

AULAS PRÁTICAS

IC



Lecture 1

Exercício 1

$$x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{similar nas restantes}$$

$$z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \Rightarrow \text{Estado } |+\rangle$$

similar para $H|1\rangle \Rightarrow \text{Estado } |-\rangle$

$$H\left(\frac{1}{\sqrt{2}}(|1\rangle - |0\rangle)\right) = (\text{Acabar})$$

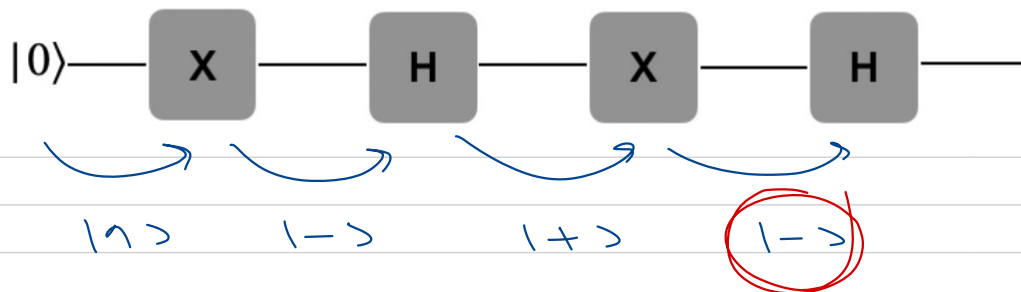
Exercício 2

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \text{Obtemos o estado I (estado identidade)}$$

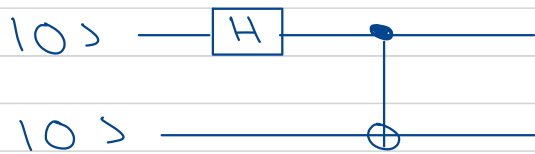
Exercício 3



CNOT



Exercício 9



$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

Gate Ry

$$\begin{pmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\frac{\theta}{2}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \sin(\frac{\theta}{2}) \end{pmatrix}$$

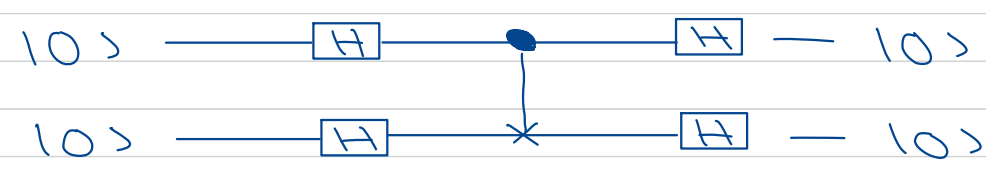
$$= \cos(\frac{\theta}{2}) |0\rangle + \sin(\frac{\theta}{2}) |1\rangle$$



sobreposição

mais exers

Exercício 10



$$H = H^+$$

Algoritmo de Grover



$$H^{\otimes \log_2 N} = H \otimes H \otimes \dots \otimes H$$

$$H^{\otimes \log_2 N} = |000\dots 0\rangle$$

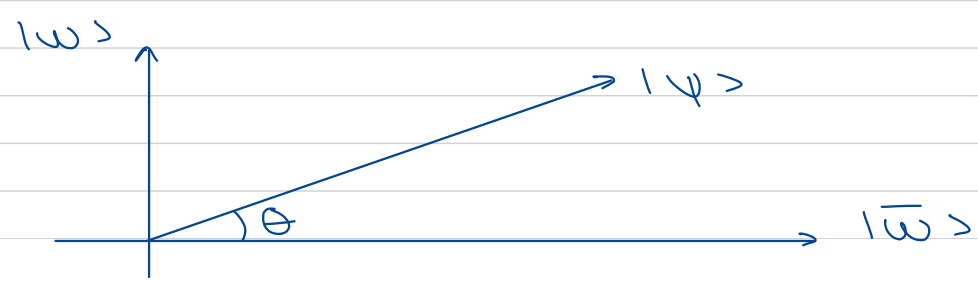
$$= |+\dots +\dots +\dots +\rangle$$

$$= \frac{1}{\sqrt{2^{\log_2 N}}} \sum_{j=0}^{\dots} |j\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2^{\log_2 N}}} [|000\dots 0\rangle + |000\dots 1\rangle + \dots + |111\dots 1\rangle]$$

$$|\psi\rangle = \cos \theta |\bar{w}\rangle + \sin \theta |w\rangle$$

w é o q queremos encontrar



2) Oracle (função)

