Consensus-Free Spreadsheet Integration

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Abstract

We describe a method for merging multiple spreadsheets into one sheet, and/or exchanging data among the sheets, by expressing each sheet's formulae as an algebraic (equational) theory and each sheet's values as a model of its theory, expressing the overlap between the sheets as theory and model morphisms, and then performing "colimit", "lifting", and "Kan-extension" constructions from category theory to compute a canonically "universal" integrated theory and model, which can then be expressed as a spreadsheet. Our motivation is to find methods of merging engineering models that do not require consensus (agreement) among the authors of the models being merged, a condition fulfilled by our method because theory and model morphisms can be mechanically checked to be semantics-preserving. We describe a case study of this methodology on a real-world oil and gas calculation at a major energy company, describing the theories and models that arise when integrating two different casing pressure test (MASP) calculation spreadsheets constructed by two non-interacting engineers. We also describe the automated theorem proving burden associated with both verifying the semantics preservation of the overlap mappings as well as verifying the "conservativity"/"consistency" of the resulting integrated sheet. We conclude with thoughts on how to apply the methodology to scale engineering efforts across the enterprise.

1 Introduction

As the energy industry moves to model-based software for performing day-to-day engineering tasks, it becomes more and more important to ensure the semantic consistency of models composed of related models (integrated models). For example, we don't want errors to propagate from one model to another or to try to integrate models with conflicting requirements (e.g. requiring positive and negative voltages on the same wire at the same time). Commonly, to ensure semantic consistency of integrated models the human subject matter experts that created them communicate informally with each other about their respective understandings of the integrated model and check for consistency of the integrated model with respect to their original models (think NASA mission control with its various 'functions'). Although using groups of human experts to certify/construct integrated models works for small numbers of input models, this methodology is both costly and not scalable, because in principle, all the humans may need to communicate with each other (not everything can pass through mission control), and moreover, people may disagree about the meaning of the input and/or integrated models. Therefore, to construct/certify engineering models in a scalable way, we must make sure that when models are merged, the original authors of the models need not be involved.

1.1 Contributions

In this paper, we propose a methodology that does precisely the above for many spreadsheet-based engineering models, based on treating each sheet as both an algebraic (equational) theory [2] and a model of that theory (a so-called "olog" [13]) and using techniques from automated theorem proving [2] and category theory [1] (summarized in [11]) to construct composed ologs (theories and models), which can then be exported as spreadsheets. Relationships between input ologs are captured as theory and model "morphisms" [8] (olog morphisms), for which we generate and solve [11] verification conditions to ensure they are semantics-preserving without recourse to the original spreadsheet authors. We demonstrate the practicality of our methodology on a maximum anticipated surface pressure (MASP) calculation done in two different spreadsheets by two non-interacting engineers, describing the associated ologs as well as the verification conditions generated by both relating the models and integrating them. Specifically, our research contributions (new results) are:

- a method to represent some spreadsheets as algebraic theories [2] and models ('ologs' [13]), and vice versa; and,
- a case study about two independently-developed ologs corresponding to a standard spreadsheet-based MASP calculation, as well as another olog and two olog morphisms [8] capturing the relationship of the first two ologs to each other; and,
- a use case about how to apply the technology demonstrated in the case study to reducing costs in auditing spreadsheet-based models for requirements compliance.

1.2 Outline

In the remainder of the introduction, we provide background on MASP calculations, their applications, and why integrating/exchanging data between them can be useful. The rest of this paper is structured as follows:

1. In section 2 we describe how spreadsheets can be represented as ologs and vice-versa.
2. In section 3 we describe the two source sheets/theories/models of MASP calculations to be merged, the overlap between them, and integrated result and its applications. We also discuss the practicalities of the case study.
3. In section 4 we describe the verification conditions generated by relating the sources to each other and during construction of the integrated result. This section will be of interest primarily to computer scientists.
4. In section 5 we conclude by discussing applications of our methodology across the enterprise.

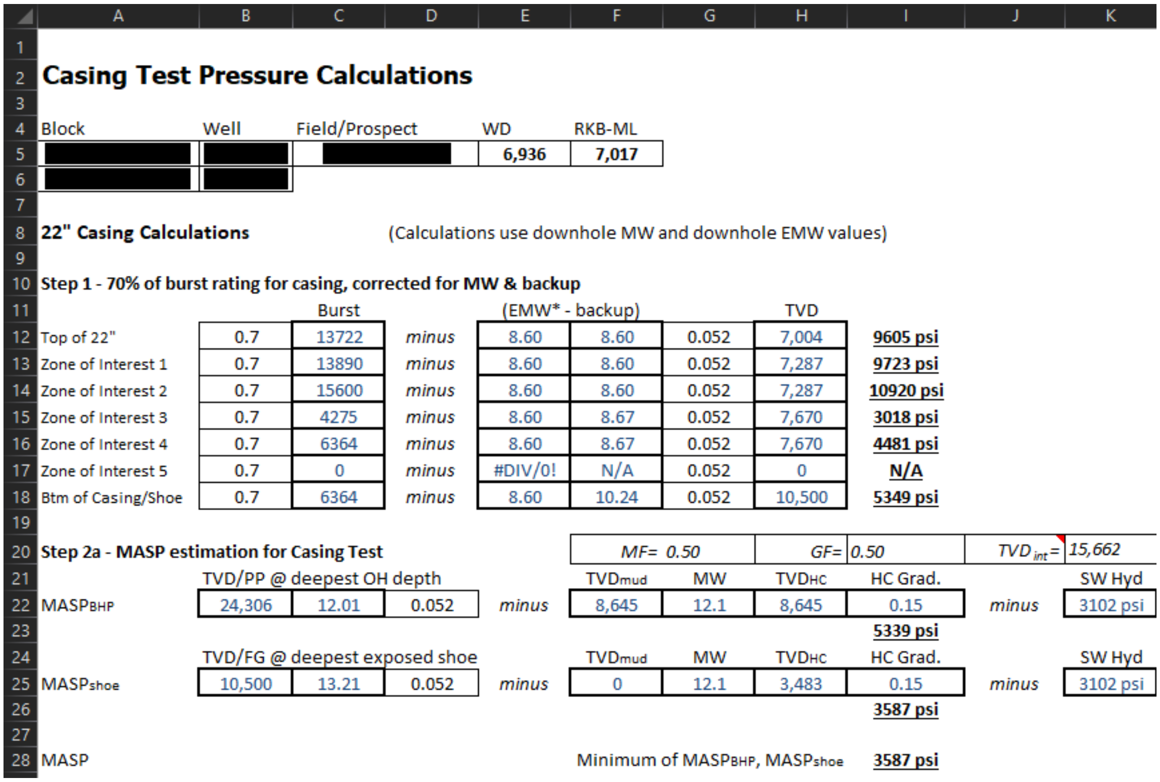
It is our hope that our methodology can be applied by engineers without any training in category theory, logic, or algebra, and that this paper should be readable without such knowledge as well. However, to fully understand this paper, knowledge of all three areas is required, and to that end, we have included appendices that review them (Appendices 6, 7, and 8, respectively).

1.3 MASP Background

Maximum anticipated surface pressure (MASP) is the highest (worst case) surface pressure that is expected to be encountered throughout the well construction process [7]. MASP calculation is a key engineering activity for both preliminary and detailed well design because it impacts well control equipment specifications and pressure test plans. In particular, MASP determines the sizing and ratings for pressure control/containment equipment such as blowout preventers and strings of casing [6]. The results of the MASP calculation are also used to derive numerical values to test said pressure control/containment devices in the field. These are key aspects of maintaining process safety and integrity of oil and gas wells during drilling, completion, or intervention operations.

MASP is normally calculated for each casing section as part of the well design process. This typically assumes a shut-in well and various types of fluid in the wellbore (e.g., mud or gas). Guidance for determining the appropriate calculation scenarios (e.g., the correct ratio of fluid in the well) are often provided by global standards and technical requirements. Other inputs include pore pressure and fracture gradient data (obtained from offset wells or regional analogues), fluid density values, well depths, and casing specifications.

The determination of MASP typically includes pore pressure uncertainties, new geologic zones of interest being added/encountered, regulatory requirements, and other unanticipated factors. For these reasons there are verification activities to assure/audit that engineering calculations are sound and compliant with technical requirements. Moreover, there are often several engineers working on singular aspects of the well plan and then collaborating to ensure an optimal design. In either case, auditors or engineers may want to integrate their respective viewpoints to generate new insights and ensure explicit alignment between each other, suggesting use cases described in the conclusion.



	A	B	C	D	E	F	G	H	I	J	K
1											
2	**Casing Test Pressure Calculations**										
3											
4	Block	Well	Field/Prospect		WD	RKB-ML					
5	[redacted]	[redacted]	[redacted]		6,936	7,017					
6	[redacted]	[redacted]									
7											
8	**22" Casing Calculations**			(Calculations use downhole MW and downhole EMW values)							
9											
10	**Step 1 - 70% of burst rating for casing, corrected for MW & backup**										
11			Burst		(EMW* - backup)			TVD			
12	Top of 22"	0.7	13722	*minus*	8.60	8.60	0.052	7,004	9605 psi		
13	Zone of Interest 1	0.7	13890	*minus*	8.60	8.60	0.052	7,287	9723 psi		
14	Zone of Interest 2	0.7	15600	*minus*	8.60	8.60	0.052	7,287	10920 psi		
15	Zone of Interest 3	0.7	4275	*minus*	8.60	8.67	0.052	7,670	3018 psi		
16	Zone of Interest 4	0.7	6364	*minus*	8.60	8.67	0.052	7,670	4481 psi		
17	Zone of Interest 5	0.7	0	*minus*	#DIV/0!	N/A	0.052	0	N/A		
18	Btm of Casing/Shoe	0.7	6364	*minus*	8.60	10.24	0.052	10,500	5349 psi		
19											
20	**Step 2a - MASP estimation for Casing Test**					*MF= 0.50*		*GF=*	*0.50*	*TVD$_{int}$=*	*15,662*
21		TVD/PP @ deepest OH depth				TVD$_{mud}$	MW	TVD$_{HC}$	HC Grad.		SW Hyd
22	MASP$_{BHP}$	24,306	12.01	0.052	*minus*	8,645	12.1	8,645	0.15	*minus*	3102 psi
23									5339 psi		
24		TVD/FG @ deepest exposed shoe				TVD$_{mud}$	MW	TVD$_{HC}$	HC Grad.		SW Hyd
25	MASP$_{shoe}$	10,500	13.21	0.052	*minus*	0	12.1	3,483	0.15	*minus*	3102 psi
26									3587 psi		
27											
28	MASP					Minimum of MASP$_{BHP}$, MASP$_{shoe}$			3587 psi		

Figure 1: Original MASP Excel Sheet

2 Ologs and Spreadsheets

The primary research contribution of this paper, besides the case study, is a methodology for representing certain spreadsheets as ologs [13] (theories and models), which we describe in this section. We begin by observing that it is trivial to represent every spreadsheet as a model [11] of a signature containing one constant symbol for every used cell. For example, the initial model of the theory:

```
b17 c12 : Float    b17 = .07    c12 = 13722
```

formalizes a fragment of the MASP sheet in Figure 1. In contrast, this paper's method is based on leveraging the typical, natural decomposition of most sheets into smaller tables into order to provide additional semantics (sorts and symbols and equations). First, we define a notion of *categorical normal form* for a sheet, from [12]. Then we define translations to/from olog sheets in this normal form. A spreadsheet is in this form when:

- the sheet is composed entirely of rectangular sub-tables, each with a name, a set of named columns, and a distinguished "primary key" column. The cells in the primary key column are called the row-ids of t and semantically row-ids are meaningless identifiers, not data;
- for every column c of a table t there is either some table t' such that the value in each cell of c refers to some row-id of t' (in which case we say that c is a "foreign key" column from t to t'; or column c of table t is a "pure data" / "value" column of t with values in some non-row-id type such as Float.

Of course, most spreadsheets are not in categorical normal form to begin with. Fortunately, we have found that human subject matter experts (not computer scientists) can rapidly and accurately create new spreadsheet tabs that are in the above form and that reference back into the original sheets. These new tabs provide an auditable "olog view" of the original sheet, and heuristic methods to automatically generate these olog tabs are a subject of current work. In our case study, two engineers (paper co-authors Brandon and

James) both independently created olog tabs from the government-provided MASP calculation spreadsheet shown in Figure 1. In practice, each engineer may start with a different spreadsheet, but the olog views of the two engineers for this one government sheet are so different that for the purposes of this paper there is no difference between starting from one sheet or two: what matters is starting from two different ologs.

