed{f(z) = -1}$

b) $y'' - 8y' + 15y = 0$

① $k^2 - 8k + 15 = 0$

$\Delta = 64 - 60 = 4 > 0 \Rightarrow k_1 \neq k_2$

$k_1 = \frac{8-2}{2} = \underline{3}$ $k_2 = \frac{8+2}{2} = \underline{5}$

② $\boxed{y_0 = C_1 e^{3x} + C_2 e^{5x}}$

c) $y'' + 4y' + 4y = 24xe^{-2x}$

① $k^2 + 4k + 4 = 0$

$\Delta = 16 - 16 = 0 \Rightarrow k_1 = k_2$

$k = \frac{-4-0}{2} = -2$

$\boxed{y_0 = C_1 e^{-2x} + C_2 x e^{-2x}}$

③ $\cancel{\begin{cases} C_1'(x)e^{-2x} + C_2'(x)xe^{-2x} = 0 \\ C_1'(x)e^{-2x} + C_2'(x)xe^{-2x} = 24xe^{-2x} \end{cases}}$

$f(x) = 24xe^{-2x}$

$y_p = x^2 \cdot Axe^{-2x} \Rightarrow y_p = Ax^3e^{-2x}$

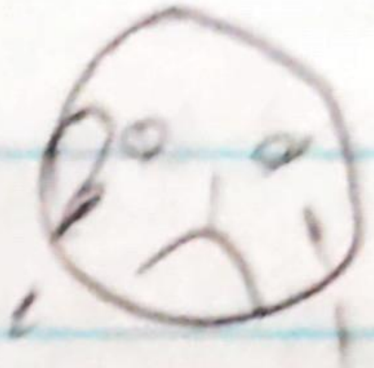
$A\cancel{x^3}x^2 e^{-2x} = 24xe^{-2x}$

$A = \frac{24}{x^2}$

$\boxed{y_n = C_1 e^{-2x} + C_2 x e^{-2x} + \frac{24}{x^2} \cdot x \cdot e^{-2x}}$

Ex. 2.

$$tg\left(-\frac{\pi}{3} + i \ln 2\right).$$



$$tg\underbrace{\left(-\frac{\pi}{3} + i \ln e\right)}_{\text{formă algebrică}}$$

Ex3.

b) $u(x,y) = -2xy + 3x + 1$; $f(i) = 1+2i$

① ~~[illegible]~~ cercetăm dacă u - armonică.

$u'_x = -2y+3$ $\quad u''_{xx} = 0$

$u'_y = -2x$ $\quad u''_{yy} = 0$

$u''_{xx} + u''_{yy} = 0$ -?

$0 + 0 = 0 \Rightarrow u$ - armonică $\Rightarrow$

$f(z)$ poate fi restabilită.

② C-R
$$\begin{cases} u'_x = v'_y \\ u'_y = -v'_x \end{cases}$$

$$\begin{cases} -2y+3 = v'_x \quad (**) \\ -2x = -v'_y \quad (*) \quad / \cdot (-1) \end{cases}$$

(*) $v'_y = 2x \Rightarrow$ ~~$v = \int 2x\,dx = x^2 + C$~~

$\frac{dv}{dy} = 2x \Rightarrow dv = 2x\,dy \Rightarrow \boxed{v = 2xy} + C(x)$

(**) $(2xy + C(x))'_x = -2y + 3$

$2y + C'(x) = -2y + 3$

$C'(x) = -4y + 3$

$C(x) = \boxed{-2y^2 + 3y + C}$

$f(z) = u \cdot v = (-2x \cdot y + 3x + 1)(2x - 2y^2 + 3y + C) =$

$$= -4x^2y + 4xy^3 - 6xy^2 + 6x^2 - 6xy^2 + 2x - 2y^2 + 3y + C =$$

$$= -4x^2y + 4xy^3 - 12xy^2 + 6x^2 - 2y^2 + 3y + 2x + C.$$

$$f(1+2i) = -4 \cdot 2i + 4 \cdot (2i)^3 - 12(2i)^2 + 6 - 2 \cdot (2i)^2 + 3 \cdot (2i) + 2 + C =$$

$$= -8i + \underbrace{32i^3}_{-i} - \underbrace{24i^2}_{-i} + 6 - \underbrace{8i^2}_{-1} + 6i + 2 =$$

$$= -8i - 32i + 24 + 6 + 8 + 6i + 2 =$$

$$= \underline{40 \cancel{+} - 34i}$$