Aula 4: Milner's Calculus of Communicating Systems (cont.)

Interaction & Concurrency Course Unit: Reactive Systems Module

April 19, 2023

Recommended reading

Chapters 2 and 3 of Aceto et al. 2007 and Chapter 2 of Groote and Mousavi 2014.

Concepts introduced and discussed:

- Weak transition,
- Weak bisimulation and observational equivalence (or weak bisimilarity),
- Game characterization of weak bisimilarity.

Definitions, theorems and examples (from Aceto et al. 2007):

- Definition 3.3,
- Definition 3.4 (Weak bisimulation and observational equivalence),
- Example 3.4,
- Example: protocol of table 3.2,
- Theorem 3.3,
- Definition 3.6 (Weak bisimulation game),
- Proposition 3.4,
- Example 3.8.

Exercises suggested (from Aceto et al. 2007):

- Exercise 3.21,

Bissimilação Fraca

Relacao de Transicao Fraca

$$\overset{a}{\Rightarrow} = \begin{cases} (\xrightarrow{\tau})^* \circ \xrightarrow{a} \circ (\xrightarrow{\tau})^* & \text{if } a \neq \tau \\ (\xrightarrow{\tau})^* & \text{if } a = \tau \end{cases}$$

existe pelo menos uma transicao por a.

Bissimulacao Fraca

se $s \xrightarrow{a} s'$ entao $t \overset{a}{\Rightarrow} t'$ $(s', t') \in R$

se $t \xrightarrow{a} t'$ entao $s \overset{a}{\Rightarrow} s'$ $(s', t') \in R$

transicao fraca

tudo resto é igual ao game em strong Bisimul.

Definition

All the same except that

- defender can now answer using $\overset{a}{\Longrightarrow}$ moves.

The attacker is still using only $\xrightarrow{a}$ moves.

$\approx$ → relacao de equiv.

$s \approx t$

Se $a \neq \tau$ então $s \overset{a}{\Longrightarrow} t$ significa que

- de s podemos chegar a t fazendo zero ou mais ações τ, seguido pela ação a, seguido por zero ou mais ações τ.

Se $a = \tau$ então $s \overset{a}{\Longrightarrow} t$ significa que

- de s podemos chegar a t fazendo zero ou mais ações τ.

Dois estados de um LTS sao fracamente bissimilares sse o defensor tem uma estratégia vencedora universal.
Se o atacante tiver uma estratégia vencedora universal, os estados nao sao frac. bissimilares.

pg 68

- Exercise 3.22, → fazer prova que está em cima Protocol $\approx$ ProtocolSpec
- Exercise 3.25,
- ~~Exercise 3.27~~.

Exercises suggested (from Groote and Mousavi 2014):

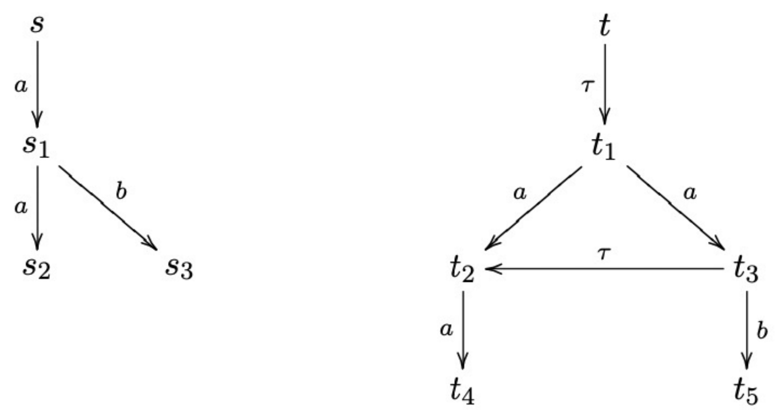
- Exercise 2.4.11,
- Exercise 2.4.12

Other exercises

Decide whether the following claims are true or false. Support your claims either by using weak bisimulation games or the definition of weak bisimilarity.

