PREDICTING BURNED AREAS OF FOREST FIRES: AN ARTIFICIAL INTELLIGENCE APPROACH

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ABSTRACT

Forest fires importantly influence our environment and lives. The ability of accurately predicting the area that may be involved in a forest fire event may help in optimizing fire management efforts. Given the complexity of the task, powerful computational tools are needed for predicting the amount of area that will be burned during a forest fire. The purpose of this study was to develop an intelligent system based on genetic programming for the prediction of burned areas, using only data related to the forest under analysis and meteorological data. We used geometric semantic genetic programming based on recently defined geometric semantic genetic operators for genetic programming. Experimental results, achieved using a database of 517 forest fire events between 2000 and 2003, showed the appropriateness of the proposed system for the prediction of the burned areas. In particular, results obtained with geometric semantic genetic programming were significantly better than those produced by standard genetic programming and other state of the art machine learning methods on both training and out-of-sample data. This

RESUMEN

Los incendios forestales influencian de manera importante nuestro ambiente y vidas. La habilidad para predecir con precisión el área que podría estar implicada en un evento de incendio puede ayudar a optimizar los esfuerzos para su manejo. Dada la complejidad de esta tarea, son necesarias herramientas computacionales poderosas para predecir el tamaño del área que podría quemarse durante un incendio forestal. El propósito de este estudio fue desarrollar un sistema inteligente basado en programación genética para la predicción de áreas quemadas, usando solamente datos relacionados con el bosque bajo análisis y datos meteorológicos. Nosotros usamos programas geométrico-semánticos y genéticos basados en operadores geométrico-semánticos y genéticos recientemente definidos para programación genética. Los resultados experimentales, usando una base de datos de 517 eventos de incendios forestales entre 2000 y 2003, mostraron lo adecuado del sistema propuesto para la predicción de las áreas quemadas. En particular, los resultados obtenidos con los programas geométrico-semánticos y genéticos fueron significativamente mejores que aquellos producidos por programación genética estándar y otros métodos de aprendizaje automático que consideren tanto datos de entrenamiento como datos fuera de los muestreos. Este estudio su-

study suggests that deeper investigation of genetic programming in the field of forest fires prediction may be productive.

giere que investigaciones más profundas de programación genética en el campo de la predicción de los incendios forestales pueden ser productivas.

INTRODUCTION

Forest fires are well-known events, especially during summer. Forest fires, regularly experienced in regions with hot, dry, or mediterranean climates, represent a risk to life and extant infrastructure. In Portugal, there are typically between 15 000 and 25 000 forest fires each year (Mateus and Fernandes 2014), burning from 150 000 ha to 250 000 ha. Notwithstanding the fact that these fires can cause extensive economic damage (typically with tangible repercussions for many years to come), they also threaten human life. Furthermore, the aftermath of forest fires can have other far-reaching consequences. For example, many physical, chemical, mineralogical, and biological soil properties can be affected by forest fires (Certini 2005). Negative effects resulting from high levels of burn severity include significant removal of organic matter, deterioration of both soil structure and porosity, considerable loss of nutrients through volatilization, ash entrapment in smoke columns, leaching, and erosion. Also, the release of hazardous chemicals significantly impacts human health and increases the risk of future diseases. As suggested by Lipsett *et al.* (2008), wildfire smoke is accompanied by high concentrations of carbon dioxide, which can result in consequences such as headache, mental confusion, nausea, disorientation, coma, and even death. Even at lower concentrations, the effects of carbon dioxide should not be neglected; individuals with cardiovascular dis-

ease may experience chest pain and cardiac arrhythmia. A comprehensive study tracking wildfire firefighter deaths from 1990 to 2006 reported that 21.9 % of their deaths occurred from heart attacks (Mangan 2007).

The ability to predict fire progression and area burned is crucial to mitigating the immediate and far-reaching consequences of wildfires. Existing studies have attempted to fill this gap, mainly through mathematical models (e.g. Rothermel 1972), but predictive techniques would enable decision makers to deal with large amount of data in a more timely manner. The Wildland Fire Management Research, Development & Application Organization (2012) proposed a wildland fire decision support tool called FSPro (Fire Spread Probability). FSPro is a geospatial probabilistic model that predicts fire growth, and is designed to support long-term (more than five days) decision making. FSPro addresses fire growth beyond the timeframes of reliable weather forecasts by using historic climatological data. FSPro calculates and maps the probability that fire will spread to areas on the landscape based on the current fire perimeter or ignition point.

