COS10003 Assignment 2

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Sets:

Question 1

a/ Let U be a universal set, which is the total number of students, and let A, B ,and C denote the set of students joining a student club, eating at a cafe ,and going to a gym, respectively. We can, therefore, further denote the problem as follows:

- $A \cap B \cap C$, which is a set of students participated in all the above activities.
- $(A \cup B \cup C)^C$, which is a set of students that did not participate in any of the above activities.
- $A \cup B \cup C$, which is a set of students that participate in at least one activity.

According to the **Fundamental Laws** and **Cardinality Principle** for a finite set of number, we can denote the total number of the student as follows:

$U = (A \cup B \cup C) \cup \overline{(A \cup B \cup C)}$ **(Complement Laws)**

$\Leftrightarrow |U| = |(A \cup B \cup C)| + |\overline{(A \cup B \cup C)}|$ **(Cardinality Principle)**

$\Leftrightarrow$ 465 = $|(A \cup B \cup C)|$ + 101

$\Rightarrow$ $|(A \cup B \cup C)|$ = 465 – 101 = 364 (students)

Therefore, the number of students that participate in at least one activity is 364 students.

According to the **Inclusion – Exclusion Principle** we have the formula:

$|(A \cup B \cup C)| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$\Leftrightarrow$ 364 = 220 + 159 + 208 – 68 - $|B \cap C|$ – 126 + 32

$\Rightarrow$ $|B \cap C|$ = -364 + 220 + 159 + 208 – 68 - 126 + 32 = 61 (students)

Therefore, $|B \cap C|$, which is the number of students that ate at a café and went to a gym is the missing component, and the value of which is 61 students.

b/

In order to draw the Venn Diagram, the following information is required:

- The number of students who only joined the club:
  $|A \cap B^C \cap C^C|$ = $|A|$ – $|A \cap B \cap C|$ - $|A \cap B^C \cap C|$ - $|A \cap B \cap C^C|$ = 220 – 32 – 94 – 36 = 58 (students)
- The number of students who only ate at a café:
  $|A^C \cap B \cap C^C|$ = $|B|$ – $|A \cap B \cap C|$ - $|A^C \cap B \cap C|$ - $|A \cap B \cap C^C|$ = 159 – 32 – 29 – 36 = 62 (students)
- The number of students who only went to the gym:
  $|A^C \cap B^C \cap C|$ = $|C|$ – $|A \cap B \cap C|$ - $|A^C \cap B \cap C|$ - $|A \cap B^C \cap C|$ = 208 – 32 -29 – 94 = 53 (students)
- The number of students who joined the club and ate at a café, but did not go to the gym:
  $|A \cap B \cap C^C| = |A \cap B| - |A \cap B \cap C|$ = 68 − 32 = 36 (students)
- The number of students who joined the club and went to the gym, but did not eat at a café:
  $|A \cap B^C \cap C| = |A \cap C| - |A \cap B \cap C|$ = 126 − 32 = 94 (students)
- The number of students who ate at a café and went to the gym, but did not join the club:
  $|A^C \cap B \cap C| = |B \cap C| - |A \cap B \cap C|$ = 61 − 32 = 29 (students)

Venn diagram provides a pictorial view of sets, the Venn diagram for our problems are as follows:

