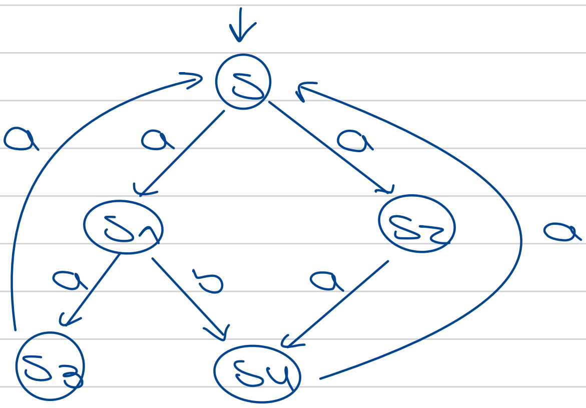
AULAS PRÁTICAS

IC

SISTEMAS REATIVOS



$LTS_1 = \{P, S_i, Act, \rightarrow\}$

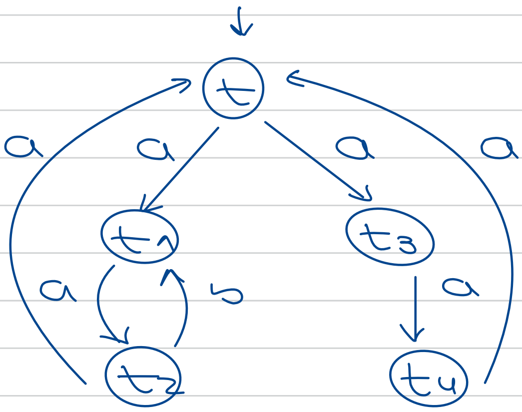
$P = \{s, s_1, s_2, s_3, s_4\}$

$S_i = s$

$Act = \{a, b\}$

$\xrightarrow{a} = \{(s, s_1), (s, s_2), (s_1, s_3), (s_2, s_4), (s_3, s), (s_4, s)\}$

$\xrightarrow{b} = \{(s_1, s_4)\}$



Estados finitos não têm estados de terminação

$lts_2 = \ldots$

Estes sistemas são Bissimilares?

Verificar se os estados iniciais são bissimilares

$s \mathrel{R} t$ se existe um $R \subseteq P \times P$ tq

$(s, t) \in R$

$\forall a \in Act$. se $s \xrightarrow{a} s'$ então $t \xrightarrow{a} t'$ e $(s', t') \in R$

$\forall a \in Act$. se $t \xrightarrow{a} t'$ então $s \xrightarrow{a} s'$ e $(s', t') \in R$

Relação de Bissimulação

$R = \{(s, t), (s_1, t_1), (s_2, t_3), (s_3, t_2), (s_4, t_2), (s_4, t_4)\}$

- $(s, t) \overset{?}{\in} R$

$s \xrightarrow{a} s_1 \qquad t \xrightarrow{a} t_1 \qquad (s_1, t_1) \overset{?}{\in} R$ ✓

$s \xrightarrow{a} s_2$ $\quad t \xrightarrow{a} t_3$ $\quad (s_2, t_3) \overset{?}{\in} R$ ✓

$t \xrightarrow{a} t_1$ $\quad s \xrightarrow{a} s_1$ $\quad (s_1, t_1) \in R$

$t \xrightarrow{a} t_3$ $\quad s \xrightarrow{a} s_2$ $\quad (s_2, t_3) \in R$

- $(s_1, t_1) \overset{?}{\in} R$ (com isto podemos verificar lá em cima)

$s_1 \xrightarrow{a} s_3$ $\quad t_1 \xrightarrow{a} t_2$ $\quad (s_3, t_2) \overset{?}{\in} R$ ✓

$s_1 \xrightarrow{b} s_4$ $\quad t_1 \xrightarrow{b} t_2$ $\quad (s_4, t_2) \overset{?}{\in} R$ ✓

$t_1 \xrightarrow{a} t_2$ $\quad s_1 \xrightarrow{a} s_3$

$t_1 \xrightarrow{b} t_2$ $\quad s_1 \xrightarrow{b} s_4$

- $(s_2, t_3) \overset{?}{\in} R$

$s_2 \xrightarrow{a} s_4$ $\quad t_3 \xrightarrow{a} t_4$ $\quad (s_4, t_4) \overset{?}{\in} R$ ✓