In this paper, we propose an intelligent system based on genetic programming for the prediction of burned areas of forest fires. In order to build predictive models, we only considered data relating to forest characteristics and meteorological data. Drawing on the idea of using computational intelligence techniques (and genetic programming in particular; e.g.

Brumby *et al.* 2001, Manson 2005), we employed recently defined geometric semantic genetic operators for genetic programming, which were able to produce results significantly better than traditional methods.

Genetic Programming

Genetic Programming (GP) (Koza 1992; Poli *et al.* 2008) belongs to the family of bio-inspired computational intelligence techniques. The main idea of GP is to mimic the biological evolutionary process in order to create, by stepwise iteration, more refined solutions to a given problem.

In GP, candidate solutions are represented using a tree structure (Figure 1). In order to create new solutions, GP uses stochastic operators called genetic operators, typically entitled crossover and mutation. In the standard version of GP, these two operators work as follows: given two solutions (called parents), crossover builds two new solutions (offspring) by swapping a subtree of the first parent with a subtree of the second parent. The subtrees are usually chosen at random. Mutation acts on one solution: given a tree, it creates a new solution by replacing a randomly chosen subtree with a newly generated subtree. These operators act on the structure (i.e., the syntax) of the individuals and ignore the information related to semantics. The application of standard crossover and mutation operators yields new tree structures (Figures 2 and 3).
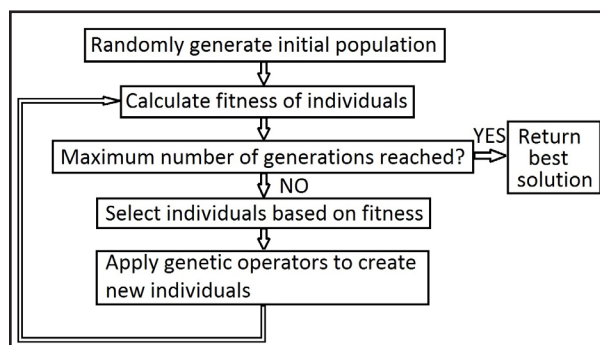
Figure 1. The genetic programming iterative search process.
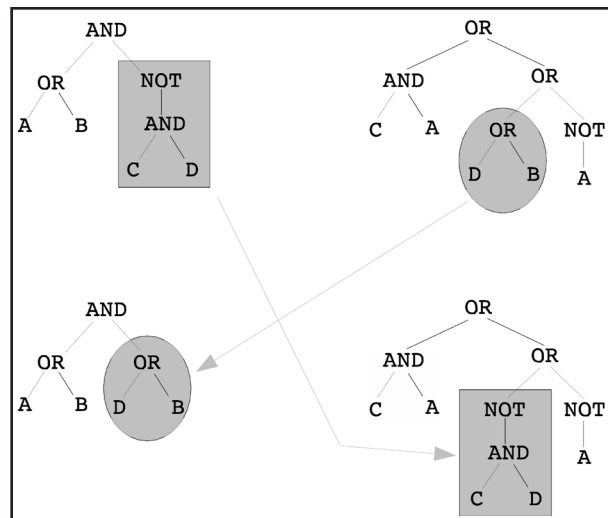
Figure 2. Example of the application of the standard, syntax-based, crossover operator. Given two solutions (called parents), the crossover operator builds two new solutions (offspring) by swapping a subtree of the first parent with a subtree of the second parent. The subtrees are usually chosen at random.

