Soluções

• Derivadas parciais

1. (a) $\lim_{t\to 0} g(t) = y^2+3$; $\lim_{t\to 0} h(t) = 2xy$.
(b) $\lim_{t\to 0} g(t) = 8x+y^3$; $\lim_{t\to 0} h(t) = 3xy^2$.

2. (a) $f_x = 3x^2y+14x$; $f_y = x^3-6y^2$;

(b) $f_x = \dfrac{(3-7y)y}{(7x+y)^2}$; $f_y = \dfrac{y^2+14xy-3x}{(7x+y)^2}$;

(c) $f_x = ye^{xy}\cos\left(1+e^{xy}\right)$;
$f_y = xe^{xy}\cos\left(1+e^{xy}\right)$;

(d) $f_x = 6x^2(x^3-y^2)$; $f_y = -4y(x^3-y^2)$;

(e) $f_x = e^y + y\cos x$; $f_y = xe^y + \operatorname{sen} x$;

(f) $f_s = e^s\left(\ln(st)+1/s\right)$; $f_t = \dfrac{e^s}{t}$;

(g) $f_x = e^x\left[\ln\left(y^2+3x\right)+3/(y^2+3x)\right]$;
$f_y = 2ye^x/(y^2+3x)$;

(h) $f_x = \cos\dfrac{x}{y} - \dfrac{x}{y}\operatorname{sen}\dfrac{x}{y}$; $f_y = \dfrac{x^2}{y^2}\operatorname{sen}\dfrac{x}{y}$;

(i) $f_x = 4x^3+6y$; $f_y = 3y^2+6x$;

(j) $f_x = 2y^3e^{2xy^3}$; $f_y = 6xy^2e^{2xy^3}$;

(k) $f_x = e^{\sqrt{xy}}\left(1+\frac{xy}{2\sqrt{xy}}\right) = e^{\sqrt{xy}}\left(1+\frac{\sqrt{xy}}{2}\right)$;
$f_y = \frac{x^2}{2\sqrt{xy}}e^{\sqrt{xy}}$;

(l) $f_x = yx^{y-1}$; $f_y = x^y\ln x$;

(m) $f_r = \dfrac{2\pi}{T}$; $f_T = -\dfrac{2\pi r}{T^2}$;

(n) $f_x = (1+xy)e^{xy}\operatorname{sen} yz$;
$f_y = xe^{xy}\left(x\operatorname{sen} yz + z\cos yz\right)$;
$f_z = xye^{xy}\cos yz$;

(o) $f_s = 2s\cos(2tu)$; $f_t = -2us^2\operatorname{sen}(2tu)$;
$f_u = -2ts^2\operatorname{sen}(2tu)$;

(p) $f_x = 2z$; $f_y = 0$; $f_z = 2(x+z)$

(q) $f_x = (1+xyz)yze^{xyz}$;
$f_y = (1+xyz)xze^{xyz}$;
$f_z = (1+xyz)xye^{xyz}$

(r) $f_x = \dfrac{1}{1+x+y^2+z^3}$;
$f_y = \dfrac{2y}{1+x+y^2+z^3}$;
$f_z = \dfrac{3z^2}{1+x+y^2+z^3}$

(s) $f_r = -2r\cos(r^2)$; $f_u = 1$; $f_v = 1$

(t) $f_x = e^x\big(\operatorname{sen}(x+y)+\cos(x+y)\big)$;
$f_y = e^x\cos(x+y) + 3\operatorname{sen}(z-3y)$;
$f_z = -\operatorname{sen}(z-3y)$

(u) $f_m = \dfrac{v^2}{r}$; $f_v = \dfrac{2mv}{r}$; $f_r = -\dfrac{mv^2}{r^2}$

(v) $f_x = \dfrac{yx^{y-1}}{e^z+x^y}$; $f_y = \dfrac{x^y\ln x}{e^z+x^y}$; $f_z = \dfrac{e^z}{e^z+x^y}$.

