Aula 3: Milner's Calculus of Communicating Systems

Interaction & Concurrency Course Unit: Reactive Systems Module

April 14, 2023

Recommended reading

Chapter 2 of Aceto et al. 2007.

Concepts introduced and discussed:

- Structural operation semantics (SOS) for CCS,
- Use of SOS rules to derive LTS whose states are processes described by CCS expressions,
- Transition trees and transition graphs,
- Use of SOS rules to prove the existence of a transition in a LTS corresponding to a CCS program.

Some relevant definitions and examples (from Aceto et al. 2007):

- Definition 2.3 (formal syntax for CCS)
- Table 2.2 (SOS rules for CCS)

Exercises suggested (from Aceto et al. 2007):

- Exercises ~~2.7~~, 2.8 and 2.9. pg 31

Other exercises discussed

1. Let $A \overset{\text{def}}{=} a \,.\, A$. Use SOS rules to prove:

$$((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil)[c/a] \xrightarrow{c} ((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil)[c/a]$$

Regras Estruturais

ACT $\dfrac{}{\alpha.P \xrightarrow{\alpha} P}$ SUM$_j$ $\dfrac{P_j \xrightarrow{\alpha} P'_j}{\sum_{i\in I} P_i \xrightarrow{\alpha} P'_j}\ j \in I$

COM1 $\dfrac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$ COM2 $\dfrac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$

COM3 $\dfrac{P \xrightarrow{a} P' \quad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$

→ Ações em paralelo

RES $\dfrac{P \xrightarrow{\alpha} P'}{P \setminus L \xrightarrow{\alpha} P' \setminus L}\ \alpha, \bar{\alpha} \notin L$ REL $\dfrac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$

CON $\dfrac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'}\ K \stackrel{\text{def}}{=} P$

→ Ações não internas

Uso de regras SOS para derivar LTS cujos estados são processos definidos descritos por expressões CCS.

Utilização de regras SOS para comprovar a existência de uma transição num LTS correspondente a um programa CCS

Processos

- **Processo nulo** (ou 0) (o único processo atômico)
- **Prefixo de ação** ($a.P$)
- **Nomes e definições recursivas** ($=^{def}$)
- **Escolha não-determinística** ($+$) ($P1 + P2$)

$P :=$

K → constantes do processo

$\alpha.P$ → prefixo ($a \in Act$)

$\sum_{i\in I} P_i$ → soma (I→ conjunto de índices arbitrários)

$P1|P2$ → composição paralela

$P\setminus L$ → restrição ($L \in A \cup \bar{A}$, L **não é visível a partir do exterior**)

$P[f]$ → renomeação ($f : Act \to Act$)

- $f(\tau) = \tau$
    - τ é uma **ação interna** ou silenciosa (não visível do exterior)
- $f(\bar{a}) = \overline{f(a)}$

Notation

$P_1 + P_2 = \sum_{i\in\{1,2\}} P_i$ $Nil = 0 = \sum_{i\in\emptyset} P_i$

2. Consider the following CCS program discussed before:

$$Univ \stackrel{\text{def}}{=} (CM \mid CS)\backslash\{coin, coffee\}$$

$$CM \stackrel{\text{def}}{=} coin \,.\, \overline{coffee} \,.\, CM$$

$$CS \stackrel{\text{def}}{=} \overline{pub} \,.\, \overline{coin} \,.\, coffee \,.\, CS$$

- Derive a LTS that describes the behavior of the system.
- Use SOS rules to prove the existence of the transitions presented.

3. Consider the alarm clock systems discussed previously.

- Present a CCS specification of each system.
- Derive a LTS for each CCS specification.
- Use SOS rules to prove the existence of (some of) the transitions presented.

SOS Rules

$A = b.a.B$

(1)

$$\frac{}{b.a.B \xrightarrow{b} a.B}\ \text{Act} \qquad \frac{}{\overline{b}.Nil \xrightarrow{\overline{b}} Nil}\ \text{Act}$$

(*) + CON

$$\frac{b.a.B \xrightarrow{b} a.B \qquad \overline{b}.Nil \xrightarrow{\overline{b}} Nil}{(A \mid \overline{b}.Nil) \xrightarrow{\tau} (a.B \mid Nil)}\ COM_3\ (*)$$

$$\frac{(A \mid \overline{b}.Nil) \xrightarrow{\tau} (a.B \mid Nil)}{(A \mid \overline{b}.Nil)\setminus\{b\} \xrightarrow{\tau} (a.B \mid Nil)\setminus\{b\}}\ Res$$