Since its definition, GP has been used to solve complex problems in several domains (Koza 2010) using only syntax-based genetic operators. Abstraction from semantics allows GP to use simple genetic operators that are easy to define and that are independent of any particular application. Hence, standard genetic operators can be used for addressing regression, classification, or even clustering problems without changing their definition. A second advantage is the existence of a solid theory that guarantees asymptotic convergence of standard GP towards optimal solutions (Poli and Langdon 1998). Nevertheless, relying on syntax-based genetic operators results in some difficulties: abstraction from the semantics will produce solutions that completely ignore the knowledge associated with the available data, and it is difficult for an expert of a particular domain to accept and adopt a model built without considering this information. To offset this limitation, researchers have recently focused on the definition of methods that are able to integrate the semantic information in the search process.
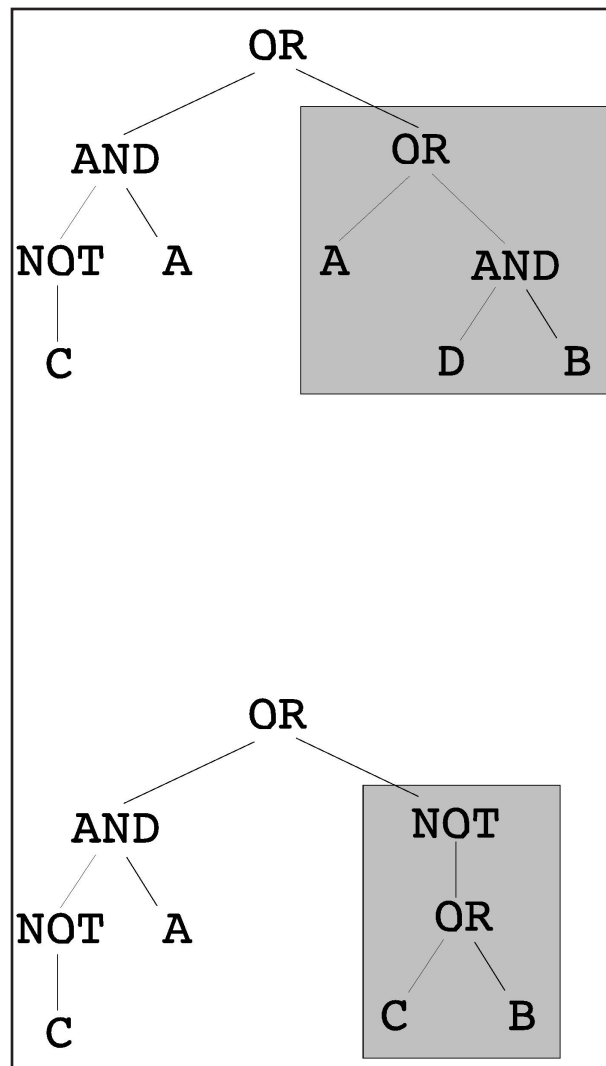
Figure 3. Example of the application of the standard, syntax-based, mutation operator. Given a tree, the mutation operator creates a new solution by replacing a randomly chosen subtree with a newly generated subtree.

Geometric Semantic Operators

This section introduces the concepts related to the definition of semantic-based methods, describing the semantic genetic operators that were used in this study. Even though the term semantics can have several different interpretations, the most common interpretation (and the one used here) is to identify the semantics of a solution with the vector of its output values on the training data (Vanneschi *et*

al. 2014). From this perspective, a GP individual can be identified with a point (its semantics) in a multidimensional space known as semantic space. The term Geometric Semantic Genetic Programming (GS-GP) indicates a variant of GP in which traditional crossover and mutation operators are replaced by so-called geometric semantic operators, which exploit semantic awareness and induce precise geometric properties on the semantic space.

Geometric semantic operators, introduced by Moraglio *et al.* (2012), are becoming more and more popular in the GP community (Vanneschi *et al.* 2014) because of their property of inducing a unimodal fitness landscape on any problem consisting of matching sets of input data into known targets (e.g., supervised learning problems such as regression and classification).

To understand this property (for a full proof see Moraglio *et al.* 2012), let us first consider a Genetic Algorithms (GAs) problem in which the unique global optimum is known and the fitness of each individual (to be minimized) corresponds to its distance to that global optimum (our reasoning holds for any employed distance). In this problem, if we use, for instance, *ball mutation* (Krawiec and Lichocki 2009) (i.e., a variation operator that slightly perturbs some of the coordinates of a solution), then any possible individual different from the global optimum has at least one fitter neighbor (another individual resulting from its mutation). Similar considerations hold also for many types of crossover, including various kinds of geometric crossover (Krawiec and Lichocki 2009). Accordingly, there are no local optima other than the global optimum, and the fitness landscape is unimodal, resulting in a problem characterized by a good evolvability.

Now, let us consider the typical GP problem of finding a function that maps sets of input data into known target values (e.g., regression and classification). The fitness of an indi-

vidual for this problem is typically considered to be represented as a distance between its predicted output values and the target ones (an error measure). Geometric semantic operators simply define transformations on the syntax of the individuals that correspond to geometric crossover and ball mutation in the semantic space, thus allowing us to map the considered GP problem into the previously discussed GA problem.

