Homework/Programming Assignment #2 **Homework/midterm Due: 03/26/2020- 5:00PM**

Name/EID:
Email:
Signature (*required*)
I/We have followed the rules in completing this Assignment.

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Email:
Signature (*required*)
I/We have followed the rules in completing this Assignment.

Question	Points	Total
HA 1	25	
HA 2	25	
HA 3	25	
HA 4	25	
PA	100	
PA. k (Bonus)	15	
PA. m (Bonus)	30	
Presentation* (Bonus)	20	

Instruction:

1. Remember that this is a graded assignment. It is the equivalent of a **midterm take-home exam**.
2. *** You should present the results of the PA in the class** and receive extra bonus depending on the quality of your presentation!
3. **For PA questions**, you need to write a report showing how you derived your equations, describes your approach, test functions, and discusses the results. You should show your test results for each function.

3. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
4. It is open book, notes, and web. But you should cite any references you consult.
5. Unless I say otherwise in class, it is due before the start of class on the due date mentioned in the P/H Assignment.
6. **Sign and append** this score sheet as the first sheet of your assignment.
7. Remember to submit your assignment in Canvas.

ASBR HW 2

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Note!!!

All the four questions we wrote the code in HW.m file in Matlab, you can check it by section. Some result are very complex which contain a lot of sin and cos function in the Jacobian, so we do not write all the details here. Please check the code for these details.

Problem 1

The end-effector zero position configuration M is the same, which is $M = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_3 + L_4 \\ 0 & 0 & 1 & -L_5 - L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The screw axes S_i in $\{0\}$ are:

i	w_i	q_i	$v_i = -w_i \times q_i$
1	(1, 0, 0)	(0, 0, 0)	(0, 0, 0)
2	(0, 0, -1)	$(L_1,\ 0,\ 0)$	$(0,\ L_1,\ 0)$
3	(0, 1, 0)	$(L_1,\ L_3,\ L_2)$	$(-L_2,\ 0,\ L_1)$
4	(1, 0, 0)	$(L_1,\ L_3,\ 0)$	$(0,\ 0,\ L_3)$
5	(0, 0, 0)	$(L_1,\ L_3 + L_4,\ 0)$	(0, 1, 0)
6	(0, 1, 0)	$(L_1,\ L_3 + L_4,\ -L_5)$	$(L_5,\ 0,\ L_1)$

Therefore, $S_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, S_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ L_1 \\ 0 \end{bmatrix}, S_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -L_2 \\ 0 \\ L_1 \end{bmatrix}, S_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ L_3 \end{bmatrix}, S_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, S_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ L_5 \\ 0 \\ L_1 \end{bmatrix}$

The screw axes B_i in $\{b\}$ are:

i	w_i	q_i	$v_i = -w_i \times q_i$
1	(1, 0, 0)	$(-L_1,\ -L_3 - L_4,\ L_5 + L_6)$	$(0,\ L_5 + L_6,\ L_3 + L_4)$
2	(0, 0, -1)	$(0,\ -L_3 - L_4,\ L_5 + L_6)$	$(L_3 + L_4,\ 0,\ 0)$
3	(0, 1, 0)	$(0,\ -L_3 - L_4,\ L_2 + L_5 + L_6)$	$(-L_2 - L_5 - L_6,\ 0,\ 0)$
4	(1, 0, 0)	$(0,\ -L_4,\ L_5 + L_6)$	$(0,\ L_5 + L_6,\ L_4)$
5	(0, 0, 0)	$(0,\ 0,\ L_5 + L_6)$	(0, 1, 0)
6	(0, 1, 0)	$(0,\ 0,\ L_6)$	$(-L_6,\ 0,\ 0)$

Therefore, $B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ L_5 + L_6 \\ L_3 + L_4 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ L_3 + L_4 \\ 0 \\ 0 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -L_2 - L_5 - L_6 \\ 0 \\ 0 \end{bmatrix}$, $B_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ L_5 + L_6 \\ L_4 \end{bmatrix}$, $B_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $B_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -L_6 \\ 0 \\ 0 \end{bmatrix}$

Problem 2

The end-effector zero position configuration M is the same, which is $M = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The screw axes S_i in $\{0\}$ are:

i	w_i	q_i	$v_i = -w_i \times v_i$
1	(0, 0, 1)	(0, 0, -1)	(0, 0, 0)
2	(0, 0, 0)	(0, 0, 0)	(1, 0, 0)
3	(0, 0, 1)	(1, 0, 0)	(0, -1, 0)
4	(0, -1, 0)	(1, 0, -1)	(-1, 0, -1)
5	(-$\frac{\sqrt{2}}{2}$, 0, $\frac{\sqrt{2}}{2}$)	(2, 0, -1)	(0, $-\frac{\sqrt{2}}{2}$, 0)

Therefore, $S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $S_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $S_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $S_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix}$, $S_5 = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$

