①

```
:load Nat
cataN g = g . (id -|- (cataN g)) . outNat

rep a = cataN (either nil (a:))

f = p2 . aux where aux = for (split (succ.p1) mul) (1,1)
```

(rep a) é uma função que está à espera de receber um número inteiro, devolvendo uma lista com o valor a repetido n vezes em que n é o número inteiro recebido por parâmetro.

rep "abc" 2 = ["abc", "abc"]

$$\begin{tikzcd} \mathbb{N}_0 \arrow[d, "rep\ a"'] & 1+\mathbb{N}_0 \arrow[l, "in"'] \arrow[d, "id+(rep\ a)"] \\ A^* & 1+A^* \arrow[l, "{[nil,(a:)]}"] \end{tikzcd}$$

$$\left.\begin{array}{l} f\ 0 = 0 \\ f\ 1 = 1 \\ f\ 2 = 2 \\ f\ 3 = 6 \\ f\ 4 = 24 \\ \vdots \end{array}\right\} \boxed{f\ n = n!} \quad \underline{\text{Fatorial}}$$

② a)

$$\begin{tikzcd} A^* \arrow[d, "K"'] & 1+A\times A^* \arrow[l, "in"'] \arrow[d, "id+id\times K"] \\ A & 1+A\times A \arrow[l, "{[\underline{1},mul]}"] \end{tikzcd}$$

$$\boxed{K = (\![\,[\underline{1}, mul]\,]\!)}$$

b)

$$\begin{tikzcd} A^* \arrow[d, "K"'] & 1+A\times A^* \arrow[l, "in"'] \arrow[d, "id+id\times K"] \\ A^* & 1+A\times A^* \arrow[l, "{[nil,(++)\cdot swap\cdot((:[])\times id)]}"] \end{tikzcd}$$

$$K = (\![\,[nil, (\hat{+\!+})\cdot swap\cdot((:[])\times id)]\,]\!)$$

c)

$$\begin{tikzcd} (A^*)^* \arrow[d, "K"'] & 1+A^*\times(A^*)^* \arrow[l, "in"'] \arrow[d, "id+id\times K"] \\ A^* & 1+A^*\times A^* \arrow[l, "{[nil,(\hat{+\!+})]}"] \end{tikzcd}$$

$$K = (\![\,[nil, (\hat{+\!+})]\,]\!)$$

d)

$$\begin{tikzcd} A^* \arrow[d, "K"'] & 1+A\times A^* \arrow[l, "in"'] \arrow[d, "id+id\times K"] \\ B^* & 1+A\times B^* \arrow[l, "{[nil,cons\cdot(f\times id)]}"] \end{tikzcd}$$

$$K = (\![\,[nil, cons\cdot(f\times id)]\,]\!)$$

Scanned with CamScanner

e)

$$\begin{array}{ccc}
\mathbb{N}_0^* & \xleftarrow{\ in\ } & 1 + \mathbb{N}_0 \times \mathbb{N}_0^* \\
{\scriptstyle k}\downarrow & & \downarrow{\scriptstyle id + id \times k} \\
\mathbb{N}_0 & \xleftarrow[{[\underline{0},\ \widehat{max}]}]{} & 1 + \mathbb{N}_0 \times \mathbb{N}_0
\end{array}$$

$$k = (\!|[\underline{0}, \widehat{max}]|\!)$$

f)

$$\begin{array}{ccc}
A^* & \xleftarrow{\ in\ } & 1 + A \times A^* \\
{\scriptstyle k}\downarrow & & \downarrow{\scriptstyle id + id \times k} \\
A^* & \xleftarrow[{[nil,\ (\widehat{++}) \cdot ((p \rightarrow :[\,], nil) \times id)]}]{} & 1 + A \times A^*
\end{array}$$

$$k = (\!|[nil, (\widehat{++}) \cdot ((p \rightarrow :[\,], nil) \times id)]|\!)$$

3.

$sumprod\ a = (a*) \cdot sum$

$\Leftrightarrow (\!|[zero, add \cdot ((a*) \times id)]|\!) = (a*) \cdot (\!|[zero, add]|\!)$

$\Leftarrow (a*) \cdot [zero, add] = [zero, add \cdot ((a*) \times id)] \cdot id + id \times (a*)$ $\{(49)\}$

$\Leftrightarrow [(a*) \cdot zero, (a*) \cdot add] = [zero, add \cdot ((a*) \times id) \cdot (id \times (a*))]$ $\{(20), (22), (1)\}$

$\Leftrightarrow \begin{cases} (a*) \cdot zero = zero \\ (a*) \cdot add = add \cdot ((a*) \times id) \cdot (id \times (a*)) \end{cases}$ $\{(27)\}$

$\Leftrightarrow \begin{cases} (a*) \cdot zero = zero \\ (a*) \cdot add = add \cdot ((a*) \times (a*)) \end{cases}$ $\{(14)\}$

$\Leftrightarrow \begin{cases} \forall x \mid ((a*) \cdot zero)\ x = zero\ x \\ \forall x, y \mid ((a*) \cdot add)(x, y) = (add \cdot ((a*) \times (a*)))(x, y) \end{cases}$ $\{(72)\ (x2)\}$

$\Leftrightarrow \begin{cases} \forall x \mid a \times 0 = 0 \\ \forall x \mid a \times (x + y) = (a \times x) + (a \times y) \end{cases}$ $\{(73)(x3), (75)(x2), (78)\}$

$\Leftrightarrow$ True $\{$ Absorção e Distributividade da Multiplicação $\}$

4.