The definitions of semantic crossover and semantic mutation follow.

Geometric semantic crossover. Given two parent functions T_1, T_2: $R^n \rightarrow R$, the geometric semantic crossover returns the real function

$$T_{XO} = (T_1 \cdot T_R) + ((1 - T_R) \cdot T_2) , \qquad (1)$$

where T_R is a random function such that T_R: $R^n \rightarrow [0, 1]$.

To constrain T_R in producing values in [0; 1] we use the sigmoid function

$$T_R = \frac{1}{1 + e^{-Trand}} , \qquad (2)$$

where *Trand* is a random tree with no constraints on the output values.

Geometric semantic mutation. Given a parent function T: $R^n \rightarrow R$, the geometric semantic mutation with mutation step *ms* returns the real function

$$T_M = T + ms \cdot (T_{R1} - T_{R2}) , \qquad (3)$$

where T_{R1} and T_{R2} are random real functions.

Moraglio *et al.* (2012) showed that geometric semantic crossover corresponds to geometric crossover in semantic space (i.e., the point representing the offspring stands on the segment joining the points representing the parents), and geometric semantic mutation corresponds to ball mutation on the semantic space (and thus induces a unimodal fitness

landscape on the above mentioned types of problem). Moraglio *et al.* (2012) further showed that these operators create much larger offspring than their parents and the fast growth of the individuals in the population makes fitness evaluation unbearably slow, making the system unusable. Vanneschi *et al.* (2013) and Castelli *et al.* (2014) proposed a possible solution to this problem, consisting of an implementation of the Moraglio *et al.* (2012) operators that makes them not only usable in practice, but also very efficient. Their implementation is based on the idea that, besides storing the initial trees, at every generation it is enough to maintain in memory, for each individual, its semantics and a reference to its parents. Vanneschi *et al.* (2013) showed that the computational cost of evolving a population of *n* individuals for *g* generations is *O(ng)*, while the cost of evaluating a new, unseen instance is *O(g)*.

Geometric semantic operators have a known limitation (Castelli *et al.* 2014; Vanneschi *et al.* 2014): the reconstruction of the best individual at the end of a run can be a difficult (and sometimes even impossible) task, due to its large size. As a result, the interpretation of the optimal GP individual can be difficult, and the system can come to resemble a black box .

