

1.  $(3 \ominus 2) \ominus 3$

$= (1 + (3 \ominus (2+1))) \ominus 3$

$= (1 + (3 \ominus 3)) \ominus 3$

$= (1 + 0) \ominus 3$

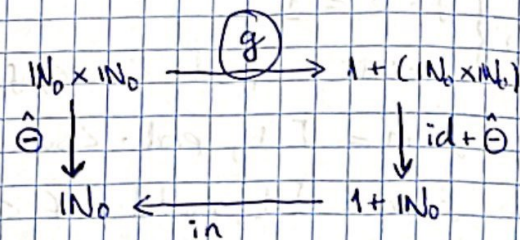
$= 1 \ominus 3$

$= 0$

$(3 \ominus 4) + 4$

$= 0 + 4$

$= 4$



$x \ominus y = \text{if } x \leq y \text{ then } 0 \text{ else } 1 + x \ominus (y+1)$

$\Rightarrow \hat{\Theta}(x, y) = \text{if } \hat{\leq}(x, y) \text{ then } \underline{0}(x, y) \text{ else } \text{succ}(x \ominus (y+1))$  {85, 75}

$\Rightarrow \hat{\Theta}(x, y) = \text{if } \hat{\leq}(x, y) \text{ then } \underline{0}(x, y) \text{ else } \text{succ}(\hat{\Theta}(x, \text{succ } y))$  {85}

$\Rightarrow \hat{\Theta}(x, y) = \text{if } \hat{\leq}(x, y) \text{ then } \underline{0}(x, y) \text{ else } \text{succ}(\hat{\Theta} \cdot (\text{id} \times \text{succ}))(x, y)$  {73, 74, 78}

$\Rightarrow \hat{\Theta}(x, y) = \text{if } \hat{\leq}(x, y) \text{ then } \underline{0}(x, y) \text{ else } (\text{succ} \cdot \hat{\Theta} \cdot (\text{id} \times \text{succ}))(x, y)$  {73}

$\Rightarrow \hat{\Theta}(x, y) = (\hat{\leq} \rightarrow \underline{0}, \text{succ} \cdot \hat{\Theta} \cdot (\text{id} \times \text{succ}))(x, y)$  {79}

$\Rightarrow \hat{\Theta} = \hat{\leq} \rightarrow \underline{0}, \text{succ} \cdot \hat{\Theta} \cdot (\text{id} \times \text{succ})$  {72}

$\Rightarrow \hat{\Theta} = [\underline{0}, \text{succ} \cdot \hat{\Theta} \cdot (\text{id} \times \text{succ})] \cdot (\hat{\leq}?)$  {30}

~~$\Rightarrow \hat{\Theta} = \text{in} \cdot (! + (\hat{\Theta} \cdot (\text{id} \times \text{succ}))) \cdot (\hat{\leq}?)$~~

$\Rightarrow \hat{\Theta} = \text{in} \cdot (! + (\hat{\Theta} \cdot (\text{id} \times \text{succ}))) \cdot (\hat{\leq}?)$  {3, 22}

$\Rightarrow \hat{\Theta} = \text{in} \cdot ((\text{id} \circ !) + (\hat{\Theta} \cdot (\text{id} \times \text{succ}))) \cdot (\hat{\leq}?)$  {11}

$\Rightarrow \hat{\Theta} = \text{in} \cdot ((\text{id} + \hat{\Theta}) \cdot (! + (\text{id} \times \text{succ}))) \cdot (\hat{\leq}?)$  {25}

$\Rightarrow \hat{\Theta} = \text{in} \cdot F \hat{\Theta} \cdot (! + (\text{id} \times \text{succ})) \cdot (\hat{\leq}?)$  {24 F, 24}

$= \text{out} \cdot \hat{\Theta} = F \hat{\Theta} \cdot (! + (\text{id} \times \text{succ})) \cdot (\hat{\leq}?)$  {34}

$= \boxed{\hat{\Theta} = [(! + (\text{id} \times \text{succ})) \cdot (\hat{\leq}?)]}$  {55}

Logo, o gene  $\hat{g}$  é igual à função  $(! + (\text{id} \times \text{succ})) \cdot (\hat{\leq}?)$