3. (a) $w_{xy} = (w_x)_y = 4y^3-12xy^2 = w_{yx}$;
(b) $w_{xy} = (w_x)_y = -6x^2e^{-2y}+2y^{-3}\operatorname{sen} x = w_{yx}$;
(c) $w_{xy} = (w_x)_y = -\dfrac{2xz}{y^2}\operatorname{sen}\dfrac{z}{y} = w_{yx}$.

4. $w_{xyz} = (w_x)_{yz} = (6xy^3z+2y^4z^2)_{yz} = (18xy^2z+8y^3z^2)_z = 18xy^2+16y^3z$.

5. $\dfrac{\partial^3 w}{\partial x\partial y\partial z} = 1$.

6. $w_{rrs} = w_{rsr} = w_{srr} = 36r^2s^2t - 6st^2e^{rt}$.

7. $\dfrac{\partial v}{\partial t} = -\frac{1}{2}t^{-\frac{3}{2}}e^{-\frac{x^2}{4t}} + t^{-\frac{1}{2}}\dfrac{x^2}{4t^2}e^{-\frac{x^2}{4t}} = \frac{1}{2}t^{-\frac{1}{2}}e^{-\frac{x^2}{4t}}\left(t^{-1}+\dfrac{x^2}{2t^2}\right)$;

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\frac{x}{2t}t^{-\frac{1}{2}}e^{-\frac{x^2}{4t}}\right) = \frac{\partial}{\partial x}\left(-\frac{x}{2}t^{-\frac{3}{2}}e^{-\frac{x^2}{4t}}\right) = \frac{\partial v}{\partial t}.$$

8. (a) $\dfrac{\partial^2 f}{\partial x^2} = k^2e^{kx}\cos(ky) = -\dfrac{\partial^2 f}{\partial y^2}$
(b) $\dfrac{\partial^2 f}{\partial x^2} = 6y = -\dfrac{\partial^2 f}{\partial y^2}$
(c) $\dfrac{\partial^2 f}{\partial y^2} = f$; $\dfrac{\partial^2 f}{\partial x^2} = -f$

9. $\lambda = -1$.

10. $\dfrac{\partial^2 w}{\partial x^2} = -\cos(x-y) + \dfrac{1}{(x+y)^2} = \dfrac{\partial^2 w}{\partial y^2}$.

11. (a) $\dfrac{\partial T}{\partial x}(1,2) = 40x(x^2+y^2)\big|_{(1,2)} = 200$; (b) $\dfrac{\partial T}{\partial y}(1,2) = 40y(x^2+y^2)\big|_{(1,2)} = 400$.

12. (a) $\dfrac{\partial V}{\partial x}(2,-1,1) = -\dfrac{200x}{\left(x^2+y^2+z^2\right)^2}\Bigg|_{(2,-1,1)} = -\dfrac{100}{9}$;

(b) $\dfrac{\partial V}{\partial y}(2,-1,1) = -\dfrac{200y}{\left(x^2+y^2+z^2\right)^2}\Bigg|_{(2,-1,1)} = \dfrac{50}{9}$;

(c) $\dfrac{\partial V}{\partial z}(2,-1,1) = -\dfrac{200z}{\left(x^2+y^2+z^2\right)^2}\Bigg|_{(2,-1,1)} = -\dfrac{50}{9}$;

- **Planos tangentes e diferenciais**

13. (a) $-4x - 8y + z + 8 = 0$.
(b) $-6x - 4y + z + 5 = 0$.
(c) $-\mathrm{e}^3\, y + z + \mathrm{e}^3 = 0$.
(d) $-x - y + z = 0$.
(e) $\frac{3}{2}x - \frac{3}{4}y + z - \frac{5}{2} = 0$.
(f) $-x - y + z = 0$.

14. (a) $dz = (2x+3y)dx + (3x-2y)dy$;
(Observação: dado (x,y), dz é uma função de dx e dy; em notação completa, escrevemos $dz = dz_{(x,y)}(dx,dy)$).
(b) $(x,y) = (2,3)$; $(x+\Delta x, y+\Delta y) = (2.05, 2.96)$;
$dx = \Delta x = 2.05 - 2 = 0.05$; $dy = \Delta y = 2.96 - 3 = -0.04$;
$dz = (2x+3y)|_{(2,3)} \times 0.05 + (3x-2y)|_{(2,3)} \times (-0.04) = 0.65$;
$\Delta z = f(2.05, 2.96) - f(2,3) = 0.6449 \simeq dz$.