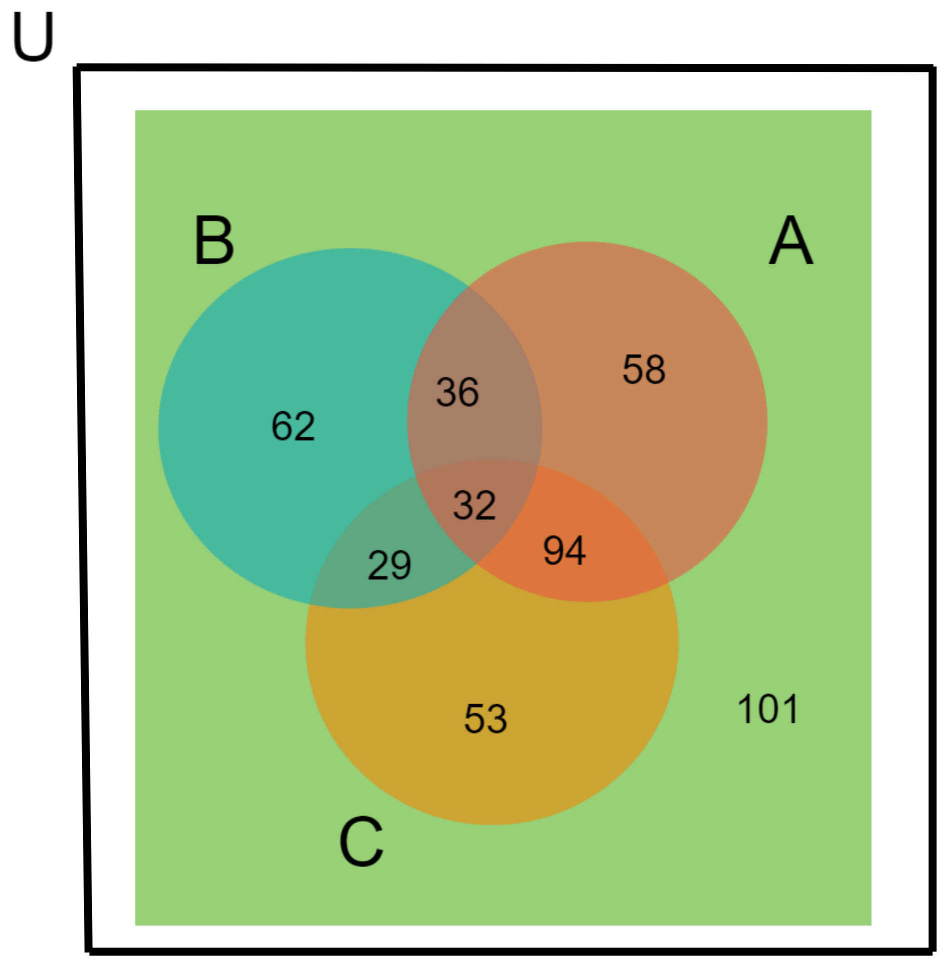


Figure 1. Venn Diagram represent the activities

c/

i. The number of students that did not go to a gym would be the equivalent of $|\bar{C}|$, therefore, according to the Fundamental Laws :

$U = C \cup \bar{C}$ **(Complement Laws)**

⇔ $|U| = |C| + |\bar{C}|$ **(Cardinality Principle)**

⇔ $465 = 208 + |\bar{C}|$

⇨ $|\bar{C}| = 465 – 208 = 257$ (students)

Therefore, there are 257 students who did not go to the gym.

ii. The number of students that joined a student club but did not eat at a café would be the equivalent of $|A \cap \bar{B}|$, according to set difference, this can also be written as $|A \setminus B|$.

$|A \setminus B|$ would mean the number of values inside *A*, but not inside *B*, so we have the following formula:
$|A \setminus B| = |A| - |A \cap B| = 220 – 68 = 152$ (students)

Therefore, there are 152 students who joined a student club but did not eat at a café.

iii. The number of students who only joined a student club, or ate at a café, or went to a gym can be written as follows:
$|A \cap B^C \cap C^C| + |A^C \cap B \cap C^C| + |A^C \cap B^C \cap C| = 58 + 62 + 53 = 173 \text{ (students)}$

Therefore, there are 173 students who only joined a student club, or ate at a café, or went to a gym

d/

i. As the set of students who went to the gym, ate at a café or both is B ∪ C
Therefore, the set of students who did not go to the gym and did not eat at a café would be $(\boldsymbol{B} \cup \boldsymbol{C})^{\boldsymbol{C}}$.

ii. As the set of students who joined a student club, ate at a café ,and went to the gym is $A \cup B \cup C$.
Therefore, the set of students who joined a student club, ate at a café ,but did not go to the gym would be $(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C}^{\boldsymbol{C}}$

Logic

Question 2

a/

i. ***h* ∧ *(d* ∨*a):*** Huyen plays cricket and David plays esports or Adita plays esports.