- $a.\tau.Nil \overset{?}{\approx} \tau.a.Nil$
- $\tau.a.A + b.B \overset{?}{\approx} \tau.(a.A + b.B)$
- $\tau.Nil + (a.Nil \mid \overline{a}.Nil) \setminus \{a, b\} \overset{?}{\approx} \tau.Nil$
- $a.(\tau.Nil + b.B) \overset{?}{\approx} a.Nil + a.b.B$

Bissimulação Fraca



(s, t)

A

$s \xrightarrow{a} s_1$ ① 1. $t \overset{a}{\Rightarrow} t_2$ (s_1, t_2)

2. $t \overset{a}{\Rightarrow} t_3$ (s_1, t_3)

$t \xrightarrow{a} t_2$ $s \overset{a}{\Rightarrow} s_1$ (s_1, t_2)

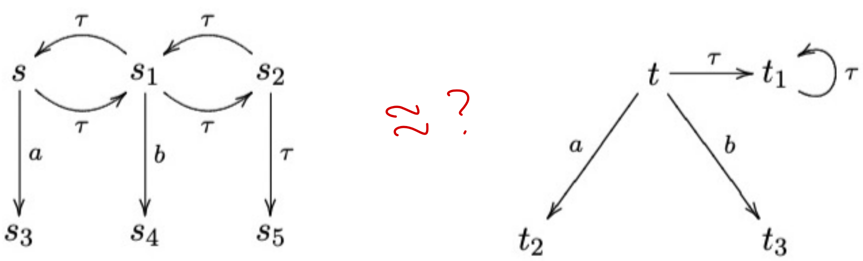
$t \xrightarrow{a} t_3$ $s \overset{a}{\Rightarrow} s_1$ (s_1, t_3)

(s_1, t_2)

$s_1 \xrightarrow{a} s_2$ $t_2 \overset{a}{\Rightarrow} t_4$

$s_1 \xrightarrow{b} s_3$ $t_2 \overset{b}{\not\Rightarrow}$ $\therefore\ s \not\approx t$

$t_2 \xrightarrow{a} t_4$



$\approx$?

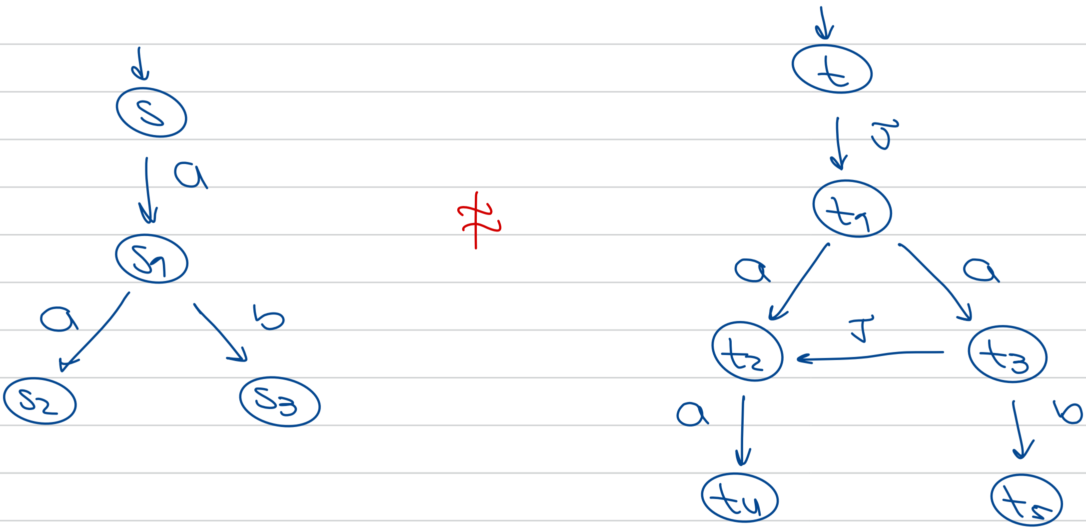
(s, t)

$s \xrightarrow{a} s_3$ $t \overset{a}{\Rightarrow} t_2$ (s_3, t_2)

$s \xrightarrow{b} s_4$ $t \overset{b}{\Rightarrow} t_3$ (s_4, t_3)

$\therefore\ s \approx t$

Game



A: $s \xrightarrow{a} s_1$

D: $t \overset{a}{\Rightarrow} t_3$ (s_1, t_3)

A: $t_3 \xrightarrow{\tau} t_2$

A: $s_1 \xrightarrow{b} s_3$

D: $s_1 \overset{\tau}{\Rightarrow} s_1$ (s_1, t_2)