$t_3 \xrightarrow{a} t_4$ $\quad s_2 \xrightarrow{a} s_4$

- $(s_3, t_2) \overset{?}{\in} R$

$s_3 \xrightarrow{a} s$ $\quad t_2 \xrightarrow{a} t$ $\quad (s, t) \in R$

$t_2 \xrightarrow{a} t$ $\quad s_3 \xrightarrow{a} s$ $\quad (s, t) \in R$

- $(s_4, t_2) \overset{?}{\in} R$

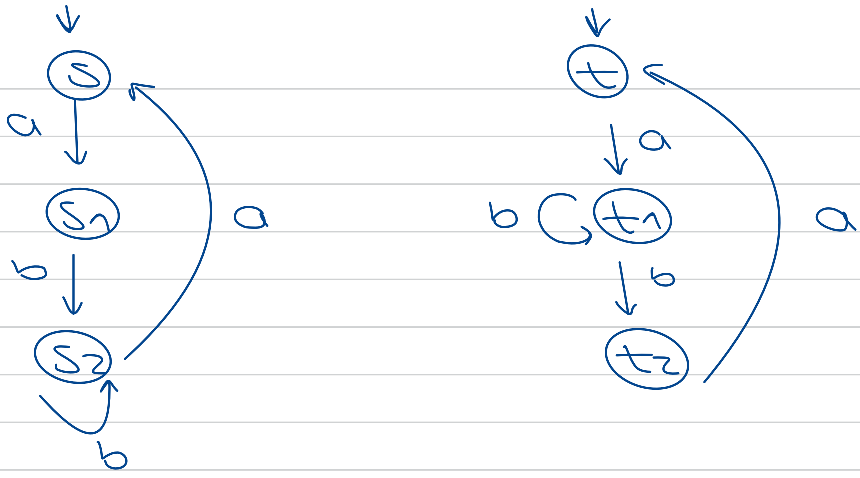
$s_4 \xrightarrow{a} s$ $\quad t_2 \xrightarrow{a} t$ $\quad (s, t) \in R$

$t_2 \xrightarrow{a} t$ $\quad s_4 \xrightarrow{a} s$ $\quad (s, t) \in R$

- $(s_4, t_4) \overset{?}{\in} R$

$s_4 \xrightarrow{a} s$ $\quad t_4 \xrightarrow{a} t$ $\quad (s, t) \in R$

$t_4 \xrightarrow{a} t$ $\quad s_4 \xrightarrow{a} s$ $\quad \therefore\ sRt$



Verificar se são Biss.

Método atacante / defensor

$(s, t) \overset{?}{\in} R$

A escolhe $s \xrightarrow{a} s_1$

D escolhe $t \xrightarrow{a} t_1$

tem que ser capaz de responder a todas as escolhas do atacante

$(s_1, t_1) \overset{?}{\in} R$

A $s_1 \xrightarrow{b} s_2$

D $t_1 \xrightarrow{b} t_1$ → resultados a) $t_1 \xrightarrow{b} t_1$ → (s_2, t_1)
b) $t_1 \xrightarrow{b} t_2$ → (s_2, t_2)