ii. ***d* → ¬*a* ∨*h***: If David plays esports then Adita doesn't play esports and Huyen plays cricket.

iii. ***¬(h* ∨*d*)*:*** Neither does Huyen play cricket nor David play esport.

b/

i. The conditional proposition "If David plays esports, then Adita plays esports" would be symbolized as: ***d* → *a***

ii. The proposition "neither Adita nor David play esports" would be symbolized as: ***¬(a* ∨ *d)***

iii. The biconditional proposition "Adita plays esports if and only if Huyen plays cricket and David plays esports" would be symbolized as: ***a* ↔ *(h* ∧ *d)***

Question 3

Let *h* represents the proposition "height is larger than 100" and *w* represents the proposition "width is larger than 10".

Therefore, the proposition "height is lesser or equal to 100" would be represented as ¬ *h.* Hence, our problem can be represented symbolically as:

$$(\neg h \vee w) \wedge (h \vee w) \wedge \neg h$$

$$\equiv (\neg h \vee w) \wedge \neg h \wedge (h \vee w) \qquad (\textbf{\textit{Commutative Law}})$$

$$\equiv \neg h \wedge (h \vee w) \qquad (\textbf{\textit{Absorption Law}})$$

$$\equiv (\neg h \wedge h) \vee (\neg h \wedge w) \qquad (\textbf{\textit{Distributive Law}})$$

$$\equiv F \vee (\neg h \wedge w) \qquad (\textbf{\textit{Negation Law}})$$

$$\equiv \neg h \wedge w \qquad (\textbf{\textit{Identity Law}})$$

$$\equiv w \wedge \neg h \qquad (\textbf{\textit{Commutative Law}})$$

After multiple steps of simplification, we got the expression $w \wedge \neg h$ which can be translated to "If width > 10 and height <= 100". Therefore, we have successfully simplified the initial statement.

Relations and functions

Question 4:

Suppose we have a set called A = {x,y,z,k,m}, A can be modeled as a directed graph.

We can also use directed graph to represent properties of a relation of a set, for example we have as follows:

- Reflexivity of a Relation R: Every node in the graph has a loop.

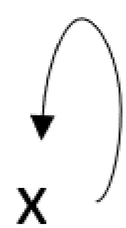


Figure 2. Reflexivity of a Relation

That is, if R is reflexive, then: ∀x ∈ A. xRx, or accordingly (x, x) ∈ R.

- Symmetry of a Relation R: If there is an edge from x to y in the graph, then there is also an edge from y to x in the graph as well.

Figure 3. Symmetry of a Relation

That is, if R is symmetric, then: ∀x, y ∈ A. xRy IMPLIES yRx, or accordingly (x, y) ∈ R => (y, x) ∈ R.

- Transitivity of a Relation R: For any walk $v_0, v_1, \ldots, v_k$ in the graph where $k \geq 2$, $v_0 \rightarrow v_k$ is in the graph (and, hence, $v_i \rightarrow v_j$ is also in the graph for all $i < j$.

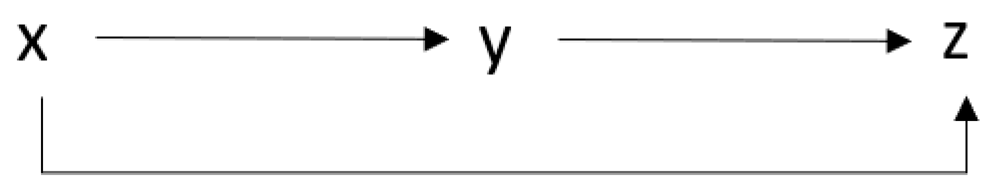


Figure 4. Transitivity of a Relation

That is, if R is transitive, then: ∀x, y, z ∈ A. xRy, yRz IMPLIES xRz, or accordingly (x, y) ∈ R, (y, z) ∈ R => (x, z) ∈ R.