Remark. In this paper we will always be working with spreadsheets that correspond to models in the sense of logic [11], but the CQL software that we are using [4] allows us to work with "presentations" of spreadsheets (those that are missing values in foreign key columns; missing values in data columns don't matter here) by automatically constructing initial models of equational theories for us.

2.1 Spreadsheet to Olog

The above categorical normal form suggests an easy way to generate a signature (sorts and symbols) from a spreadsheet, especially when column headers are used to indicate the foreign keys directly in Excel. However, categorical normal form doesn't suggest any axioms along with the generated signature. The way in which we recover axioms is by examining the formulae of the sheet. In particular, to be imported as a universally quantified equation, we require that a column's rows all contain "the same" formula– a formula that is a function of only its row – which itself must be written entirely of spreadsheet functions applied to "lookups" of foreign-key columns. For example, in the MASP olog from Source A, the MASP Calc. Step 1 table contains a column called 70% Burst (not-corrected) (column C) whose rows have the form:

```
C76 = LOOKUP(E76, $A$13:$A$41,$F$13:$F$41)*B76
C77 = LOOKUP(E77, $A$13:$A$41,$F$13:$F$41)*B77
...
```

where column E has "Casing Section" as a header and column B has "De-rated Percent" as a header. Moreover, the indicated ranges are those of the ID and "Burst Rating" column in another table. As such we import the following equation (also seen in Figure 4):

```
forall x:"MASP Calc. Step 1", "70% Burst (not corrected)"(x) =
    "Burst Rating"("Casing Section"(x)) * "De-Rated Percent"(x)
```

In the above example our columns all produce numbers. In general, however, columns may also produce boolean values indicating the equivalence of two arbitrary expressions. We say that such columns are not "definitional" in the sense of the 70% burst column in the example above. The input MASP calculations for this paper contain entirely "definitional" formulae, but as described later the integrated result contains some columns that are not definitional (the integrated sheet contains some boolean-valued columns that constrain other columns).

2.2 Olog to Spreadsheet

With one caveat, to convert from ologs to spreadsheets it is necessary to merely invert the representation in the above section. The caveat is that the above discussion does not describe how to encode "type algebras" [11] – missing values and equations between missing values that can appear in ologs but not in spreadsheets that people construct naturally. To encode a non-trivial type algebra, we must include a single-column table in the spreadsheet for each type, such as integer and string. Then, each "skolem variable"/missing value at that type becomes a row in this new table, and formulae may refer to this new cell (such formulae will compute as N/A in Excel, of course). Finally, for each type we also require another table with two columns wherein ground equations between these missing values are encoded as in the preceding section. An example is shown below.

```
  A         B    C   D         E  F
1 Person    Age      Integer      IntegerEqs
2 p1        20       x            =(D2+D3=20)
3 p2        =D2      y
4 p3        =D3
```

Of course, when we import a spreadsheet containing a type algebra as encoded above, we must be careful to import the type tables as a type algebra, and not as "user tables". In this paper's example, the integrated result does have a non-trivial type algebra, because it contains blank cells in calculated columns in the original sheets. Whether our result's type algebra is contradictory is discussed in Section 4.

Remark. Our olog to spreadsheet and reverse translations fail on empty ologs because there are no corresponding spreadsheet cells with which to record equations. This problem can be remedied by including, for each empty table, a "phantom" row that records the equations with non-formulae cells left blank. Or, by writing the equations down as additional column headers.

2.3 Free Theories Not Closed Under Colimits

We might ask, "if we start with two definitional/free sheets, turn them into ologs, compose them using colimits as described later, and then turn the result back into a sheet, will the resulting sheet be entirely definitional/free as well"? That is, if we start with sheets where every column is or computes a number will we end up with a sheet where every column is or computes a number? The answer is no: in mathematical terms, a free theory such as $f(x) = x^2$ and a free theory such as $g(y) = y^3$ can be merged along the free theory $f(z) = g(z)$ to give the theory in which $x^2 = x^3$, which is non-free (non-definitional). A non-free theory requires boolean-valued columns to represent as a spreadsheet and requires more effort to reason about than a free theory but is otherwise benign[1]. In our MASP example, failure of freeness preservation under colimits manifests when we have multiple equations that define the same symbol, such as "Well" below:

```
forall x,      x.Well = x."TVD Shoe".Well
forall x,      x.Well = x."TVD Deepest OH".Well
```

The theory above is not free because it contains non-trivial equations such as

```
forall x,   x."TVD Shoe".Well = x."TVD Deepest OH".Well
```

and so when exported as a spreadsheet will contain a boolean-valued column witnessing the truth of the above equation; this boolean-valued column is considered part of the schema, not part of the data, of the sheet's corresponding olog. For example, any sheet containing the equations above would be exported as

	Step1	Well	TVD Shoe	TVD Deepest OH	x.Well = x."TVD Shoe".Well	x.Well = x."TVD Deepest OH".Well
1	s1	w1	x1	y1	true	true
2	s2	w2	x2	t2	true	true
3	...					

where each `true` is obtained by evaluating the formula in the column it appears in, where that formula will contain lookups etc as described earlier.

A related question is whether the merge of two finite sheets is always finite, where the answer is also no, such as when the schema $\cdot \rightarrow_p \cdot$ is merged with the schema $\cdot \leftarrow_q \cdot$ along the left and right dots, resulting in a schema with a cycle that is not "broken" by equations $p(q(x)) = x$ and $q(p(x)) = x$ and thus which will force the integrated sheet to be infinite when there is no data level overlap of the sources (having e.g. a row c, and a row $c.p$, and a row $c.p.q$ not equal to c, and so on). In such a case it is customary to add additional equations to break the cycle and force a finite model.

2.4 Axiomatizing Spreadsheet Functions

In this subsection we axiomatize a small collection of spreadsheet functions that are sufficient for our case study; the resulting theory is the `Type` input in the algebraic integration design pattern in Figure 2. We start with an infinite ground (variable-free) equational theory for addition `+`, subtraction `-`, multiplication `*`, maximum `MAX`, and minimum `MIN`, all of which are binary operations on floating point numbers:

$$0{+}0 = 0 \quad 0{+}1 = 1 \quad 1{+}2.2 = 3.2 \quad \ldots \quad 0{*}0 = 0 \quad 0{*}1 = 0 \quad 1{*}2.2 = 2.2 \quad \ldots \quad \texttt{MAX}(3.5, 2) = 3.5 \ \ldots$$

[1] For example, spreadsheets can only compute initial models of free/definitional theories, implying that there is no spreadsheet that can compute the algebraic merge of two other spreadsheets– the merge must be done externally to Excel (where it can, for example, create an infinite number of rows or fail with contradictions) and then re-expressed as a sheet.

$$
\begin{tikzcd}
S_O \arrow[rrrr, "M_A"] \arrow[dddd, "M_B"'] \arrow[dr, dash] & & & & S_A \arrow[dddd, "N_A"] \arrow[dl, dash] \\
& I_O \arrow[rr, Rightarrow, "h_A"] \arrow[dd, Rightarrow, "h_B"'] \arrow[dr] & & I_A \arrow[dd, Rightarrow, "k_A"] \arrow[dl] & \\
& & \mathsf{Type} & & \\
& I_B \arrow[rr, Rightarrow, "k_B"'] \arrow[ur] & & I \arrow[ul] & \\
S_B \arrow[rrrr, "N_B"'] \arrow[ur, dash] & & & & S \arrow[ul, dash]
\end{tikzcd}
$$

Universality guarantees that for any other S', N'_A, N'_B, I', k'_A, k'_B making the above diagram commute, there is a unique schema mapping $M : S \to S'$ and data mapping $k : I \to I'$ such that $N'_A = N_A; M$ and $N'_B = N_B; M$ and $k'_A = k_A; k$ and $k'_B = k_B; k$.

- Type is an algebraic (equational) theory representing types and functions (addition, subtraction, etc.).
- S_O, S_A, S_B are input schemas/theories (with S_O the overlap) and S is the output integrated schema/theory (excel formulae).
- $I_O : S_O \to \mathsf{Type}$, $I_A : S_A \to \mathsf{Type}$, and $I_B : S_B \to \mathsf{Type}$ are input models/databases (with I_O the overlap) and $I : S \to \mathsf{Type}$ the output integrated model/database (excel values).
- $M_A : S_O \to S_A$ and $M_B : S_O \to S_B$ are input schema mappings/theory morphisms, and $N_A : S_A \to S$ and $N_B : S_B \to S$ are output schema mappings/theory morphisms (functors).
- $h_A : I_0 \Rightarrow M_A; I_A$ and $h_B : I_O \Rightarrow M_B; I_B$ are input data mappings/modem morphisms, and $k_A : I_B \Rightarrow N_A; I$ and $k_B : N_B; I$ are output data mappings/model morphisms (natural transformations).

Figure 2: Algebraic Schema and Data Integration

From there, we add the non-ground (universally quantified) axioms stating that $(0, 1, +, \times)$ form a commutative ring [1], as well as some axioms stating basic properties of how `MAX` and `MIN` interact with arithmetic.

Remark. In this paper, we have associated a type with each spreadsheet cell, either floating point or string. However, in many spreadsheets, cells are not typed, and functions such as addition will throw errors on non-numeric inputs. This dynamically-typed situation can also be represented in the manner of this section, with a "universal type". Multiple types also allow for the possibility of representing different units (feet, inches, etc) as different types, reducing risk of error.

Remark. One of the original sheets used division in a single column, which for expediency we removed in favor of treating the column as plain data so as not to need to deal with division at all in this case study.

3 MASP Sheet Formalizations and Integration

We now recall the algebraic data integration design pattern [11], displayed in Figure 2. Our goal in this section is to describe all of the inputs and some of the outputs in the above diagram, including how they were constructed, either by human or machine, for our MASP example.

Remark. Intuitively, the guarantee provided by algebraic data integration is that the output of our methodology "is at least as good as every solution" to the problem of integrating the input ologs and overlap. Universality also guarantees a complete "lineage" or "provenance" of how every output row was constructed, although we do not elaborate further in this paper.

3.1 Schema Integration

In this section, we assume that the input spreadsheets have been converted to ologs as described in the previous sub-sections and discuss the resulting schema (theory part) of the olog conversion. Recall that schema integration is the process of taking input schemas (theories) S_O, S_A, S_B and computing output schema (theory) S, according to the usual algebraic integration diagram in Figure 2.

3.1.1 Schema A

The signature (sorts and symbols) of schema A are defined in Figure 3. Each edge denotes a unary function between tables and each row in the table associated to a node is a unary function targeting a type (String, Int, etc) (i.e., a "data column"). The equations for Schema A are defined in Figure 4.