METHODS

To test the GP-GS method on a fire-frequent region, we selected Montesinho Natural Park, a protected area located in the municipalities of Vinhais and Bragança, in the mountainous region of northeast Portugal (Figure 4). The park consists of 748 000 ha of natural wooded landscape and traditional mountain agricultural landscape, with highly variable gradients. The park lies in the northeast Trás-os-Montes plateau, part of the northern Iberian Meseta, with elevations generally from 750 m to 900 m (Castro *et al.* 2010). However, in Montesinho, the elevation range is more than 1000 m: from the lowest point in the River
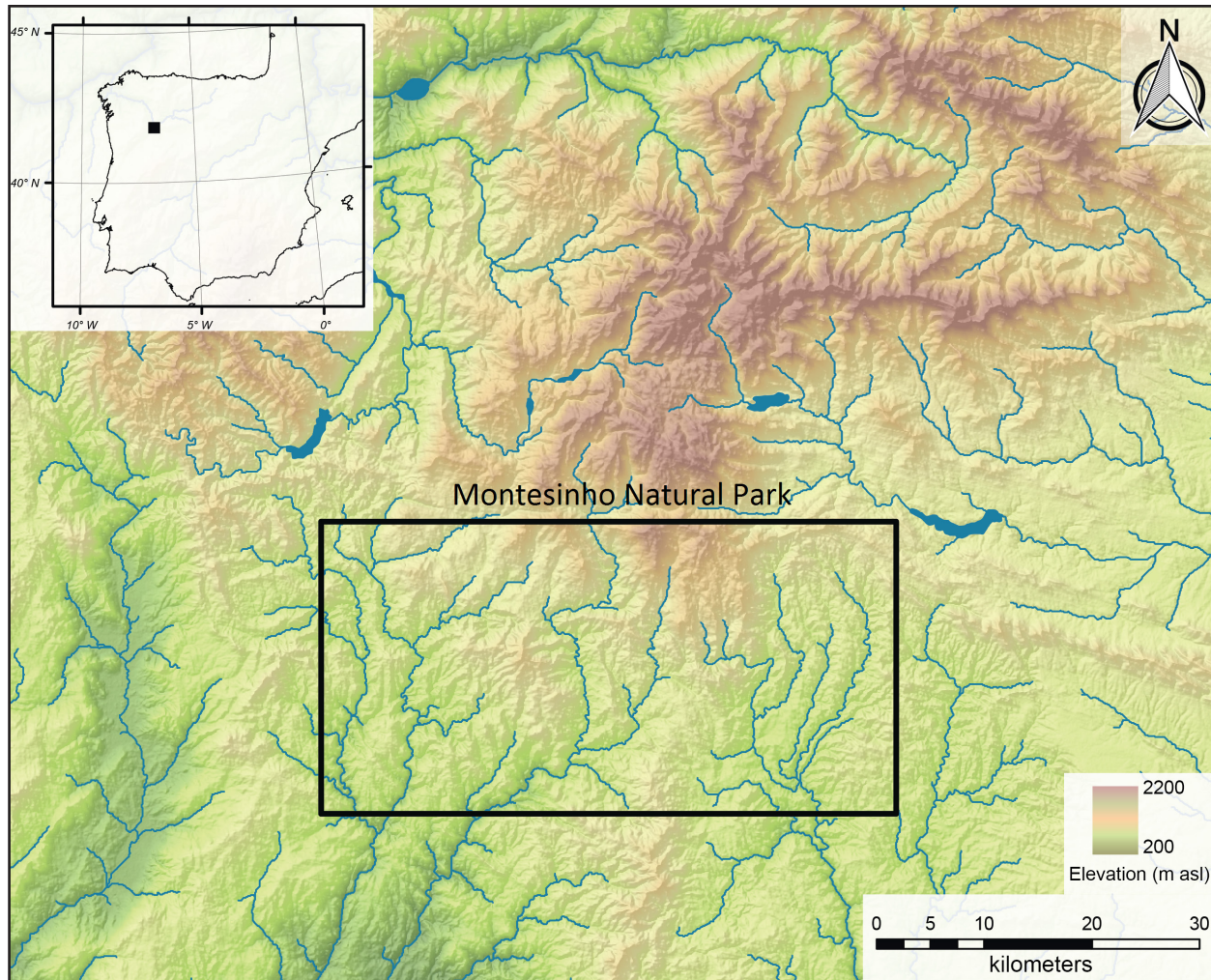
Figure 4. Location of the Montesinho Natural Park in Portugal.

Mente (436 m), which is the park's western border, to peak of Sierra de Montesinho, at 1487 m. The main altitudinal belts correspond to the main landforms found in the area. Climatic diversity within the park is high, with a mean annual rainfall of 800 mm to 1500 mm and an average annual temperature of 8 °C to 13 °C; this variation follows continental and altitudinal gradients (Raínha and Fernandes 2002). The summer drought period is usually four months. Vegetation is dominated by extensive shrub land areas, with important and flammable pine plantations (*Pinus pinaster* Aiton, *P. nigra* J.F. Arnold, and *P. sylvestris* L.). Natural hardwood stands (*Quercus rotundifolia* Lam., *Q. faginea* Lam., and *Q. pyrenaica*

Willd.) occur as residual patches in the landscape (Castro *et al.* 2010). Schist is the most widely represented soil parent material in the area, but basic rocks, ultramafic rocks, granites, and migmatites are also important lithological groups (Fonseca *et al.* 2012). The spatial distribution of the soil groups is characterized by the strong dominance of Leptosols (77.1 %), followed by Cambisols (20.1 %), with the well-developed soils (Luvisols and Alisols) covering 2 % of the territory.

The park includes 92 small villages inhabited by less than 8000 people. Intensive grazing takes place from May to August when about 5000 sheep are transported from the surrounding lowlands to graze in the highlands.

The non-regulated use of fire is common and related to agricultural and pastoral activities. Consequently, this area is very often subjected to wildfires, either naturally ignited or as a result of escaped human ignitions.