In the context of this problem the graph below will demonstrate the properties of the relation S:

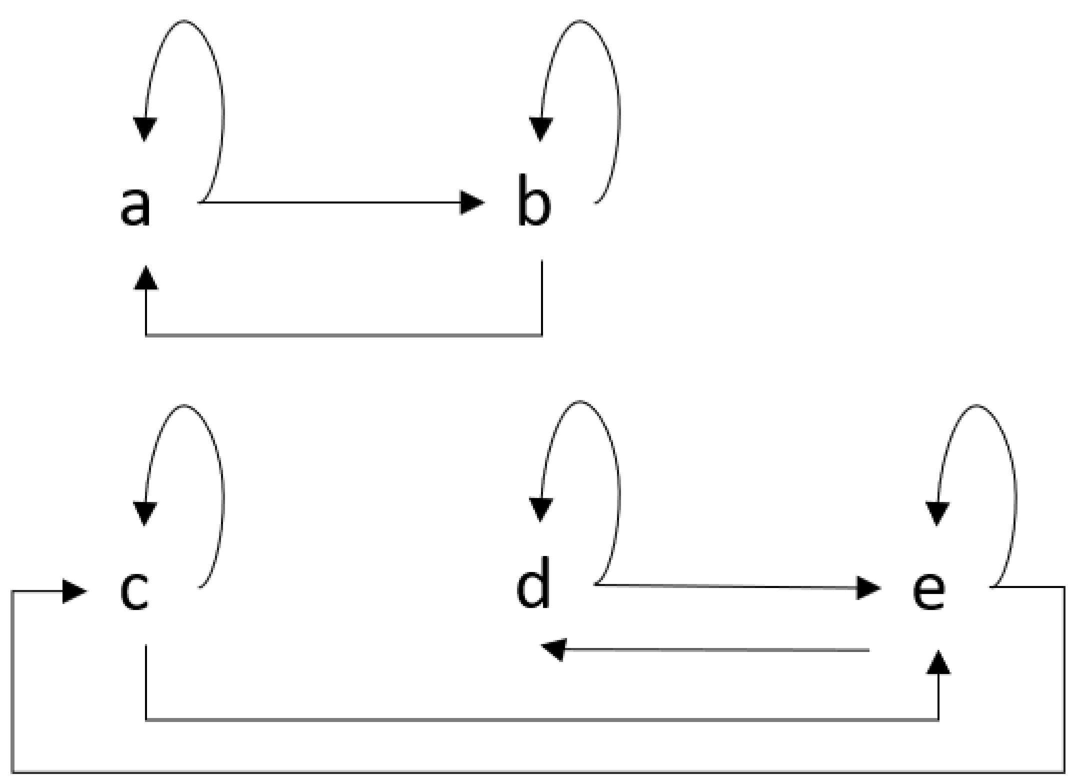


Figure 5. Directed graph of Question 4

From the graph we can see that the relation S is **Reflexive** as there is a loop in every node or specifically:

$$(a, a), (b, b), (c, c), (d, d), (e, e) \ \in \ S.$$

The graph can also show that the relation is **Symmetric** as if there is an edge from one entity to another, for example (a, b). There is also an edge from the other entity to the initial one , for example (b, a). Specifically, as follows:

$$(a, b) \ \in \ S \; and \; (b, a) \ \in \ S$$

$$(c, e) \ \in \ S \; and \; (e, c) \ \in \ S$$

$$(d, e) \ \in \ S \; and \; (e, d) \ \in \ S$$

However, the graph show that the relation S is **not Transitive**. Because, although $v_d \rightarrow v_e$ and $v_c \rightarrow v_e$, $v_c \nRightarrow v_d$ Specifically:

$$(c, e) \ \in S$$

$$(e, d) \ \in \ S$$

$$But: (c, d) \ \notin \ S$$

A relation is said to be an equivalence relation if it is reflexive, symmetric, and transitive .Therefore, the relation S is not an equivalence relation.

