

$$35 f) X = (ab^*)^* a X + ab$$

$$r = (ab^*)^* a$$

$$s = ab$$

$$X = r^* s = ((ab^*)^* a)^* ab$$

Como  $\varepsilon \notin L((ab^*)^* a)$ , a solução é única.

$$36 a) \rightarrow \begin{cases} X_1 = \overbrace{bX_1}^n + \overbrace{a^*X_2 + a}^s \\ X_2 = a^*X_1 + abX_2 + \varepsilon \end{cases} \Leftrightarrow \begin{cases} X_1 = b^*(a^*X_2 + a) \\ X_2 = a^*(\underbrace{b(a^*X_2 + a)}_{X_1}) + abX_2 + \varepsilon \end{cases}$$

$$\Leftrightarrow \begin{cases} X_2 = a^*b^*a^*X_2 + a^*b^*a + abX_2 + \varepsilon \end{cases}$$

$$\Leftrightarrow \begin{cases} X_2 = (a^*b^*a^* + ab)X_2 + a^*b^*a + \varepsilon \end{cases}$$

$$\begin{aligned} L(a^*b^*a^* + ab) &= L(a^*b^*a^*) \cup L(ab) = \{a^n b^m a^p : n, m, p \in \mathbb{N}_0\} \cup \{ab\} \\ &= \{a^n b^m a^p : n, m, p \in \mathbb{N}_0\} \\ &= L(a^*b^*a^*) \end{aligned}$$

( $n=1, m=1, p=0 \rightarrow ab$ )

$$a^*b^*a^* + ab = a^*b^*a^*$$

$$\Leftrightarrow \begin{cases} X_2 = a^*b^*a^*X_2 + a^*b^*a + \varepsilon \end{cases}$$

$$\Leftrightarrow \begin{cases} X_2 = (a^*b^*a^*)^* (a^*b^*a + \varepsilon) \\ = (a^*b^*a^*)^* a^*b^*a + (a^*b^*a^*)^* \end{cases}$$

$$\begin{aligned} &\underbrace{a^3 b^2 a^2}_{a^3 b^2 a^2} \cdot \underbrace{a^2 b^4 a}_{a^2 b^4 a} \\ &\underbrace{a^3 b^4 a^5}_{a^3 b^4 a^5} \end{aligned}$$

$ab a$

Verifica-se que

$$(a^*b^*a^*)^* a^*b^*a \leq (a^*b^*a^*)^*$$

$$X_1 = b^*(a^*(a^*b^*a^*)^* + a)$$

$$\Leftrightarrow \begin{cases} X_2 = (a^*b^*a^*)^* \end{cases}$$

$$\Leftrightarrow \begin{cases} X_1 = b^*((a^*b^*a^*)^* + a) \\ X_2 = (a^*b^*a^*)^* \end{cases}$$

$$\begin{aligned} a^*(a^*b^*a^*)^* \\ = (a^*b^*a^*)^* \end{aligned}$$

$$\Leftrightarrow \begin{cases} x_2 = (a^* b^* a^*)^* \\ x_1 = b^* (a^* b^* a^*)^* \\ x_2 = (a^* b^* a^*)^* \\ \dots \end{cases}$$

$$= (a^* b^* a^*)^*$$

$$\Leftrightarrow \begin{cases} x_1 = (a+b)^* \\ x_2 = (a+b)^* \end{cases}$$

A soluz e'  $((a+b)^*, (a+b)^*)$