

$$\text{flip } f = \hat{f} \cdot \text{Swap}$$

$$\Rightarrow \hat{f} \cdot \text{Swap} = \text{ap} \cdot (\text{flip } f \times \text{id})$$

{(35)}

$$\Rightarrow \forall x, y \mid (\hat{f} \cdot \text{Swap})(x, y) = (\text{ap} \cdot (\text{flip } f \times \text{id}))(x, y) \quad \{(72)\}$$

$$\Rightarrow \forall x, y \mid \hat{f}(\text{Swap}(x, y)) = \text{ap}((\text{flip } f \times \text{id})(x, y)) \quad \{(73)(x2)\}$$

$$\Rightarrow \forall x, y \mid \hat{f}(y, x) = \text{ap}(\text{flip } f \ x, y) \quad \{\text{Def Swap}, (72), (71)\}$$

$$\Rightarrow \forall x, y \mid \hat{f}(y, x) = \text{flip } f \ x \ y \quad \{(73)\}$$

$$\Rightarrow \forall x, y \mid f \ y \ x = \text{flip } f \ x \ y \quad \{(75)\}$$

(6.)

$$\left\{ \begin{array}{l} \text{for } b \ i \ 0 = i \\ \text{for } b \ i \ (n+1) = b \ (\text{for } b \ i \ n) \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \text{for } b \ i \ (\text{zero } n) = \underline{i} \ n \\ \text{for } b \ i \ (\text{succ } n) = b \ (\text{for } b \ i \ n) \end{array} \right.$$

{(75)(x2),  
Def succ}

$$\Rightarrow \left\{ \begin{array}{l} ((\text{for } b \ i) \cdot \text{zero}) \ n = \underline{i} \ n \\ ((\text{for } b \ i) \cdot \text{succ}) \ n = (b \cdot (\text{for } b \ i)) \ n \end{array} \right. \quad \{(73)(x3)\}$$

$$\Rightarrow \left\{ \begin{array}{l} (\text{for } b \ i) \cdot \text{zero} = \underline{i} \\ (\text{for } b \ i) \cdot \text{succ} = b \cdot (\text{for } b \ i) \end{array} \right. \quad \{(72)(x2)\}$$

$$\Rightarrow \left\{ \begin{array}{l} (\text{for } b \ i) \cdot \text{in} \cdot i_1 = \underline{i} \\ (\text{for } b \ i) \cdot \text{in} \cdot i_2 = b \cdot (\text{for } b \ i) \end{array} \right. \quad \{(18)(x2), \text{Def in}\}$$

$$\Rightarrow (\text{for } b \ i) \cdot \text{in} = [\underline{i}, b \cdot (\text{for } b \ i)] \quad \{(17)\}$$

$$\Rightarrow (\text{for } b \ i) \cdot \text{in} = [\underline{i} \cdot \text{id}, b \cdot (\text{for } b \ i)] \quad \{(1)\}$$

$$\Rightarrow (\text{for } b \ i) \cdot \text{in} = [\underline{i}, b] \cdot \text{id} + (\text{for } b \ i) \quad \{(22)\}$$

$$\Rightarrow \text{for } b \ i = ([\underline{i}, b]) \quad \{(46)\}$$

Logo, neste caso  $\boxed{g = [\underline{i}, b]}$



7.

$$\begin{cases} a + 0 = a \\ a + (n+1) = 1 + (a+n) \end{cases}$$

$$\Rightarrow \begin{cases} (a+) (\underline{0}) = \underline{a} \\ (a+) (\text{succ } n) = 1 + ((a+) n) \end{cases}$$

{(75) (x2),  
Def succ}

$$\Rightarrow \begin{cases} ((a+) \cdot \underline{0}) n = \underline{a} n \\ ((a+) \cdot \text{succ}) n = (\text{succ} - (a+)) n \end{cases}$$

{(73) (x3),  
Def succ}

$$\Rightarrow \begin{cases} ((a+) \cdot \text{zero}) = \underline{a} \\ ((a+) \cdot \text{succ}) = \text{succ} \cdot (a+) \end{cases}$$

{(72) (x2)}

$$\Rightarrow \begin{cases} (a+) \cdot \text{in} \cdot i_1 = \underline{a} \\ (a+) \cdot \text{in} \cdot i_2 = \text{succ} \cdot (a+) \end{cases}$$

{Def in, (72) (x2)}

$$\Rightarrow (a+) \cdot \text{in} = [\underline{a}, \text{succ} \cdot (a+)]$$

{(77)}

$$\Rightarrow (a+) \cdot \text{in} = [\underline{a} \cdot \text{id}, \text{succ} \cdot (a+)]$$