$(s_2, t_1) \overset{?}{\in} R$

A $s_2 \xrightarrow{a} s$

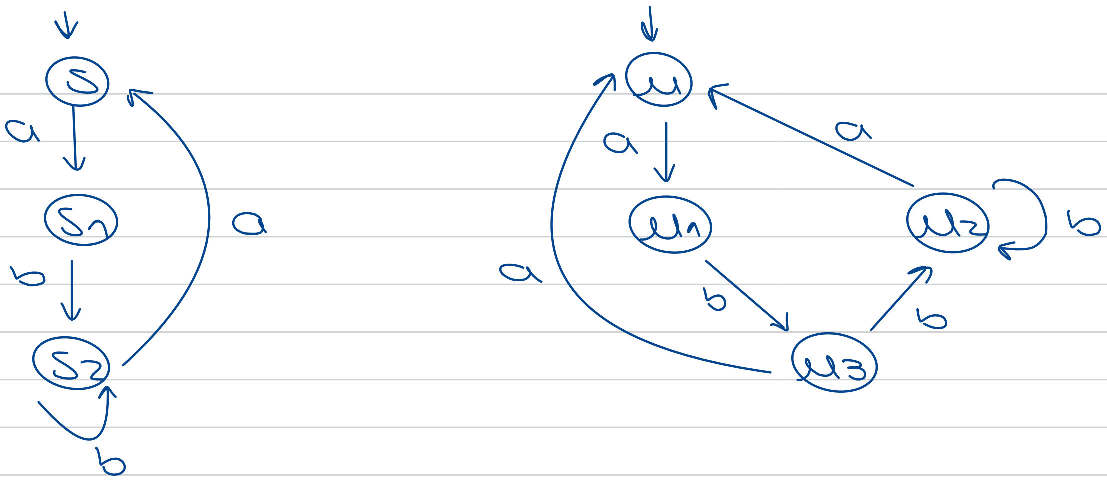
D $t_1 \not\xrightarrow{a}$

$(s_2, t_2) \overset{?}{\in} R$

A $s_2 \xrightarrow{b} s_2$

D $t_2 \not\xrightarrow{b}$

Portanto não são bissimilares. $s \not\sim t$



$(s, u) \overset{?}{\in} R$

A $\quad s \xrightarrow{a} s_1$

D $\quad u \xrightarrow{a} u_1$

(s_1, u_1)

A $\quad s_1 \xrightarrow{b} s_2$

D $\quad u_1 \xrightarrow{b} u_3$

(s_2, u_3)

A
a) $s_2 \xrightarrow{b} s_2$
b) $s_2 \xrightarrow{a} s$
c) $u_3 \xrightarrow{b} u_2$
d) $u_3 \xrightarrow{a} u$

a) A $\quad s_2 \xrightarrow{b} s_2$

D $\quad u_3 \xrightarrow{b} u_2$

(s_2, u_2)

(s_2, u_2)

A
e) $s_2 \xrightarrow{b} s_2$
f) $s_2 \xrightarrow{a} s$
g) $u_3 \xrightarrow{b} u_2$

D $\quad u_2 \xrightarrow{b} u_2 \quad (s_2, u_2)$ ✓
D $\quad u_2 \xrightarrow{a} u \quad (s, u)$ ✓
D $\quad s_2 \xrightarrow{b} s_2 \quad (s_2, u_2)$ ✓

b) (s_2, u_3)

A $\quad s_2 \xrightarrow{a} s \qquad$ D $\quad u_3 \xrightarrow{a} u \qquad (s, u)$ ✓

A $\quad s_2 \xrightarrow{b} s_2 \qquad$ D $\quad u_3 \xrightarrow{b} u_2 \qquad (s_2, u_2)$ ✓

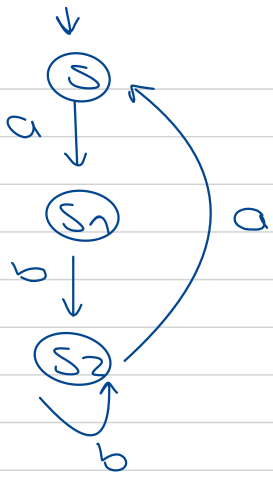
c) (s_2, u_3)

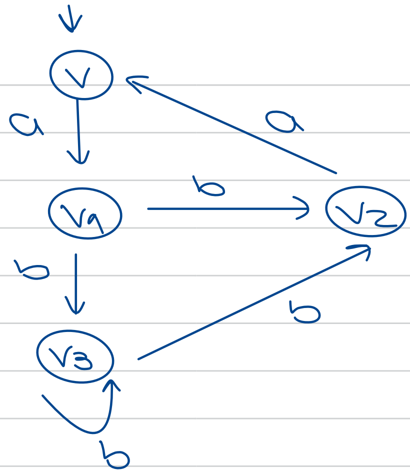
A $\quad u_3 \xrightarrow{b} u_2 \qquad$ D $\quad s_2 \xrightarrow{b} s_2 \qquad (u_2, s_2)$ ✓

A $\quad u_3 \xrightarrow{a} u \qquad$ D $\quad s_2 \xrightarrow{b} s \qquad (u, s)$ ✓

d) (s_2, u_3)

A $\quad u_3 \xrightarrow{a} u \qquad$ D $\quad s_2 \xrightarrow{a} s \qquad (u, s)$ ✓





(s, v)

A $s \xrightarrow{a} s_1$ D $v \xrightarrow{a} v_1$ (s_1, v_1)

(s_1, v_1)

A $s_1 \xrightarrow{b} s_2$ D $v_1 \xrightarrow{b} v_3$ (s_2, v_3)

. . .