Question 5

a/ Supposedly, we have two set A and B:

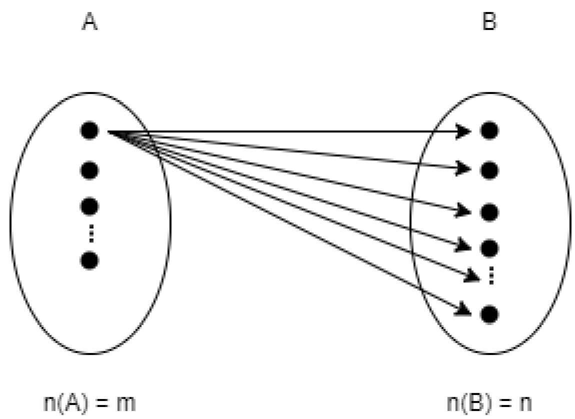


Figure 6. Number of functions

The total number of functions that we can generate from the two sets will be:

$$n\ x\ n\ x\ n\ \ldots\ldots\ldots.(m - time)\ or, equivalently,\ \ n^m$$

In the context of our problem, we have the domain X = {x, y} ,and the co-domain Y = {x, y, z}, then m will be 2 and n will be 3. Therefore, the number of possible functions will be 3^2 or 9 functions.

b/

Let A = $\{a_1, a_2, a_3 \ldots. a_m\}$ and B = $\{b_1, b_2, b_3 \ldots b_n\}$ where m $\leq$ n.

If $f: A \rightarrow B$ *is an injective mapping* then:

- For $a_1 \in A, a_1$ *can form n possible injective functions with* B.
- For $a_2 \in A, a_2$ *can form* $n - 1$ *possible injective functions with* B.
- Similarly, for $a_m \in A$, *there are* $(n - m - 1)$ *choices for* $f(a_m) \in B$.

Therefore, there are *n(n - 1)(n -2)……(n – m – 1) or* $\frac{n!}{(n-m)!}$ Injective functions from A to B.

In the context of our problem, m = 2 and n = 3, so m is lesser than n. Therefore, there are $\frac{3!}{(3-2)!}$ = 6 injective functions.

The examples of injective functions are as follows:

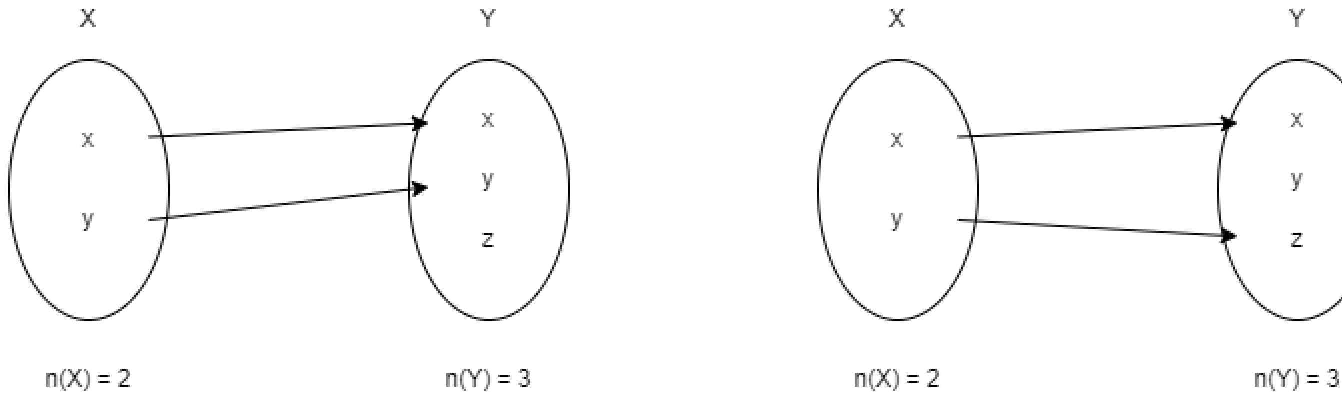


Figure 7. Injection

The examples of functions that are not injective are as follows:

Figure 8. Not an injection

c/

We have the domain X = {x, y} ,and the co-domain Y = {x, y, z}.
A bijection is a function which is both an injection and surjection. Therefore, every element of the co-domain is the image of exactly one element from the domain. In other words, both sets must have the same number of elements or |X| = |Y|. Specifically, according to the Mapping Rules for two finite sets:

$$If\ there\ is\ a\ surjection\ from\ X\ to\ Y, then\ |X|\ \geq\ |Y|.$$

$$If\ there\ is\ an\ injection\ from\ X\ to\ Y, then\ |X|\ \leq\ |Y|.$$

$$If\ there\ is\ a\ bijection\ between\ X\ and\ B, then\ |X|\ =\ |Y|.$$

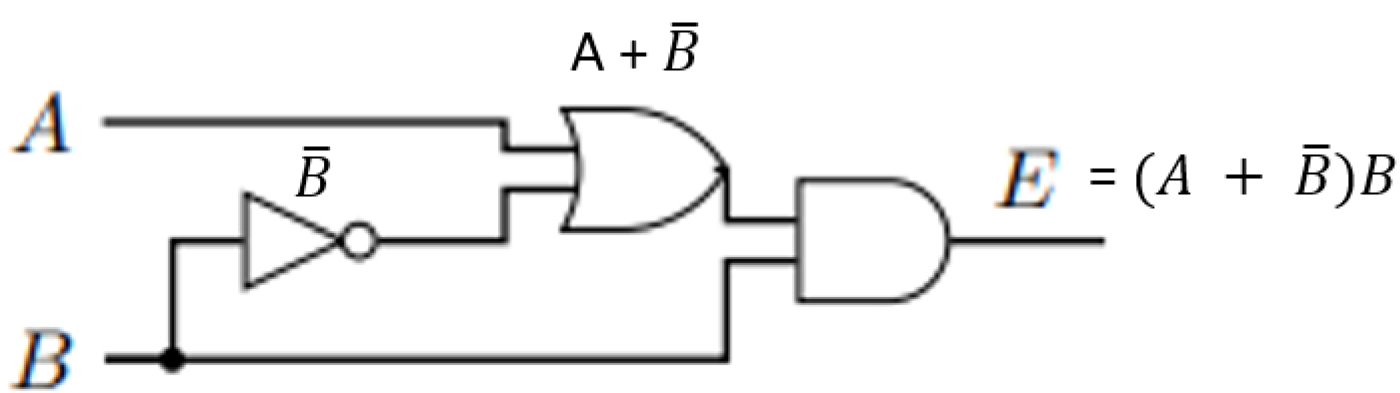
In the context of our problem, however, there is no surjection from X to Y as |X| ≤ |Y|. Therefore, there would not be any bijective function as well.

CIRCUITS

Question 6:

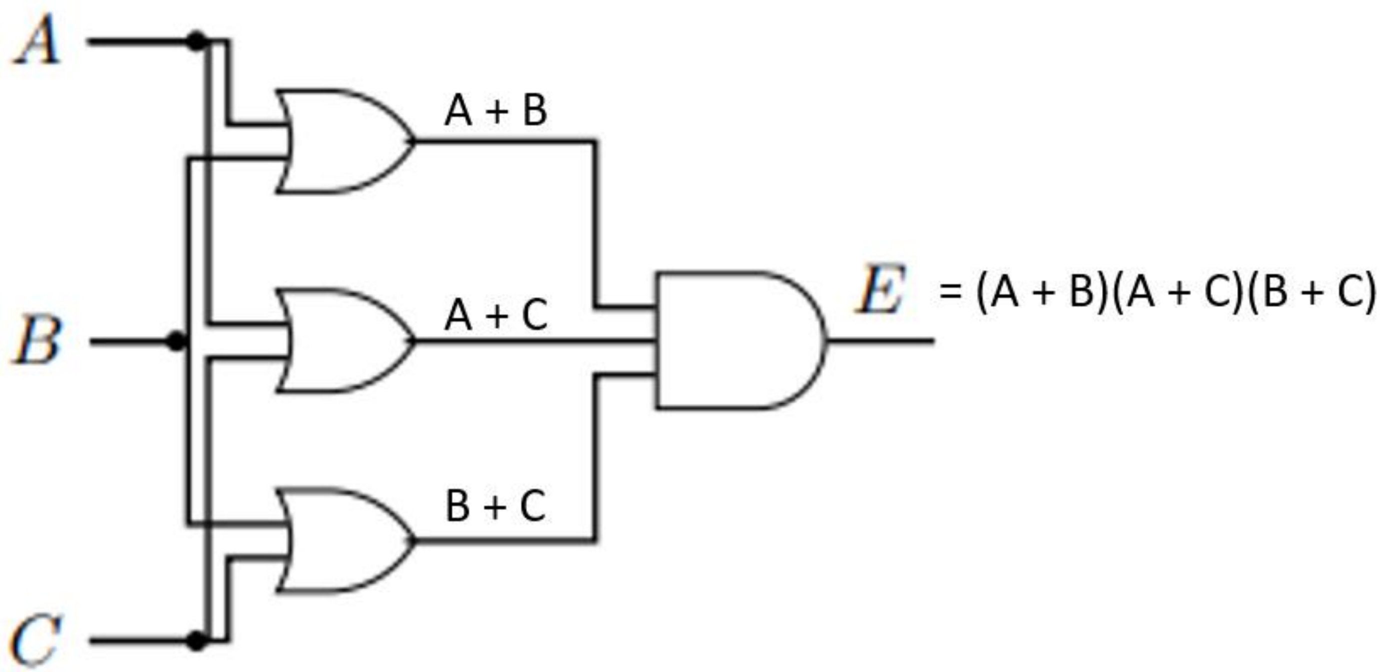
i.

