TP5 | Ficha 1 | 26/09/2022

1. a) $\begin{cases} f\,x = 2 \times x \\ g\,x = x+1 \end{cases}$ $\begin{cases} f = succ \\ g = length \end{cases}$

$(f \cdot g)\,x = f\,(g\,x)$
$= f\,(x+1)$
$= 2\,(x+1)$
$= 2x+2$

$(f \cdot g)\,5 = 12$
$(f \cdot g)\,2 = 6$

$(f \cdot g)\,x = f\,(g\,x)$
$= \begin{cases} (f \cdot g)\,[\,] = f\,(g\,[\,]) \\ (f \cdot g)\,(h:t) = f\,(g\,(h:t)) \end{cases}$
$= \begin{cases} (f \cdot g)\,[\,] = succ\,(0) \\ (f \cdot g)\,(h:t) = succ\,(1 + length\,t) \end{cases}$
$= \begin{cases} (f \cdot g)\,[\,] = 1 \\ (f \cdot g)\,(h:t) = 2 + length\,t \end{cases}$

$(f \cdot g)\,[1,2,3] = 4$

$\begin{cases} g\,(x,y) = x+y \\ f = succ \cdot (2*) \end{cases}$

$(f \cdot g)\,(x,y) = f\,(g\,(x,y))$
$\cancel{= f\,(succ \cdot (2*))\,x}$
$= f\,(x+y)$
$= (succ \cdot (2*))\,(x+y)$
$= succ\,(2\,(x+y))$
$= 2x+2y+1$

$(f \cdot g)\,(1,1) = 5$
$(f \cdot g)\,(2,0) = 5$

b)
$(f \cdot g) \cdot h = f \cdot (g \cdot h)$
$\Leftrightarrow \forall x \mid ((f \cdot g) \cdot h)\,x = (f \cdot (g \cdot h))\,x$ {(72)}
$\Leftrightarrow \forall x \mid ((f \cdot g) \cdot h)\,x = f\,((g \cdot h)\,x)$ {(73)}
$\Leftrightarrow \forall x \mid ((f \cdot g) \cdot h)\,x = f\,(g\,(h\,x))$ {(73)}
$\Leftrightarrow \forall x \mid ((f \cdot g) \cdot h)\,x = (f \cdot g)\,(h\,x)$ {(73)}
$\Leftrightarrow \forall x \mid ((f \cdot g) \cdot h)\,x = ((f \cdot g) \cdot h)\,x$ {(73)}
$\Leftrightarrow$ True

e)

$f \cdot id = id \cdot f = f$

$\Leftrightarrow \forall x \mid (f \cdot id)\, x = (id \cdot f)\, x = f\, x$ { (72) }

$\Leftrightarrow \forall x \mid f\, (id\, x) = id\, (f\, x) = f\, x$ { (73) (a) }

$\Leftrightarrow \forall x \mid f\, x = f\, x = f\, x$ { (74) (x2) }

$\Leftrightarrow$ True

(2) a) store 7 [1..10] = [7, 1, 2, 3, 4, 5, 6, 8, 9, 10]

store 11 [1..10] = [11, 1, 2, 3, 4, 5, 6, 7, 8, 9]

b) Viola o requisito c), o que se verifica pelo resultado de store 11 [1..10] que retorna uma lista com 11 elementos.

c) Neste caso, não respeita o requisito a), porque a inserção do novo elemento à cabeça é seguida pelo filtro.

d) Funciona perfeitamente, devido ao Polimorfismo de função (e do Haskell).

(3)

length [] = 0

length (x:xs) = 1 + length xs

reverse [] = []

reverse (x:xs) = (reverse xs) ++ [x]

(4) TPC

(5) a)

(F4) $id \times id = id$

$\Leftrightarrow \forall (x,y) \mid (id \times id)\,(x,y) = id\,(x,y)$ { (72) }

$\Leftrightarrow \forall (x,y) \mid (id\, x, id\, y) = id\,(x,y)$ { F3 }

$\Leftrightarrow \forall (x,y) \mid (x,y) = (x,y)$ { (74) (x3) }

$\Leftrightarrow$ True

(F5) $(f \times g) \cdot (h \times K) = (f \cdot h) \times (g \cdot K)$

$\Leftrightarrow \forall (x,y) \mid ((f \times g) \cdot (h \times K))\,(x,y) = ((f \cdot h) \times (g \cdot K))\,(x,y)$ { 72 }

$\Leftrightarrow \forall (x,y) \mid (f \times g)\,((h \times K)\,(x,y)) = ((f \cdot h)\, x, (g \cdot K)\, y)$ { 73, F3 }

$\Leftrightarrow \forall (x,y) \mid (f \times g)\,(h\, x, K\, y) = (f\,(h\, x), g\,(K\, y))$ { 73, F3 }

$\Leftrightarrow \forall (x,y) \mid (f\,(h\, x), g\,(K\, y)) = (f\,(h\, x), g\,(K\, y))$ { F3 }

$\Leftrightarrow$ True

b)

(F7) $\pi_1 \circ (f \times g) = f \cdot \pi_1$

$\Leftrightarrow \forall (x,y) \mid (\pi_1 \cdot (f \times g))(x,y) = (f \cdot \pi_1)(x,y)$ {F72}

$\Leftrightarrow \forall (x,y) \mid \pi_1 ((f \times g)(x,y)) = f(\pi_1 (x,y))$ {F3 (x2)}

$\Leftrightarrow \forall (x,y) \mid \pi_1 (f x, g y) = f(\pi_1 (x,y))$ {F3}

$\Leftrightarrow \forall (x,y) \mid f x = f x$ {Def π_1 (x2)}

$\Leftrightarrow$ True

(F8) (F9) TPC

6. uncurry :: (a -> b -> c) -> (a,b) -> c

uncurry f (a,b) = f a b

curry, flip -> TPC