Data

We created a database of wildfire activity within the boundaries of the Montesinho Natural Park from January 2000 to December 2003, comprising 517 wildfires. Fuel and meteorological data related to the fires included the forest Fire Weather Index (FWI) (Taylor and Alexander 2006), which is the Canadian system for rating fire danger and includes five components: Fine Fuel Moisture Code (FFMC), Duff Moisture Code (DMC), Drought Code (DC), Initial Spread Index (ISI), and Buildup Index (BUI). The first three are related to fuel codes and are measured at the beginning of the year by means of a ground survey—the FFMC denotes the moisture con-

tent of surface litter and influences ignition and fire spread, while the DMC and DC represent the moisture content of shallow and deep organic layers, which affect fire intensity. The ISI is a score that correlates with fire spread, while BUI represents the amount of available fuel. Although different scales are used for each of the FWI elements, high values suggest more severe burning conditions (Taylor and Alexander 2006).

For each forest fire, several attributes were registered on a daily basis, such as the time, date, spatial location, the type of vegetation involved, the five components of the FWI system, and the total burned area (Table 1). More precisely, the areas affected by the fires were assessed by the park's personnel, using ground survey with GPS and false color aerial photography. The FWI components were calculated using meteorological information measured every 30 min by an automatic weather station located in the center of Montesinho Natural Park. Temperature, relative humidity, and

Table 1. Description of input data and prediction data (burned area).

Variable	Description
X	*x*-axis coordinate (from 1 to 9). It indicates one of the 9 sub-areas obtained from the division of the area of study along the X axis. All the areas have the same size.
Y	*y*-axis coordinate (from 1 to 9). It indicates one of the 9 sub-areas obtained from the division of the area of study along the Y axis. All the areas have the same size.
MONTH	Month of the year (from 1 to 12)
DAY	Day of the week (from 1 to 7)
FFMC	Fine Fuel Moisture Code (from 18.7 to 96.20)
DMC	Duff Moisture Code (from 1.1 to 291.3)
DC	Drought Code (from 7.9 to 860.6)
ISI	Initial Spread Index (from 0 to 56.10)
TEMP	Temperature (ºC) (from 2.2 to 33.30)
RH	Relative humidity (%) (from 15.0 to 100)
WIND	Wind speed (km hr^{-1}) (from 0.40 to 9.40)
RAIN	Rain (mm) (from 0.0 to 6.4)
BURNED AREA	Total burned area (ha) (from 0 to 1090.84)

wind speed were averages calculated during the time the fire was burning. Rain information was the total amount of rain during the time the fire burned. The full dataset of 517 instances is available at: https://archive.ics.uci.edu/ml/datasets/Forest+Fires, and a complete description of the data can be found in Cortez and Morais (2007).

Following the same procedure reported in Cortez and Morais (2007), month and day of the week have been chosen as temporal variables. The day of the week could influence forest fires (e.g., workdays vs. weekends), considering that most fires have a human cause. The BUI was discarded, since it is dependent on the other values (FFMC, DMC, DC, and ISI). Regarding the meteorological data, only the weather attributes used by the FWI system have been considered.

Experimental Settings

We tested the proposed implementation of GP with geometric semantic operators (GS-GP from now on), and we compared it to a standard GP system (ST-GP) (i.e., to the system that was originally defined in Koza 1992). All of the parameters were obtained by means of a preliminary tuning experimental analysis. We performed a total of 50 runs with each technique. In each run, a different partition between training and test data was considered. All the runs used populations of 100 individuals and the evolution was stopped after 500 generations. Tree initialization was performed with the Ramped Half-and-Half method (Koza 1992) with a maximum initial depth of six. The function set contained arithmetic operators, including division protected by returning a numeric constant when the denominator was equal to zero, a well-known method proposed in Koza (1992) to avoid system failures due to failures in the evaluation of the individuals. Fitness was calculated as the mean absolute error (MAE) between predicted and target values, defined as:

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |y_i - t_i| \, , \qquad (4)$$

where y_i is the predicted value given input i (output of the generated model, evaluated on the training data), and t_i is the corresponding target value.

The terminal set contained 12 variables, each one corresponding to a different feature in the dataset. To create new individuals, ST-GP used standard (subtree swapping) crossover and subtree mutation (Koza 1992) with probabilities equal to 0.9 and 0.1, respectively. For GS-GP, the crossover rate was 0.7, and the mutation rate was 0.3. The mutation step parameter was 0.1. Survival from one generation to the other was always guaranteed to the best individual of the population (elitism). No maximum tree depth limit was imposed during the evolution.