Schema A contains information about MASP as defined by the Header Info table in Figure 3. This sets a broad context for how MASP is being calculated along with high level data that applies to all hole sections

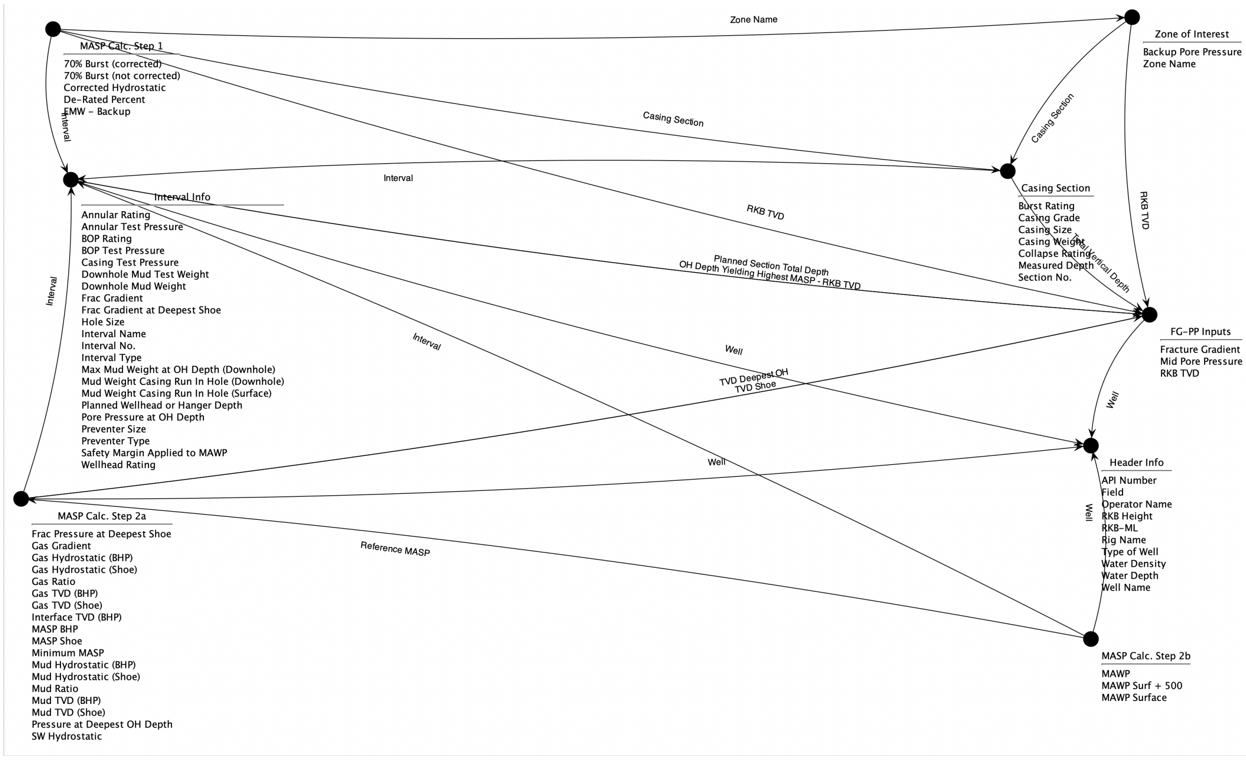


Figure 3: Signature for Schema A

where the engineering analysis is performed. These specific hole sections are defined by multiple tables: "Casing Section", "Zone of Interest", and "Interval Info". These tables reflect the general well construction process and ensure alignment between equipment specifications (such as casing) and their associated depths (where formation pressure inputs apply). The specific pressure inputs, correlated by well depth, are defined in the FG-PP Inputs table and are usually provided by teams working in the subsurface domain. Finally, the sequence of the MASP calculation itself is defined by the following tables: "MASP Calc. Step 1", "MASP Calc. Step 2a", and "MASP Calc. Step 2b". In addition to the computations, these tables include attributes for the appropriate calculation scenarios (such as the ratio of mud or gas in the well).

3.1.2 Schema B

The signature (sorts and symbols) of schema B are defined in Figure 5. The equations for schema B are defined in Figure 6.

Schema B follows a similar pattern as schema A because they share the same MASP sheet as a starting point. However, because the input models were independently developed there are key differences. Much of the disparity is driven by how the engineers chose to approach the original MASP calculation sheet. Schema A models five hole sections and began olog conversion from the Header Info table. Schema B, on the other hand, approached the olog conversion from the bottom-up and modeled only one hole section starting from the MASP computation itself.

Schema B defines the high level well information in the "Well Data Key" table. The particular depths of interest, where the MASP calculation was performed for the lone hole section, are defined by the "Item of Intereest Key" table[2]. Schema B also captures a series of tables to differentiate information related to the various steps of the MASP calculation: "Exposed Shoe Key", "OH Key", and "Casing Burst Key". The subsurface depth and pressure inputs are defined in the "Pore Pressure Frac Pressure Key" table, and the mud/gas scenarios are defined in the "Mud Gradient Key" table. The MASP computation is decomposed across the following tables: "Burst Calculation Key", "MASP Open Hole Key", "MASP Shoe Key", and

[2]Our data sources contain typos in their original schemas and data which are reflected in our development.

```
forall x:"Header Info", "RKB-ML"(x) = "Water Depth"(x) + "RKB Height"(x)

forall x:"MASP Calc. Step 1", "70% Burst (corrected)"(x) =
     "70% Burst (not corrected)"(x) - "Corrected Hydrostatic"(x)

forall x:"MASP Calc. Step 1", "70% Burst (not corrected)"(x) =
    "Burst Rating"("Casing Section"(x)) * "De-Rated Percent"(x)

forall x:"MASP Calc. Step 1", "EMW - Backup"(x) =
    "Downhole Mud Weight"("Interval"(x)) - "Backup Pore Pressure"("Zone Name"(x))

forall x:"MASP Calc. Step 1", "Corrected Hydrostatic"(x) = .052 * ("RKB TVD"("RKB TVD"(x)) * "EMW - Backup"(x))

forall x:"MASP Calc. Step 2a", "MASP BHP"(x) = (("Pressure at Deepest OH Depth"(x) - "SW Hydrostatic"(x))
        - "Gas Hydrostatic (BHP)"(x)) - "Mud Hydrostatic (BHP)"(x)

forall x:"MASP Calc. Step 2a", "MASP Shoe"(x) = (("Frac Pressure at Deepest Shoe"(x) - "Mud Hydrostatic (Shoe)"(x))
        - "Gas Hydrostatic (Shoe)"(x)) - "SW Hydrostatic"(x)

forall x:"MASP Calc. Step 2a", "Mud Hydrostatic (BHP)"(x) =
        .052 * "Mud TVD (BHP)"(x) * "Max Mud Weight at OH Depth (Downhole)"(Interval(x)))

forall x:"MASP Calc. Step 2a", "Gas Hydrostatic (BHP)"(x) = "Gas Gradient"(x) * "Gas TVD (BHP)"(x)

forall x:"MASP Calc. Step 2a", "SW Hydrostatic"(x) = 0.052 * "Water Depth"("Well"(x)) * "Water Density"("Well"(x))

forall x:"MASP Calc. Step 2a", "Pressure at Deepest OH Depth"(x) =
        "RKB TVD"("TVD Deepest OH"(x)) * .052 * "Pore Pressure at OH Depth"("Interval"(x))

forall x:"MASP Calc. Step 2a", "Interface TVD (BHP)"(x) =
        "RKB-ML"("Well"(x)) + ("Gas Ratio"(x) * ("RKB TVD"("TVD Deepest OH"(x)) - "RKB-ML"("Well"(x))))

forall x:"MASP Calc. Step 2a", "Mud TVD (BHP)"(x) =
        ("RKB TVD"("TVD Deepest OH"(x)) - "RKB-ML"("Well"(x))) * "Mud Ratio"(x)

forall x:"MASP Calc. Step 2a", "Gas TVD (BHP)"(x) =
        ("RKB TVD"("TVD Deepest OH"(x)) - "RKB-ML"("Well"(x))) * "Gas Ratio"(x)

forall x:"MASP Calc. Step 2a", "Frac Pressure at Deepest Shoe"(x) =
        "RKB TVD"("TVD Shoe"(x)) * .052 * "Frac Gradient at Deepest Shoe"("Interval"(x))

forall x:"MASP Calc. Step 2a", "Mud TVD (Shoe)"(x) =
        MAX(0,("RKB TVD"("TVD Shoe"(x)) - "Gas TVD (Shoe)"(x)) - "RKB-ML"("Well"(x)))

forall x:"MASP Calc. Step 2a", "Gas TVD (Shoe)"(x) =
        MIN("RKB TVD"("TVD Shoe"(x)) - "RKB-ML"("Well"(x)), "Interface TVD (BHP)"(x) - "RKB-ML"("Well"(x)))

forall x:"MASP Calc. Step 2a", "Minimum MASP"(x) = MIN("MASP BHP"(x),"MASP Shoe"(x))

forall x:"MASP Calc. Step 2b", "MAWP"(x) =
        "Minimum MASP"("Reference MASP"(x)) + "SW Hydrostatic"("Reference MASP"(x))

forall x:"MASP Calc. Step 2b", "MAWP Surface"(x) =
        "MAWP"(x) - ((.052 * "RKB-ML"("Well"(x))) * "Downhole Mud Weight"("Interval"(x)))

forall x:"MASP Calc. Step 2b", "MAWP Surf + 500"(x) = "MAWP Surface"(x) + 500
```

Figure 4: Equations for Schema A

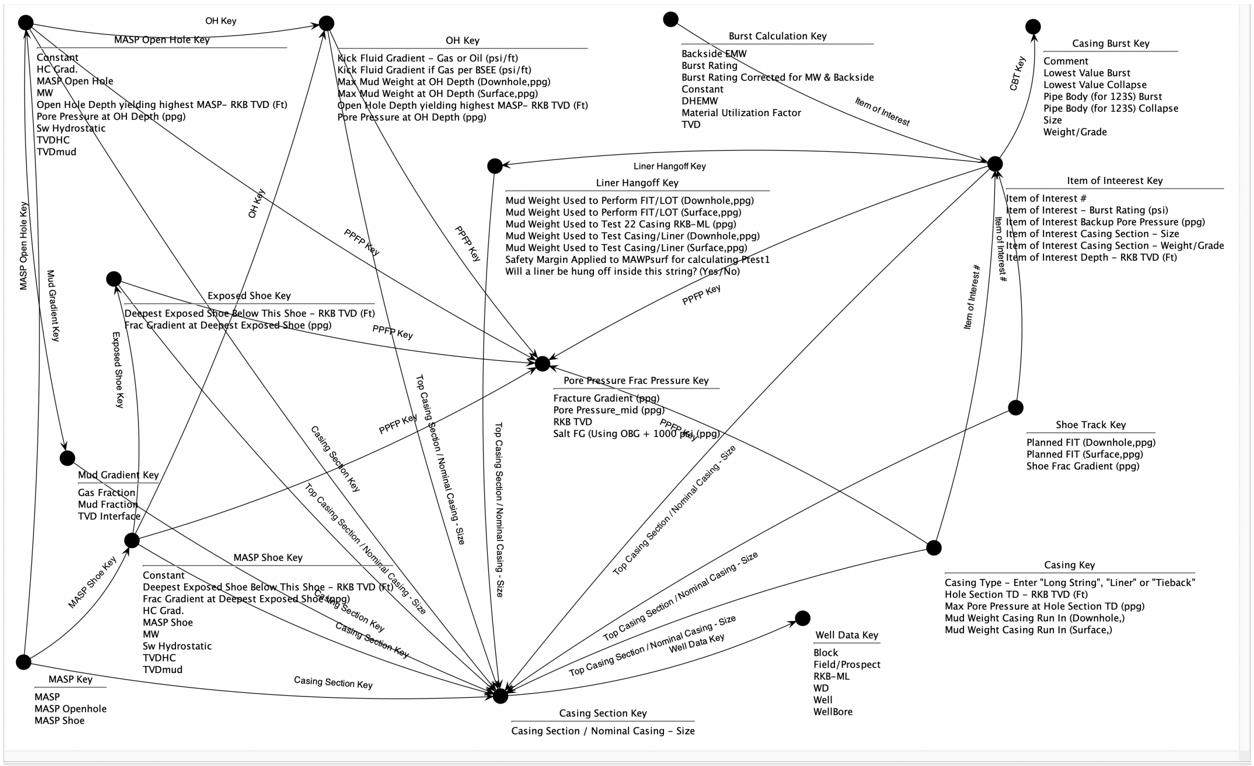


Figure 5: Signature for Schema B

"MASP Key". Schema B also includes the "Exposed Shoe Key" table, which informs the fracture gradient variables in the MASP shoe calculation. The different MASP uses (open hole and shoe) are separated in Schema B for transparency. Finally, schema B contains tables that contain a few data points that serve more as an efficient modeling structure to map keys between tables ("Casing Section Key" and "Casing Key"); this approach was taken to ensure each critical "variable" received its own table to better inform future auditing of results.

3.1.3 Overlap Schema and Mappings

The signature (sorts and symbols) of schema O, the overlap schema, are defined in Figure 7, along with its equations. The overlap schema is inspired by the English definition of the MASP calculation included with the original Excel sheet, shown in Figure 11.

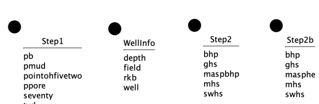
The three equations in the overlap schema generate the verification conditions described in section 4; our methodology checks that the equations in the overlap schema are entailed by the equations of each source schema. In this way, we need not appeal to the authors of the source schemas; we can check if the equations (semantics) we desire (in the overlap) are already present in the source schemas. It is for this reason we describe our methodology as "consensus-free".

The schema mappings from the overlap schema to source A's schema and source B's schema are shown in Figures 8 and 9 respectively. As required by algebraic data integration, for each overlap table, we give a table in source A (or B, respectively), and for each overlap column, we give an expression in source A (or B, respectively).