15. $(x,y) = (1,2)$; $(x+\Delta x, y+\Delta y) = (1.05, 2.1)$;
$dx = \Delta x = 1.05 - 1 = 0.05$; $dy = \Delta y = 2.1 - 1 = 0.1$;
$dz = (10x)|_{(1,2)} \times 0.05 + (2y)|_{(1,2)} \times 0.1 = 0.9$;
$\Delta z = f(1.05, 2.1) - f(1,2) = 0.9225 \simeq dz$.

16. Utilize diferenciais para calcular um valor aproximado de

(a) Seja $f(x,y) = \sqrt{9x^2+y^2}$. Assim,

$$\sqrt{9(1.95)^2 + (8.1)^2} = f(1.95, 8.1) = f(2+\Delta x, 8+\Delta y)$$

com $(\Delta x, \Delta y) = (-0.05, 0.1)$. Usando diferenciais, sabemos que

$$f(2+\Delta x, 8+\Delta y) \simeq f(2,8) + df_{(2,8)}(\Delta x, \Delta y)$$

onde

$$df_{(x,y)}(\Delta x, \Delta y) = \frac{9x\,\Delta x}{\sqrt{9x^2+y^2}} + \frac{y\,\Delta y}{\sqrt{9x^2+y^2}}.$$

Assim,

$$\sqrt{9(1.95)^2 + (8.1)^2} \simeq \sqrt{9\times 2^2 + 8^2} + \frac{9\times 2\times(-0.05)}{\sqrt{9\times 2^2+8^2}} + \frac{8\times 0.1}{\sqrt{9\times 2^2+8^2}} = 9.99$$

(b) Definindo $f(x,y) = \sqrt{x}\,\mathrm{e}^y$, temos

$$df_{(x,y)}(\Delta x, \Delta y) = \frac{\mathrm{e}^y\,\Delta x}{2\sqrt{x}} + \sqrt{x}\,\mathrm{e}^y\,\Delta y$$

e

$$\sqrt{99}\,\mathrm{e}^{0.02} = f(99, 0.02) \simeq f(100,0) + df_{(100,0)}(\Delta x, \Delta y)$$

com $(\Delta x, \Delta y) = (-1, 0.02)$. Assim,

$$\sqrt{99}\,\mathrm{e}^{0.02} \simeq 10 + \frac{-1}{20} + 10 \times 0.02 = 10.15.$$

(c) Para $f(x,y) = x^2 - y \ln \dfrac{y}{x}$, temos

$$df_{(x,y)}(\Delta x, \Delta y) = \left(2x + \frac{y}{x}\right) \Delta x + \left(-\ln \frac{y}{x} - 1\right) \Delta y.$$

Assim, com $(x,y) = (1,1)$ e $(\Delta x, \Delta y) = (-0.02, 0.01)$, podemos escrever

$$(0.98)^2 - 1.01 \ln \frac{1.01}{0.98} = f(0.98, 1.01) \simeq f(1,1) + df_{(1,1)}(-0.02, 0.01) = 1 - 0.07 = 0.93.$$

(d) Considere-se $f(x,y) = x^{1/3}y^{1/2}$, $(x,y) = (27,36)$ e $(\Delta x, \Delta y) = (-0.02, 0.04)$. Temos

$$df_{(x,y)}(\Delta x, \Delta y) = \frac{1}{3}x^{-2/3}y^{1/2}\Delta x + \frac{1}{2}x^{1/3}y^{-1/2}\Delta y$$

e

$$26.98^{1/3} \times 36.04^{1/2} = f(26.98, 36.04) \simeq f(27,36) + df_{(27,36)}(-0.02, 0.04) = 18 + 0.0056 = 18.0056.$$

17. (a) $dw = (3x^2 - 2xy)dx + (-x^2 + 6y)dy$;

(b) $dw = xe^{xy}(2 + xy)dx + \left(x^3 e^{xy} - \frac{2}{y^3}\right) dy$;

(c) $dz = \mathrm{e}^x \left(\cos(xy) - y \operatorname{sen}(xy)\right)dx - x\,\mathrm{e}^x \operatorname{sen}(xy)dy$.

