Ficha 6

(1)

$$Point :: a \to a \to a \to Point\ a$$

Point é uma função parecida a in, a única diferença é que Point recebe os 3 a's um de cada vez, enquanto que in os recebe numa só vez num tuplo do tipo $(A \times A) \times A$.

Qual a função que nos permite transformar Point em in?

uncurry

$$Point \in (((Point\ A)^A)^A)^A$$

$$(uncurry\ Point) \in ((Point\ A)^A)^{A \times A}$$

$$uncurry\ (uncurry\ Point) \in (Point\ A)^{(A \times A) \times A}$$

$$\boxed{in = uncurry(uncurry\ Point) = (\widehat{Point})}$$

(2)

$$ap \cdot (\overline{f} \times id) = f$$

$\Leftrightarrow \forall a,b \mid (ap \cdot (\overline{f} \times id))(a,b) = f(a,b)$ $\quad \{(72)\}$

$\Leftrightarrow \forall a,b \mid ap((\overline{f} \times id)(a,b)) = f(a,b)$ $\quad \{(73)\}$

$\Leftrightarrow \forall a,b \mid ap(\overline{f}\,a, id\ b) = f(a,b)$ $\quad \{(78)\}$

$\Leftrightarrow \forall a,b \mid ap(\overline{f}\,a, b) = f(a,b)$ $\quad \{(74)\}$

$\Leftrightarrow \forall a,b \mid \overline{f}\ a\ b = f(a,b)$ $\quad \{(85)\}$

$\Leftrightarrow \forall a,b \mid curry\ f\ a\ b = f(a,b)$ $\quad \{\overline{f} = curry\ f\}$

(3)

$$\overline{f \cdot g \times h} = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$$

$\Leftrightarrow \overline{f \cdot ((id \cdot g) \times (h \cdot id))} = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(1)\ (\times 2)\}$

$\Leftrightarrow \overline{f \cdot (id \times h) \cdot (g \times id)} = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(14)\}$

$\Leftrightarrow \overline{f \cdot (id \times h)} \cdot g = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(38)\}$

$\Leftrightarrow \overline{ap \cdot (\overline{f} \times id) \cdot (id \times h)} \cdot g = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(36)\}$

$\Leftrightarrow \overline{ap \cdot (\overline{f} \cdot id) \times (id \cdot h)} \cdot g = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(14)\}$

$\Leftrightarrow \overline{ap \cdot (id \cdot \overline{f}) \times (h \cdot id)} \cdot g = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(1)\ (\times 2)\}$

$\Leftrightarrow \overline{ap \cdot (id \times h) \cdot (\overline{f} \times id)} \cdot g = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(14)\}$

$\Leftrightarrow \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$ $\quad \{(38)\}$

$\Leftrightarrow$ True $\quad$ { Prop. Reflexiva = Igualdade }

④

june · unjune = id

$\Leftrightarrow \forall K \mid (\text{june} \cdot \text{unjune})\, K = \text{id}\, K$ {(72)}

$\Leftrightarrow \forall K \mid \text{june}\,(\text{unjune}\; K) = K$ {(73),(74)}

$\Leftrightarrow \forall K \mid \text{june}\,(K \cdot i_1, K \cdot i_2) = K$ {Def unjune}

$\Leftrightarrow \forall K \mid [K \cdot i_1, K \cdot i_2] = K$ {Def june}

$\Leftrightarrow \forall K \mid K \cdot i_1 = K \cdot i_1 \wedge K \cdot i_2 = K \cdot i_2$ {(17)}

$\Leftrightarrow$ True {Prop. Reflexiva Igualdade}

unjune · june = id

$\Leftrightarrow \forall f, g \mid (\text{unjune} \cdot \text{june})(f, g) = \text{id}\,(f, g)$ {(72)}

$\Leftrightarrow \forall f, g \mid \text{unjune}\,(\text{june}\,(f, g)) = (f, g)$ {(74),(73)}

$\Leftrightarrow \forall f, g \mid \text{unjune}\,([f, g]) = (f, g)$ {Def june}

$\Leftrightarrow \forall f, g \mid ([f, g] \cdot i_1, [f, g] \cdot i_2) = (f, g)$ {Def unjune}

$\Leftrightarrow \forall f, g \mid (f, g) = (f, g)$ {(18) (x2)}

$\Leftrightarrow$ True {Prop. Reflexiva Igualdade}

⑤

$\text{flip}\,(\text{flip}\; f) = f$

$\Leftrightarrow \text{flip}\,(\hat{f} \cdot \text{swap}) = f$ {(F6)}

$\Leftrightarrow \left(\widehat{\overline{\hat{f} \cdot \text{swap}}}\right) \cdot \text{swap} = f$ {(F6)}

$\Leftrightarrow \hat{f} \cdot \text{swap} \cdot \text{swap} = f$ {uncurry · curry = id}

$\Leftrightarrow \hat{f} \cdot \text{id} = f$ {swap · swap = id}

$\Leftrightarrow \hat{f} = f$ {(1)}

$\Leftrightarrow f = f$ {curry · uncurry = id}

$\Leftrightarrow$ True {Prop. Reflexiva Igualdade}

$$flip\ f = \overline{\hat{f} \cdot swap}$$

$$\Leftrightarrow \hat{f} \cdot swap = ap \cdot (flip\ f \times id) \qquad \{(35)\}$$

$$\Leftrightarrow \forall x,y \mid (\hat{f} \cdot swap)(x,y) = (ap \cdot (flip\ f \times id))(x,y) \qquad \{(72)\}$$

$$\Leftrightarrow \forall x,y \mid \hat{f}(swap(x,y)) = ap((flip\ f \times id)(x,y)) \qquad \{(73)(\times 2)\}$$

$$\Leftrightarrow \forall x,y \mid \hat{f}(y,x) = ap(flip\ f\ x, y) \qquad \{\text{Def } swap, (78), (71)\}$$

$$\Leftrightarrow \forall x,y \mid \hat{f}(y,x) = flip\ f\ x\ y \qquad \{(83)\}$$

$$\Leftrightarrow \forall x,y \mid f\ y\ x = flip\ f\ x\ y \qquad \{(85)\}$$