RESULTS

GS-GP vs. Standard GP

GS-GP outperformed ST-GP both on training and on out-of-samples data (Figure 5). GS-GP returned a MAE of 12.0 on the training set, whereas ST-GP produced a MAE of 13.8. GS-GP was more explanatory on the test data, with a MAE of 12.9, compared to ST-GP, which produced a MAE of 21.0.

To examine the statistical significance of these results, we tested the median errors. Preliminary analysis using the Kolmogorov-Smirnov test showed that the data were not normally distributed and hence a rank-based statistic was used. The Wilcoxon rank-sum test for pairwise data comparison was used with the alternative hypothesis that the samples do not have equal medians of burned area ($P < 0.001$ for training data, $P = 0.002$ for test data).
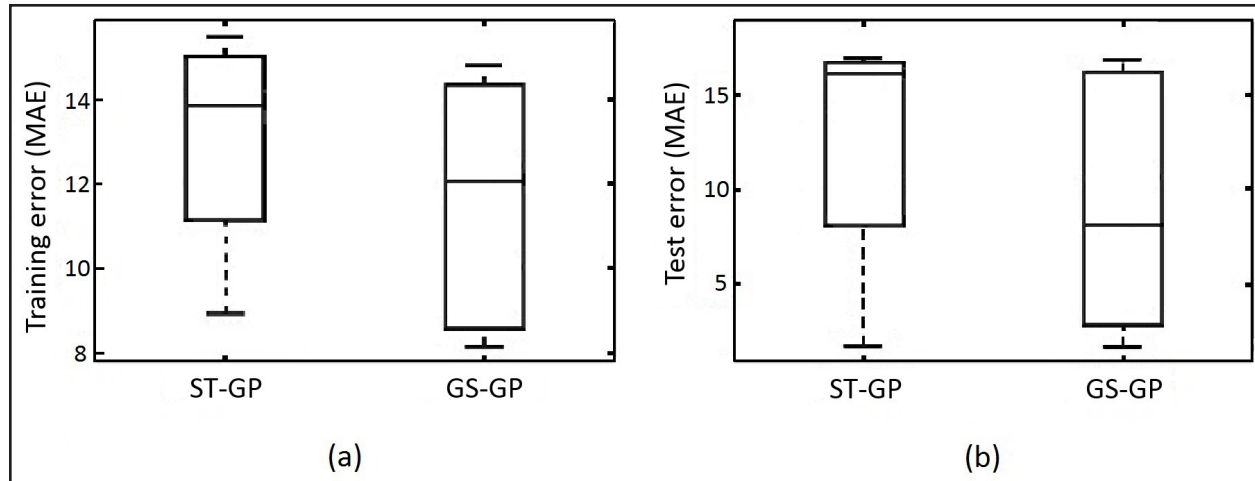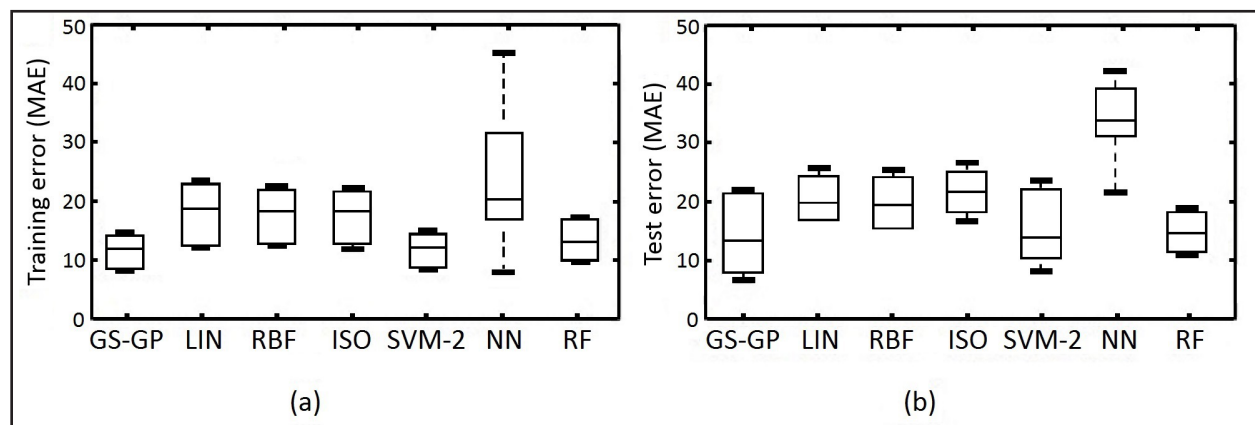
Figure 5. Boxplots of mean absolute error for (a) training and (b) test fitness at the end of the evolution. On each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points that are not considered outliers.