The screw axes B_i in $\{b\}$ are:

i	w_i	q_i	$v_i = -w_i \times q_i$
1	(0, 0, 1)	(-3, 0, -1)	(0, 3, 0)
2	(0, 0, 0)	(-3, 0, 0)	(1, 0, 0)
3	(0, 0, 1)	(-2, 0, 0)	(0, 2, 0)
4	(0, -1, 0)	(-2, 0, -1)	(-1, 0, 2)
5	(-$\frac{\sqrt{2}}{2}$, 0, $\frac{\sqrt{2}}{2}$)	(-1, 0, -1)	(0, $\sqrt{2}$, 0)

Therefore, $B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, $B_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ 2 \end{bmatrix}$, $B_5 = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \\ \sqrt{2} \\ 0 \end{bmatrix}$

Problem 3

(a)

The screw axes B_i are: , $B_i = [w_i, v_i]$

i	w_i	q_i	$v_i = -w_i \times q_i$
1	(0, 0, 1)	(0, -L, -L)	(-L, 0, 0)
2	(1, 0, 0)	(0, -L, 0)	(0, 0, L)
3	(0, 0, 1)	(0, 0, 0)	(0, 0, 0)
4	(0, 0, 0)	(0, 1, 0)	(0, 1, 0)

The Body Jacobian is given by:

$$J_b(\theta) = [Ad_{e^{-[B_4]\theta_4}e^{-[B_3]\theta_3}e^{-[B_2]\theta_2}} B_1, Ad_{e^{-[B_4]\theta_4}e^{-[B_3]\theta_3}} B_2, Ad_{e^{-[B_4]\theta_4}} B_3, B_4]$$

When $\theta = (0, 0, \pi/2, L)$, the $J_b = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -L & 0 & -L & 0 \\ L & 0 & 0 & 1 \\ 0 & L & 0 & 0 \end{bmatrix}$

(b)

The first joint axis is in the direction $w_{s1} = (0, 0, 1)$. Choosing $q_1 = (0, 0, L)$. We get $v_{s1} = -w_{s1} \times q_1 = (0, 0, 0)$. $S_1 = [0, 0, 1, 0, 0, 0]^T$
The second joint axis is in the direction $w_{s2} = Rot(z, \theta_1) \cdot [1, 0, 0]^T = (c_1, s_1, 0)$. Choosing $q_2 = (0, 0, L)$. We get $v_{s2} = -w_s \times q_2 = (-Ls_1, Lc_1, 0)$. $S_2 = [c_1, s_1, 0, -Ls_1, Lc_1, 0]^T$
The third joint axis is in the direction $w_{s3} = Rot(z, \theta_1) \cdot Rot(x, \theta_2) \cdot [0, 0, 1]^T = (s_1 s_2, -c_1 s_2, c_2)$. Choosing $q_3 = q_2 + Rot(z, \theta_1) \cdot Rot(x, \theta_2) \cdot [0, L, 0]^T = (-Lc_2 s_1, Lc_1 s_2, L + Ls_2)$. We get $v_{s3} = -w_{s3} \times q_3 = (Lc_1 c_2^2 + (L + Ls_2)c_1 s_2, Ls_1 c_2^2 + (L + Ls_2)s_1 s_2, 0)$. $S_3 = [w_{s3}, v_{s3}]$
The forth joint is prismatic, so $w_{s4} = (0, 0, 0)$. The direction of the prismatic joint axis is given by $v_{s4} = Rot(z, \theta_1) \cdot Rot(x, \theta_2) \cdot Rot(z, \theta_3) \cdot [0, 1, 0]^T = [-c_1 s_3 - c_2 c_3 s_1, c_1 c_2 c_3 - s_1 s_3, c_3 s_2]^T$. $S_4 = [w_{s4}, v_{s4}]$
Thus, the Space Jocabian is given by:

$$J_s(\theta) = [S_1, S_2, S_3, S_4] = \begin{bmatrix} 0 & c_1 & s_1 s_2 & 0 \\ 0 & s_1 & -c_1 s_2 & 0 \\ 1 & 0 & c_2 & 0 \\ 0 & -Ls_1 & Lc_1(s_2 + 1) & -c_1 s_3 - c_2 c_3 s_1 \\ 0 & Lc_1 & Ls_1(s_2 + 1) & c_1 c_2 c_3 - s_1 s_3 \\ 0 & 0 & 0 & c_3 s_2 \end{bmatrix}$$

Therefore, $\dot{p} = J_s(\theta)\dot{\theta} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & L & -1 \\ 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ L - 1 \\ L \\ 0 \end{bmatrix}$

Problem 4

(a)