D: $t_2 \overset{b}{\not\Rightarrow}$ x

- $a.\tau.Nil \overset{?}{\approx} \tau.a.Nil$ ✓
- $\tau.a.A + b.B \overset{?}{\not\approx} \tau.(a.A + b.B)$
- $\tau.Nil + (a.Nil \mid \overline{a}.Nil) \setminus \{a, b\} \overset{?}{\approx} \tau.Nil$ ✓
- $a.(\tau.Nil + b.B) \overset{?}{\not\approx} a.Nil + a.b.B$

(verificar)

Dizer sempre a relação resultante começar com s e t

$$
\begin{tikzcd}
s \arrow[d, "a"] & t \arrow[d, "\tau"] \\
s_1 \arrow[d, "\tau"] & t_1 \arrow[d, "a"] \\
s_2 & t_2
\end{tikzcd}
$$

(s, t) ✓

$s \xrightarrow{a} s_1$ $\quad$ $t \overset{a}{\Rightarrow} t_2$ $\quad$ (s_1, t_2) ✓

$t \xrightarrow{\tau} t_1$ $\quad$ $s \overset{\tau}{\Rightarrow} s$ $\quad$ (s, t_1) ✓

(s_1, t_2)

$s_1 \xrightarrow{\tau} s_2$ $\quad$ $t_2 \overset{\tau}{\Rightarrow} t_2$ $\quad$ (s_2, t_2) ✓

(s, t_1)

$s \xrightarrow{a} s_1$ $\quad$ $t_1 \overset{a}{\Rightarrow} t_2$

$t_1 \xrightarrow{a} t_2$ $\quad$ $s \overset{a}{\Rightarrow} s_1$

$R = \{(s, t), (s_1, t_2), (s, t_1), (s_2, t_2)\}$

$\tau.a.A + b.B \not\approx \tau.(a.A + b.B)$ Game

$$
\begin{tikzcd}
 & \tau.a.A + b.B \ (s) \arrow[dl, "\tau"'] \arrow[dr, "b"] & \\
(s_1)\ a.A \arrow[d, "a"'] & & B\ (s_2) \\
A\ (s_3) & &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & \tau.(a.A + b.B)\ (t) \arrow[d, "\tau"] & \\
 & a.A + b.B\ (t_1) \arrow[dl, "a"'] \arrow[dr, "b"] & \\
(t_2)\ A & & B\ (t_3)
\end{tikzcd}
$$

Atacante joga de forma a que o defensor perca.

A: $t \xrightarrow{\tau} t_1$ $\quad$ D: $s \overset{\tau}{\Rightarrow} s_1$ $\quad$ (s_1, t_1)

A: $t_1 \xrightarrow{b} t_3$ $\quad$ D: $s_1 \overset{b}{\not\Rightarrow}$

Muito IMP

$\tau . Nil + (a . Nil \mid \bar{a} . Nil) \setminus \{a, b\} \approx \tau . Nil$ ✓

$\tau . Nil + (a . Nil \mid \bar{a} . Nil) \setminus \{a, b\}$

$\tau . Nil + (a . Nil \mid \bar{a} . Nil) \setminus \{a, b\} \xrightarrow{\tau} Nil$

$\tau . Nil + (a . Nil \mid \bar{a} . Nil) \setminus \{a, b\} \xrightarrow{\tau} Nil \mid Nil$

$\tau . Nil \xrightarrow{\tau} Nil$

$a . (\tau . Nil + b . B) \not\approx a . Nil + a . b . B$

(s) $a . (\tau . Nil + b . B)$

$a . (\tau . Nil + b . B) \xrightarrow{a}$ (s1) $\tau . Nil + b . B$

(s1) $\tau . Nil + b . B \xrightarrow{\tau}$ (s2) Nil

(s1) $\tau . Nil + b . B \xrightarrow{b} B$ (s3)

(t) $a . Nil + a . b . B$

$a . Nil + a . b . B \xrightarrow{a}$ (t1) Nil

$a . Nil + a . b . B \xrightarrow{a} b . B$ (t2)

$b . B \xrightarrow{b} B$ (t3)

Normal

(s, t)