The four tables that make up the overlap schema are defined as follows. The "WellInfo" table captures general information about the subject well to allow the input models to integrate data for calculations being performed on the same well. The first step of the MASP calculation, defined in the "Step1" table, represents the de-rated burst pressure rating of the subject casing strings after being corrected for mud weight and pore pressure backup values at the depth of interest. This is a requirement based on government regulations, which stipulate that the subject casing string should be de-rated to 70 percent of the casing burst pressure rating.

```
forall x:"Casing Key",  x."Max Pore Pressure at Hole Section TD (ppg)" = x."PPFP Key"."Pore Pressure_mid (ppg)"

forall x:"Item of Inteerest Key", x."Item of Interest Backup Pore Pressure (ppg)" = x."PPFP Key"."Pore Pressure_mid (ppg)"

forall x:"OH Key",  x."Pore Pressure at OH Depth (ppg)" = x."PPFP Key"."Pore Pressure_mid (ppg)"

forall x:"Burst Calculation Key", x."Burst Rating" = x."Item of Interest"."Item of Interest - Burst Rating (psi)"
forall x:"Burst Calculation Key", x."Backside EMW" = x."Item of Interest"."Item of Interest Backup Pore Pressure (ppg)"
forall x:"Burst Calculation Key", x."TVD" = x."Item of Interest"."Item of Interest Depth - RKB TVD (Ft)"
forall x:"Burst Calculation Key", x."Burst Rating Corrected for MW & Backside" =
    (x."Material Utilization Factor" * x."Burst Rating") - (x.DHEMW - x."Backside EMW" * x.Constant * x.TVD)

forall x:"MASP Open Hole Key", x."Pore Pressure at OH Depth (ppg)" = x."OH Key"."Pore Pressure at OH Depth (ppg)"
forall x:"MASP Open Hole Key", x."MW" = x."OH Key"."Max Mud Weight at OH Depth (Downhole,ppg)"
forall x:"MASP Open Hole Key", x."HC Grad." = x."OH Key"."Kick Fluid Gradient if Gas per BSEE (psi/ft)"

forall x:"MASP Open Hole Key", x."MASP Open Hole" =
        -(-(-((x."Open Hole Depth yielding highest MASP- RKB TVD (Ft)"
        * x."Pore Pressure at OH Depth (ppg)" * x.Constant),
        x.TVDmud * x.MW * x.Constant), x.TVDHC * x."HC Grad."), x."Sw Hydrostatic")
forall x:"MASP Open Hole Key", x."TVDmud" = MAX(0, *(-(x."Open Hole Depth yielding highest MASP- RKB TVD (Ft)",
        x."Casing Section Key"."Well Data Key"."RKB-ML"), x."Mud Gradient Key"."Mud Fraction"))
forall x:"MASP Open Hole Key", x."TVDHC" = MAX(0, *(-(x."Open Hole Depth yielding highest MASP- RKB TVD (Ft)",
        x."Casing Section Key"."Well Data Key"."RKB-ML"), x."Mud Gradient Key"."Gas Fraction"))

forall x:"MASP Shoe Key", x."MASP Shoe" =
        -(-(-(x."Deepest Exposed Shoe Below This Shoe - RKB TVD (Ft)" *
        x."Frac Gradient at Deepest Exposed Shoe (ppg)" * x.Constant),
        x.TVDmud * x.MW * x.Constant, x.TVDHC * x."HC Grad."), x."Sw Hydrostatic")
forall x:"MASP Shoe Key", x."MW" = x."OH Key"."Max Mud Weight at OH Depth (Downhole,ppg)"
forall x:"MASP Shoe Key", x."Frac Gradient at Deepest Exposed Shoe (ppg)" =
        x."Exposed Shoe Key"."Frac Gradient at Deepest Exposed Shoe (ppg)"

forall x:"MASP Key", x.MASP = MIN(x."MASP Openhole", x."MASP Shoe")
forall x:"MASP Key", x."MASP Openhole" = x."MASP Open Hole Key"."MASP Open Hole"
forall x:"MASP Key", x."MASP Shoe" = x."MASP Shoe Key"."MASP Shoe"

forall x:"Exposed Shoe Key", x."Frac Gradient at Deepest Exposed Shoe (ppg)" =
        x."PPFP Key"."Salt FG (Using OBG + 1000 psi (ppg)"

forall x:"Burst Calculation Key",   x.Constant=.052
```

Figure 6: Equations for Schema B




```
forall i:Step1, casingburst(i) = (i.seventy * i.pb)
        - i.tvd * i.pointohfivetwo * (i.pmud - i.ppore)
forall i:Step2 , maspbhp (i) = i.bhp - (i.mhs + i.ghs + i.swhs)
forall i:Step2b, maspshoe(i) = i.bhp - (i.mhs + i.ghs + i.swhs)
```

Figure 7: Signature and Equations for Overlap Schema

```
Step1 -> "MASP Calc. Step 1"                                pointohfivetwo -> lambda x, .052
pmud -> lambda x, x."Interval"."Downhole Mud Weight"        pb -> lambda x, x."Casing Section"."Burst Rating"
ppore -> lambda x, x."Zone Name"."Backup Pore Pressure"     tvd -> lambda x, x."RKB TVD"."RKB TVD"
casingburst -> lambda x, x."70% Burst (corrected)"          seventy -> lambda x, x."De-Rated Percent"

Step2 -> "MASP Calc. Step 2a"                  bhp -> lambda x, x."Pressure at Deepest OH Depth"
mhs -> lambda x, x."Mud Hydrostatic (BHP)"      ghs -> lambda x, x."Gas Hydrostatic (BHP)"
swhs -> lambda x, x."SW Hydrostatic"            maspbhp -> lambda x, x."MASP BHP"

Step2b -> "MASP Calc. Step 2a"                 bhp ->  lambda x, x."Frac Pressure at Deepest Shoe"
mhs ->  lambda x, x."Mud Hydrostatic (Shoe)"    ghs ->  lambda x, x."Gas Hydrostatic (Shoe)"
swhs -> lambda x, x."SW Hydrostatic"            maspshoe ->  lambda x, x."MASP Shoe"

WellInfo -> "Header Info"       well -> lambda x, x."Well Name"
rkb -> lambda x, x."RKB-ML"     depth -> lambda x, x."WD"       field -> lambda x, x."Field"
```

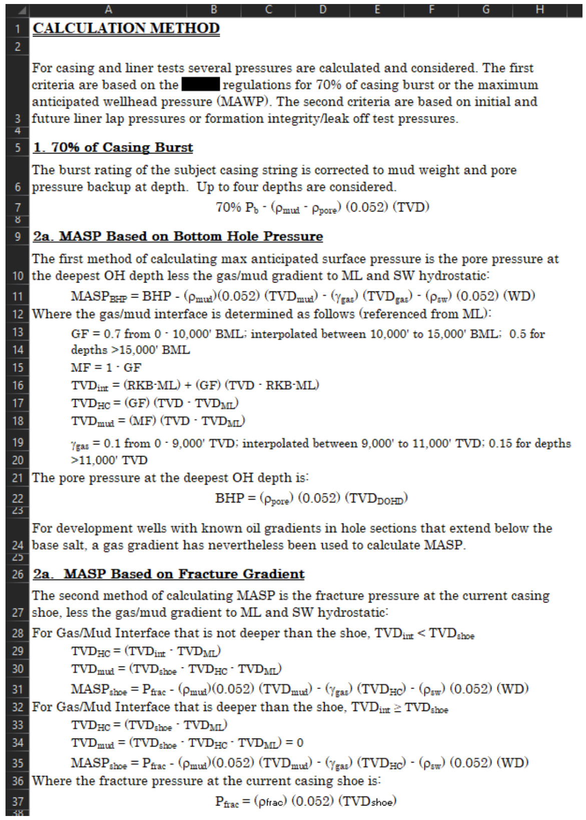
Figure 8: Schema Mapping from Overlap to A

```
Step1 -> "Burst Calculation Key"        pointohfivetwo -> lambda x, .052        tvd -> lambda x, x.TVD
pb -> lambda x, x."Item of Interest"."Item of Interest - Burst Rating (psi)"
pmud -> lambda x, x.DHEMW               ppore -> lambda x, x."Backside EMW"
casingburst -> lambda x, x."Burst Rating Corrected for MW & Backside"
seventy -> lambda x, x."Material Utilization Factor"

Step2 -> "MASP Key"     swhs -> lambda x, x."MASP Open Hole Key"."Sw Hydrostatic"
mhs -> lambda x, x."MASP Open Hole Key"."MW" * x."MASP Open Hole Key"."TVDmud" * x."MASP Open Hole Key"."Constant"
ghs -> lambda x, x."MASP Open Hole Key"."HC Grad." * x."MASP Open Hole Key"."TVDHC"
maspbhp -> lambda x, x."MASP Open Hole Key"."MASP Open Hole"
bhp -> lambda x, *(x."MASP Open Hole Key"."Pore Pressure at OH Depth (ppg)",
  *(x."MASP Open Hole Key"."Constant", x."MASP Open Hole Key"."Open Hole Depth yielding highest MASP- RKB TVD (Ft)"))

Step2b -> "MASP Key"        swhs -> lambda x,  x."MASP Shoe Key"."Sw Hydrostatic"
mhs -> lambda x, x."MASP Shoe Key"."TVDmud" * x."MASP Shoe Key"."MW" * x."MASP Shoe Key"."Constant"
ghs -> lambda x, x."MASP Shoe Key"."HC Grad." * x."MASP Shoe Key"."TVDHC"
maspshoe -> lambda x,  x."MASP Shoe Key"."MASP Shoe"
bhp -> lambda x, *(*(x."MASP Shoe Key"."Deepest Exposed Shoe Below This Shoe - RKB TVD (Ft)",
    x."MASP Shoe Key"."Frac Gradient at Deepest Exposed Shoe (ppg)"), x."MASP Shoe Key"."Constant")

WellInfo -> "Well Data Key"     well -> lambda x, x."Well"      rkb -> lambda x, x."RKB-ML"
depth -> lambda x, x."Water Depth"      field -> lambda x, x."Field/Prospect"
```

Figure 9: Schema Mapping from Overlap to B

```
////////////////// source A /////////////////
forall bc:"MASP Calc. Step 1",  bc."Casing Section" =  bc."Zone Name"."Casing Section"
forall bc:"MASP Calc. Step 1",  bc."Casing Section".Interval = bc.Interval
forall bc:"MASP Calc. Step 1", bc.Interval.Well = bc."RKB TVD".Well
forall bc:"MASP Calc. Step 1", bc."Casing Section"."Total Vertical Depth".Well = bc."RKB TVD".Well
forall bc:"MASP Calc. Step 1",  bc.Interval."Planned Section Total Depth".Well = bc."Zone Name"."RKB TVD".Well

forall x:"MASP Calc. Step 2a", x.Well = x."TVD Shoe".Well
forall x:"MASP Calc. Step 2a", x.Well = x."TVD Deepest OH".Well
forall x:"MASP Calc. Step 2a", x.Well = x."Interval"."Planned Section Total Depth".Well

////////////////// source B /////////////////
forall x:"Item of Inteerest Key", x."Liner Hangoff Key"."Top Casing Section / Nominal Casing - Size" =
    x."Top Casing Section / Nominal Casing - Size"

 forall x:"MASP Key", x."Casing Section Key" = x."MASP Open Hole Key"."Casing Section Key"
 forall x:"MASP Key", x."Casing Section Key" = x."MASP Open Hole Key"."Mud Gradient Key"."Casing Section Key"
 forall x:"MASP Key", x."Casing Section Key" =
    x."MASP Open Hole Key"."OH Key"."Top Casing Section / Nominal Casing - Size"
 forall x:"MASP Key", x."MASP Open Hole Key"."Mud Gradient Key"."Casing Section Key" =
    x."MASP Shoe Key"."Casing Section Key"
 forall x:"MASP Key",x."MASP Shoe Key"."Exposed Shoe Key"."Top Casing Section / Nominal Casing - Size" =
    x."MASP Shoe Key"."Casing Section Key"
 forall x:"MASP Key",x."MASP Open Hole Key"."OH Key"."Top Casing Section / Nominal Casing - Size" =
     x."MASP Shoe Key"."Casing Section Key"

////////////////// both sources /////////////////
forall x:"Interval Info", x.Interval.Well."Well Name" =
    x."Item of Interest"."Top Casing Section / Nominal Casing - Size"."Well Data Key"."Well"

forall x:"MASP Calc. Step 2a", x."MASP Open Hole Key"."Mud Gradient Key"."Casing Section Key"."Well Data Key" =
    x."MASP Shoe Key"."OH Key"."Top Casing Section / Nominal Casing - Size"."Well Data Key"

forall x:"MASP Calc. Step 1", x."Item of Interest"."Liner Hangoff Key"."Top Casing Section / Nominal Casing - Size"
    ."Well Data Key" = x."Casing Section"."Interval".Well

forall x:"MASP Calc. Step 1", x."Item of Interest"."Liner Hangoff Key"."Top Casing Section / Nominal Casing - Size"
    ."Well Data Key" = x."Zone Name"."Casing Section"."Interval".Well

forall x:"MASP Calc. Step 1", x."Item of Interest"."Liner Hangoff Key"."Top Casing Section / Nominal Casing - Size"
    ."Well Data Key" = x."Interval".Well

forall x:"MASP Calc. Step 1", x."Item of Interest"."Liner Hangoff Key"."Top Casing Section / Nominal Casing - Size"
    ."Well Data Key" = x."Zone Name"."Casing Section"."Interval".Well

forall x:"MASP Calc. Step 2a", x.Interval.Well =
    x."MASP Open Hole Key"."Mud Gradient Key"."Casing Section Key"."Well Data Key"

forall x:"MASP Calc. Step 2a", x.Well =
    x."MASP Open Hole Key"."Mud Gradient Key"."Casing Section Key"."Well Data Key"

forall x:"Item of Inteerest Key", x."Top Casing Section / Nominal Casing - Size"."Well Data Key" =
    x."Liner Hangoff Key"."Top Casing Section / Nominal Casing - Size"."Well Data Key"
```

Figure 10: Additional Equations for Better Data Integration



CALCULATION METHOD

For casing and liner tests several pressures are calculated and considered. The first criteria are based on the [redacted] regulations for 70% of casing burst or the maximum anticipated wellhead pressure (MAWP). The second criteria are based on initial and future liner lap pressures or formation integrity/leak off test pressures.