(d) $dw = 2x \ln(y^2 + z^2)dx + \dfrac{2x^2y}{y^2+z^2}dy + \dfrac{2x^2z}{y^2+z^2}dz$;

(e) $dw = \dfrac{yz(y+z)}{(x+y+z)^2}dx + \dfrac{xz(x+z)}{(x+y+z)^2}dy + \dfrac{xy(x+y)}{(x+y+z)^2}dz$;

(f) $dw = (2xz - z^2t)dx + 4t^3dy + (x^2 - 2xzt)dz + (12yt^2 - xz^2)dt$.

18. (a) $dz = \left.\dfrac{1}{x-3y}\right|_{(7,2)} \times (-0.1) - \left.\dfrac{3}{x-3y}\right|_{(7,2)} \times 0.06 = -0.28 \simeq \Delta z = \ln(6.9 - 3 \times 2.06) - \ln 1$

(b) $dw = y^2 \operatorname{sen} \pi z\big|_{(4,5,4)} \times (-0.01) + 2xy \operatorname{sen} \pi z\big|_{(4,5,4)} \times (-0.02) + \pi x y^2 \cos \pi z\big|_{(4,5,4)} \times 0.03 = 9.4248$
$\simeq \Delta w$

(c) $dw = \left.\dfrac{-x}{\sqrt{20 - x^2 - 7y^2}}\right|_{(2,1)} \times (-0.05) + \left.\dfrac{-7y}{\sqrt{20 - x^2 - 7y^2}}\right|_{(2,1)} \times 0.08 = -0.1533 \simeq \Delta w$

19. A área de um retângulo é dada por $A(x,y) = xy$, onde x e y representam o comprimento da base e a altura do retângulo, respetivamente.

Pretende-se estimar o valor máximo de $|A(10 + \Delta x, 5 + \Delta y) - A(10,5)|$ quando $|\Delta x| \leq 0.1$ e $|\Delta y| \leq 0.1$.

Usando diferenciais, tem-se $dA = y\Delta x + x\Delta y$ e

$$|A(10 + \Delta x, 5 + \Delta y) - A(10,5)| \simeq |5\Delta x + 10\Delta y| \leq 5 \times 0.1 + 10 \times 0.1 = 1.5\, cm^2.$$

20. Temos $|\Delta x| \leq 0.2$, $|\Delta y| \leq 0.2$, $|\Delta z| \leq 0.2$ e $dV = yz\Delta x + xz\Delta y + xy\Delta z$. Assim,

$$\begin{aligned} |\Delta V| &= |V(75 + \Delta x, 60 + \Delta y, 40 + \Delta z) - V(75, 60, 40)| \\ &\simeq |60 \times 40 \times \Delta x + 75 \times 40 \times \Delta y + 75 \times 60 \times \Delta z| \leq 1980\, cm^3. \end{aligned}$$

- **Derivadas de funções compostas**

21. $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s} = -2xy \operatorname{sen}(x^2y)3s^2t^2 - x^2 \operatorname{sen}(x^2y)2s = -2xs \operatorname{sen}(x^2y)\left(3yst^2 + x\right)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} = -2xy \operatorname{sen}(x^2y)2s^3t - x^2 \operatorname{sen}(x^2y) \left(-\frac{1}{t^2}\right) = -x \operatorname{sen}(x^2y) \left(4ys^3t - \frac{x}{t^2}\right)$$

22. $$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial x} = 6ux^2 \operatorname{sen} v + u^2y^2 \cos v$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial y} = -12uy^2 \operatorname{sen} v + 2u^2xy \cos v$$

23. $$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial r} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial r} = \frac{u\,\mathrm{e}^{-s}}{\sqrt{u^2+v^2}} - \frac{vs^2\,\mathrm{e}^{-r}}{\sqrt{u^2+v^2}}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial s} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial s} = \frac{-ur\,\mathrm{e}^{-s}}{\sqrt{u^2+v^2}} + \frac{2vs\,\mathrm{e}^{-r}}{\sqrt{u^2+v^2}}$$