$g = (! + (\text{id} \times \text{succ})) \cdot (\hat{\leq}?)$

$\Rightarrow g = [i_1 \cdot !, i_2 \cdot (\text{id} \times \text{succ})] \cdot (\hat{\leq}?)$  {21}

$\Rightarrow g = \hat{\leq} \rightarrow i_1 \cdot !, i_2 \cdot (\text{id} \times \text{succ})$  {30}

$\Rightarrow g(x, y) = \text{if } x \leq y \text{ then } i_1() \text{ else } i_2(x, y+1)$  {72, 73, 73, 78}



(2)

a)

$$\begin{cases} f(0) = 1 \\ f(n+1) = (n+1) \times f(n) \end{cases}$$

$$\Rightarrow \begin{cases} f \cdot 0 = 1 \\ f \cdot \text{succ} = \text{prod} \cdot \langle \text{succ}, f \rangle \end{cases} \quad \begin{matrix} \{75, 73, 72\} \\ \{73, 77\} \end{matrix}$$

$$\Rightarrow f \cdot \text{in} = [1, \text{prod} \cdot \langle \text{succ}, f \rangle] \quad \{17\}$$

$$\Rightarrow f \cdot \text{in} = [1, \text{prod}] \cdot F \langle \text{succ}, f \rangle \quad \{1, 22\}$$

$$\Rightarrow f \cdot \text{in} = [1, \text{prod}] \cdot F \langle \text{succ-id}, \text{id} \cdot f \rangle \quad \{1^{(2)}\}$$

$$\Rightarrow f \cdot \text{in} = [1, \text{prod}] \cdot F((\text{succ} \times \text{id}) \cdot \langle \text{id}, f \rangle) \quad \{11\}$$

$$\Rightarrow f \cdot \text{in} = [1, \text{prod}] \cdot F(\text{succ} \times \text{id}) \cdot f \langle \text{id}, f \rangle \quad \{44\}$$

$$\Rightarrow f \cdot \text{in} = \underbrace{[1, \text{prod} \cdot (\text{succ} \times \text{id})]}_g \cdot \underbrace{F \langle \text{id}, f \rangle}_G \quad \{22, 1\}$$

$$\text{Logo } \text{in} = \text{in}_{w_0}; \quad g = [1, \text{prod} \cdot (\text{succ} \times \text{id})]; \quad Gg = \text{id} \cdot g$$

b)

$$f = \boxed{(g) \cdot [G \langle \text{id}, \text{id} \rangle \cdot \text{out}]} \quad \{27, 56, 34, 33\}$$

$$\Rightarrow f = g \cdot F(g) \cdot \text{out} \cdot \text{in} \cdot F[G \langle \text{id}, \text{id} \rangle \cdot \text{out}] \cdot G \langle \text{id}, \text{id} \rangle \cdot \text{out} \quad \{27, 56, 34, 33\}$$

$$\Rightarrow f = g \cdot F(g) \cdot F[G \langle \text{id}, \text{id} \rangle \cdot \text{out}] \cdot G \langle \text{id}, \text{id} \rangle \cdot \text{out} \quad \{\text{out} \cdot \text{in} = \text{id}\}$$

$$\Rightarrow f = g \cdot F(\boxed{(g) \cdot [G \langle \text{id}, \text{id} \rangle \cdot \text{out}]}) \cdot G \langle \text{id}, \text{id} \rangle \cdot \text{out} \quad \{44\}$$

$$\Rightarrow f = g \cdot Ff \cdot G \langle \text{id}, \text{id} \rangle \cdot \text{out} \quad \{\text{Passo 1}\}$$

$$\Rightarrow f = g \cdot B(\text{id}, f) \cdot G \langle \text{id}, \text{id} \rangle \cdot \text{out} \quad \{50\}$$

$$\Rightarrow f = g \cdot B(\text{id} \times f) \cdot G \langle \text{id}, \text{id} \rangle \cdot \text{out} \quad \{B(x, y) = B(x \times y)\}$$

$$\Rightarrow f = g \cdot B((\text{id} \times f) \cdot \langle \text{id}, \text{id} \rangle) \cdot \text{out} \quad \{\text{função de aridade}\}$$

$$\Rightarrow f = g \cdot B \langle \text{id}, f \rangle \cdot \text{out} \quad \{11, 1^{(2)}\}$$

$$\Rightarrow f \cdot \text{in} = g \cdot B \langle \text{id}, f \rangle \quad \{33\}$$