1. 70% of Casing Burst

The burst rating of the subject casing string is corrected to mud weight and pore pressure backup at depth. Up to four depths are considered.

$$70\%\ P_b - (\rho_{mud} - \rho_{pore})\ (0.052)\ (TVD)$$

2a. MASP Based on Bottom Hole Pressure

The first method of calculating max anticipated surface pressure is the pore pressure at the deepest OH depth less the gas/mud gradient to ML and SW hydrostatic:

$$MASP_{BHP} = BHP - (\rho_{mud})(0.052)\ (TVD_{mud}) - (\gamma_{gas})\ (TVD_{gas}) - (\rho_{sw})\ (0.052)\ (WD)$$

Where the gas/mud interface is determined as follows (referenced from ML):

GF = 0.7 from 0 - 10,000' BML; interpolated between 10,000' to 15,000' BML; 0.5 for depths >15,000' BML

MF = 1 - GF

$TVD_{int} = (RKB\text{-}ML) + (GF)\ (TVD - RKB\text{-}ML)$

$TVD_{HC} = (GF)\ (TVD - TVD_{ML})$

$TVD_{mud} = (MF)\ (TVD - TVD_{ML})$

γ_{gas} = 0.1 from 0 - 9,000' TVD; interpolated between 9,000' to 11,000' TVD; 0.15 for depths >11,000' TVD

The pore pressure at the deepest OH depth is:

$$BHP = (\rho_{pore})\ (0.052)\ (TVD_{DOHD})$$

For development wells with known oil gradients in hole sections that extend below the base salt, a gas gradient has nevertheless been used to calculate MASP.

2a. MASP Based on Fracture Gradient

The second method of calculating MASP is the fracture pressure at the current casing shoe, less the gas/mud gradient to ML and SW hydrostatic:

For Gas/Mud Interface that is not deeper than the shoe, $TVD_{int} < TVD_{shoe}$

$TVD_{HC} = (TVD_{int} - TVD_{ML})$

$TVD_{mud} = (TVD_{shoe} - TVD_{HC} - TVD_{ML})$

$MASP_{shoe} = P_{frac} - (\rho_{mud})(0.052)\ (TVD_{mud}) - (\gamma_{gas})\ (TVD_{HC}) - (\rho_{sw})\ (0.052)\ (WD)$

For Gas/Mud Interface that is deeper than the shoe, $TVD_{int} \geq TVD_{shoe}$

$TVD_{HC} = (TVD_{shoe} - TVD_{ML})$

$TVD_{mud} = (TVD_{shoe} - TVD_{HC} - TVD_{ML}) = 0$

$MASP_{shoe} = P_{frac} - (\rho_{mud})(0.052)\ (TVD_{mud}) - (\gamma_{gas})\ (TVD_{HC}) - (\rho_{sw})\ (0.052)\ (WD)$

Where the fracture pressure at the current casing shoe is:

$$P_{frac} = (\rho_{frac})\ (0.052)\ (TVD_{shoe})$$

Figure 11: MASP Definition as Overlap

The next requirement in the MASP calculation determines the maximum anticipated surface pressure. This is performed for both bottomhole pressure and fracture gradient scenarios and is defined by the "Step2" and "Step2b" tables, respectively. The mappings from this overlap model to schema A and schema B define columns where each component of the overlap computation is contained within the input models.

These tables were chosen as overlap, and the schema mappings designed as they are, because the overlapping tables represent engineering requirements shared by many business units and engineering teams around the world. Mathematically verifying that an expression of these requirements is entailed by the equations in each source schema opens a door to new forms of well design assurance and improving the quality and cycle time of well engineering activities, as discussed in the conclusion.

3.2 Suggesting rules

For large spreadsheets, it is impractical for users to start from a blank screen and simply type hundreds or thousands of formulae to relate them. Therefore data integration tools, including ours, provide heuristic suggestions as a starting point. Rules that well names are preserved as foreign keys are traversed are good examples of rules that are easy to guess based on column names and on data. The paper [10] describes some heuristic rule suggestion algorithms for our particular algebraic data model, and many traditional heuristic algorithms [5] can be re-used off-the-shelf in our system.

3.2.1 Result

The result of schema integration can be easily described: we start by taking the disjoint union $S_A + S_B$ of the source schemas; then, for every table o in the overlap schema O, we merge tables $M_A(o)$ and $M_B(o)$ and add equations $M_A(f) = M_B(f)$ for every f with domain o, ultimately resulting in a "colimit" [11] schema (theory) denoted $S_A +_{O,M_A,M_B} S_B$. This schema is large and somewhat redundant in our case study because we are only merging three tables ("step 1", "step 2a", and "header/well info"), so we do not display it here. However, in Figure 10 we display additional equations that we added to the colimit schema to aid data quality during data integration– arguably, some of these equations were forgotten in the original schemas. Additionally, the integrated schema can be read off the integrated data from the full result in the appendix.

Remark. The result of schema integration is only defined up to unique isomorphism, and so for example the names of tables in the result, and even the number of columns and equations in it, is not determined uniquely– the CQL software has many options to allow users to select which resulting schema (theory) they want from among many choices. In fact, under mild assumptions it is possible to prove that there is no "best" table and column naming scheme. In practice, the integrated schema is often customized with user-specific naming of tables and columns.

3.3 Data Integration and Exchange

We continue to follow the algebraic data integration pattern from Figure 2 and in this section define the data sources and their overlap that complete the pattern started in the previous section. We now walk through the data contained in each table below.

3.3.1 Data A

Input data for source A is shown in Figure 12.

- Casing Section Table
  - Multiple casing intervals are included (e.g. 22", 16", etc).
  - Question marks in the Casing Weight column reflect missing values in the original Excel MASP spreadsheet.
- FG-PP Inputs Table
  - Over 4,000 rows were originally included in this table's sheet, but not all are imported into CQL for expediency.
  - This table represents geologic inputs from cross-domain stakeholders who govern the reservoir characteristics and Earth model.
- Header Info Table
  - Only one well was modeled, so there is only one row.
- Interval Info Table
  - Five rows reflect the casing/hole sections that are included.
  - The question mark in the table reflects a missing fracture gradient value in the original Excel MASP spreadsheet.
- MASP Calc. Step 1 Table
  - The first criteria for calculating MASP is based on government regulations and includes calculating 70 percent of the burst pressure rating for each casing string.
  - This step also considers the burst pressure rating of the subject casing string when corrected to mud weight and pore pressure backup at various depths or zones of interest.
- MASP Calc. Step 2a Table
  - Five rows reflect the MASP result (for each casing section) based on bottom hole pressure, which is calculated using the deepest open hole depth.

Casing Section (29)

Row	Burst Rating	Casing Grade	Casing Size	Casing Weight	Collapse Rating	Measured Depth	Section No.	Interval	Total Vertical Depth
0	13722	23x2 HPWH Extension Jt	23"	?0	12545.00000000	7015	1.00000000	81	37
1	13890	X-80Q (H-100DM/QT)	23"	453.04	4000.00000000	7276	2.00000000	81	38
2	15600	16" 12.5K SA Upper	23"	?1	14200.00000000	7279	3.00000000	81	38
3	7540	16" 12.5K SA Lower	22"	?2	7200.00000000	7282	4.00000000	81	39
4	6364	X-80 (S-90DM/QT-CR)	22"	224.21	3873.00000000	7600	5.00000000	81	40
5	4275	4275psi Burst Disk	22"	?3	3873.00000000	7670	6.00000000	81	41
6	6364	X-80 (S-90DM/QT-CR)	22"	224.21	3873.00000000	10500	7.00000000	81	42
7	12350	Dril-Quip CsgHgr 16.25" ...	16"	?4	5000.00000000	7293	1.00000000	82	43
8	12350	Q-125 HP	16.25"	136.04	7680.00000000	7593	2.00000000	82	44
9	10100	Q-125 ICY	16.04"	109.61	4800.00000000	14700	3.00000000	82	45
10	10880	Q-125 XHP	16.15"	119.23	6120.00000000	23842	4.00000000	82	46
11	10100	Innovex Centramax Sub	16.04"	109.61	4670.00000000	24298	5.00000000	82	47
12	10100	Q-125 ICY	16.04"	109.61	4800.00000000	24342	6.00000000	82	48
13	10328	14x16 VF LH,125.58# SL...	14"	125.58	9928.00000000	23856	1.00000000	83	49
14	14760	VM125 HYHC WLSF	14"	115.53	12500.00000000	26847	2.00000000	83	33
15	9070	11-7/8x13-3/8 VF XG Li...	10-1/8"	?5	8730.00000000	25407	1.00000000	84	50
16	18190	TN Q125-ICY WLSF	10-1/8"	75.9	16370.00000000	26380	2.00000000	84	51
17	14370	145K Cflex	10-1/8"	?6	13741.00000000	26430	3.00000000	84	52
18	18190	TN Q125-ICY WLSF	10-1/8"	75.9	16370.00000000	27093	4.00000000	84	53
19	14370	145K Cflex	10-1/8"	?7	13741.00000000	27143	5.00000000	84	54
20	18190	TN Q125-ICY WLSF	10-1/8"	75.9	16370.00000000	28502	6.00000000	84	35
21	15000	VM125SS VAM CSG HGR	10-3/4"	85.3	10000.00000000	7008	1.00000000	85	55
22	16680	VM125SS SLIJ-II-KT	10-3/4"	85.3	15530.00000000	7028	2.00000000	85	35
23	16680	VM125SS Wedge 624	10-3/4"	85.3	15530.00000000	8048	3.00000000	85	56
24	18020	Q125 ICY Wedge 623RW	10-3/4"	85.3	17250.00000000	13500	4.00000000	85	57
25	15550	Q125 ICY Wedge 623RW	9-7/8"	65.1	13900.00000000	22842	5.00000000	85	58
26	18870	Q125X HP SLIJ-II	10.175"	81	18870.00000000	24857	6.00000000	85	59
27	18190	Q125 ICY WLSF	10-1/8"	75.9	16370.00000000	25357	7.00000000	85	60
28	15580	10.125 x 8.520 ID C140 ...	10-1/8"	75.9	14820.00000000	25400	8.00000000	85	61

FG-PP Inputs (51)