(s_2, v_3)

A $s_2 \xrightarrow{a} s$ D $v_3 \stackrel{a}{\not\to}$ deadlock

$\therefore s \not\leq t$

$$
\left.
\begin{aligned}
Uni &= (CM \mid CS) \setminus \{coin, coffee\} \\
CM &= coin \,.\, CM_1 \\
CM_1 &= \overline{coffee} \,.\, CM \\
CS &= \overline{pub} \,.\, CS_1 \\
CS_1 &= \overline{coin} \,.\, CS_2 \\
CS_2 &= coffee \,.\, CS
\end{aligned}
\right]
\quad CCS
$$

$$
\begin{tikzcd}
① \quad Uni \arrow[d, "\overline{pub}"] \\
② \quad (CM \mid CS_1) \setminus \{coin, coffee\} \arrow[d, "\tau"] \\
③ \quad (CM_1 \mid CS_2) \setminus \{coin, coffee\} \arrow[d, "\tau"] \\
④ \quad (CM \mid CS) \setminus \{coin, coffee\} \arrow[uuu, bend right=60, "\overline{pub}"']
\end{tikzcd}
$$

$\not\sim$

$$
\begin{tikzcd}
Spec \arrow[d, "\overline{pub}"] \\
Spec_1 \arrow[loop right, "\overline{pub}"]
\end{tikzcd}
$$

$Spec \xrightarrow{\overline{pub}} Spec_1$

$① \xrightarrow{\overline{pub}} ②$

$(Spec_1, ②)$

$\sim$

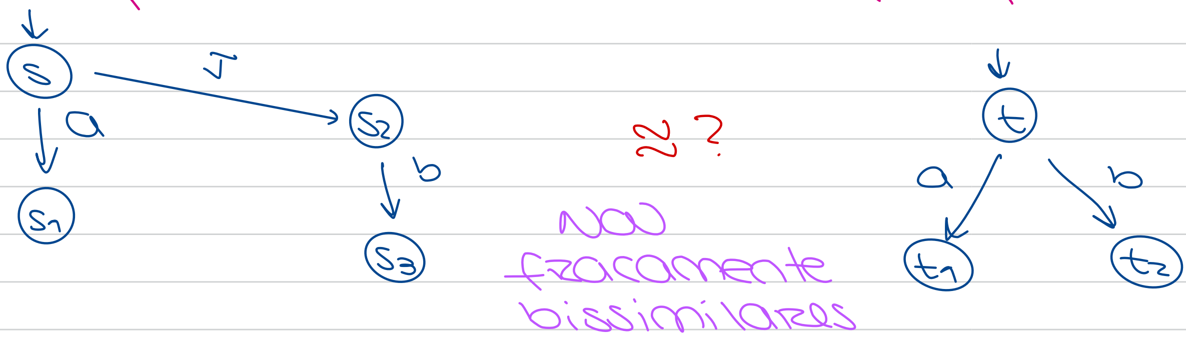
$$
\begin{tikzcd}
⑤ \quad Spec \arrow[loop right, "\overline{pub}"]
\end{tikzcd}
$$

$① \approx ⑤$

$R = \{(①,⑤), (②,⑤), (③,⑤), (④,⑤)\}$

Accabar

verificação de bissimulação fraca



(s, t)