24. $\dfrac{\partial z}{\partial x} = \dfrac{\partial w}{\partial r}\cdot\dfrac{\partial r}{\partial x} + \dfrac{\partial w}{\partial s}\cdot\dfrac{\partial s}{\partial x} + \dfrac{\partial w}{\partial v}\cdot\dfrac{\partial v}{\partial x} = \dfrac{\cos y}{v} + \dfrac{y\cos x}{v} - \dfrac{2x(r+s)\,\mathrm{e}^{-y}}{v^2}$

$\dfrac{\partial z}{\partial y} = \dfrac{\partial w}{\partial r}\cdot\dfrac{\partial r}{\partial y} + \dfrac{\partial w}{\partial s}\cdot\dfrac{\partial s}{\partial y} + \dfrac{\partial w}{\partial v}\cdot\dfrac{\partial v}{\partial y} = \dfrac{-x\,\mathrm{sen}\,y}{v} + \dfrac{\mathrm{sen}\,x}{v} + \dfrac{x^2(r+s)\,\mathrm{e}^{-y}}{v^2}$

25. (a) $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\cdot\dfrac{dy}{dt} = -\dfrac{3x^2}{(1+t)^2} - \dfrac{3y^2}{(1+t)^2}$;

(b) $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial u}\cdot\dfrac{du}{dt} + \dfrac{\partial w}{\partial v}\cdot\dfrac{dv}{dt} = \dfrac{2\,\mathrm{e}^{2t}}{u+v} + \dfrac{3t^2-2t}{u+v}$;

(c) $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial r}\cdot\dfrac{dr}{dt} + \dfrac{\partial w}{\partial s}\cdot\dfrac{ds}{dt} + \dfrac{\partial w}{\partial s}\cdot\dfrac{dv}{dt} = 2r\cos t + v\,\mathrm{sen}\,t - 4s$;

(d) $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\cdot\dfrac{dy}{dt} + \dfrac{\partial w}{\partial z}\cdot\dfrac{dz}{dt} = 4xy^3z^4 + 9x^2y^2z^4 + 20x^2y^3z^3$.

26. $\dfrac{\partial z}{\partial t} = xy^2$.

$\dfrac{dz}{dt} = \left[t + \ln\left(\mathrm{e}^t + t^2\right)\right]\mathrm{e}^{2t} + t\left(1 + \dfrac{\mathrm{e}^t + 2t}{\mathrm{e}^t + t^2}\right)\mathrm{e}^{2t} + 2t\left[t + \ln\left(\mathrm{e}^t + t^2\right)\right]\mathrm{e}^{2t}$.

Sugestão: fazer a composição $z = t\left[t + \ln\left(\mathrm{e}^t + t^2\right)\right]\mathrm{e}^{2t}$ e usar as regras de derivação para funções de uma variável.

27. $\dfrac{d^2u}{dt^2} = \mathrm{e}^{\sin t - 2t^3}\left[-\sin t - 12t + \left(\cos t - 6t^2\right)^2\right]$

Sugestão: fazer a composição $u = \mathrm{e}^{\sin t - 2t^3}$ e usar as regras de derivação para funções de uma variável.

28. $\dfrac{\partial u}{\partial s} = \left[4x^3yr\,\mathrm{e}^t + 2(x^4 + 2yz^3)rs\,\mathrm{e}^{-t} + 3y^2z^2r^2\,\mathrm{sen}\,t\right]_{(r=2,s=1,t=0)} = 2^7 + 2^6$.

29. $T = \dfrac{1}{c}pV$; $\dfrac{dT}{dt} = \dfrac{\partial T}{\partial p}\cdot\dfrac{dp}{dt} + \dfrac{\partial T}{\partial V}\cdot\dfrac{dV}{dt} = \dfrac{1}{c}V\dfrac{dp}{dt} + \dfrac{1}{c}p\dfrac{dV}{dt} = \dfrac{1}{c}\left(V\dfrac{dp}{dt} + p\dfrac{dV}{dt}\right)$.

