Seminarul 2

Algoritmul simplex. Metoda celor 2 faze

1) Se consideră problema:

$$\inf\{3x_1 - x_2 - x_3 + x_4\}$$
$$\begin{cases} x_1 + x_2 - x_3 + 3x_4 = 2 \\ -2x_1 + x_2 + 2x_3 - x_4 = -1 \end{cases}$$
$$x_i \geq 0, \quad i = \overline{1,4}.$$

a) Să se verifice dacă baza $B = \left(A^2 A^3\right)$ este primal admisibilă.

$B = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$, inverse: $\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \not\geq 0.$

b) Să se scrie tabloul simplex pentru $B = \left(A^1 A^2\right)$ şi să se rezolve problema.

	$\bar{x}$	x_1	x_2	x_3	x_4
x_1	1	1	0	−1	$\frac{4}{3}$
x_2	1	0	1	0	$\frac{5}{3}$
	2	0	0	−2	$\frac{4}{3}$

	$\bar{x}$	x_1	x_2	x_3	x_4
x_1	$\frac{1}{5}$	1	$-\frac{4}{5}$	−1	0
x_4	$\frac{3}{5}$	0	$\frac{3}{5}$	0	1
	$\frac{6}{5}$	0	$-\frac{4}{5}$	−2	0

2) Să se rezolve problema

$$\inf\{-2x_1 - x_2 + 2x_3\}$$
$$\begin{cases} x_1 - x_3 \leq 2 \\ -x_1 + x_2 + 2x_3 = 1 \end{cases}$$
$$x_i \geq 0, \quad i = \overline{1,3}.$$

	$\bar{x}$	x_1	x_2	x_3	x_4
x_4	2	1	0	−1	1
x_2	1	−1	1	2	0
	−1	3	0	−4	0

	$\bar{x}$	x_1	x_2	x_3	x_4
x_1	2	1	0	−1	1
x_2	3	0	1	1	1
	−7	0	0	−1	−3

3) Să se rezolve problema cu metoda celor două faze:

$$\inf\{x_1 - 2x_2 + 2x_3 - 3x_4\}$$
$$\begin{cases} 2x_1 + x_2 - 3x_3 - 3x_4 = -1 \\ -x_1 + 3x_2 + 2x_3 + x_4 = 1 \end{cases}$$
$$x_i \geq 0, \quad i = \overline{1,4}.$$

Faza I

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5	x_6
x_5	1	−2	−1	3	3	1	0
x_6	1	−1	3	2	1	0	1
	2	−3	2	5	4	0	0

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5	x_6
x_3	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	1	1	$\frac{1}{3}$	0
x_6	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{11}{3}$	0	−1	$-\frac{2}{3}$	1
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{11}{3}$	0	−1	$-\frac{5}{3}$	0

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5	x_6
x_3	$\frac{4}{11}$	$-\frac{7}{11}$	0	1	$\frac{10}{11}$	$\frac{3}{11}$	$\frac{1}{11}$
x_2	$\frac{1}{11}$	$\frac{1}{11}$	1	0	$-\frac{3}{11}$	$-\frac{2}{11}$	$\frac{3}{11}$
	0	0	0	0	0	−1	−1

Faza II

	$\bar{x}$	x_1	x_2	x_3	x_4
x_3	$\frac{4}{11}$	$-\frac{7}{11}$	0	1	$\frac{10}{11}$
x_2	$\frac{1}{11}$	$\frac{1}{11}$	1	0	$-\frac{3}{11}$
	$\frac{6}{11}$	$-\frac{27}{11}$	0	0	$\frac{59}{11}$

	$\bar{x}$	x_1	x_2	x_3	x_4
x_4	$\frac{2}{5}$	$-\frac{7}{10}$	0	$\frac{11}{10}$	1
x_2	$\frac{1}{5}$	$-\frac{1}{10}$	1	$\frac{3}{10}$	0
	$-\frac{8}{5}$	$\frac{13}{10}$	0	$-\frac{59}{10}$	0

Problema are optimul $-\infty$.

4) Să se rezolve problema cu metoda celor două faze:

$$\inf\left\{2x_1 - 3x_2 + x_3\right\}$$
$$\begin{cases} 2x_1 + x_2 - 3x_3 = 6 \\ x_1 + x_2 + 2x_3 = -2 \end{cases}$$
$$x_i \geq 0, \quad i = \overline{1,3}.$$

Faza I

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5
x_4	6	2	1	−3	1	0
x_5	2	−1	−1	−2	0	1
	8	1	0	−5	0	0

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5
x_1	3	1	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	0
x_5	5	0	$-\frac{1}{2}$	$-\frac{7}{2}$	$\frac{1}{2}$	1
	5	0	$-\frac{1}{2}$	$-\frac{7}{2}$	$-\frac{1}{2}$	0

Problema nu admite soluţii.

5) Să se rezolve problema cu metoda celor două faze:

$$\inf\left\{-x_1+x_2-2x_3-3x_4\right\}$$
$$\begin{cases} 4x_1-x_2-3x_3+2x_4=8 \\ x_1+3x_2+2x_3+x_4=3 \\ 2x_1-7x_2-7x_3=2 \end{cases}$$
$$x_i\geq 0,\quad i=\overline{1,4}.$$

Faza I

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	8	4	–1	–3	2	1	0	0
x_6	3	1	3	2	1	0	1	0
x_7	2	2	–7	–7	0	0	0	1
	13	7	–5	–8	3	0	0	0

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	4	0	13	11	2	1	0	–2
x_6	2	0	$\frac{13}{2}$	$\frac{11}{2}$	1	0	1	$-\frac{1}{2}$
x_1	1	1	$-\frac{7}{2}$	$-\frac{7}{2}$	0	0	0	$\frac{1}{2}$
	6	0	$\frac{39}{2}$	$\frac{33}{2}$	3	0	0	$-\frac{7}{2}$

	$\bar{x}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	0	0	0	0	0	1	–2	–1
x_4	2	0	$\frac{13}{2}$	$\frac{11}{2}$	1	0	1	$-\frac{1}{2}$
x_1	1	1	$-\frac{7}{2}$	$-\frac{7}{2}$	0	0	0	$\frac{1}{2}$
	0	0	0	0	0	0	–3	–2

Faza II

	$\bar{x}$	x_1	x_2	x_3	x_4
x_4	2	0	$\frac{13}{2}$	$\frac{11}{2}$	1
x_1	1	1	$-\frac{7}{2}$	$-\frac{7}{2}$	0
	–7	0	–17	–11	0