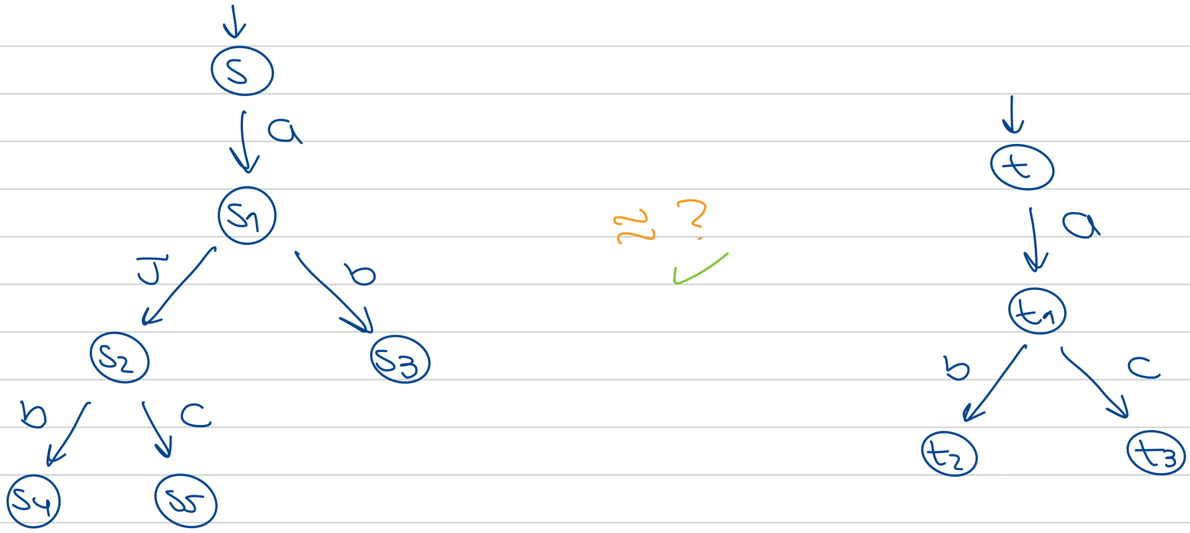
$s \xrightarrow{a} s_1$ $\quad t \overset{a}{\Rightarrow} t_1$ $\quad (s_1, t_1)$ ✓

$s \xrightarrow{\tau} s_2$ $\quad t \overset{\tau}{\Rightarrow} t$ $\quad (s_2, t) \overset{?}{\in} R$

(s_2, t)

$s_2 \xrightarrow{b} s_3$ $\quad t \overset{b}{\Rightarrow} t_2$ $\quad (s_3, t_2)$

$t \xrightarrow{a} t_1$ $\quad s_2 \overset{a}{\not\Rightarrow}$ ×



(s, t)

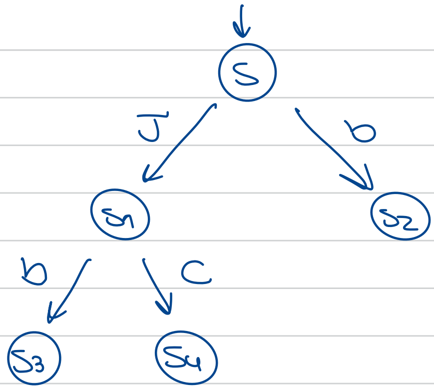
$s \xrightarrow{a} s_1$ $\quad t \overset{a}{\Rightarrow} t_1$ $\quad (s_1, t_1)$

(s_1, t_1)

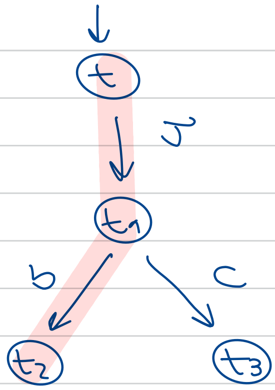
$s_1 \xrightarrow{\tau} s_2$ $\quad t_1 \overset{\tau}{\Rightarrow} t_1$ $\quad (s_2, t_1)$

(. . .)

$s_1 \xrightarrow{b} s_3$ $\quad t_1 \overset{b}{\Rightarrow} t_2$ $\quad (s_3, t_2)$



≈ ? ✓



(s, t)

