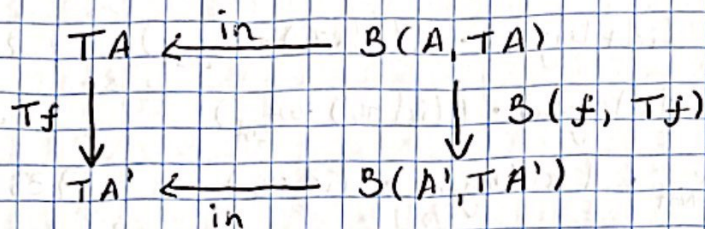


1.



$$\cancel{(T_f) \cdot \text{in} = \text{in} \cdot B(f, T_f)} \quad \boxed{(T_f) \cdot \text{in} = \text{in} \cdot B(f, T_f)}$$

Propriedade básica de in

Para listas:

$$(T_f) \cdot \text{in} = \text{in} \cdot B(f, T_f)$$

$$\Rightarrow (\text{map } f) \cdot [\text{nil}, \text{cons}] = [\text{nil}, \text{cons}] \cdot (\text{id} + f \times (\text{map } f))$$

$$\Rightarrow \begin{cases} (\text{map } f) \cdot \text{nil} = \text{nil} \\ (\text{map } f) \cdot \text{cons} = \text{cons} \cdot (f \times (\text{map } f)) \end{cases} \quad \begin{matrix} \{ (20), (22), (1), (27) \} \\ \{ 72(x2) \} \end{matrix}$$

$$\Rightarrow \begin{cases} \forall x \mid ((\text{map } f) \cdot \text{nil}) x = \text{nil } x \\ \forall h, t \mid ((\text{map } f) \cdot \text{cons})(h, t) = (\text{cons} \cdot (f \times (\text{map } f)))(h, t) \end{cases} \quad \begin{matrix} \{ 72(x2) \} \\ \{ 73(x3), 75(x2), 78 \} \end{matrix}$$

$$\Rightarrow \begin{cases} \forall x \mid \text{map } f [] = [] \\ \forall h, t \mid \text{map } f (h:t) = (f h) : (\text{map } f t) \end{cases} \quad \begin{matrix} \{ 73(x3), 75(x2), 78 \} \end{matrix}$$

Logo, a propriedade natural de in é a definição de map.

2.

$$\begin{aligned}
 T_f &= \text{id} \cdot \text{in} \cdot B(f, \text{id}) \quad \{ 51 - \text{def-map-cons} \} \\
 &= \{ 46 \}
 \end{aligned}$$

$$\begin{aligned}
 T_f \cdot \text{in} &= \text{in} \cdot B(f, \text{id}) \cdot F(T_f) \equiv \text{in} \cdot B(f, \text{id}) \cdot B(\text{id}, T_f) \quad \{ 50 \} \\
 &\equiv \text{in} \cdot B(f \cdot \text{id}, \text{id} \cdot T_f) \quad \{ 91 | 3.57 \} \\
 &\equiv \{ 50, 1(x2), B(g \cdot h, i \cdot j) = B(g, i) \cdot B(h, j) \} \equiv \text{in} \cdot B(\text{id} \cdot f, (T_f) \cdot \text{id}) \quad \{ 1(x2) \} \\
 &\equiv \text{in} \cdot B(\text{id}, T_f) \cdot B(f, \text{id}) \quad \{ 91 | 3.57 \}
 \end{aligned}$$

$$\begin{aligned}
 T_f \cdot \text{in} &= \text{in} \cdot B(\text{id}, T_f) \cdot B(f, \text{id}) \\
 &= \{ 33, 34, 50/59 \}
 \end{aligned}$$

$$\begin{aligned}
 \text{out} \cdot T_f &= F(T_f) \cdot B(f, \text{id}) \cdot \text{out} \\
 &= \{ 55 \}
 \end{aligned}$$

$$T_f = [B(f, \text{id}) \cdot \text{out}] \quad \{ 60 - \text{def-map-cons} \}$$