Row	Fracture Gradient	Mid Pore Pressure	RKB TVD	Well
29	14.478	12.08	24302	80
30	13.256	10.298	10602	80
31	15.465	13.973	26437	80
32	14.486	12.014	24307	80
33	15.592	13.879	26707	80
34	15.608	13.875	26737	80
35	15.748	13.459	28322	80
36	15.739	13.48	28232	80
37	8.5	8.6	7017	80
38	8.707	8.6	7277	80
39	8.711	8.6	7282	80
40	8.969	8.643	7602	80
41	8.944	8.633	7572	80
42	13.209	10.238	10502	80
43	8.719	8.6	7292	80
44	8.961	8.64	7592	80
45	14.634	11.993	14702	80
46	14.291	11.467	23767	80
47	14.389	11.913	24212	80
48	14.403	11.961	24257	80
49	14.293	11.48	23782	80
50	15.279	13.399	25297	80
51	15.443	13.934	26247	80
52	15.449	13.948	26297	80
53	15.62	8.764	26947	80
54	15.067	10.962	26997	80
55	8.5	8.6	7004	80
56	9.333	8.791	8047	80
57	14.317	11.604	13502	80
58	13.885	10.723	22792	80
59	15.085	12.913	24762	80
60	15.273	13.41	25247	80
61	15.279	13.4	25292	80
62	8.715	8.6	7287	80
63	9.026	8.666	7672	80
64	12.756	9.388	18332	80
65	14.292	11.476	23777	80
66	14.436	11.982	24277	80
67	14.294	11.485	23787	80
68	14.295	11.489	23792	80
69	14.296	11.494	23797	80
70	14.297	11.498	23802	80
71	14.298	11.503	23807	80
72	14.299	11.507	23812	80
73	14.71	7.85	27517	80
74	9.341	8.794	8057	80
75	13.875	10.693	22737	80
76	15.079	12.89	24752	80
77	15.277	13.402	25282	80
78	9.024	8.666	7670	80
79	15.274	13.408	25257	80

Header Info (1)

Row	API Number	Field	Operator Name	RKB Height	RKB-ML	Rig Name	Type of Well	Water Density	Water Depth	Well Name
80	123456789	Field 1	Chevron	81	7017	Rig 2	Development	8.6	6936	XYZ

Interval Info (5)

Row	Annular ...	Annular ...	BOP Rating	BOP Test...	Casing T...	Downhol...	Downhol...	Frac Gra...	Frac Gra...	Hole Size	Interval ...	Interval ...	Interval ...	Max Mud...	Mud Wei...	Mud Wei...	Planned ...	Pore Pre...	Prevente...	Prevente...	Safety M...	Wellhead...	OH Dept...	Planned ...	Well
81	10000	6800	15000	11800	3100	8.6	8.6	13.2	13.209	26	Surface	1.0000...	Casing	12.1	12.5	12.5	7004	12.008	18.75	Blowout	500	15000	29	30	80
82	10000	5600	15000	9100	5300	12.1	12.1	14.4	15.748	21	Interme...	2.0000...	Liner	13.6	12.1	11.9	7287	13.973	18.75	Blowout	500	15000	31	32	80
83	10000	4800	15000	8300	2700	14.1	14.1	15.59	15.748	16.5	Interme...	3.0000...	Liner	13.6	14.1	13.9	23766	13.879	18.75	Blowout	500	15000	33	34	80
84	10000	4900	15000	8300	3600	13.4	13.4	15.75	15.748	12.25	Production	4.0000...	Liner	13.6	13.4	13.2	25251	13.459	18.75	Blowout	500	15000	35	35	80
85	10000	4900	15000	8300	7200	13.9	13.9	?8	15.748	12.25	Tieback	5.0000...	Casing	13.9	13.9	13.7	7007	13.459	18.75	Blowout	500	15000	35	36	80

MASP Calc. Step 1 (30)

Row	70% Burst (corrected)	70% Burst (not corrected)	Corrected Hydrostatic	De-Rated Percent	EMW - Backup	Casing Section	Interval	RKB TVD	Zone Name
86	9605.4	9605.4	0	0.7	0	0	81	55	126
87	9723	9723	0	0.7	0	1	81	62	127
88	10920	10920	0	0.7	0	2	81	62	128
89	3018.82344	2992.5	-26.32344	0.7	-0.066	5	81	78	129
90	4481.12344	4454.8	-26.32344	0.7	-0.066	4	81	78	130
91	5349.318352	4454.8	-894.518352	0.7	-1.638	4	81	42	131
92	7318.766	8645	1326.234	0.7	3.5	7	82	62	132
93	7317.856	8645	1327.144	0.7	3.5	8	82	43	133
94	6987.433568	7070	82.566432	0.7	0.108	9	82	45	134
95	5030.748032	7616	2585.251968	0.7	2.712	10	82	64	135
96	7856.174248	8645	788.825752	0.7	0.638	8	82	65	136
97	6921.036328	7070	148.963672	0.7	0.118	11	82	66	137
98	6894.670404	7070	175.329596	0.7	0.139	12	82	48	138
99	3969.338008	7229.6	3260.261992	0.7	2.638	13	83	46	139
100	7086.993664	10332	3245.006336	0.7	2.624	14	83	49	140
101	10059.87672	10332	272.12328	0.7	0.22	14	83	67	141
102	6361.37184	6349	-12.37184	0.7	-0.01	15	84	68	142
103	12742.899552	12733	-9.899552	0.7	-0.008	16	84	69	143
104	10719.933936	10059	-660.933936	0.7	-0.534	17	84	70	144
105	13410.166308	12733	-677.166308	0.7	-0.547	18	84	71	145
106	9485.702288	10059	573.297712	0.7	0.463	19	84	72	146
107	4791.5938	12733	7941.4062	0.7	5.55	20	84	73	147
108	12819.891896	12733	-86.891896	0.7	-0.059	20	84	35	148
109	8569.6976	10500	1930.3024	0.7	5.3	21	85	55	149
110	9742.1148	11676	1933.8852	0.7	5.3	23	85	37	150
111	9536.769816	11676	2139.230184	0.7	5.106	23	85	74	151
112	11000.565008	12614	1613.434992	0.7	2.298	24	85	57	152
113	7121.662708	10885	3763.337292	0.7	3.183	25	85	75	153
114	11894.866816	13209	1314.133184	0.7	1.021	26	85	76	154
115	10251.297328	10906	654.702672	0.7	0.498	28	85	77	155

MASP Calc. Step 2a (5)

Row	Frac Press...	Gas Gradi...	Gas Hydro...	Gas Hydro...	Gas Ratio	Gas TVD (...	Gas TVD (...	Interface T...	MASP BHP	MASP Shoe	Minimum ...	Mud Hydr...	Mud Hydr...	Mud Ratio	Mud TVD (...	Mud TVD (...	Pressure a...	SW Hydros...	Interval	TVD Deep...	TVD Shoe	Well
116	7213.48...	0.15	1296.75	522.75	0.5	8645	3485	15662	5339.71...	3588.95...	3588.95...	5439.434	0	0.5	8645	0	15177.6...	3101.7792	81	32	42	80
117	23192.7...	0.15	1456.5	1456.5	0.5	9710	9710	16727	7783.82...	10434.5...	7783.82...	6866.912	8199.984	0.5	9710	11595	19209.0...	3101.7792	82	31	35	80
118	23192.7...	0.15	1476.75	1476.75	0.5	9845	9845	16862	7733.74...	10509.7...	7733.74...	6962.384	8104.512	0.5	9845	11460	19274.6...	3101.7792	83	33	35	80
119	23192.7...	0.15	1597.875	1597.875	0.5	10652.5	10652.5	17669.5	7588.55...	10959.6...	7588.55...	7533.448	7533.448	0.5	10652.5	10652.5	19821.6...	3101.7792	84	35	35	80
120	23192.7...	0.361	7691.105	7691.105	1	21305	21305	28322	9028.77...	12399.8...	9028.77...	0	0	0	0	0	19821.6...	3101.7792	85	35	35	80

MASP Calc. Step 2b (5)

Row	MAWP	MAWP Surf + 500	MAWP Surface	Interval	Reference MASP	Well
121	6690.737736	4052.735336	3552.735336	81	116	80
122	10885.606452	6970.510052	6470.510052	82	117	80
123	10835.521556	6190.657156	5690.657156	83	118	80
124	10690.338496	6300.892896	5800.892896	84	119	80
125	12130.556496	7558.668896	7058.668896	85	120	80

Zone of Interest (30)

Row	Backup Pore Pressure	Zone Name	Casing Section	RKB TVD
126	8.6	Top Section Backup Pore Pressure (ppg)	0	55
127	8.6	Zone of Interest 1 Backup Pore Pressure (ppg)	1	62
128	8.6	Zone of Interest 2 Backup Pore Pressure (ppg)	2	63
129	8.666	Zone of Interest 3 Backup Pore Pressure (ppg)	5	63
130	8.666	Zone of Interest 4 Backup Pore Pressure (ppg)	4	63
131	10.238	Bottom Section Backup Pore Pressure (ppg)	4	42
132	8.6	Top Section Backup Pore Pressure (ppg)	7	62
133	8.6	Zone of Interest 1 Backup Pore Pressure (ppg)	8	43
134	11.992	Zone of Interest 2 Backup Pore Pressure (ppg)	9	45
135	9.388	Zone of Interest 3 Backup Pore Pressure (ppg)	10	64
136	11.462	Zone of Interest 4 Backup Pore Pressure (ppg)	8	65
137	11.982	Zone of Interest 5 Backup Pore Pressure (ppg)	11	66
138	11.961	Bottom Section Backup Pore Pressure (ppg)	12	48
139	11.462	Top Section Backup Pore Pressure (ppg)	13	46
140	11.476	Zone of Interest 1 Backup Pore Pressure (ppg)	14	49
141	13.88	Bottom Section Backup Pore Pressure (ppg)	14	67
142	13.41	Top Section Backup Pore Pressure (ppg)	15	68
143	13.408	Zone of Interest 1 Backup Pore Pressure (ppg)	16	69
144	13.934	Zone of Interest 2 Backup Pore Pressure (ppg)	17	70
145	13.947	Zone of Interest 3 Backup Pore Pressure (ppg)	18	71
146	12.937	Zone of Interest 4 Backup Pore Pressure (ppg)	19	72
147	7.85	Zone of Interest 5 Backup Pore Pressure (ppg)	20	73
148	13.459	Bottom Section Backup Pore Pressure (ppg)	20	35
149	8.6	Top Section Backup Pore Pressure (ppg)	21	55
150	8.6	Zone of Interest 1 Backup Pore Pressure (ppg)	23	37
151	8.794	Zone of Interest 2 Backup Pore Pressure (ppg)	23	74
152	11.602	Zone of Interest 3 Backup Pore Pressure (ppg)	24	57
153	10.717	Zone of Interest 4 Backup Pore Pressure (ppg)	25	75
154	12.879	Zone of Interest 5 Backup Pore Pressure (ppg)	26	76
155	13.402	Bottom Section Backup Pore Pressure (ppg)	28	77

Figure 12: Data for Schema A

Burst Calculation Key (6)

Row	Backside EMW	Burst Rating	Burst Rating Corrected for MW ...	Constant	DHEMW	Material Utilization Factor	TVD	Item of Interest
0	8.6	13722	9605.4	0.052	8.6	0.7	7004	14
1	8.6	13890	9723	0.052	8.6	0.7	7287	15
2	8.6	15600	10920	0.052	8.6	0.7	7287	16
3	8.666	4275	3018.82344	0.052	8.6	0.7	7670	17
4	8.666	6364	4481.12344	0.052	8.6	0.7	7670	18
5	10.238	6364	5349.318352	0.052	8.6	0.7	10502	19

Casing Burst Key (5)

Row	Comment	Lowest Value Burst	Lowest Value Collapse	Pipe Body (for 123S) Burst	Pipe Body (for 123S) Collapse	Size	Weight/Grade
6	Casing Hanger, Supplemental Ad...	13722	12545	13722	12545	23	23x2 HPWH Extension Jt
7	Formerly 448.41 ppf X-80	13890	4000	13890	12690	23	453.04ppf X-80Q (H-100DM/QT)
8	From TDS 2-413383-02 Rev B, ...	15600	14200	15600	14200	23	16" 12.5K SA Upper
9	Burst Disk Use ONLY - 4000psi -...	4275	3873	?0	?1	22	4275psi Burst Disk
10	Dual Metal-to-Metal, Quick Thre...	6364	3873	6364	3873	22	224.21ppf X-80 (S-90DM/QT-...