a/



Result: E = $(A + \bar{B}).B$

b/



Result: E = (A + B)(A + C)(B + C)

ii.

a/

A	B	$\bar{B}$	A + $\bar{B}$	$(A + \bar{B}).B$
1	1	0	1	0

b/

A	B	C	A + B	A + C	B + C	(A + B)(A + C)(B + C)
1	1	0	1	1	1	1

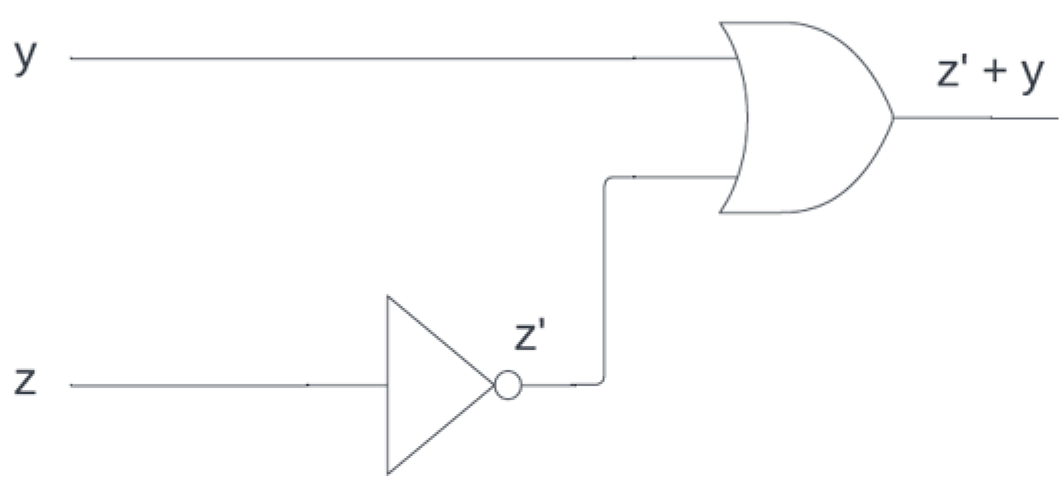
Question 7:

a/ We are given the following expression:

$$E = (z' + y)yz$$

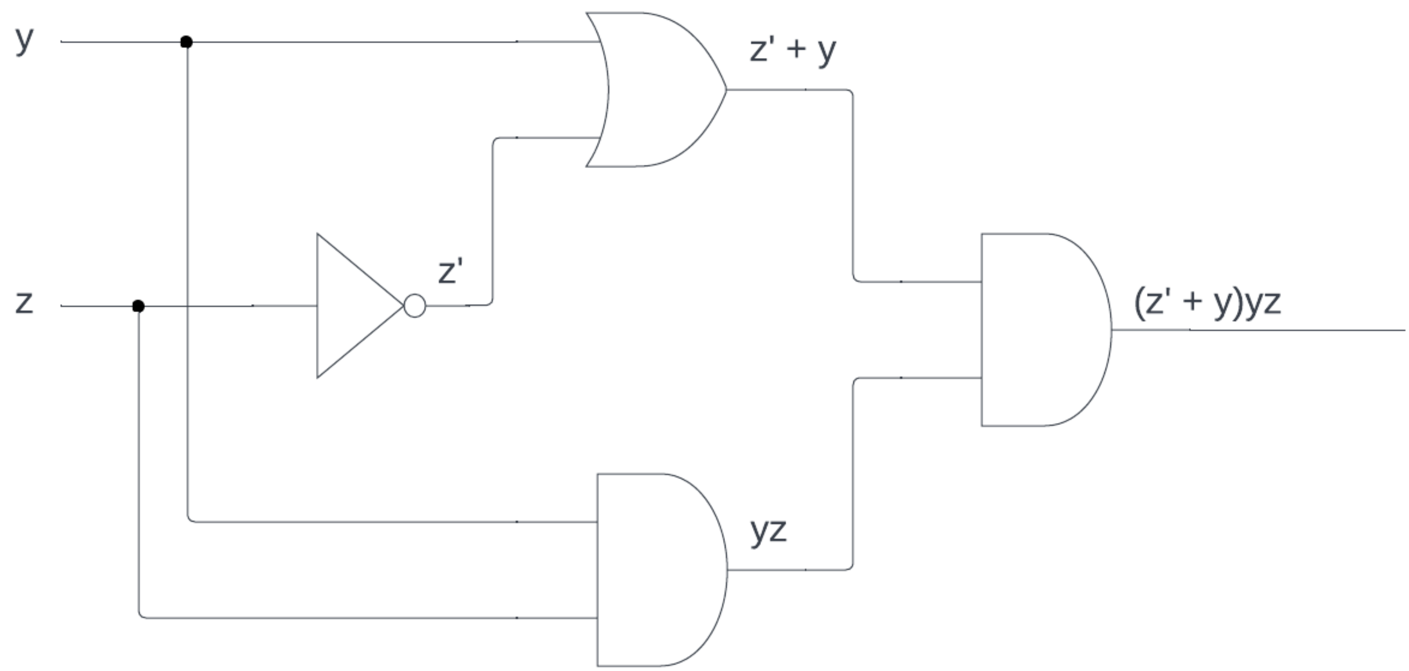
Let’s break down the expression into smaller chunks so as to draw the circuit more effectively.

Firstly, we have (z’ + y) , which includes two components z’ and y, for z to be converted to z’, it has to go through a NOT gate first. After that, z’ and y are fed into an OR gate. According to the process, we will have the following circuit:



Secondly, we have yz, which also includes two components y and z, as an AND operation is similar to the multiplication in ordinary algebra, a logical operation performed by AND gate will result in a product of the two inputs. Therefore, we have yz and according to the provided explanation, we will also have the following circuit:

Eventually, we have our final circuit, which is a product of the above circuits, (z' + y)yz. Because the final circuit is a product of the two circuits, it will be fed into another AND gate. Consequently, the final circuit are as follows:



b/

The statement can be simplified using the laws of Boolean Algebra

$E = (z' + y)yz$	
$\Leftrightarrow E = z'zy + yyz$	**Distributive Law**
$\Leftrightarrow E = 0y + yyz$	**Complement Law**
$\Leftrightarrow E = 0 + yyz$	**Boundedness Law**
$\Leftrightarrow E = yyz$	**Identity Law**
$\Leftrightarrow E = yz$	**Idempotent Law**

Therefore, after steps of simplification, we have the final statement as: E = yz

c/

The original circuit has an input that goes through 3 gates ,which is z, it is also the longest path from input to output. Therefore, the initial depth is 3. However, the simplified circuit's depth is only 1. **As a result, the depth has changed from 3 to 1.**

The original circuit has 4 gates in total, which included 1 NOT gate, 1 OR gate ,and 2 AND gates, therefore the initial size is 4. However, the simplified circuit's size is only 1. **As a result, the size has changed from 4 to 1.**