$$\frac{1}{\sqrt{2}} [|0\rangle + |1\rangle]$$

inverter

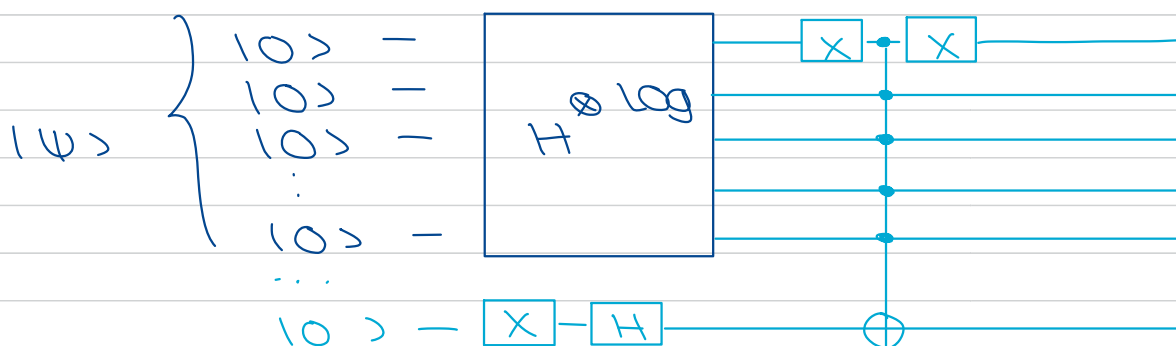
$$Z \times \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = \frac{1}{2} (|0\rangle - |1\rangle)$$

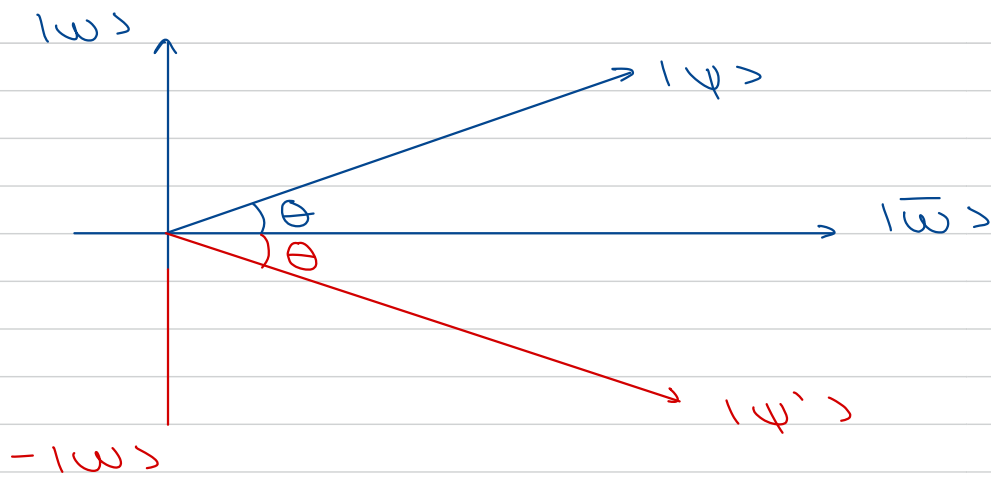
$$\frac{1}{\sqrt{2^n}} [|000\dots 0\rangle + |000\dots 1\rangle + \dots + |111\dots 1\rangle]$$

$$\begin{aligned} |\psi\rangle |-\rangle &= |\psi\rangle \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \\ &= (\cos\theta |\bar{w}\rangle + \sin\theta |w\rangle) \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \\ &= \cos\theta |\bar{w}\rangle |0\rangle - \cos\theta |\bar{w}\rangle |1\rangle + \\ &\quad + \sin\theta |w\rangle |1\rangle - \sin\theta |w\rangle |0\rangle \end{aligned}$$

Oracle é uma multicontrol CNOT

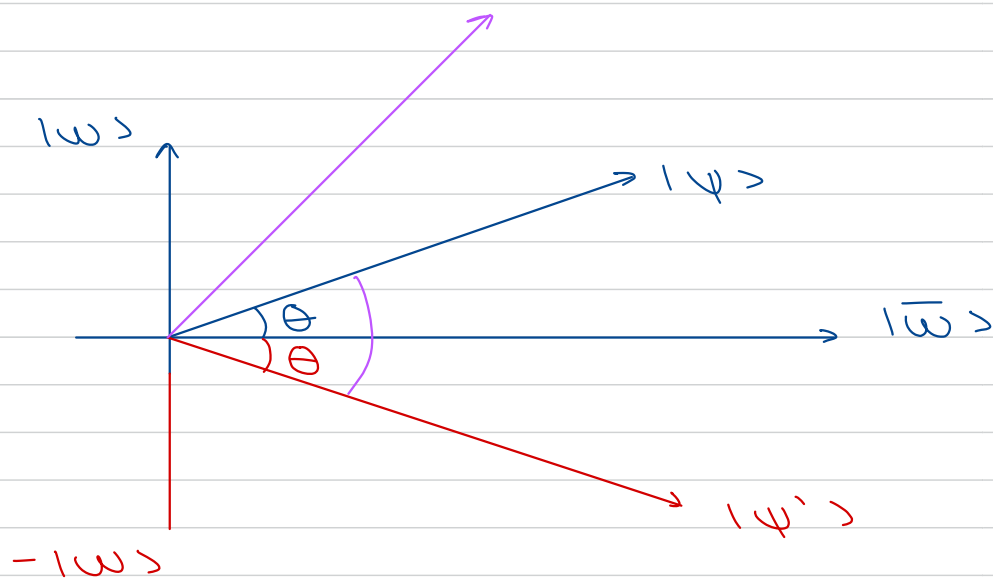
$$\begin{aligned} &= \cos\theta |\bar{w}\rangle |0\rangle - \sin\theta |w\rangle |0\rangle + \sin\theta |w\rangle |1\rangle - \cos\theta |\bar{w}\rangle |1\rangle \\ &= (\cos\theta |\bar{w}\rangle |0\rangle - \sin\theta |w\rangle |0\rangle) - (\cos\theta |\bar{w}\rangle |1\rangle - \sin\theta |w\rangle |1\rangle) \\ &= (\cos\theta |\bar{w}\rangle - \sin\theta |w\rangle) |-\rangle \end{aligned}$$





