

①

$$\text{out} \cdot \text{in} = \text{id}$$

$$\Rightarrow \text{out} \cdot [0, \text{succ}] = \text{id} \quad \{ \text{def in} \}$$

$$\Rightarrow [ \text{out} \cdot 0, \text{out} \cdot \text{succ} ] = \text{id} \quad \{ (20) \}$$

$$\Rightarrow \begin{cases} \text{id} \cdot i_1 = \text{out} \cdot 0 \\ \text{id} \cdot i_2 = \text{out} \cdot \text{succ} \end{cases} \quad \{ (17) \}$$

$$\Rightarrow \begin{cases} \text{out} \cdot 0 = i_1 \\ \text{out} \cdot \text{succ} = i_2 \end{cases} \quad \{ (1) (x_2) \}$$

$$\Rightarrow \begin{cases} \forall n \mid (\text{out} \cdot 0) n = i_1 n \\ \forall n \mid (\text{out} \cdot \text{succ}) n = i_2 n \end{cases} \quad \{ (72) (x_2) \}$$

$$\Rightarrow \begin{cases} \forall n \mid \text{out } 0 = i_1 () \\ \forall n \mid \text{out } (n+1) = i_2 n \end{cases} \quad \{ (73) (x_2), (75) \}$$

②

$$f \cdot [0, \text{succ}] = [0, \text{add}] \cdot (\text{id} + \langle \text{odd}, f \rangle)$$

$$\Rightarrow [f \cdot 0, f \cdot \text{succ}] = [0, \text{add} \cdot \langle \text{odd}, f \rangle] \quad \{ (20), (22) \}$$

$$\Rightarrow \begin{cases} f \cdot 0 = 0 \\ f \cdot \text{succ} = \text{add} \cdot \langle \text{odd}, f \rangle \end{cases} \quad \{ (27) \}$$

$$\Rightarrow \begin{cases} \forall n \mid (f \cdot 0) n = 0 n \\ \forall n \mid (f \cdot \text{succ}) n = (\text{add} \cdot \langle \text{odd}, f \rangle) n \end{cases} \quad \{ (72) (x_2) \}$$

$$\Rightarrow \begin{cases} \forall n \mid f 0 = 0 \\ \forall n \mid f (n+1) = \text{add} ((\text{odd } n, f n)) \end{cases} \quad \{ (73) (x_3), (75) (x_2), (77) \}$$

$$\Rightarrow \begin{cases} \forall n \mid f 0 = 0 \\ \forall n \mid f (n+1) = (2n+1) + f n \end{cases} \quad \{ \text{def add, odd} \}$$

$$f 0 = 0$$

$$f 1 = 1$$

$$f 2 = 4$$

$$f 3 = 9$$

$$f 4 = 16$$

$$f 5 = 25$$

$$\boxed{f n = n^2}$$



③

$$\alpha = (\text{id} + \pi_1) \cdot i_2 \cdot \pi_2$$

$$(A \times B) \xrightarrow{\pi_2} B \xrightarrow{i_2} C + B \rightarrow C +$$

$$(A \times (B \times C)) \xrightarrow{\pi_2} B \times C \xrightarrow{i_2} D + (B \times C) \xrightarrow{\text{idem}_1} D + B$$

$$\boxed{(A \times (B \times C)) \xrightarrow{\alpha} D + B}$$

④

$$\boxed{(p \rightarrow f, g) \cdot h = (p \cdot h) \rightarrow (f \cdot h), (g \cdot h)}$$