$$\begin{aligned}
 (3) \quad & [Zero, sree - \pi_2] = [(id + \pi_2) \cdot out_{list}] \\
 \Rightarrow & out_{Nat} \cdot length = (id + length) \cdot ((id + \pi_2) \cdot out_{list}) \quad \{55\} \\
 \Rightarrow & length = in_{Nat} \cdot (id + length) \cdot ((id + \pi_2) \cdot out_{list}) \quad \{34\} \\
 \Rightarrow & length \cdot in_{list} = in_{Nat} \cdot (id + length) \cdot (id + \pi_2) \quad \{33, 2\} \\
 \Rightarrow & \begin{cases} length \cdot nil = Zero \\ length \cdot cons = sree \cdot length \cdot \pi_2 \end{cases} \quad \{20, 27, 22(x2), 1(x2)\} \\
 \Rightarrow & \begin{cases} length [] = 0 \\ length (h:t) = (length t) + 1 \end{cases} \quad \{72(x2), 73(x2), 75(x2)\} \\
 \Rightarrow & True \quad \{Def length\}
 \end{aligned}$$

(4)

$$\begin{aligned}
 & (LTree f) \cdot mirror = mirror \cdot (LTree f) \\
 \Rightarrow & (LTree f) \cdot mirror = (in \cdot (id + swap) \cdot B(f, id)) \quad \{52\} \\
 \Leftarrow & (LTree f) \cdot (in \cdot (id + swap)) = in \cdot (id + swap) \cdot (f + id^2) \cdot F(LTree f) \quad \{49\} \\
 \Rightarrow & (LTree f) \cdot (in \cdot (id + swap)) = in \cdot (f + swap) \cdot B(id, LTree f) \quad \{27, 1(x2), 15, 50\} \\
 \Rightarrow & (LTree f) \cdot (in \cdot (id + swap)) = in \cdot (f + swap) \cdot (id + (LTree f)^2) \quad \{B LTree\} \\
 \Rightarrow & (LTree f) \cdot [leaf, fork \cdot swap] = [leaf \cdot f, fork \cdot swap] \cdot (id + (LTree f)^2) \quad \{20(x2), 22(x2), 1\} \\
 \Rightarrow & \begin{cases} (LTree f) \cdot leaf = leaf \cdot f \\ (LTree f) \cdot fork \cdot swap = fork \cdot swap \cdot (LTree f)^2 \end{cases} \quad \{20, 22, 1, 23\} \\
 \Rightarrow & \begin{cases} (LTree f) (leaf a) = leaf (f a) \\ (LTree f) \cdot fork \cdot swap = fork \cdot (LTree f)^2 \cdot swap \end{cases} \quad \{72, 73(x2)\} \quad \{Natural swap\} \\
 \Rightarrow & \begin{cases} (LTree f) (leaf a) = leaf (f a) \\ (LTree f) \cdot fork = fork \cdot (LTree f)^2 \end{cases} \quad \{33, swap \cdot swap = id\} \\
 \Rightarrow & \begin{cases} (LTree f) (leaf a) = leaf (f a) \\ LTree f (fork (x, y)) = fork (LTree f x, LTree f y) \end{cases} \quad \{72, 73(x2), 78\} \\
 \Rightarrow & True \quad \{Def LTree f\}
 \end{aligned}$$

5

$$\text{Suffixes} = [(id + \langle \text{cons}, \pi_2 \rangle) \cdot \text{out}]$$

$$\Rightarrow \text{out} \cdot \text{Suffixes} = (f \text{ Suffixes}) \cdot (id + \langle \text{cons}, \pi_2 \rangle) \cdot \text{out} \quad \{55\}$$

$$\Rightarrow \text{Suffixes} = in \cdot (f \text{ Suffixes}) \cdot (id + \langle \text{cons}, \pi_2 \rangle) \cdot \text{out} \quad \{34\}$$

$$\Rightarrow \text{Suffixes} \cdot in = in \cdot (id + id \times \text{Suffixes}) \cdot (id + \langle \text{cons}, \pi_2 \rangle) \quad \{33\}$$

$$\Rightarrow \text{Suffixes} \cdot in = in \cdot (id + ((id \times \text{Suffixes}) \cdot \langle \text{cons}, \pi_2 \rangle)) \quad \{25, 1\}$$