4.

$$\text{While } p \text{ f } g = \text{tailr } ((g+f) \cdot (1-p)?)$$

$$\Rightarrow \text{While } p \text{ f } g = \llbracket \text{join}, ((g+f) \cdot (1-p)?) \rrbracket \quad \{F4\}$$

$$\Rightarrow \text{While } p \text{ f } g = \text{join} \cdot F(\text{While } p \text{ f } g) \cdot ((g+f) \cdot (1-p)?) \quad \{22, 1\}$$

$$\Rightarrow \text{While } p \text{ f } g = [\text{id}, \text{id}] \cdot (\text{id} + (\text{While } p \text{ f } g)) \cdot ((g+f) \cdot (1-p)?) \quad \{22, 1\}$$

$$\Rightarrow \text{While } p \text{ f } g = [\text{id}, \text{While } p \text{ f } g] \cdot ((g+f) \cdot (1-p)?) \quad \{22, 1\}$$

$$\Rightarrow \text{While } p \text{ f } g = [g, (\text{While } p \text{ f } g) \cdot f] \cdot (1-p)? \quad \{22, 1\}$$

$$\Rightarrow \text{While } p \text{ f } g = (1-p) \rightarrow g, (\text{While } p \text{ f } g) \cdot f \quad \{30\}$$

$$\Rightarrow \text{if } x \mid \text{While } p \text{ f } g \text{ } x = \text{if not } (p \ x) \text{ then } g \ x \text{ else } \text{While } p \text{ f } g \ (f \ x)$$

5.

$$(\text{tailr } g) \cdot f = \text{tailr } h$$

$$\{ \text{tailr } f = \llbracket \nabla, f \rrbracket = (\nabla) \cdot [f] \}$$

$$\Rightarrow (\nabla) \cdot [g] \cdot f = (\nabla) \cdot [h]$$

$$\{ \text{tailr } f = \llbracket \nabla, f \rrbracket = (\nabla) \cdot [f] \}$$

$$\leq [g] \cdot f = [h]$$

$$\{5\}$$

$$\Rightarrow g \cdot f = f \cdot h$$

$$\{58\}$$

$$\Rightarrow g \cdot f = (\text{id} + f) \cdot h$$

$$\{ \text{Def } F(B(x, y) = x + y) \}$$

6.

$$f \cdot [g, h] = [f \cdot g, f \cdot h]$$

$$= [\mu \cdot \tau f \cdot g, \mu \cdot \tau f \cdot h]$$

$$\{66(x2)\}$$

$$= \mu \cdot \tau f \cdot [g, h]$$

$$\{20\}$$

$$= f \cdot [g, h]$$

$$\{66\}$$

7.

$$\text{sequence} = ([\text{return}, \text{id}] \cdot (\text{nil} + \text{L cons}))$$

$$\Rightarrow \text{sequence} \cdot \text{in} = [\text{return}, \text{id}] \cdot (\text{nil} + \text{L cons}) \cdot f_{\text{sequence}} \quad \{46\}$$

$$\Rightarrow \text{sequence} \cdot \text{in} = [\text{return} \cdot \text{nil}, \text{L cons}] \cdot (\text{id} + \text{id} \times \text{sequence}) \quad \{22, 1\}$$

$$\Rightarrow \begin{cases} \text{sequence} \cdot \text{nil} = \text{return} \cdot \text{nil} \\ \text{sequence} \cdot \text{cons} = \text{L cons} \cdot (\text{id} \times \text{sequence}) \end{cases} \quad \{22, 1, 27\}$$

$$\Rightarrow \begin{cases} \text{sequence} [i] = \text{return} [i] \\ \text{sequence} (h:t) = \text{do } \{ a \leftarrow h; b \leftarrow \text{sequence } t; \text{return } (\text{cons } (a, b)) \} \end{cases} \quad \{72, 73(x2), 75(x2)\}$$

$$\{72, 78, 43(x2)\}$$



⑧.