(2)

$$\frac{}{\overline{b}\ A[a/b] \xrightarrow{\overline{a}} A[a/b]}\ \text{Act}$$

$$\frac{\overline{b}\ A[a/b] \xrightarrow{\overline{a}} A[a/b]}{(\overline{b}.A)[a/b] \xrightarrow{\overline{a}} A[a/b]}\ REL$$

$$\frac{(\overline{b}.A)[a/b] \xrightarrow{\overline{a}} A[a/b]}{(A \mid (\overline{b}.a.B)) + (\overline{b}.A)[a/b] \xrightarrow{\overline{a}} A[a/b]}\ SUM_2$$

1. Let $A \stackrel{\text{def}}{=} a \,.\, A$. Use SOS rules to prove:

$$((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil)[c/a] \xrightarrow{c} ((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil)[c/a]$$

$$\frac{}{a \,.\, A \xrightarrow{c} A}\ \text{ACT}$$

CON, $A \stackrel{\text{def}}{=} a \,.\, A$

$$A \xrightarrow{c} A$$

COM1

$$A \mid \overline{a} \,.\, Nil \xrightarrow{c} A \mid \overline{a} \,.\, Nil$$

COM1

$$((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil \xrightarrow{c} ((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil)$$

REL

$$((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil)[c/a] \xrightarrow{c} ((A \mid \overline{a} \,.\, Nil) \mid b \,.\, Nil)[c/a]$$

Só substituimos A pela sua definicao, na última "derivação"

2. Consider the following CCS program discussed before:

$$Univ \stackrel{\text{def}}{=} (CM \mid CS) \backslash \{coin, coffee\}$$

$$CM \stackrel{\text{def}}{=} coin \,.\, \overline{coffee} \,.\, CM$$

$$CS \stackrel{\text{def}}{=} \overline{pub} \,.\, \overline{coin} \,.\, coffee \,.\, CS$$

- Obtenha um LTS que descreva o comportamento do sistema.
- Use regras SOS para provar a existência das transições apresentadas.

LTS

Ações internas

$$Univ \xrightarrow{\overline{pub}} (CM \mid \overline{coin} \,.\, coffee \,.\, CS) \backslash \{coin, coffee\}$$

$$(CM \mid \overline{coin} \,.\, coffee \,.\, CS) \backslash \{coin, coffee\} \xrightarrow{\tau} (\overline{coffee} \,.\, CM \mid coffee \,.\, CS) \backslash \{\dots\}$$

$$(\overline{coffee} \,.\, CM \mid coffee \,.\, CS) \backslash \{\dots\} \xrightarrow{\tau} Univ$$

τ representa as ações internas, neste caso coin, coffee.

Provar a existência das transições apresentadas usando as regras SOS:

$$\dfrac{}{\overline{pub} \,.\, (\overline{coin} \,.\, coffee \,.\, CS) \xrightarrow{\overline{pub}} (coin \,.\, coffee \,.\, CS)}\ \text{ACT}$$

$$\dfrac{\overline{pub} \,.\, (\overline{coin} \,.\, coffee \,.\, CS) \xrightarrow{\overline{pub}} (coin \,.\, coffee \,.\, CS)}{(CM \mid \overline{pub} \,.\, \overline{coin} \,.\, coffee \,.\, CS) \backslash \{\dots\} \xrightarrow{\overline{pub}} (CM \mid \overline{coin} \,.\, coffee \,.\, CS) \backslash \{\dots\}}\ \text{COM}_2$$

$$\dfrac{(CM \mid \overline{pub} \,.\, \overline{coin} \,.\, coffee \,.\, CS) \backslash \{\dots\} \xrightarrow{\overline{pub}} (CM \mid \overline{coin} \,.\, coffee \,.\, CS) \backslash \{\dots\}}{(CM \mid CS) \backslash \{\dots\} \xrightarrow{\overline{pub}} (CM \mid \overline{coin} \,.\, coffee \,.\, CS) \backslash \{\dots\}}\ \text{CON}$$

$$\dfrac{(CM \mid CS) \backslash \{\dots\} \xrightarrow{\overline{pub}} (CM \mid \overline{coin} \,.\, coffee \,.\, CS) \backslash \{\dots\}}{Univ \xrightarrow{\overline{pub}} (CM \mid \overline{coin} \,.\, coffee \,.\, CS) \backslash \{\dots\}}\ \text{RES}$$