$$(p \rightarrow f, g) \cdot h = ([f, g] \cdot p?) \cdot h \quad \{(30)\}$$

$$= [f, g] \cdot (p? \cdot h) \quad \{(2)\}$$

$$= [f, g] \cdot ([h + h] \cdot (p \cdot h)?) \quad \{(29)\}$$

$$= ([f, g] \cdot (h + h)) \cdot (p \cdot h)? \quad \{(2)\}$$

$$= [f \cdot h, g \cdot h] \cdot (p \cdot h)? \quad \{(22)\}$$

$$= (p \cdot h) \rightarrow (f \cdot h), (g \cdot h) \quad \{(30)\}$$

⑤

$$\langle (p \rightarrow f, h), (p \rightarrow g, i) \rangle$$

$$= \langle [f, h] \cdot p?, [g, i] \cdot p? \rangle \quad \{(30)(x2)\}$$

$$= \langle [f, h], [g, i] \rangle \cdot p? \quad \{(9)\}$$

$$= [\langle f, g \rangle, \langle h, i \rangle] \cdot p? \quad \{(28)\}$$

$$= p \rightarrow \langle f, g \rangle, \langle h, i \rangle \quad \{(30)\}$$

$\boxed{F7}$

$$p \rightarrow \langle f, g \rangle, \langle f, h \rangle$$

$$= [\langle f, g \rangle, \langle f, h \rangle] \cdot p? \quad \{(30)\}$$

$$= \langle [f, f], [g, h] \rangle \cdot p? \quad \{(28)\}$$

$$= \langle [f, f] \cdot p?, [g, h] \cdot p? \rangle \quad \{(9)\}$$

$$= \langle p \rightarrow f, f, p \rightarrow g, h \rangle \quad \{(30)(x2)\}$$

$$= \langle f, p \rightarrow g, h \rangle \quad \{(F2)\}$$

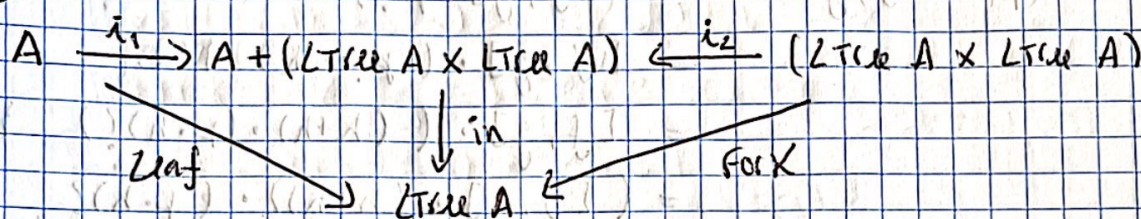
$\boxed{F2}$



$$\begin{aligned}
& p \rightarrow (p \rightarrow a, b), (p \rightarrow e, d) \\
&= [p \rightarrow a, b, p \rightarrow e, d] \cdot p? \quad \{(30)\} \\
&= [[a, b] \cdot p?, [e, d] \cdot p?] \cdot p? \quad \{(30)(x2)\} \\
&= ([ [a, b], [e, d] ] \cdot (p? + p?)) \cdot p? \quad \{(22)\} \\
&= [[a, b], [e, d]] \cdot (i_1 + i_2) \cdot p? \quad \{(46)\} \\
&= [ [a, b] \cdot i_1, [e, d] \cdot i_2 ] \cdot p? \quad \{(22)\} \\
&= [a, d] \cdot p? \quad \{(17)(x2)\} \\
&= p \rightarrow a, d \quad \{(30)\}
\end{aligned}$$

F9

⑥



$$out \cdot in = id$$

$$\Rightarrow out \cdot [leaf, fork] = id$$

$$\Rightarrow \begin{cases} out \cdot leaf = i_1 \\ out \cdot fork = i_2 \end{cases} \quad \{(20), (1)(x2), (17)\}$$

$$\Rightarrow \begin{cases} \forall a \mid (out \cdot leaf) a = i_1 a \\ \forall a, b \mid (out \cdot fork) (a, b) = i_2 (a, b) \end{cases} \quad \{(72)(x2)\}$$

$$\Rightarrow \begin{cases} \forall a \mid out(leaf a) = i_1 a \\ \forall a, b \mid out(fork(a, b)) = i_2(a, b) \end{cases} \quad \{(73)(x2)\}$$