$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

$$\left(e^{-x}\cos(y) + e^{-y}\cos(x)\right) + \left(-e^{-x}\cos(y) - e^{-y}\cos(x)\right) = 0$$

$$e^{-x}\cancel{\cos(y)} + e^{-y}\cancel{\cos(x)} - e^{-x}\cancel{\cos(y)} - e^{-y}\cancel{\cos(x)} = 0$$

$$0 = 0 \qquad c.q.m.$$

Assim, concluímos que a função é harmônica.

4) $\frac{dz}{dt}$

a) $z = x^3 + y^2$, $x = \cos t$ e $y = \frac{1}{t}$

$$d'\left(\frac{1}{t}\right) = -\frac{1}{t^2}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = (3x^2) \cdot (-\operatorname{sen} t) + (2y) \cdot \left(-\frac{1}{t^2}\right)$$

$$\frac{dz}{dt} = -3x^2 \operatorname{sen} t - \frac{2y}{t^2}$$

Substituir o $x = \cos t$ e $y = \frac{1}{t}$

$$\frac{dz}{dt} = -3\cos^2 t \operatorname{sen} t - \frac{2}{t^3}$$

b) $z = \ln(x + y^2)$, $x = e^{t^2}$ e $y = t^3 + t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = \left(\frac{1}{x+y^2}\right) 2t e^{t^2} + \left(\frac{2y}{x+y^2}\right) \times (3t^2 + 1)$$