$T(t) = \dfrac{1}{c}p(t)V(t)$; $T'(t) = \dfrac{1}{c}\big(p'(t)V(t) + p(t)V'(t)\big)$.

30. $\dfrac{\partial w}{\partial x} = \dfrac{dw}{du}\dfrac{\partial u}{\partial x} = 2x\dfrac{dw}{du}$ e $\dfrac{\partial w}{\partial y} = \dfrac{dw}{du}\dfrac{\partial u}{\partial y} = 2y\dfrac{dw}{du}$.

31. $\dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial u}\cdot\dfrac{\partial u}{\partial s} + \dfrac{\partial w}{\partial v}\cdot\dfrac{\partial v}{\partial s} = 2s\dfrac{\partial w}{\partial u} - 2s\dfrac{\partial w}{\partial v}$; $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial u}\cdot\dfrac{\partial u}{\partial t} + \dfrac{\partial w}{\partial v}\cdot\dfrac{\partial v}{\partial t} = -2t\dfrac{\partial w}{\partial u} + 2t\dfrac{\partial w}{\partial v}$;

32. Fazendo $u = x - y$, vem $\dfrac{\partial z}{\partial x} = \dfrac{dz}{du}\cdot\dfrac{\partial u}{\partial x} = \dfrac{dz}{du}$ e $\dfrac{\partial z}{\partial y} = \dfrac{dz}{du}\cdot\dfrac{\partial u}{\partial y} = -\dfrac{dz}{du}$.

- **Derivada da função implícita**

33. (a) Seja $F(x,y) = 2x^3 + x^2y + y^3 - 1$.

$\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = -\dfrac{6x^2 + 2xy}{x^2 + 3y^2}$.

(b) Seja $F(x,y) = 6x + \sqrt{xy} - 3y + 4$.

$\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = -\dfrac{6 + \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - 3} = -\dfrac{12\sqrt{xy} + y}{x - 6\sqrt{xy}}$.

(c) $\dfrac{dy}{dx} = -\dfrac{3x^2 - 6y}{3y^2 - 6x}$.

(d) $\dfrac{dy}{dx} = -\dfrac{2xy^2 + 1}{2x^2y - 6y^2}$.

34. $\dfrac{dy}{dx}(x) = -\dfrac{(1 + y - x^2 + \ln y)_x}{(1 + y - x^2 + \ln y)_y} = -\dfrac{-2x}{1 + \frac{1}{y}} = \dfrac{2xy}{y+1}$; $\dfrac{dy}{dx}(\sqrt{2}) = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$.

35. (a) Seja $F(x,y,z) = 2xz^3 - 3yz^2 + x^2y^2 + 4z$.

$\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z} = -\dfrac{2z^3 + 2xy^2}{6xz^2 - 6yz + 4}$;

$\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z} = -\dfrac{-3z^2 + 2x^2y}{6xz^2 - 6yz + 4}$.

(b) $\dfrac{\partial z}{\partial x} = -\dfrac{e^{yz} - 2yze^{xz} + 3yze^{xy}}{xye^{yz} - 2xye^{xz} + 3e^{xy}}$;

$\dfrac{\partial z}{\partial y} = -\dfrac{xze^{yz} - 2e^{xz} + 3xze^{xy}}{xye^{yz} - 2xye^{xz} + 3e^{xy}}$.

(c) $\dfrac{\partial z}{\partial x} = -\dfrac{2xy - yz\,\mathrm{sen}(xyz)}{2z - xy\,\mathrm{sen}(xyz)}$;

$\dfrac{\partial z}{\partial y} = -\dfrac{x^2 - xz\,\mathrm{sen}(xyz)}{2z - xy\,\mathrm{sen}(xyz)}$.

(d) $\dfrac{\partial z}{\partial x} = -\dfrac{z^2 + 4xy}{2xz - 4y^2}$;

$\dfrac{\partial z}{\partial y} = -\dfrac{2x^2 - 8yz + 3}{2xz - 4y^2}$.