Ficha 9 22/23

1.

$$K = [\![(id + \langle f, id \rangle) \cdot out_{\mathbb{N}_0}]\!]$$

$\Leftrightarrow K = in_{\mathbb{N}_0^*} \cdot (id + id \times K) \cdot ((id + \langle f, id \rangle) \cdot out_{\mathbb{N}_0})$ {(F1)}

$\Leftrightarrow K \cdot in_{\mathbb{N}_0} = in_{\mathbb{N}_0^*} \cdot (id + id \times K) \cdot (id + \langle f, id \rangle)$ {(2), (33)}

$\Leftrightarrow K \cdot in_{\mathbb{N}_0} = in_{\mathbb{N}_0^*} \cdot (id + ((id \times K) \cdot \langle f, id \rangle))$ {(1), (25)}

$\Leftrightarrow K \cdot in_{\mathbb{N}_0} = in_{\mathbb{N}_0^*} \cdot (id + \langle f, K \rangle)$ {(11)}

$\Leftrightarrow [K \cdot \underline{0}, K \cdot succ] = [nil, cons \cdot \langle f, K \rangle]$ {(20), (22)}

$\Leftrightarrow \begin{cases} K \cdot \underline{0} = nil \\ K \cdot succ = cons \cdot \langle f, K \rangle \end{cases}$ {(27)}

$\Leftarrow \begin{cases} (K \cdot \underline{0})\, x = nil\, x \\ (K \cdot succ)\, n = (cons \cdot \langle f, K \rangle)\, n \end{cases}$ {(72) (X2)}

$\Rightarrow \begin{cases} K\ 0 = [] \\ K\ (n+1) = (f\ n) : (K\ n) \end{cases}$ {(73)(X3), (75)(X2), (77)}

Substituindo $f\ n$ por $2n+1$ temos

$$\begin{cases} K\ 0 = [] \\ K(n+1) = (2n+1) : (K\ n) \end{cases}$$

$K\ 1 = [1]$

$K\ 2 = [3, 1]$

$K\ 3 = [5, 3, 1]$

Logo, K é a função que devolve numa lista os n primeiros números ímpares, por ordem decrescente.

(2.)

length · ecnecat = sum · map length

$\equiv$ { def ecnecat , def sum }

length · ⦇[nil, cone]⦈ = ⦇[0, add]⦈ · map length

$\equiv$ { (5x) }

length · ⦇[nil, cone]⦈ = ⦇[0, add] · (id + length x id)⦈

$\Leftarrow$ { (49) }

length · [nil, cone] = ([0, add] · (id + length x id)) · (id + id x length)

$\equiv$ { (22) }

length · [nil, cone] = [0, add · (length x id)] · id + id x length

$\equiv$ { (20), (22), (1) }

$$\begin{cases} length \cdot nil = 0 \\ length \cdot cone = add \cdot (length \times id) \cdot (id \times length) \end{cases}$$

$\equiv$ { length [] = 0, (14), (1) (x2), 72, 73, 75 }

length · cone = add · (length x length)

$\equiv$ { length (x ++ y) = (length x) + (length y), 72, 73 (x2), 78 }

True

③

FS length = sum · (map 1)

⇔ length = ⦇[0, add]⦈ · (map 1) {Def sum}

⇔ length = ⦇[0, add] · (id + 1 × id)⦈ {(52)}

⇔ length · in = [0, add · (1 × id)] · (id + id × length) {(22), (1), (26)}

⇔ { length · nil = 0
  { length · cons = add · (1 × id) · (id × length) {(20), (27), (1), (22)}

⇔ { length · nil = 0
  { length · cons = add · (1 × length) {(14), (1)(22)}

⇔ { length [] = 0
  { length (h:t) = 1 + length t {72(x2), 73(x3), 75(x3), (78)}

{Def length}

⇔ True

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length = length · (map f)

⇔ length = ⦇[0, add]⦈ · (map f) {Def sum}

⇔ length = ⦇[0, add] · (id + f × id)⦈ {(52)}

⇔ length · in = [0, add · (f × id)] · (id + id × length) {(22), (1), (46)}

⇔ [0, add] · (id + id × length) = [0, add · (f × length)] {(47), (1)(23), (14), (22)}

⇔ { 0 = 0
  { add · (id × length) = add · (f × length)

length = length · (map f)

⇔ length = ⦇[0, add · (1 × id)]⦈ · (map f) {Def length}

⇔ length = ⦇[0, add · (1 × id)] · (id + f × id)⦈ {(52)}

⇔ length · in = [0, add · ((1 · f) × id)] · id + id × length {(46), (22), (1)(22), (14)}

⇔ { length · nil = 0
  { length · cons = add · (1 × length) {(20), (27), 622), (1)(23), (14), (3)}

⇔ { length [] = 0
  { length (h:t) = 1 + length t {(72)(x2), (73)(x3), 75(x3), (78)}

{Def length}

⇔ True

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④

$fmap\ f = T f = (|in \cdot B(f, id)|)$

$\Leftrightarrow (fmap\ f) \cdot in = in \cdot B(f, id) \cdot F(fmap\ f)$ {(9.6)}

$\Leftrightarrow (fmap\ f) \cdot [\underline{Empty}, Node] = [\underline{Empty}, Node] \cdot (id + f \times id^2) \cdot id + id \times (fmap\ f)^2$ {Def in, B, F}

$\Leftrightarrow \begin{cases} (fmap\ f) \cdot \underline{Empty} = \underline{Empty} \\ (fmap\ f) \cdot Node = Node \cdot (f \times id^2) \cdot (id \times (fmap\ f)^2) \end{cases}$ {20, (27), (27)(x1), 1(x2)}

$\Leftrightarrow \begin{cases} (fmap\ f) \cdot \underline{Empty} = \underline{Empty} \\ (fmap\ f) \cdot Node = Node \cdot (f \times (id^2 \cdot (fmap\ f)^2)) \end{cases}$ {14, 11}

$\Leftrightarrow \begin{cases} (fmap\ f) \cdot \underline{Empty} = \underline{Empty} \\ (fmap\ f) \cdot Node = Node \cdot (f \times (fmap\ f)^2) \end{cases}$ {14}

$\Leftrightarrow \begin{cases} fmap\ f\ Empty = Empty \\ fmap\ f\ (Node\ (a, (x, y)) = Node\ (f\ a, (fmap\ f\ x, fmap\ f\ y)) \end{cases}$ {72(x2), 73(x3), 75(x2), 78(x2)}

⑤

$depth \cdot LTree\ f = depth$

$\Leftrightarrow (|[one, succ \cdot umax]| \cdot LTree\ f = depth$ {Def depth}

$\Leftrightarrow depth = (|[one, succ \cdot umax] \cdot (f + id^2)|)$ {52}

$\Leftrightarrow depth \cdot in = [one \cdot f, succ \cdot umax \cdot id^2] \cdot id + depth^2$ {46}

$\Leftrightarrow depth \cdot in = [one, succ \cdot umax \cdot id^2 \cdot depth^2]$ {22, 1, 31}

$\Leftrightarrow \begin{cases} depth \cdot Leaf = one \\ depth \cdot Fork = succ \cdot umax \cdot depth^2 \end{cases}$ {20, 27, 14, 1(x2)}

$\Leftrightarrow \begin{cases} depth\ (Leaf\ a) = 1 \\ depth\ (Fork(x, y)) = 1 + umax\ (depth\ x, depth\ y) \end{cases}$ {72(x2), 73(x3), 78, 75}

$\Leftrightarrow$ True {Def depth}