$s \xrightarrow{b} s_2$ $\quad t \xrightarrow{b} t_2$ $\quad (s_2, t_2) \in R$

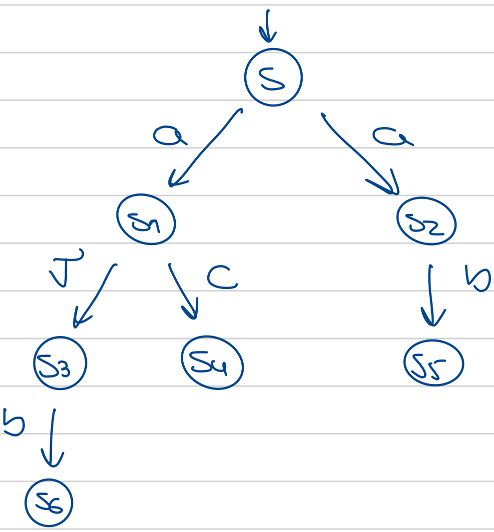
$s \xrightarrow{J} s_1$ $\quad t \xrightarrow{J} t_1$ $\quad (s_1, t_1) \in R$

(s_1, t_1)

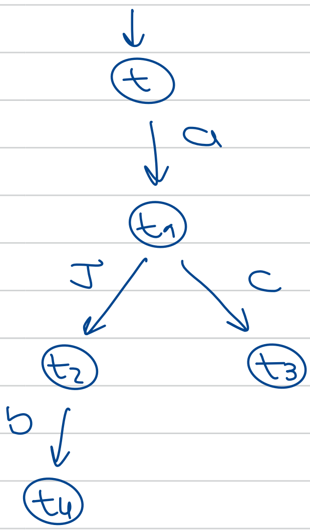
$s_1 \xrightarrow{b} s_3$ $\quad t_1 \xrightarrow{b} t_2$ $\quad (s_3, t_2) \in R$

$s_1 \xrightarrow{c} s_4$ $\quad t_1 \xrightarrow{c} t_3$ $\quad (s_4, t_3) \in R$

$R = \{(s, t), (s_2, t_2), (s_1, t_1), (s_3, t_2), (s_4, t_3)\}$



≈ ?



$s \xrightarrow{a} s_2$ $\quad t \xrightarrow{a} t_1$ $\quad (s_2, t_1)$

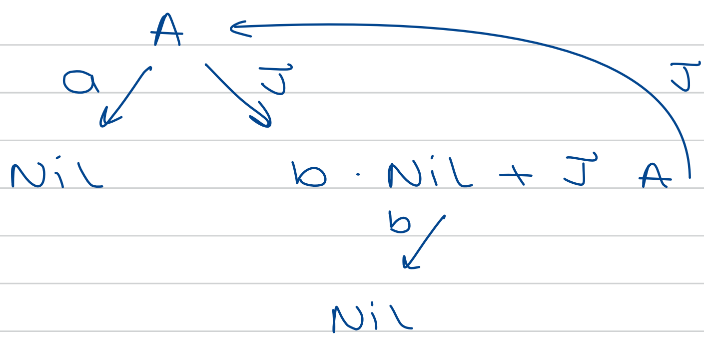
(fazer)

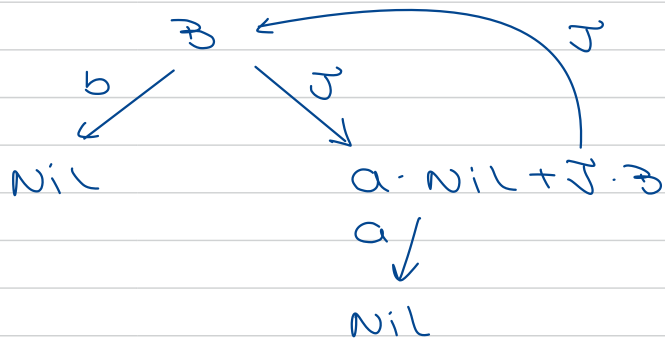
$A = a \cdot Nil + \tau \cdot Nil$

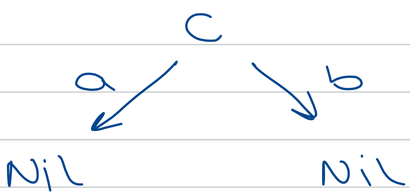
$B = b \cdot Nil + \tau \cdot A$

$C = a \cdot Nil + b \cdot Nil$

$A \approx B \approx C$







$R = \{(C, B), (A, C), (Nil, Nil)\}$

$C \xrightarrow{a} Nil \qquad B \overset{a}{\Rightarrow} Nil$

$C \xrightarrow{b} Nil \qquad B \overset{b}{\Rightarrow} Nil$

$B \xrightarrow{\tau} A \qquad C \overset{\tau}{\Rightarrow} C$

$\{(A, C), (Nil, Nil)\}$

(A, C)