We use two ways to calculate the Jacobian.
First Method:
The first joint axis is in the direction $w_{s1} = (0,0,1)$. Choosing $q_1 = (0,0,0)$. We get $v_{s1} = -w_{s1} \times q_1 = (0,0,0)$. $S_1 = [0,0,1,0,0,0]^T$
The second joint axis is in the direction $w_{s2} = Rot(z,\theta_1) \cdot [0,1,0]^T = (-s_1, c_1, 0)$. Choosing $q_2 = (0,0,0)$. We get $v_{s2} = -w_s \times q_2 = (0,0,0)$. $S_2 = [-s_1, c_1, 0, 0, 0, 0]^T$
The third joint axis is in the direction $w_{s3} = Rot(z,\theta_1) \cdot Rot(y,\theta_2) \cdot [-1,0,0]^T = (-c_1c_2, -c_2s_1, s_2)$. Choosing $q_3 = (0,0,0)$. We get $v_{s3} = -w_{s3} \times q_3 = (0,0,0)$. $S_3 = [w_3, v_3]$
The forth joint axis is in the direction $w_{s4} = Rot(z,\theta_1) \cdot Rot(y,\theta_2) \cdot Rot(x,-\theta_3) \cdot [-1,0,0]^T = (-c_1c_2, -c_2s_1, s_2)$. Choosing $q_4 = Rot(z,\theta_1) \cdot Rot(y,\theta_2) \cdot Rot(x,-\theta_3) \cdot [0,L,0]^T$. We get $v_{s4} = -w_{s4} \times q_4$. $S_4 = [w_4, v_4]$
The fifth joint axis is in the direction $w_{s5} = Rot(z,\theta_1) \cdot Rot(y,\theta_2) \cdot Rot(x,-\theta_3) \cdot Rot(x,-\theta_4) \cdot [-1,0,0]^T = (-c_1c_2, -c_2s_1, s_2)$. Choosing $q_5 = q_4 + Rot(z,\theta_1) \cdot Rot(y,\theta_2) \cdot Rot(x,-\theta_3) \cdot Rot(x,-\theta_4) \cdot [0,L,0]^T$. We get $v_{s5} = -w_{s5} \times q_5$. $S_5 = [w_5, v_5]$
The sixth joint axis is in the direction $w_{s6} = Rot(z,\theta_1) \cdot Rot(y,\theta_2) \cdot Rot(x,-\theta_3) \cdot Rot(x,-\theta_4) \cdot Rot(x,-\theta_5) \cdot [0,1,0]^T = (s_5(c_4(s_1s_3 - c_1c_3s_2) + s_4(c_3s_1 + c_1s_2s_3))) - c_5(c_4(c_3s_1 + c_1s_2s_3) - s_4(s_1s_3 - c_1c_3s_2)), c_5(c_4(c_1c_3 - s_1s_3s_2) - s_4(c_1s_3 + c_3s_1s_2))) - s_5(c_4(c_1s_3 + c_3s_1s_2) + s_4(c_1c_3 - s_1s_2s_3)), -s_{345}c_2)$. Choosing $q_6 = q_5 + Rot(z,\theta_1) \cdot Rot(y,\theta_2) \cdot Rot(x,-\theta_3) \cdot Rot(x,-\theta_4) \cdot Rot(x,-\theta_5) \cdot [0,L,0]^T$. We get $v_{s6} = -w_{s6} \times q_6$. $S_6 = [w_6, v_6]$
Then the Jacobian should be:

$$J_s(\theta) = [S_1, S_2, S_3, S_4, S_5, S_6]$$

Second Method:

The screw axes S_i are:

i	w_i	q_i	$v_i = -w_i \times v_i$
1	(0, 0, 1)	(0, 0, 0)	(0, 0, 0)
2	(0, 1, 0)	(0, 0, 0)	(0, 0, 0)
3	(-1, 0, 0)	(0, 0, 0)	(0, 0, 0)
4	(-1, 0, 0)	(0, L, 0)	(0, 0, L)
5	(-1, 0, 0)	(0, 2L, 0)	(0, 0, 2L)
6	(0, 1, 0)	(0, 3L, 0)	(0, 0, 0)

, $s_i = [w_i, v_i]$

Therefore, the Jacobian:

$$J_s(\theta) = [s_1, Ad_{e^{[s_1]\theta_1}} s_2, Ad_{e^{[s_1]\theta_1} e^{[s_2]\theta_2}} s_3, ..., Ad_{e^{[s_1]\theta_1} e^{[s_2]\theta_2} \cdots e^{[s_5]\theta_5}} s_6]$$

(b)

Let $det(J(\theta)) = 0$, we can get $det(J(\theta)) = L^3 c_2 s_4 (s_5 + s_{45}) = 0$. Here, $s_{45} = sin(\theta_4 + \theta_5)$.
Therefore, the singularity configurations are:
1) $L = 0$, loss link length
2) $cos(\theta_2) = 0, \theta_2 = \pi/2$, the joint 3 and joint 1's axes are collinear, thus, the joint 1 loss the ability to move joint 3's position.
3) $sin(\theta_4) = 0, \theta_4 = 0$, the joint 5 and joint 3's revolute joint axes are parallel.

4) $sin(\theta_4 + \theta_5) + sin(\theta_5) = 0$, joint 6's screw axis will always be the $y - axis$ of joint 3, thus, joint 4 loss the ability to move joint 6's position.