12.03.2021

Testul 22

① $\sum_{n=1}^{\infty} \frac{1}{49n^2-35n-6}$

$a_n = \frac{1}{49n^2-35n-6}$

$49n^2-35n-6=0$

$\Delta = b^2-4ac = 2401 = 49^2$

$n_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{+35-49}{2\cdot 49} = \frac{-14}{98} = \frac{-1}{7}$

$n_2 = \frac{35+49}{98} = \frac{84}{98} = \frac{42}{49}$

$49n^2-35n-6 = 49\left(n-\frac{1}{7}\right)\left(n+\frac{42}{49}\right)$

$\sum a_n = \frac{1}{49\left(n-\frac{1}{7}\right)\left(n+\frac{42}{49}\right)} = \frac{1}{\left(n-\frac{1}{7}\right)\left(49n+42\right)}$

$a_n = \frac{1}{\left(n-\frac{1}{7}\right)\left(49n+42\right)} = \cancel{\frac{A}{\left(n-\frac{1}{7}\right)} + \frac{B}{\left(49n+42\right)}}$

$\frac{A(49n+42)+B\left(n-\frac{1}{7}\right)}{\left(n-\frac{1}{7}\right)\left(49n+42\right)}$

$A(49n+42)+B\left(n-\frac{1}{7}\right)=1$

$49n+42=0 \Rightarrow n=\frac{-42}{49} \Rightarrow B\left(-\frac{42}{49}-\frac{1}{7}\right)=1 \Rightarrow$

$B(-1)=1 \Rightarrow B=-1$

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$$\left(n-\frac{1}{7}\right) = 0 \Rightarrow n = \frac{1}{7} \Rightarrow A\left(49\frac{1}{7} + 42 = 1 \Rightarrow\right.$$

$$49A = 1 \Rightarrow A = \frac{1}{49}$$

$$= \frac{1}{n-\frac{1}{7}} - \frac{1}{49n+42} = \frac{(49n+42)-\left(n-\frac{1}{7}\right)}{\left(n-\frac{1}{7}\right)(49n+42)} =$$

$$= \frac{48n + 41\frac{6}{7}}{\left(n-\frac{1}{7}\right)(49n+42)}$$

$$a_n = \frac{\frac{1}{49}}{n-\frac{1}{7}} - \frac{1}{49n+42}$$

$$S_n = \left[\frac{1-\frac{1}{7}}{49} - \frac{1}{49+42} + \frac{2-\frac{1}{7}}{49} - \frac{1}{98+42} \dots\right]$$

$$S = \lim_{n\to\infty} S_n$$

(2)

(a) $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot(2n-1)}{5^n}$

d'alambert

$a_n = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{5^n}$

$a_{n+1} = \frac{1\cdot 3\cdot 5\ldots(2(n+1)-1)}{5^{n+1}}$

$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \frac{\cancel{1\cdot 3\cdot 5\cdots}(2n+1)}{\cancel{5^{n+1}}\ 5} \cdot \frac{\cancel{5^n}}{\cancel{1\cdot 3\cdot 5\cdots(2n-}}$

$= \lim_{n\to\infty} \frac{2n+1}{5} = \infty > 1$

$\sum$ - div

(b) $\sum_{n=1}^{\infty} \left(\frac{n-2}{3n+2}\right)^{3n}$

rad. Chauchy

$a_n = \left(\frac{n-2}{3n+2}\right)^{3n}$

$l = \lim_{n\to\infty} \sqrt[n]{\left(\frac{n-2}{3n+2}\right)^{3n}} = \lim \left(\underbrace{\frac{n-2}{3n+2}}_{\frac{1}{3}}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{9} <$

$\sum$ - conv.

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}\left(1-\cos\frac{1}{\sqrt{n+1}}\right) \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

$a_n = \frac{1}{\sqrt[4]{n^3}} = \frac{1}{n^{3/4}}$

Dirichet, $\alpha = \frac{3}{4} \leq 1 \Rightarrow \sum_6$ div.

$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}\left(1-\cos\frac{1}{\sqrt{n+1}}\right)$ - div.

(3) $\sum_{n=1} \frac{(-1)^n x^n}{3^n(n^3+3n)}$

$a_n = \frac{\cancel{(-1)^n}\ 1}{3^n(n^3+3n)}$

$a_{n+1} = \frac{1}{3^{n+1}(n+1)^3+3(n+1)}$

$R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim \left|\frac{1}{\cancel{3^n}(n^3+3n)} \cdot \frac{\cancel{3^{n+1}}^{3}\left[(n+1)^3+3(n+1)\right]}{1}\right| =$

$= \lim \frac{3\left((n+1)^3+3(n+1)\right)}{n^3+3n} = \lim \frac{3(n+1)^3+9(n+1)}{n^3+3n} = 3$

$R = \pm 3$

$x = 3; \sum \frac{(-1)^n}{\cancel{3^n}(n^3+2n)} \cancel{3^n}$

leibniz

$a_n = \frac{1}{n^3+2n}$

$a_1 \geq a_2$ -?