$A \xrightarrow{a} Nil \qquad C \overset{a}{\Rightarrow} Nil$

$A \xrightarrow{\tau} B \qquad C \overset{\tau}{\Rightarrow} C$

(B, C)

$A \approx B$

$A \xrightarrow{a} Nil$ $\quad B \xRightarrow{a} Nil$ $\quad (Nil, Nil)$

$A \xrightarrow{\tau} \underbrace{(b \cdot Nil + \tau \cdot A)}_{B}$ $\quad B \xRightarrow{\tau} \underbrace{(a \cdot Nil + \tau \cdot B)}_{A}$

$B \xrightarrow{b} Nil$ $\quad A \xRightarrow{b} Nil$ $\quad ((b \cdot Nil + \tau \cdot A),$
$(a \cdot Nil + \tau \cdot B))$

$(Nil, Nil) \in R$

Exercício: (usando game ou a outra)

$t \xrightarrow{b} t_2$ $\quad s \xRightarrow{b} s_4$

Casos de Estudo - Aula 4 - 3B

① $\text{Protocol} \stackrel{def}{=} (\text{Sender} \mid \text{Receiver}) \setminus \{a, b\}$

$\text{Sender} \stackrel{def}{=} in \cdot \bar{a} \cdot b \cdot \text{Sender}$

$\text{Receiver} \stackrel{def}{=} a \; out \cdot \bar{b} \cdot \text{Receiver}$

→ Processos em paralelo (comunicação entre si)

① $(\text{Sender} \mid \text{Receiver}) \setminus \{a, b\}$

$\downarrow in$

$(\bar{a} \cdot b \cdot \text{Sender} \mid \text{Receiver}) \setminus \{a, b\}$

$\downarrow \tau$

Nunca poderia por uma trans. por a ou $\bar{a}$ pois estes não são visíveis do ext.

$(b \cdot \text{Sender} \mid out \cdot \bar{b} \cdot \text{Receiver}) \setminus \{a, b\}$

Outra forma:

$\text{Protocol} \stackrel{def}{=} (\text{Sender} \mid \text{Receiver}) \setminus \{a, b\}$

$\text{Sender} \stackrel{def}{=} in \cdot S_1$

$S_1 \stackrel{def}{=} \bar{a} \cdot S_2$

$S_2 \stackrel{def}{=} b \cdot \text{Sender}$

$\text{Receiver} \stackrel{def}{=} a \cdot R_1$

$R_1 \stackrel{def}{=} out \cdot R_2$

$R_2 \stackrel{def}{=} \bar{b} \cdot \text{Receiver}$

$$
\begin{tikzcd}
1 \quad (Sender \mid Receiver) \setminus \{a,b\} \arrow[d, "in"] \\
2 \quad (S_1 \mid Receiver) \setminus \{a,b\} \arrow[d, "\tau"] \\
3 \quad (S_2 \mid R_1) \setminus \{a,b\} \arrow[d, "out"] \\
4 \quad (b.Sender \mid R_2) \setminus \{a,b\} \arrow[uuu, bend right=60, "\tau"']
\end{tikzcd}
$$

$Pspec = in.out.Pspec$

$$
\begin{tikzcd}
5 \quad Pspec \arrow[d, "in"] \\
6 \quad out.Pspec \arrow[u, bend right=60, "out"']
\end{tikzcd}
$$

uma relação de Bissimilação não tem que ser simétrica

$R = \{(1,5), (2,6), (3,6), (4,5)\}$

$(1,5) \in R$

$1 \xrightarrow{in} 2 \qquad 5 \xRightarrow{in} 6 \qquad (2,6) \in R$ (*)

$(2,6) \in R$

$2 \xrightarrow{\tau} 3 \qquad 6 \xRightarrow{\tau} 6 \qquad (3,6) \in R$?

$(3,6) \in R$

$3 \xrightarrow{out} 4 \qquad 6 \xRightarrow{out} 5 \qquad (4,5) \in R$?