Casing Key (1)

Row	Casing Type - Enter "Long Strin...	Hole Section TD - RKB TVD (Ft)	Max Pore Pressure at Hole Secti...	Mud Weight Casing Run In (Dow...	Mud Weight Casing Run In (Surf...	Item of Interest #	PPFP Key	Top Casing Section / Nominal ...
11	Long String	10600	10.281	12.5	12.5	14	27	12

Casing Section Key (1)

Row	Casing Section / Nominal Casing - Size	Well Data Key
12	Top 22 Surface Csg	34

Exposed Shoe Key (1)

Row	Deepest Exposed Shoe Below This Shoe - RKB TVD (Ft)	Frac Gradient at Deepest Exposed Shoe (ppg)	PPFP Key	Top Casing Section / Nominal Casing - Size
13	10502	13.209	30	12

Item of Inteerest Key (6)

Row	Item of Interest #	Item of Interest - Burst R...	Item of Interest Backup P...	Item of Interest Casing Se...	Item of Interest Casing Se...	Item of Interest Depth - R...	CBT Key	Liner Hangoff Key	PPFP Key	Top Casing Section / No...
14	Top 22 Surface Csg	13722	8.6	23	23x2 HPWH Extension Jt	7004	6	20	26	12
15	Zone of Interest 1	13890	8.6	23	453.04ppf X-80Q (H-...	7287	7	20	28	12
16	Zone of Interest 2	15600	8.6	23	16" 12.5K SA Upper	7287	8	20	28	12
17	Zone of Interest 3	4275	8.666	22	4275psi Burst Disk	7670	9	20	29	12
18	Zone of Interest 4	6364	8.666	22	224.21ppf X-80 (S-90...	7670	10	20	29	12
19	Btm of Casing/Shoe	6364	10.238	22	224.21ppf X-80 (S-90...	10502	10	20	30	12

Liner Hangoff Key (1)

Row	Mud Weight Used to Perform Fl...	Mud Weight Used to Perform Fl...	Mud Weight Used to Test 22 Ca...	Mud Weight Used to Test Casin...	Mud Weight Used to Test Casin...	Safety Margin Applied to MAWP...	Will a liner be hung off inside th...	Top Casing Section / Nominal ...
20	11.3	11.1	8.6	8.6	8.6	500	Yes	12

MASP Key (1)

Row	MASP	MASP Openhole	MASP Shoe	Casing Section Key	MASP Open Hole Key	MASP Shoe Key
21	3588.958536	5339.716512	3588.958536	12	22	23

MASP Open Hole Key (1)

Row	Constant	HC Grad.	MASP Open Hole	MW	Open Hole Depth yi...	Pore Pressure at O...	Sw Hydrostatic	TVDHC	TVDmud	Casing Section Key	Mud Gradient Key	OH Key	PPFP Key
22	0.052	0.15	5339.716512	12.1	24307	12.008	3101.7792	8645	8645	12	24	25	32

MASP Shoe Key (1)

Row	Constant	Deepest Exposed S...	Frac Gradient at De...	HC Grad.	MASP Shoe	MW	Sw Hydrostatic	TVDHC	TVDmud	Casing Section Key	Exposed Shoe Key	OH Key	PPFP Key
23	0.052	10502	13.209	0.15	3588.958536	12.1	3101.7792	3485	0	12	13	25	30

Mud Gradient Key (1)

Row	Gas Fraction	Mud Fraction	TVD Interface	Casing Section Key
24	0.5	0.5	15661.5	12

OH Key (1)

Row	Kick Fluid Gradient - Gas or Oil (psi/ft)	Kick Fluid Gradient if Gas per BS...	Max Mud Weight at OH Depth (...	Max Mud Weight at OH Depth (S...	Open Hole Depth yielding highe...	Pore Pressure at OH Depth (ppg)	PPFP Key	Top Casing Section / Nominal Casin...
25	0.15	0.15	12.1	11.9	24307	12.008	31	12

Pore Pressure Frac Pressure Key (7)

Row	Fracture Gradient (ppg)	Pore Pressure_mid (ppg)	RKB TVD	Salt FG (Using OBG + 1000 psi (ppg)
26	8.5	8.6	7004	8.5
27	-999.25	10.281	10572	13.24
28	8.715	8.6	7287	8.715
29	9.026	8.666	7672	9.026
30	-999.25	10.238	10502	13.209
31	14.478	12.008	24302	14.478
32	14.486	12.014	24307	14.486

Shoe Track Key (1)

Row	Planned FIT (Downhole,ppg)	Planned FIT (Surface,ppg)	Shoe Frac Gradient (ppg)	Item of Interest #	Top Casing Section / Nominal Casing - Size
33	13.2	13	13.207	19	12

Well Data Key (1)

Row	Block	Field/Prospect	RKB-ML	WD	Well	WellBore
34	Block ABC	Field 1	7017	6936	XYZ	ST00BP00

Figure 13: Data for Schema B

- MASP Calc. Step 2b Table
  - Five rows reflect the MASP result (for each casing section) based on the fracture gradient, which is calculated using the depth of the deepest exposed casing string (known as the casing shoe).
- Zone of Interest Table
  - This table reflects key attributes associated with each casing string, the components that make up that casing string, and the associated geological characteristics at a particular depth.

3.3.2 Data B

The input data for source B is shown in Figure 13. B's data includes the same source data as A, except the MASP calculation is only performed on a single casing section (as opposed to five). As we will see later, this is the primary reason why B's data will be extended significantly in the case study's resulting data exchange, but A's data will be unchanged. Further, both A and B include ancillary columns from the source sheet that were not used in the MASP calculation, resulting in additional differences at the data level. We now walk through the data contained in each table below.

- Burst Calculation Key Table
  - This table reflects the depths of interest considered for the MASP calculation of the 22" casing section.

- Key variables for step 1 of the MASP calculation (i.e., 70 percent burst pressure rating of the casing, de-rated and corrected for backup) are also included.

- Casing Burst Key Table
  - This table contains casing specifications for the 22" hole section.
  - Source B captured this data from a separate tab in the original Excel MASP sheet, whereas Source A did not use this table at all for casing data.
  - Question marks represent missing values in the original Excel MASP sheet.
- Casing Key Table
  - Only one well is modeled, hence there is only one row.
- Casing Section Key Table
  - This table contains a basic description of the 22" surface casing.
- Exposed Shoe Key Table
  - This table informs the fracture gradient at the deepest exposed shoe, which is a key input for the MASP shoe calculation.
- Item of Intereest [sic] Table
  - Similar to the Burst Calculation Key table, this table reflects the depths of interest where the MASP calculations were performed.
- Liner Hangoff Key Table
  - Data for this table is included in anticipation of MASP calculation steps beyond the scope of the case study. So while the data are not used for step 1 or step 2 for MASP as defined in Figure 11, they are typically used in subsequent steps of the casing design analysis.
- MASP Key Table
  - This table contains a summary of results from the MASP open hole and MASP shoe calculations.
- MASP Open Hole Key Table
  - This table contains the key inputs and calculations for the MASP open hole step of the analysis.
- MASP Shoe Key Table
  - This table contains the key inputs and calculations for the MASP shoe step of the analysis.
- Mud Gradient Key Table
  - This table informs the fluid column scenario for the MASP calculation (i.e., the ratio of gas-to-mud in the wellbore).
- OH Key Table
  - This table contains key information for the MASP open hole scenario, such as pore pressures and mud weights.
- Pore Pressure Frac Pressure Key Table

```
forall x y : "MASP Calc. Step 1", x."70% Burst (corrected)" = y."Burst Rating Corrected for MW & Backside" -> x = y
forall x y : "MASP Calc. Step 2a", x."MASP BHP" = y."MASP Openhole" -> x = y
forall x y : "Header Info", x."Well Name" = y."Well Name" -> x = y
forall x y : "Header Info", x."Well" = y."Well" -> x = y
```

Figure 14: Row merge rules

- The original source table contains over 4,000 rows of data, but not all are imported into CQL for expediency.
- This table represents geologic inputs from cross-domain stakeholders who govern the reservoir characteristics and earth model.

- Shoe Track Key Table
  - This table contains information for MASP shoe scenario, specifically the fracture gradient at the shoe depth.
- Well Data Key Table
  - Only one well is modeled, so there is a single row of data reflecting the general well header information.

3.3.3 Overlap Data and Mappings

Although the algebraic data integration design pattern requires an overlap instance I_O on the overlap schema S_O, it can be unfeasible to generate such instances by hand, because they are often large and difficult to describe as models. So, instead we describe rules–Horn clauses; implications of equations–that implicitly specify the overlap, including the required data mappings out of the overlap data. Figure 14 states that step 1 rows, now coming from both source A and source B, should be merged when they agree on two particular columns, and similarly for step 1a. If such a (recursive) merge is contradictory, CQL reports this, as described in Section 4.

Remark. Not all sets of Horn clauses (implications of equations) induce an overlap instance and two data mappings as required for algebraic integration. If we were to follow the algebraic integration pattern precisely [11], we would instead give a "query" of a specific form that materializes the matching pairs of source A rows and source B rows as an overlap instance. Such a query is a re-statement of our Horn clauses.

Remark. As an alternative to thinking of row merge rules as specifying an overlap instance for which a colimit is taken, we can also think of them as specifying a "lifting problem" that we are solving. If we do this, as occurs often in practice, then we can generalize our rule language to include existential quantifiers on the right-hand side of implications [11], dramatically increasing their expressive power.

3.3.4 Result

The the integrated result in algebraic data integration is given by taking a colimit of the input databases, each suitably pushed forward onto the colimit schema (plus additional user-defined equations, shown in Figure 10) via "left Kan extension" [11]. However, because this integrated result subsumes both spreadsheets, it is too large to conveniently display here (we do so in the appendix). So instead, we will focus on data exchange between the source sheets. That is, we project the integrated result back on to each source and analyze the difference with the original source. When we do this on this example, we find that source A is unchanged, but source B is extended significantly. Sometimes, the extensions to source B are with concrete values, such as in the "Burst Calculation Key" table, but other times the extensions are with blank values (labeled nulls), indicating that the source B is incomplete with respect to source A as defined by the overlap and mappings. Intuitively, this phenomenon happens because source B is in some sense contained source A, which has more rows to begin with. The "round-tripped" B data is displayed in Figures 15 and 16. The source owners and subject matter experts made the following observations about each table after the data exchange:

Burst Calculation Key (30)

Row	Backside EMW	Burst Rating	Burst Rating Corrected for MW ...	Constant	DHEMW	Material Utilization Factor	TVD	Item of Interest
0	8.666	6364	4481.123440	0.052	8.6	0.7	7670	97
1	8.666	4275	3018.823440	0.052	8.6	0.7	7670	98
2	8.6	15600	10920.0000	0.052	8.6	0.7	7287	99
3	10.238	6364	5349.318352	0.052	8.6	0.7	10502	100
4	8.6	13722	9605.4000	0.052	8.6	0.7	7004	101
5	8.6	13890	9723.0000	0.052	8.6	0.7	7287	102
6	13.459	18190	12819.891896	0.052	13.4	0.7	28322	103
7	7.85	18190	4791.59380	0.052	13.4	0.7	27517	104
8	12.937	14370	9485.702288	0.052	13.4	0.7	23812	105
9	13.947	18190	13410.166308	0.052	13.4	0.7	23807	106
10	13.934	14370	10719.933936	0.052	13.4	0.7	23802	107
11	13.408	18190	12742.899552	0.052	13.4	0.7	23797	108
12	13.41	9070	6361.37184	0.052	13.4	0.7	23792	109
13	13.88	14760	10059.87672	0.052	14.1	0.7	23787	110
14	11.476	14760	7086.993664	0.052	14.1	0.7	23782	111
15	11.462	10328	3969.338008	0.052	14.1	0.7	23767	112
16	11.961	10100	6894.670404	0.052	12.1	0.7	24257	113
17	11.982	10100	6921.036328	0.052	12.1	0.7	24277	114
18	11.462	12350	7856.174248	0.052	12.1	0.7	23777	115
19	13.402	15580	10251.297328	0.052	13.9	0.7	25282	116
20	9.388	10880	5030.748032	0.052	12.1	0.7	18332	117
21	12.879	18870	11894.866816	0.052	13.9	0.7	24752	118
22	11.992	10100	6987.433568	0.052	12.1	0.7	14702	119
23	10.717	15550	7121.662708	0.052	13.9	0.7	22737	120
24	8.6	12350	7317.8560	0.052	12.1	0.7	7292	121
25	11.602	18020	11000.565008	0.052	13.9	0.7	13502	122
26	8.6	12350	7318.7660	0.052	12.1	0.7	7287	123
27	8.794	16680	9536.769816	0.052	13.9	0.7	8057	124
28	8.6	16680	9742.1148	0.052	13.9	0.7	7017	125
29	8.6	15000	8569.6976	0.052	13.9	0.7	7004	126

Casing Burst Key (29)