$$a) \text{ report} = \llbracket \langle \text{id}, \text{id} \rangle \rrbracket$$

$$\Rightarrow \text{out} \cdot \text{report} = f \text{ report} \cdot \langle \text{id}, \text{id} \rangle$$

$$\Rightarrow \text{report} = \text{in} \cdot (\text{id} \times \text{report}) \cdot \langle \text{id}, \text{id} \rangle$$

$$\Rightarrow \text{report} = \text{cons} \cdot \langle \text{id}, \text{report} \rangle$$

$$\Rightarrow \text{report } a = a : \text{report } a =$$

{55}

{34}

{11, 1(x2)}

{72, 73, 77}

b)

$$(\text{map } f) \cdot \text{report} = \text{report} \cdot f$$

$$\Rightarrow \llbracket B(f, \text{id}) \cdot \langle \text{id}, \text{id} \rangle \rrbracket = \llbracket \langle \text{id}, \text{id} \rangle \rrbracket \cdot f$$

$$\Rightarrow \llbracket (f \times \text{id}) \cdot \langle \text{id}, \text{id} \rangle \rrbracket = \llbracket \langle \text{id}, \text{id} \rangle \rrbracket \cdot f$$

$$\Rightarrow \llbracket \langle f, \text{id} \rangle \rrbracket = \llbracket \langle \text{id}, \text{id} \rangle \rrbracket \cdot f$$

$$\Rightarrow \langle \text{id}, \text{id} \rangle \cdot f = f \cdot \langle \text{id}, \text{id} \rangle$$

$$\Rightarrow \langle f, f \rangle = (\text{id} \times f) \cdot \langle f, \text{id} \rangle$$

$$\Rightarrow \langle f, f \rangle = \langle f, f \rangle$$

$$\Rightarrow \text{True}$$

{(1)}

{Def B}

{11, 1(x2)}

{55}

{9, Def f}

{11, 1(x2)}

{Prop. Extensional de typage}

③.

$$\text{divide } (x : y : xs)$$

$$| x > y = i_2(y, (x : xs))$$

$$| \text{otherwise} = i_2(x, (y : xs))$$

$$\text{divide } x = i_1 x$$

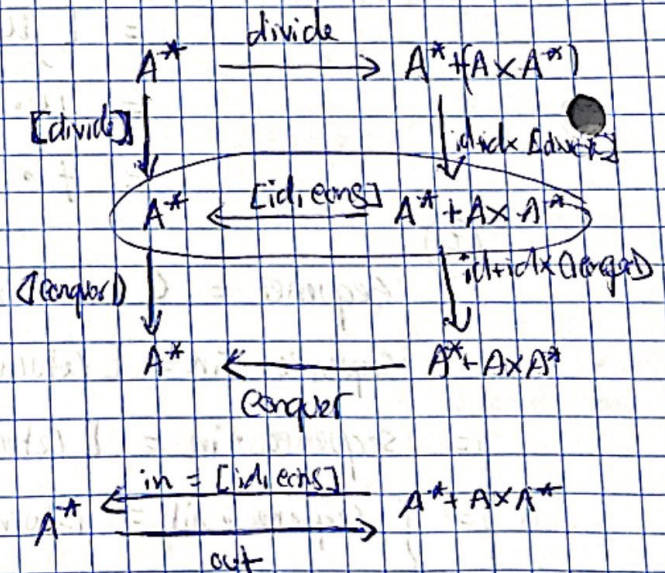
$$\text{conquer} = \llbracket \text{id}, \text{cons} \rrbracket$$

$$\text{bubble} = \llbracket \text{conquer}, \text{divide} \rrbracket$$

$$\llbracket f, g \rrbracket = f \cdot \llbracket f, g \rrbracket \cdot g$$

$$\text{base } f \cdot g = \text{id} + (f \times g)$$

$$f \cdot = \text{base id}$$



~~out~~

out (h:t)

length t > 0 = Right (h,t)

otherwise = Left (h:t)