$(4,5) \in R$

$4 \xrightarrow{\tau} 1 \qquad 5 \xRightarrow{\tau} 5 \qquad (1,5) \in R$

Faltava analisar no Pspec

$5 \xrightarrow{in} 6 \qquad 4 \xRightarrow{in} 2 \qquad (6,2) \in R$ (*) (=)

case 2

$$Road = car\ up \cdot \overline{ccross} \cdot \overline{dw} \cdot Road$$

$$Rail = train \cdot green \cdot \overline{tcross} \cdot \overline{red} \cdot Rail$$

$$Signal = \overline{green} \cdot red \cdot Signal + \overline{up}\ dw \cdot Signal$$

$$C = (Road \mid Rail \mid Signal) \setminus \{green, red, up, down\}$$

$$C \xrightarrow{car} (up \cdot \overline{ccross} \cdot \overline{dw} \cdot Road \mid Rail \mid Signal) \setminus \{\ldots\}$$

$$C \xrightarrow{train} \text{(fazer)}$$

$$(up \cdot \overline{ccross} \cdot \overline{dw} \cdot Road \mid Rail \mid Signal) \setminus \{\ldots\} \xrightarrow{\tau} (\overline{ccross} \cdot dw \cdot Road \mid Rail \mid dw \cdot Signal) \setminus \{\ldots\}$$

$$\xrightarrow{\overline{ccross}} \overline{dw} \cdot Road \mid Rail \mid dw \cdot Signal$$

Case 3

$$\overset{P}{} \qquad \overset{S}{} \qquad \overset{P}{}$$

$$Mutex = (PROC \mid sem \mid PROC) \setminus \{p, v\}$$

$$PROC = \bar{p} \cdot enter \cdot exit \cdot \bar{v} \cdot PROC$$

$$sem = p \cdot v \cdot sem$$

$$(P \mid S \mid P) \setminus \{p, v\}$$

$$\xrightarrow{\tau} (enter \cdot exit \cdot \bar{v} \cdot PROC) \mid v \cdot sem \mid P$$

$$\xrightarrow{\tau} (P \mid v \cdot sem \mid enter \cdot exit \cdot PROC)$$

$$(enter \cdot exit \cdot \bar{v} \cdot PROC) \mid v \cdot sem \mid P \xrightarrow{enter} exit \cdot \bar{v} \cdot PROC \mid v \cdot spec \mid P$$

$$\xrightarrow{exit} \bar{v} \cdot PROC \mid v \cdot spec \mid P \xrightarrow{\tau} (P \mid S \mid P) \setminus \{p, v\}$$

Case 4

$$FMutex = (PROC \mid sem \mid FPROC) \setminus \{\ldots\}$$

$$FPROC = \bar{p} \cdot enter \cdot exit \cdot (Nil + \bar{v} \cdot FPROC)$$

$$(PROC \mid sem \mid FPROC) \setminus \{\ldots\}$$

$$\downarrow \tau$$

$$PROC \mid v \cdot sem \mid enter \cdot exit \cdot (Nil + \bar{v} \cdot FPROC)$$

$$\downarrow enter$$

$$\ldots$$

$$\downarrow exist$$

$$(PROC \mid v \cdot sem \mid \bar{v} \;\; Nil + \bar{v} \cdot FPROC)$$

$$\xrightarrow{J} \qquad\qquad \xrightarrow{\bar{J}}$$

$$(PROC \mid sem \mid Nil) \qquad\qquad (PROC \mid sem \mid FPROC)$$

$$\Big\downarrow {\scriptstyle ?}$$

$$exit \cdot (Nil + \bar{J} \cdot FPROC)$$

$$\Big\downarrow {\scriptstyle exit}$$

$$(PROC \mid V \cdot sem \mid Nil + \bar{J} \cdot FPROC)$$

$$\Big\downarrow {\scriptstyle \bar{J}}$$

$$PROC \mid V \cdot sem \mid FPROC$$

Case study: communication Protocol

$Send = acc \cdot sending$
$sending = send \cdot wait$
$wait = ack \cdot Send + error \cdot sending$

$Rec = trans \cdot Del$
$Del = \overline{del} \cdot ACK$
$ACK = \overline{ack}\ Rec$

$Med = send \cdot Med'$
$Med' = \tau \cdot Err + trans \cdot Med$
$Err = \overline{error} \cdot Med$

Grafo de transições