ORI 397 **Optimization Under Uncertainty** **Spring 2020**

Assignment 1

Due: Thursday, January 30

Problem 1. Derive the dual *linear program* to the maximization problem

$$\begin{aligned} z^\star = \sup_{\boldsymbol{x}} \quad & \boldsymbol{c}^\top \boldsymbol{x} \\ \text{s.t.} \quad & \|\boldsymbol{x}\|_1 \leq f \\ & \|\boldsymbol{x}\|_\infty \leq g. \end{aligned}$$

Here, for any vector $\boldsymbol{x} \in \mathbb{R}^n$ its 1-norm is defined as $\|\boldsymbol{x}\|_1 = |x_1| + \ldots + |x_n|$, while its ∞-norm is defined as $\max_{i=1,\ldots,n} |x_i|$.

Problem 2. A standard normal random variable $\tilde{\xi}$ has a probability density function given by

$$p(\xi) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\xi^2} \qquad \forall \xi \in \mathbb{R}.$$

Show that $\tilde{\xi}$ has mean 0 and variance 1.

Problem 3. At the beginning of the day, a newsvendor purchases newspapers from the publisher at a unit cost c. Throughout the remainder of the day, the newsvendor sells the newspapers to the customers at a unit cost d. The demand of the newspapers is a non-negative continuous random variable $\tilde{\xi}$ with a fixed probability distribution $\mathbb{P}$. The newsvendor seeks to maximize the expected profit and therefore solves the optimization problem

$$\begin{aligned} \text{maximize} \quad & -cx + \mathbb{E}_{\mathbb{P}}\left[d \min\{x, \tilde{\xi}\} \right] \\ \text{subject to} \quad & x \in \mathbb{R}_+. \end{aligned}$$

Let x^* be the optimal solution of this problem. Show that $\mathbb{P}(\tilde{\xi} \leq x^*) = \frac{d-c}{d}$.