{(1)}

$$\Rightarrow (a+) \cdot \text{in} = [\underline{a}, \text{succ}] \cdot (\text{id} + (a+))$$

{(22)}

$$\Rightarrow (a+) = \mathbb{D} [\underline{a}, \text{succ}] \mathbb{D}$$

{(46)}

Logo, pelo exercício anterior,  $\boxed{(a+) = \text{for succ } a}$

8.

$$f \cdot \mathbb{D} g \mathbb{D} = \mathbb{D} h \mathbb{D}$$

$$\Rightarrow f \cdot \mathbb{D} g \mathbb{D} \cdot \text{id} = h \cdot (\text{id} + f \cdot \mathbb{D} g \mathbb{D}) \quad \{(46)\}$$

$$\Rightarrow f \cdot g \cdot (\text{id} + \mathbb{D} g \mathbb{D}) = h \cdot (\text{id} + f \cdot \mathbb{D} g \mathbb{D}) \quad \{(47)\}$$

$$\Rightarrow f \cdot g \cdot (\text{id} + \mathbb{D} g \mathbb{D}) = h \cdot (\text{id} \cdot \text{id} + f \cdot \mathbb{D} g \mathbb{D}) \quad \{(1)\}$$

$$\Rightarrow f \cdot g \cdot (\text{id} + \mathbb{D} g \mathbb{D}) = h \cdot (\text{id} + f) \cdot (\text{id} + \mathbb{D} g \mathbb{D}) \quad \{(25)\}$$

$$\Rightarrow f \cdot g = h \cdot (\text{id} + f) \quad \{(5) \text{ Leibnitz}\}$$



(9)

$$f \cdot (\text{for } f \ i) = \text{for } f \ (f \ i)$$

$$\Rightarrow f \cdot ([i, f]) = ([f \ i], f)$$

$$\Leftarrow f \cdot [i, f] = [f \ i, f] \cdot \text{id} + f$$

$$\Rightarrow [f \cdot i, f \cdot f] = [f \ i \cdot \text{id}, f \cdot f]$$

$$\Rightarrow \begin{cases} f \cdot i = f \ i \\ f \cdot f = f \cdot f \end{cases}$$

$$\Rightarrow \begin{cases} \text{True} \\ \text{True} \end{cases}$$

$$\Rightarrow \text{True} \wedge \text{True}$$

$$\Rightarrow \text{True}$$

$$\{ \text{for } f \ i = ([i, f]) \}$$

$$\{ (49) \}$$

$$\{ (22) \}$$

$$\{ (1), (27) \}$$

{Propriedade Reflexiva de  
igualdade, (4)}

$$(10) \ f = g$$

$$\Rightarrow \text{for id } i = \text{for } i \ i$$

$$\Rightarrow ([i, \text{id}]) = ([i, i])$$

$$\Rightarrow ([i, \text{id}]) \cdot \text{id} = [i, i] \cdot (\text{id} + ([i, \text{id}]))$$

$$\Rightarrow [i, \text{id}] \cdot (\text{id} + ([i, \text{id}])) = [i, i]$$

$$\Rightarrow [i, ([i, \text{id}])] = [i, i]$$

$$\Rightarrow \begin{cases} i = i \\ ([i, \text{id}]) = i \end{cases}$$

$$\Rightarrow \text{True} \wedge i \cdot \text{id} = [i, \text{id}] \cdot (\text{id} + i)$$

$$\Rightarrow i = [i, i]$$

$$\Rightarrow i \cdot i_1 = i \wedge i \cdot i_2 = i$$

$$\Rightarrow i = i \wedge i = i$$

$$\Rightarrow \text{True}$$

$$\{ \text{Def } f, g \}$$

$$\{ \text{Def for } (x2) \}$$

$$\{ (46) \}$$

$$\{ (47), (22), (3) (x2) \}$$

$$\{ (22), (3), (1) \}$$

$$\{ (27) \}$$

$$\{ (46) \}$$

$$\{ (3) (x2), (1), (22) \}$$

$$\{ (17) \}$$

$$\{ (3) (x2) \}$$

$$\text{for } i \ i \ i \Rightarrow \begin{cases} \text{for } i \ i \ 0 = i \\ \text{for } i \ i \ (n+1) = i \ (\text{for } i \ i \ n) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \text{for } i \ i \ 0 = i \\ \text{for } i \ i \ (n+1) = i \end{cases} \Rightarrow i$$

Logo,  $f$  e  $g$  são a função constante  $i$ .