$s \xrightarrow{a} s_1$ $\quad$ $t \overset{a}{\Rightarrow} t_1$ $\quad$ (s_1, t_1)

$\quad$ $t \overset{a}{\Rightarrow} t_2$ $\quad$ (s_1, t_2)

$t \xrightarrow{a} t_1$ $\quad$ $s \overset{a}{\Rightarrow} s_1$

$t \xrightarrow{a} t_2$ $\quad$ $s \overset{a}{\Rightarrow} s_1$

(s_1, t_1)

$s_1 \xrightarrow{\tau} s_2$ $\quad$ $t_1 \overset{\tau}{\Rightarrow} t_1$ $\quad$ $\therefore\ s \not\approx t$

$s_1 \xrightarrow{b} s_3$ $\quad$ $t_1 \overset{b}{\not\Rightarrow}$

Game

A: $s \xrightarrow{a} s_1$ D: $t \xrightarrow{a} t_1$ (s_1, t_1)

$t \xrightarrow{a} t_2$ (s_1, t_2)

(s_1, t_1)

A: $s_1 \xrightarrow{b} s_3$ D: $t_1 \not\xrightarrow{b}$

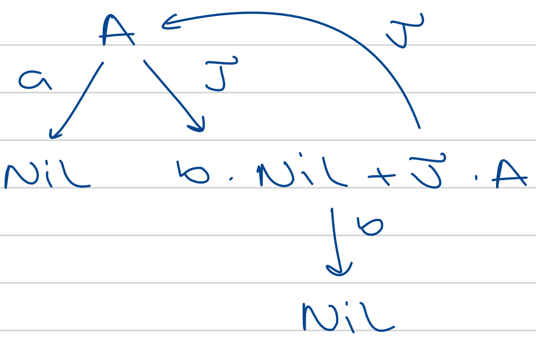
Nao precisamos de verificar para (s_1, t_2) pq já falhou.

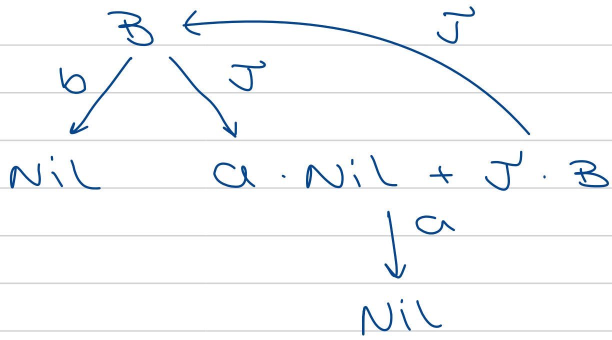
outro ex:

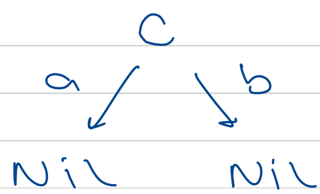
$A = a . Nil + \tau . B$

$B = b . Nil + \tau . A$ $A \approx B \approx C$

$C = a . Nil + b . Nil$







$A \approx B$

$A \xrightarrow{a} Nil$ $\qquad$ $B \xRightarrow{a} Nil$ $\qquad$ (Nil, Nil)

$A \xrightarrow{\tau} (\underbrace{b \cdot Nil + \tau \cdot A}_{B})$ $\qquad$ $B \xRightarrow{\tau} (\underbrace{a \cdot Nil + \tau \cdot B}_{A})$

$B \xrightarrow{b} Nil$ $\qquad$ $A \xRightarrow{b} Nil$

$B \approx C$

$B \xrightarrow{b} Nil$ $\qquad$ $C \xRightarrow{b} Nil$

$B \xrightarrow{a} Nil$ $\qquad$ $C \xRightarrow{a} Nil$

$B \xrightarrow{\tau} (a \cdot Nil + \tau\ B)$ $\qquad$ $C \xRightarrow{\tau} C$

$A \approx C$

$A \xrightarrow{a} Nil$ $\qquad$ $C \xRightarrow{a} Nil$

$A \xrightarrow{b} Nil$ $\qquad$ $C \xRightarrow{b} Nil$

$A \xrightarrow{\tau} (b \cdot Nil + \tau \cdot A)$ $\qquad$ $C \xRightarrow{\tau} C$