Fiche 3 22/23

1.

$[i_1, i_2] = id$

$\Leftrightarrow \begin{cases} id \cdot i_1 = i_1 \\ id \cdot i_2 = i_2 \end{cases}$ {(17)}

$\Leftrightarrow \begin{cases} i_1 = i_1 \\ i_2 = i_2 \end{cases}$ {(1) (x2)}

$\Leftrightarrow$ True

2.

coSwap ∘ coSwap = id

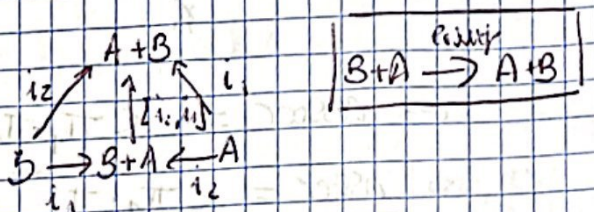
$\Leftrightarrow [i_2, i_1] \cdot [i_2, i_1] = id$ {def coSwap (x2)}

$\Leftrightarrow [[i_2, i_1] \cdot i_2, [i_2, i_1] \cdot i_1] = id$ {(20)}

$\Leftrightarrow [i_1, i_2] = id$ {(18)(x2)}

$\Leftrightarrow$ True {(19)}

3.



3.

$[\underline{K}, \underline{K}] = \underline{K}$

$\Leftrightarrow \begin{cases} \underline{K} \cdot i_1 = \underline{K} \\ \underline{K} \cdot i_2 = \underline{K} \end{cases}$ {(17)}

$\Leftrightarrow \begin{cases} \underline{K} = \underline{K} \\ \underline{K} = \underline{K} \end{cases}$ {(3) (x2)}

$\Leftrightarrow$ True

4.

$\alpha = [\langle \underline{false}, id \rangle, \langle \underline{True}, id \rangle]$

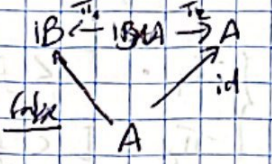
$\Leftrightarrow \begin{cases} \alpha \cdot i_1 = \langle \underline{false}, id \rangle \\ \alpha \cdot i_2 = \langle \underline{True}, id \rangle \end{cases}$ {(17)}

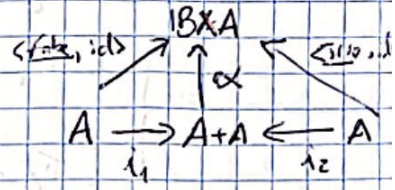
$\Leftrightarrow \begin{cases} \forall a \mid (\alpha \cdot i_1)\, a = \langle \underline{false}, id \rangle\, a \\ \forall a \mid (\alpha \cdot i_2)\, a = \langle \underline{True}, id \rangle\, a \end{cases}$ {(72)}

$$\Leftrightarrow \begin{cases} \forall a \mid \alpha\,(i_1\ a) = (\text{false}, a) \\ \forall a \mid \alpha\,(i_2\ a) = (\text{True}, a) \end{cases}$$

{(73) (x2), (77) (x2), (74) (x2), (75) (x2)}

$$\boxed{\begin{array}{l} \alpha\ (\text{Left}\ a) = (\text{false}, a) \\ \alpha\ (\text{Right}\ a) = (\text{True}, a) \end{array}}$$





5.

$$\text{fac} \cdot [\underline{0}, \text{succ}] = [\underline{1}, \text{mul} \cdot \langle \text{succ}, \text{fac} \rangle]$$

$$\Leftrightarrow [\text{fac} \cdot \underline{0}, \text{fac} \cdot \text{succ}] = [\underline{1}, \text{mul} \cdot \langle \text{succ}, \text{fac} \rangle] \quad \{(20)\}$$

$$\Leftrightarrow \begin{cases} \text{fac} \cdot \underline{0} = \underline{1} \\ \text{fac} \cdot \text{succ} = \text{mul} \cdot \langle \text{succ}, \text{fac} \rangle \end{cases} \quad \{(27)\}$$

$$\Leftrightarrow \begin{cases} \forall n \mid (\text{fac} \cdot \underline{0})\ n = \underline{1}\ n \\ \forall n \mid (\text{fac} \cdot \text{succ})\ n = (\text{mul} \cdot \langle \text{succ}, \text{fac} \rangle)\ n \end{cases} \quad \{(72) (x2)\}$$

$$\Leftrightarrow \begin{cases} \forall n \mid \text{fac}\ 0 = 1 \\ \forall n \mid f\,(n+1) = (n+1) \times (\text{fac}\ n) \end{cases} \quad \{(73) (x3), (72) (x2), (75) (x2), (77)\}$$

6. [F3] Functor-id (+)

$$id + id = id$$

$$\Leftrightarrow [i_1 \cdot id, i_2 \cdot id] = id \quad \{(21)\}$$

$$\Leftrightarrow [i_1, i_2] = id \quad \{(1) (x2)\}$$

$$\Leftrightarrow \text{True} \quad \{(3)\}$$

[F4] Natural-i_1

$$(f + g) \cdot i_1 = i_1 \cdot f$$

$$\Leftrightarrow [i_1 \cdot f, i_2 \cdot g] \cdot i_1 = i_1 \cdot f \quad \{(21)\}$$

$$\Leftrightarrow i_1 \cdot f = i_1 \cdot f \quad \{(18)\}$$

$$\Leftrightarrow \text{True}$$

(8.)

$$[\langle f, g\rangle, \langle h, k\rangle] = \langle [f, h], [g, k]\rangle$$

$$\Leftrightarrow \begin{cases} \pi_1 \cdot [\langle f, g\rangle, \langle h, k\rangle] = [f, h] \\ \pi_2 \cdot [\langle f, g\rangle, \langle h, k\rangle] = [g, k] \end{cases} \quad \{(6)\}$$

$$\Leftrightarrow \begin{cases} [\pi_1 \cdot \langle f, g\rangle, \pi_1 \cdot \langle h, k\rangle] = [f, h] \\ [\pi_2 \cdot \langle f, g\rangle, \pi_2 \cdot \langle h, k\rangle] = [g, k] \end{cases} \quad \{(20)(21)\}$$

$$\therefore \begin{cases} [f, h] = [f, h] \\ [g, k] = [g, k] \end{cases} \quad \{(7)(21)\}$$

∴ True

(9.)

$$\begin{aligned} \text{undistl} &= [i_1 \times id, i_2 \times id] && \{(10)(32)\} \\ &= [\langle i_1 \cdot \pi_1, id \cdot \pi_2\rangle, \langle i_2 \cdot \pi_1, id \cdot \pi_2\rangle] && \{(24), (1)(42)\} \\ &= \langle [i_1 \cdot \pi_1, i_2 \cdot \pi_1], [\pi_2, \pi_2]\rangle && \{(21)\} \\ &= \langle \pi_1 + \pi_1, [\pi_2, \pi_2]\rangle \end{aligned}$$