GS-GP vs. Other Machine Learning Techniques

Besides comparing GS-GP with ST-GP, we also compared GS-GP with other well-known state-of-the-art machine learning methods. To perform the comparisons with other machine learning methods, we used the implementations provided by the Weka public domain software (Machine Learning Project 2015). As we did for the previous experimental phase, we performed a preliminary analysis to tune the parameters for each considered techniques.

The results of the comparison are reported in Figure 6, while Table 2 summarizes all the results. In the figure and in the table, LIN stands for linear regression (Weisberg 2005), RBF stands for radial basis function network (Haykin 1999), ISO stands for isotonic regression (Hoffmann 2009), SVM-2 refers to the support vector machines (Schölkopf and Smola 2002) with polynomial kernel of second de-

Figure 6. Boxplots of mean absolute error for (a) training and (b) test fitness for the 50 runs of the considered machine learning techniques. On each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points that are not considered outliers.

Table 2. Experimental comparison between different non-evolutionary machine learning techniques for burned area prediction. Median of training error and test error calculated over 50 runs of each technique.

Method	Training	Test
GS-GP	12.0	12.9
SVM polynomial kernel (second degree)	12.3	13.6
Random forests	13.2	14.3
Radial basis function network	18.5	19.3
Linear regression	18.9	19.6
ST-GP	13.8	21.0
Isotonic regression	18.3	21.5
Neural networks	20.3	33.8

gree, NN refers to feed-forward artificial neural networks, trained with the backpropagation learning rule (Gurney 2003) and RF refers to random forests (Breiman 2001). Median error values for GS-GP were lower than those of all other considered methods (Table 2). There was a marked difference between the various methods: GS-GP, SVM-2, and RF performed well on this problem; LIN, RBF, and ISO performed less well; and NN performed poorly. We speculate, therefore, that the relationships hidden in the data cannot be satisfactorily approximated by a linear model. This speculation is supported by the fact that GS-GP, RF (which can generate non-linear models), and SVM-2 (which uses a quadratic kernel, and thus produces non-linear models) outperformed LIN, RBF, and ISO. The poor performance of NN on this problem deserves further discussion. One possible reason for the poor performance of NN could be due to the high number of unexpected events that affect some

particular instances of the data in the studied problem. NN has difficulty integrating these elements of discontinuities.

To assess the statistical significance of the model comparisons, the same set of tests performed in the previous section were done, but with a Bonferroni correction for the standard $\alpha = 0.05$ was applied (hence, the final value of α was 0.014). The differences in terms of training and test fitness between GS-GP and the considered machine learning techniques were significant, except for the cases when GS-GP and SVM-2 were compared as well as when GS-GP and RF were compared (Table 3).

CONCLUSIONS

The new genetic operators of genetic programming, called geometric semantic operators, have the extremely interesting property of inducing a unimodal fitness landscape for any problem consisting of matching input data into

Table 3. Comparison of geometric semantic genetic programming (GS-GP) with other machine learning methods. A *P*-value less than $\alpha = 0.014$ indicates that GS-GP is superior. LIN stands for linear regression, RBF for radial basis functions, ISO for isotonic regression, SVM-2 for support vector machines with polynomial kernel of degree 2, NN for neural networks, and RF for random forests.

		LIN	RBF	ISO	SVM-2	NN	RF
GS-GP	TRAIN	<0.001	<0.001	<0.001	0.13	<0.001	0.07
	TEST	<0.001	<0.001	<0.001	0.06	<0.001	0.004

known output values (regression and classification are instances of this general problem). Here we showed a new intelligent GP-based system that makes use of these operators to examine burned area. The main objective was the development of a system for predicting the amount of area that will be burned during a forest fire, based on explicit relationships between meteorological data, forest-related data, and the amount of burned area. The comparatively small MAE obtained from experimental results showed that geometric semantic genetic programming outperforms standard genetic programming and produces results that are better or comparable to the ones achieved with state-of-the-art machine learning methods for this application.

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