Lemma: This is a 2-approx. alg

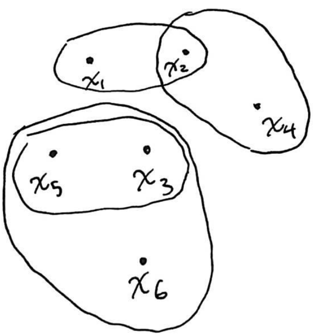
pf: Let A be the set of edges chosen in the while loop. Observe edges in A do not share any endpoints.

$|C_{opt}| \geq |A|$ $\quad |C_{ALG}| = 2|A|$, so $\rho \leq 2$.

Set Cover

input $\mathcal{C} = \{S_1, S_2, \dots, S_m\}$ collection of sets

covering elements $\{x_1, x_2, \dots, x_n\}$



Problem: find $\mathcal{C}' \subseteq \mathcal{C}$ such that $\forall x_i \ \exists S_j \in \mathcal{C}'$ where $x_i \in S_j$.

Minimize $|\mathcal{C}'|$.

Approximation alg for set-cover.

Greedy strategy: choose the next set to add to C' by picking the S_j that covers the most remaining uncovered elements.

Lemma: Greedy finds a $\ln(n)$ approximation for Set-Cover.

Pf: Let n_t be the number of uncovered elements after t iterations of greedy.

Note, $n_0 = n$. Let k be the OPT cover size.

So, at least one of these sets covers n_t/k of the remaining uncovered elements.

So, $n_{t+1} \leq n_t - \frac{n_t}{k} = \left(1 - \frac{1}{k}\right) \cdot n_t$

Repeating this, $n_t \leq \left(1 - \frac{1}{k}\right)^t \cdot n_0$

Fact $1 - x \leq e^{-x}$ for all $x \in \mathbb{R}$ (strict, unless $x=0$)

So, $1 - \frac{1}{k} \leq e^{-\frac{1}{k}}$

$n_t < n_0 \left(e^{-1/k}\right)^t = n \cdot e^{-t/k}$

When $t = k \cdot \ln(n)$, $n_t < n \cdot e^{\frac{-k \ln(n)}{k}} = 1$

Thus, $n_t = 0$ (since integer valued).

So, $|\mathcal{C}'_{greedy}| \leq k \cdot \ln(n) = \ln(n) \cdot OPT$. □

Remark: $\rho = H(\max\{|S| : S \in \mathcal{C}\})$

$$H(d) = \sum_{i=1}^{d} 1/i = \text{"}d^{th}\text{ harmonic number."}$$

Load Balancing

$M_1, M_2, \ldots, M_m$ (machines)

$j_1, j_2, \ldots, j_n$ (jobs)

t_j : time for job j.