Ficha 5 22/23

(1) $\alpha = swap \cdot (id \times swap)$

$$\begin{tikzcd}
 & (C\times B)\times A & \\
A & A\times(C\times B) \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] \arrow[u, "swap"'] & (C\times B) \\
A \arrow[u, "id"] & A\times(B\times C) \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] \arrow[u, "id\times swap"'] & (B\times C) \arrow[u, "swap"']
\end{tikzcd}$$

$$\boxed{A\times(B\times C) \xrightarrow{\alpha} (C\times B)\times A}$$

$$\begin{tikzcd}
(C\times B)\times A \arrow[d, "(h\times g)\times f"] & A\times(B\times C) \arrow[l, "\alpha"'] \arrow[d, "f\times(g\times h)"] \\
(F\times E)\times D & D\times(E\times F) \arrow[l, "\alpha"]
\end{tikzcd}$$

$$\boxed{((h\times g)\times f)\cdot\alpha = \alpha\cdot(f\times(g\times h))}$$

Propriedade Natural de α

(2) $\boxed{\alpha = dup \cdot join}$

$$\begin{tikzcd}
A & A\times A \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & A \\
 & A \arrow[ul, "id"] \arrow[u, "dup"] \arrow[ur, "id"'] & \\
A \arrow[ur, "id"] \arrow[r, "i_1"'] & A+A \arrow[u, "join"] & A \arrow[ul, "id"'] \arrow[l, "i_2"]
\end{tikzcd}$$

$$\begin{tikzcd}
A\times A \arrow[d, "f\times f"'] & A+A \arrow[l, "\alpha"'] \arrow[d, "f+f"] \\
B\times B & B+B \arrow[l, "\alpha"]
\end{tikzcd}$$

$$\boxed{(f\times f)\cdot\alpha = \alpha\cdot(f+f)}$$

Propriedade Natural de α

Para $\alpha =$ join . dup não é possível calcular, porque α não é uma função válida se seguir esta definição, já que os tipos não encaixam.

dup vai sempre retornar um produto, enquanto que join está à espera de ~~um~~ um co-produto.

3.) $\boxed{iso = \langle ! + !, [id, id] \rangle}$

$$
\begin{tikzcd}
1 \arrow[r, "i_1"] & 1+1 & 1 \arrow[l, "i_2"'] \\
A \arrow[u, "!"] \arrow[r, "i_1"'] & A+A \arrow[u, "!+!"] & A \arrow[l, "i_2"] \arrow[u, "!"']
\end{tikzcd}
\qquad
\begin{tikzcd}
 & A & \\
A \arrow[ur, "id"] \arrow[r, "i_1"'] & A+A \arrow[u, "join"] & A \arrow[l, "i_2"] \arrow[ul, "id"']
\end{tikzcd}
$$

$$
\begin{tikzcd}
1+1 & (1+1)\times(A) \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & A \\
 & A+A \arrow[ul, "!+!"] \arrow[u, "iso"] \arrow[ur, "join"'] &
\end{tikzcd}
$$

$$
\begin{tikzcd}
(1+1)\times A \arrow[d, "id \times f"'] & A+A \arrow[l, "iso"'] \arrow[d, "f+f"] \\
(1+1)\times B & B+B \arrow[l, "iso"]
\end{tikzcd}
$$

Iso testemunha o isomorfismo entre $A+A$ e $(1+1)\times A$

$(id \times f) \cdot iso = iso \cdot (f+f)$

$\Leftrightarrow (id \times f) \cdot \langle ! + !, [id, id] \rangle = \langle ! + !, [id, id] \rangle \cdot (f+f)$

$\Leftrightarrow \langle ! + !, f \cdot [id, id] \rangle = \langle ! + !, [id, id] \rangle \cdot (f+f)$ $\{(11), (11)\}$

$\Leftrightarrow \langle ! + !, [f, f] \rangle = \langle (! + !) \cdot (f+f), [id, id] \cdot (f+f) \rangle$ $\{(20)$ $(1)(cc)$ $(J)\}$

$\Leftrightarrow \begin{cases} ! + ! = (! + !) \cdot (f+f) \\ [f, f] = [f, f] \end{cases}$ $\{(16), (22), (1)(x2)\}$

$\Leftrightarrow \begin{cases} ! + ! = (! \cdot f) + (! \cdot f) \\ [f, f] = [f, f] \end{cases}$ $\{(25)\}$

$\Leftrightarrow \begin{cases} ! + ! = ! + ! \\ [f, f] = [f, f] \end{cases}$ $\{! \cdot f = !\}$