$$\Rightarrow \text{Suffixes} \cdot in = in \cdot (id + \langle \text{cons}, \text{Suffixes} \cdot \pi_2 \rangle) \quad \{11, 1\}$$

$$\Rightarrow \begin{cases} \text{Suffixes} \cdot \text{nil} = \text{nil} \\ \text{Suffixes} \cdot \text{cons} = \text{cons} \cdot \langle \text{cons}, \text{Suffixes} \cdot \pi_2 \rangle \end{cases} \quad \{20, 22, 1, 27\}$$

$$\Rightarrow \begin{cases} \text{Suffixes} [] = [] \\ \text{Suffixes} (h:t) = (h:t) : (\text{Suffixes } t) \end{cases} \quad \{72(x2), 73(x3), 75(x2), 77\}$$

6

$$\text{mirror} = [(id + \text{swap}) \cdot \text{out}]$$

$$\Rightarrow \text{out} \cdot \text{mirror} = (f \text{ mirror}) \cdot ((id + \text{swap}) \cdot \text{out}) \quad \{55\}$$

$$\Rightarrow \text{mirror} = in \cdot (f \text{ mirror}) \cdot ((id + \text{swap}) \cdot \text{out}) \quad \{34\}$$

$$\Rightarrow \text{mirror} \cdot in = in \cdot (f \text{ mirror}) \cdot (id + \text{swap}) \quad \{33\}$$

$$\Rightarrow \text{mirror} \cdot in = in \cdot (id + \text{mirror}^2) \cdot (id + \text{swap}) \quad \{24, f \text{ True}\}$$

$$\Rightarrow \text{mirror} \cdot in = in \cdot (id + (\text{mirror}^2 \cdot \text{swap})) \quad \{25, 1\}$$

$$\Rightarrow \text{mirror} \cdot in = in \cdot (id + (\text{swap} \cdot \text{mirror}^2)) \quad \{Not \text{ true swap}\}$$

$$\Rightarrow \text{mirror} \cdot in = in \cdot ((id + \text{swap}) \cdot (id + \text{mirror}^2)) \quad \{1, 25\}$$

$$\Rightarrow \text{mirror} \cdot in = in \cdot (id + \text{swap}) \cdot f \text{ mirror} \quad \{2, \text{def } f \text{ True}\}$$

$$\Rightarrow \text{mirror} = (in \cdot (id + \text{swap})) \quad \{46\}$$

$$\text{mirror} \cdot \text{mirror} = id$$

$$\Leftarrow ((id + \text{swap}) \cdot \text{out}) \cdot \text{mirror} = (f \text{ mirror}) \cdot \text{out} \quad \{57, 58\}$$

$$\Rightarrow ((id + \text{swap}) \cdot \text{out}) \cdot \text{mirror} \cdot in = f \text{ mirror} \quad \{33\}$$

$$\Rightarrow ((id + \text{swap}) \cdot \text{out}) \cdot (in \cdot (id + \text{swap})) \cdot f \text{ mirror} = f \text{ mirror} \quad \{47\}$$

$$\Leftarrow (id + \text{swap}) \cdot (id + \text{swap}) = id \quad \{5\}$$

$$\Rightarrow id + (\text{swap} \cdot \text{swap}) = id \quad \{25, 1\}$$

$$\Rightarrow id + id = id \quad \{swap \cdot swap = id\}$$

$$\Rightarrow \text{True} \quad \{26\}$$

⑦

$$K = (in_2 \cdot \alpha \cdot D)$$

$$\equiv \{46\}$$

$$K \cdot in_1 = in_2 \cdot \alpha \cdot FK$$

$$\equiv \{33, 34, F4\}$$

$$out_2 \cdot K = GK \cdot \alpha \cdot out_1$$

$$\equiv \{55\}$$

$$K = [\alpha \cdot out_1]$$

$$length = ([zero, succ \cdot \pi_2] D)$$

$$\Rightarrow length = (in_{out} \cdot \underbrace{(id + \pi_2)}_{\alpha} D)$$

$$length = [\underbrace{(id + \pi_2)}_{\alpha} \cdot out_{list}]$$

Logo, por $K = length$:

- $T_1 \rightarrow Listas \ A^*$
- $T_2 \rightarrow Naturais \ IN_0$
- $\alpha = id + \pi_2$