(acabou)

3) Difusor (amplificação da sonda)



$$D = 2 |\phi \times \phi\rangle \langle \phi \times \phi| - I$$

$$= 2 H^{\otimes \log_2 N} |0\rangle \langle 0| H^{\otimes \log_2 N} - I$$

$$= H^{\otimes \log_2 N} (2 |0\rangle \langle 0| - I) H^{\otimes \log_2 N}$$

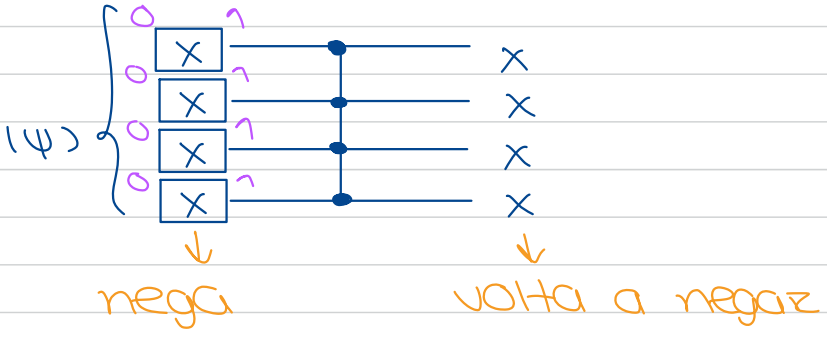
Matriz



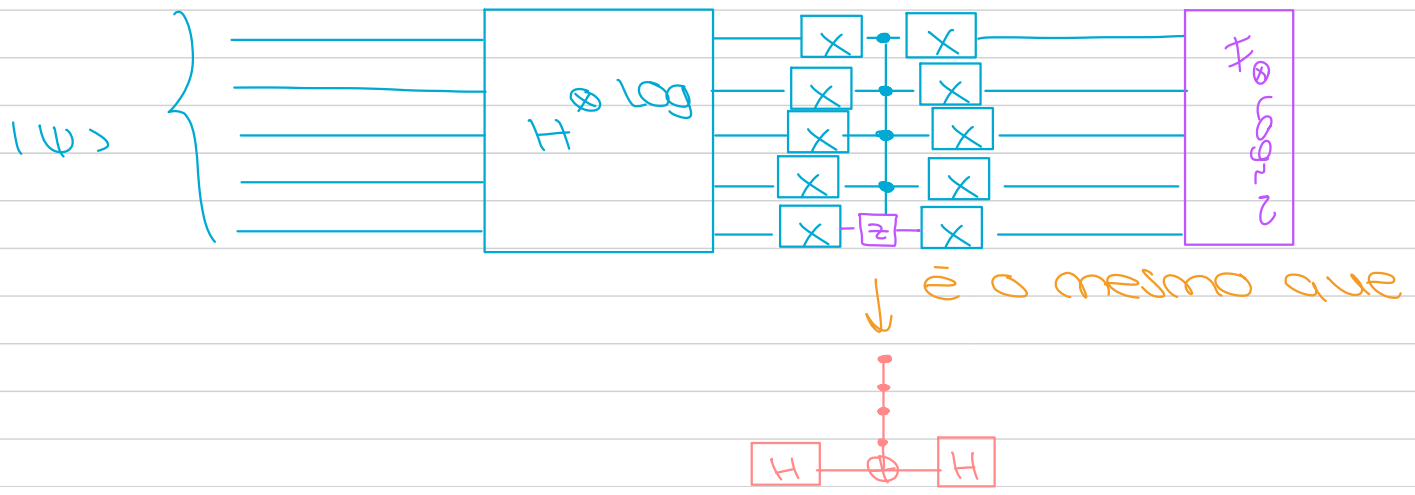
Inverte apenas $|000 \dots 0\rangle$

apenas é aplicada ao estado ψ

Cz multicontrol



construir o difusor D.



Combinar oracle com difusor