$\Leftrightarrow$ True

$$iso = \langle ! + !, [id, id] \rangle \Leftrightarrow$$

$$\Leftrightarrow iso = \langle [i_1 \cdot !, i_2 \cdot !], [id, id] \rangle \qquad \{(21)\}$$

$$\Leftrightarrow iso = [\langle i_1 \cdot !, id \rangle, \langle i_2 \cdot !, id \rangle] \qquad \{(28)\}$$

$$\Leftrightarrow \begin{cases} iso \cdot i_1 = \langle i_1 \cdot !, id \rangle \\ iso \cdot i_2 = \langle i_2 \cdot !, id \rangle \end{cases} \qquad \{(17)\}$$

$$\Leftrightarrow \begin{cases} \forall a \mid (iso \cdot i_1)\, a = \langle i_1 \cdot !, id \rangle\, a \\ \forall a \mid (iso \cdot i_2)\, a = \langle i_2 \cdot !, id \rangle\, a \end{cases} \qquad \{(72)(x2)\}$$

$$\Leftrightarrow \begin{cases} \forall a \mid iso\, (i_1\, a) = (i_1\, (), a) \\ \forall a \mid iso\, (i_2\, a) = (i_2\, (), a) \end{cases} \qquad \{(73)(x4), (77)(x2)\}$$

$$\boxed{\begin{array}{l} iso\, (Left\ a) = (Left\ (), a) \\ iso\, (Right\ a) = (Right\ (), a) \end{array}}$$

4.

$$f \cdot \nabla = \nabla \cdot (f + f) \qquad \begin{cases} \nabla \cdot i_1 = id \\ \nabla \cdot i_2 = id \end{cases} \Leftrightarrow \nabla = [id, id] \quad \{(17)\}$$

$$\cancel{\Leftrightarrow f \cdot \nabla \cdot i_1 =}$$

$$\Leftrightarrow f \cdot [id, id] = [id, id] \cdot (f + f)$$

$$\Leftrightarrow [f, f] = [f, f] \qquad \{(20), (22), (1)(x4)\}$$

$\Leftrightarrow$ True

5.

$$\begin{tikzcd} A + C \arrow[d, "f+h"'] & A + (B \times C) \arrow[l, "\alpha"'] \arrow[d, "f + (g \times h)"] \\ D + F & D + (E \times F) \arrow[l, "\alpha"'] \end{tikzcd}$$

$$\begin{tikzcd} A \arrow[r, "i_1"] & A + C & C \arrow[l, "i_2"'] \\ A \arrow[u, "id"] \arrow[r, "i_1"'] & A + (B \times C) \arrow[u, "\alpha", "id + \pi_2"'] & B \times C \arrow[l, "i_2"] \arrow[u, "\pi_2"'] \end{tikzcd}$$

$$\boxed{(f + h) \cdot \alpha = \alpha \cdot (f + (g \times h))}$$

$$(f + h) \cdot (id + \pi_2) = (id + \pi_2) \cdot (f + (g \times h))$$

$$\Leftrightarrow f + (h \cdot \pi_2) = f + (\pi_2 \cdot (g \times h)) \qquad \{(25)(x2), (1)(x2)\}$$

$$\Leftrightarrow f + (h \cdot \pi_2) = f + (h \cdot \pi_2) \qquad \{(13)\}$$

True

$$\cancel{\Leftrightarrow \begin{cases} f = i_1 \cdot f \\ h \cdot \pi_2 = \ldots \cdot \pi_2 \end{cases}}$$

Scanned with CamScanner

6.

$$\begin{array}{ccc} (A\times B)+(A\times C) & \xleftarrow{distr} & A\times(B+C) \\ \downarrow{\scriptstyle (f\times g)+(f\times h)} & & \downarrow{\scriptstyle f\times(g+h)} \\ (D\times E)+(D\times F) & \xleftarrow[distr]{} & D\times(E+F) \end{array}$$

$$\boxed{((f\times g)+(f\times h))\cdot distr = distr\cdot(f\times(g+h))}$$

Gratis distr

~~$h\cdot distr\cdot(g\times(id+\alpha)) = K$~~

~~$\Rightarrow h\cdot distr\cdot(g\times(id+\alpha))\cdot undistr = K\cdot undistr$~~

~~$\Leftrightarrow h\cdot((g\times id)+(g\times\alpha))\cdot distr\cdot undistr = K\cdot undistr$~~ ~~{Natural distr}~~

~~$\Leftrightarrow h\cdot((g\times id)+(g\times\alpha))\cdot id = K\cdot undistr$~~ ~~{ }~~

~~$\Leftrightarrow h\cdot((g\times id)+(g\times\alpha)) = K\cdot undistr$~~

$h\cdot distr\cdot(g\times(id+\alpha)) = K$

$\Leftrightarrow h\cdot((g\times id)+(g\times\alpha))\cdot distr = K$ { Natural distr }

$\Leftrightarrow h\cdot((g\times id)+(g\times\alpha)) = K\cdot undistr$ { (33) }

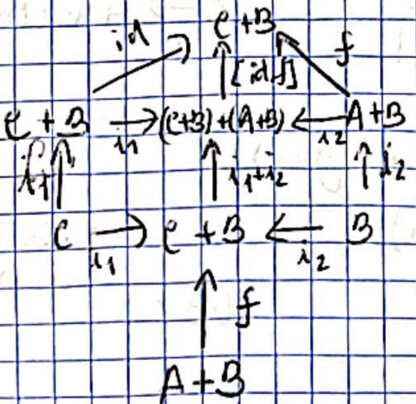
7.

$ecmb\ id = [id, id]\cdot(i_1+i_2)\cdot id$ { Def ecmb }

$= [i_1, i_2]\cdot id$ { (22), (1) (x2) }

$= [i_1, i_2]$ { (1) }

$= id$ { (19) }



$$\boxed{(C+B)^{A+B} \xrightarrow{ecmb} (C+B)^{A+B}}$$