$\frac{1}{1+2\cdot1} \geq \frac{1}{8+4}$ - adevărat.

$\lim_{n \to \infty} \frac{1}{n^3+2n} = 0$ - ✓

$\sum \frac{(-1)^n}{n^3+2n}$ - conv.

$x = -3$; $\sum \frac{(-1)^n (-3)^n}{3^n (n^3+2n)} = \frac{\cancel{3^n}}{\cancel{3^n}(n^3+2n)}$

d'alambert.

$\frac{1}{(n+1)^3+2(n+1)} \cdot n^3+2n = 1 \Rightarrow \sum$ - div

Răspuns $[-3; 3)$ - domeniu de convergență

al $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n \cdot (n^3+3n)}$

3

(4) $\sum a_n$; $\sum b_n$ - divergente

$\sum\limits_{n} (a_n - b_n)$ - ?

$\sum$ seriilor din nu există sau $= \infty$

deci $\infty - \infty = \infty$ sau nu există

pot spune că $\sum (a_n - b_n)$ - div.

(5)

$$f(x) = \begin{cases} 2, & -2 < x \le 0 \\ 0, & 0 < x < 2 \end{cases}, \quad T = 4$$

$S(6)$, $S(-11)$

$$\sum \frac{(-1)^n}{2n-1}$$

$$a_0 = \frac{1}{\cancel{\pi}} \int_{-2}^{0} 2\,dx + \underbrace{\frac{1}{\cancel{2}} \int_{0}^{\cancel{2}} 0\,dx}_{=0} =$$

$$= \frac{1}{\pi} \cdot 2x \Big|_{-2}^{0} = \frac{1}{\cancel{2}} \cdot -4 = \frac{-4}{\cancel{2}} = -2$$

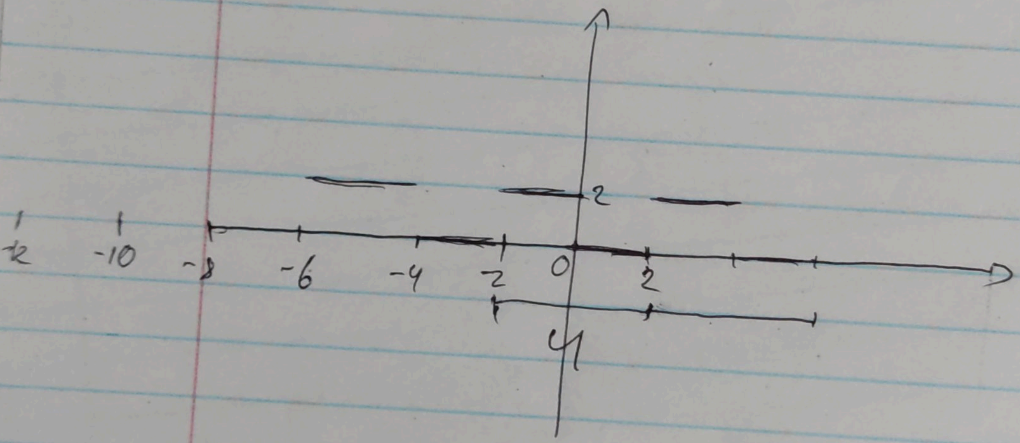
$$\cancel{a}_n = \frac{1}{\cancel{2}} \int_{-2}^{0} 2 \sin(n x)\,dx + \frac{1}{2} \int_{0}^{2} 0 \sin(nx)\,dx =$$

$$= \frac{2}{2} - \cos(n\cancel{x}) \Big|_{-2}^{0} + 0 = 1 - 2 = -1$$

$$a_n = \frac{1}{2}\int_{-2}^{0} 2\cos(nx)\,dx + \frac{1}{2}\int_{0}^{2} \underbrace{0\cos(nx)\,dx}_{0}$$

$$= \frac{1}{2}\,2\,\frac{1}{n}\underbrace{\sin nx}_{=0} = 0$$

$$f(x) = -2 + \sum_{n=1}^{\infty} 0\cdot\cos(nx) + 1$$



$$S(6) = \frac{f(6+0)+f(6-0)}{2} = \frac{2+2}{2} = 2$$

$$S(-11) = \frac{f(-11+0)+f(-11-0)}{2} = \frac{0+0}{2} = 0$$