$f = foldr\ \pi_2\ i$

$\Leftrightarrow f = (\!|[\underline{i}, \widehat{\pi_2}]|\!)$ $\{$ def foldr $\}$

$\Leftrightarrow f = (\!|[\underline{i}, \pi_2]|\!)$ $\{ uncurry \cdot curry = id \}$

$\Leftrightarrow f \cdot in = [\underline{i}, \pi_2] \cdot id + id \times f$ $\{(46)\}$

$\Leftrightarrow f \cdot in = [\underline{i}, f \cdot \pi_2]$ $\{(22), (1), (13)\}$

$\Leftrightarrow \begin{cases} f \cdot nil = \underline{i} \\ f \cdot cons = f \cdot \pi_2 \end{cases}$ $\{(20), (27)\}$

$\Leftrightarrow \begin{cases} f\ [\,] = i \\ f\ (h : t) = f\ t \end{cases}$ Logo, $\boxed{f = \underline{i}}$

5.

$\mu \cdot T\mu = id = \mu \cdot u$

$\Leftrightarrow \begin{cases} (\pi_1 \times \pi_2) \cdot (u \times u) = id \\ (\pi_1 \times \pi_2) \cdot u = id \end{cases}$ {Def μ, T}

$\Leftrightarrow \begin{cases} (\pi_1 \times \pi_2) \cdot (\langle id, id\rangle \times \langle id, id\rangle) = id \\ (\pi_1 \times \pi_2) \cdot \langle id, id\rangle = id \end{cases}$ {Def μ}

$\Leftrightarrow \begin{cases} (\pi_1 \cdot \langle id, id\rangle) \times (\pi_2 \cdot \langle id, id\rangle) = id \\ \langle \pi_1, \pi_2 \rangle = id \end{cases}$ {(14), (11), (1) (x2)}

$\Leftrightarrow \begin{cases} id \times id = id \\ \langle \pi_1, \pi_2 \rangle = id \end{cases}$ {(7) (x2)}

$\Leftrightarrow \begin{cases} \text{True} \\ \pi_1 \cdot id = \pi_1 \wedge \pi_2 \cdot id = \pi_2 \end{cases}$ {(15), (6)}

$\Leftrightarrow \pi_1 = \pi_1 \wedge \pi_2 = \pi_2$ {(1) (x2)}

$\Leftrightarrow$ True

6. F diz-se um functor se:

$F\ id_A = id_{FA}$ e $F(g \cdot h) = (F g) \cdot (F h)$

a) Provar que H é um functor

$\boxed{H\ id = id}$

$\Leftrightarrow F\ id + G\ id = id$ {Def H}

$\Leftrightarrow id + id = id$ {Def F, G}

$\Leftrightarrow$ True {(26)}

~~$H(g \cdot h) = (H g) \cdot (H h)$~~

$\Leftrightarrow$

$\boxed{H(g \cdot h) = (H g) \cdot (H h)}$

$\Leftrightarrow F(g \cdot h) + G(g \cdot h) = (F g + G g) \cdot (F h + G h)$ {Def H}

$\Leftrightarrow id + (g \cdot h) = (id + g) \cdot (id + h)$ {Def F, G}

$\Leftrightarrow id + (g \cdot h) = id + (g \cdot h)$ {(25)}

$\Leftrightarrow$ True

b) Provar que K é um funtor

$K\ id = id$

$\Leftrightarrow G\,id \times F\,id = id$ $\{ \text{Def } K \}$

$\Leftrightarrow id \times id = id$ $\{ \text{Def } G, F \}$

$\Leftrightarrow$ True $\{ (15) \}$

$K\,(f \cdot g) = (K\,f) \cdot (K\,g)$

$\Leftrightarrow G\,(f \cdot g) \times F\,(f \cdot g) = (G\,f \times F\,f) \cdot (G\,g \times F\,g)$ $\{ \text{Def } K \}$

$\Leftrightarrow f \cdot g \times id = (f \times id) \cdot (g \times id)$ $\{ \text{Def } G, F \}$

$\Leftrightarrow f \cdot g \times id = f \cdot g \times id$ $\{ (12) \}$

$\Leftrightarrow$ True

7.

$(\!|g|\!) \cdot (\!|in \cdot K|\!) = (\!|g \cdot m|\!)$

$\Leftarrow (\!|g|\!) \cdot in \cdot K = g \cdot m \cdot F\,(\!|g|\!)$ $\{ (49) \}$

$\Leftrightarrow g \cdot F\,(\!|g|\!) \cdot K = g \cdot m \cdot F\,(\!|g|\!)$ $\{ (47) \}$

$\Leftarrow F\,(\!|g|\!) \cdot K = m \cdot F\,(\!|g|\!)$ $\{ (5) \}$