Row	Comment	Lowest Value Burst	Lowest Value Collapse	Pipe Body (for 123S) Burst	Pipe Body (for 123S) Collapse	Size	Weight/Grade
30	Dual Metal-to-Metal, Quick Thre...	6364	3873	6364	3873	22	224.21ppf X-80 (S-90DM/QT-...
31	Burst Disk Use ONLY - 4000psi -...	4275	3873	?0	?1	22	4275psi Burst Disk
32	From TDS 2-413383-02 Rev B, ...	15600	14200	15600	14200	23	16" 12.5K SA Upper
33	Casing Hanger, Supplemental Ad...	13722	12545	13722	12545	23	23x2 HPWH Extension Jt
34	Formerly 448.41 ppf X-80	13890	4000	13890	12690	23	453.04ppf X-80Q (H-100DM/QT)
35	?2	?3	?4	?5	?6	?7	?8
36	?9	?10	?11	?12	?13	?14	?15
37	?16	?17	?18	?19	?20	?21	?22
38	?23	?24	?25	?26	?27	?28	?29
39	?30	?31	?32	?33	?34	?35	?36
40	?37	?38	?39	?40	?41	?42	?43
41	?44	?45	?46	?47	?48	?49	?50
42	?51	?52	?53	?54	?55	?56	?57
43	?58	?59	?60	?61	?62	?63	?64
44	?65	?66	?67	?68	?69	?70	?71
45	?72	?73	?74	?75	?76	?77	?78
46	?79	?80	?81	?82	?83	?84	?85
47	?86	?87	?88	?89	?90	?91	?92
48	?93	?94	?95	?96	?97	?98	?99
49	?100	?101	?102	?103	?104	?105	?106
50	?107	?108	?109	?110	?111	?112	?113
51	?114	?115	?116	?117	?118	?119	?120
52	?121	?122	?123	?124	?125	?126	?127
53	?128	?129	?130	?131	?132	?133	?134
54	?135	?136	?137	?138	?139	?140	?141
55	?142	?143	?144	?145	?146	?147	?148
56	?149	?150	?151	?152	?153	?154	?155
57	?156	?157	?158	?159	?160	?161	?162
58	?163	?164	?165	?166	?167	?168	?169

Casing Key (1)

Row	Casing Type - Enter "Long Strin...	Hole Section TD - RKB TVD (Ft)	Max Pore Pressure at Hole Secti...	Mud Weight Casing Run In (Dow...	Mud Weight Casing Run In (Surf...	Item of Interest #	PPFP Key	Top Casing Section / Nominal ...
59	Long String	10600	10.281	12.5	12.5	101	212	60

Casing Section Key (33)

Row	Casing Section / Nominal Casing - Size	Well Data Key
60	Top 22 Surface Csg	234
61	?170	234
62	?171	234
63	?172	234
64	?173	234
65	?174	234
66	?175	234
67	?176	234
68	?177	234
69	?178	234
70	?179	234
71	?180	234
72	?181	234
73	?182	234
74	?183	234
75	?184	234
76	?185	234
77	?186	234
78	?187	234
79	?188	234
80	?189	234
81	?190	234
82	?191	234
83	?192	234
84	?193	234
85	?194	234
86	?195	234
87	?196	234
88	?197	234
89	?198	234
90	?199	234
91	?200	234
92	?201	234

Figure 15: Data Exchange of Source A to Source B, 1 of 3

Exposed Shoe Key (5)

Row	Deepest Exposed Shoe Below This Shoe - RKB TVD (Ft)	Frac Gradient at Deepest Exposed Shoe (ppg)	PPFP Key	Top Casing Section / Nominal Casing - Size
93	10502	13.209	184	60
94	?202	?203	188	61
95	?204	?205	193	63
96	?206	?207	198	65
97	?208	?209	203	67

Item of Inteerest Key (30)

Row	Item of Interest #	Item of Interest - Burst R...	Item of Interest Backup P...	Item of Interest Casing Se...	Item of Interest Casing Se...	Item of Interest Depth - R...	CBT Key	Liner Hangoff Key	PPFP Key	Top Casing Section / No...
98	Zone of Interest 4	6364	8.666	22	224.21ppf X-80 (S-90...	7670	30	128	182	60
99	Zone of Interest 3	4275	8.666	22	4275psi Burst Disk	7670	31	128	182	60
100	Zone of Interest 2	15600	8.6	23	16" 12.5K SA Upper	7287	32	128	183	60
101	Btm of Casing/Shoe	6364	10.238	22	224.21ppf X-80 (S-90...	10502	30	128	184	60
102	Top 22 Surface Csg	13722	8.6	23	23x2 HPWH Extension Jt	7004	33	128	185	60
103	Zone of Interest 1	13890	8.6	23	453.04ppf X-80Q (H-...	7287	34	128	183	60
104	?210	18190	13.459	?211	?212	28322	35	129	208	69
105	?213	18190	7.85	?214	?215	27517	36	130	209	70
106	?216	14370	12.937	?217	?218	23812	37	131	210	71
107	?219	18190	13.947	?220	?221	23807	38	132	211	72
108	?222	14370	13.934	?223	?224	23802	39	133	212	73
109	?225	18190	13.408	?226	?227	23797	40	134	213	74
110	?228	9070	13.41	?229	?230	23792	41	135	214	75
111	?231	14760	13.88	?232	?233	23787	42	136	215	76
112	?234	14760	11.476	?235	?236	23782	43	137	216	77
113	?237	10328	11.462	?238	?239	23767	44	138	217	78
114	?240	10100	11.961	?241	?242	24257	45	139	218	79
115	?243	10100	11.982	?244	?245	24277	46	140	219	80
116	?246	12350	11.462	?247	?248	23777	47	141	220	81
117	?249	15580	13.402	?250	?251	25282	48	142	221	82
118	?252	10880	9.388	?253	?254	18332	49	143	222	83
119	?255	18870	12.879	?256	?257	24752	50	144	223	84
120	?258	10100	11.992	?259	?260	14702	51	145	224	85
121	?261	15550	10.717	?262	?263	22737	52	146	225	86
122	?264	12350	8.6	?265	?266	7292	53	147	226	87
123	?267	18020	11.602	?268	?269	13502	54	148	227	88
124	?270	12350	8.6	?271	?272	7287	55	149	228	89
125	?273	16680	8.794	?274	?275	8057	56	150	229	90
126	?276	16680	8.6	?277	?278	7017	57	151	230	91
127	?279	15000	8.6	?280	?281	7004	58	152	231	92

Liner Hangoff Key (25)

Row	Mud Weight Used to Perform Fl...	Mud Weight Used to Perform Fl...	Mud Weight Used to Test 22 Ca...	Mud Weight Used to Test Casin...	Mud Weight Used to Test Casin...	Safety Margin Applied to MAWP...	Will a liner be hung off inside th...	Top Casing Section / Nominal ...
128	11.3	11.1	8.6	8.6	8.6	500	Yes	60
129	?282	?283	?284	?285	?286	?287	?288	69
130	?289	?290	?291	?292	?293	?294	?295	70
131	?296	?297	?298	?299	?300	?301	?302	71
132	?303	?304	?305	?306	?307	?308	?309	72
133	?310	?311	?312	?313	?314	?315	?316	73
134	?317	?318	?319	?320	?321	?322	?323	74
135	?324	?325	?326	?327	?328	?329	?330	75
136	?331	?332	?333	?334	?335	?336	?337	76
137	?338	?339	?340	?341	?342	?343	?344	77
138	?345	?346	?347	?348	?349	?350	?351	78
139	?352	?353	?354	?355	?356	?357	?358	79
140	?359	?360	?361	?362	?363	?364	?365	80
141	?366	?367	?368	?369	?370	?371	?372	81
142	?373	?374	?375	?376	?377	?378	?379	82
143	?380	?381	?382	?383	?384	?385	?386	83
144	?387	?388	?389	?390	?391	?392	?393	84
145	?394	?395	?396	?397	?398	?399	?400	85
146	?401	?402	?403	?404	?405	?406	?407	86
147	?408	?409	?410	?411	?412	?413	?414	87
148	?415	?416	?417	?418	?419	?420	?421	88
149	?422	?423	?424	?425	?426	?427	?428	89
150	?429	?430	?431	?432	?433	?434	?435	90
151	?436	?437	?438	?439	?440	?441	?442	91
152	?443	?444	?445	?446	?447	?448	?449	92

MASP Key (5)

Row	MASP	MASP Openhole	MASP Shoe	Casing Section Key	MASP Open Hole Key	MASP Shoe Key
153	3588.958536	5339.716512	3588.958536	60	158	163
154	(((((?454 * (?457 * ?462)) - (((0 MAX (...	((((?454 * (?457 * ?462)) - (((0 MAX ((...	((((?459 * (?203 * ?455)) - ((?458 * ?4...	61	159	164
155	(((((?475 * (?464 * ?469)) - (((0 MAX (...	((((?475 * (?464 * ?469)) - (((0 MAX ((...	((((?466 * (?205 * ?463)) - ((?465 * ?4...	63	160	165
156	(((((?478 * (?481 * ?487)) - (((0 MAX (...	((((?478 * (?481 * ?487)) - (((0 MAX ((...	((((?483 * (?207 * ?482)) - ((?484 * ?4...	65	161	166
157	(((((?498 * (?500 * ?493)) - (((0 MAX (...	((((?498 * (?500 * ?493)) - (((0 MAX ((...	((((?490 * (?209 * ?501)) - ((?489 * ?4...	67	162	167

MASP Open Hole Key (5)

Row	Constant	HC Grad.	MASP Open Hole	MW	Open Hole Depth yi...	Pore Pressure at O...	Sw Hydrostatic	TVDHC	TVDmud	Casing Section Key	Mud Gradient Key	OH Key	PPFP Key
158	0.052	0.15	5339.716512	12.1	24307	12.008	3101.7792	8645.0	8645.0	60	168	173	187
159	?462	?451	((((?454 * (?457 *...	?461	?454	?457	3101.7792	(0 MAX ((?454 - ...	(0 MAX ((?454 - ...	61	169	175	191
160	?469	?472	((((?475 * (?464 *...	?468	?475	?464	3101.7792	(0 MAX ((?475 - ...	(0 MAX ((?475 - ...	63	170	177	196
161	?487	?476	((((?478 * (?481 *...	?479	?478	?481	3101.7792	(0 MAX ((?478 - ...	(0 MAX ((?478 - ...	65	171	179	201
162	?493	?499	((((?498 * (?500 *...	?492	?498	?500	3101.7792	(0 MAX ((?498 - ...	(0 MAX ((?498 - ...	67	172	181	206

MASP Shoe Key (5)

Row	Constant	Deepest Exposed S...	Frac Gradient at De...	HC Grad.	MASP Shoe	MW	Sw Hydrostatic	TVDHC	TVDmud	Casing Section Key	Exposed Shoe Key	OH Key	PPFP Key
163	0.052	10502	13.209	0.15	3588.958536	12.1	3101.7792	3485	0	60	93	173	184
164	?455	?459	?203	?453	((((?459 * (?203 *...	?456	3101.7792	?452	?458	61	94	174	190
165	?463	?466	?205	?474	((((?466 * (?205 *...	?473	3101.7792	?471	?465	63	95	176	195
166	?482	?483	?207	?477	((((?483 * (?207 *...	?480	3101.7792	?488	?484	65	96	178	200
167	?501	?490	?209	?497	((((?490 * (?209 *...	?496	3101.7792	?495	?489	67	97	180	205

Mud Gradient Key (5)

Row	Gas Fraction	Mud Fraction	TVD Interface	Casing Section Key
168	0.5	0.5	15661.5	60
169	?460	?450	?502	61
170	?467	?470	?503	63
171	?485	?486	?504	65
172	?491	?494	?505	67

OH Key (9)

Row	Kick Fluid Gradient - Gas or Oil ...	Kick Fluid Gradient if Gas per BS...	Max Mud Weight at OH Depth (D...	Max Mud Weight at OH Depth (S...	Open Hole Depth yielding highe...	Pore Pressure at OH Depth (ppg)	PPFP Key	Top Casing Section / Nominal ...
173	0.15	0.15	12.1	11.9	24307	12.008	186	60
174	?506	?507	?456	?508	?509	?510	189	62
175	?511	?451	?461	?512	?513	?457	192	61
176	?514	?515	?473	?516	?517	?518	194	64
177	?519	?472	?468	?520	?521	?464	197	63
178	?522	?523	?480	?524	?525	?526	199	66
179	?527	?476	?479	?528	?529	?481	202	65
180	?530	?531	?496	?532	?533	?534	204	68
181	?535	?499	?492	?536	?537	?500	207	67

Figure 16: Data Exchange of Source A to Source B, 2 of 3

Pore Pressure Frac Pressure Key (51)

Row	Fracture Gradient (ppg)	Pore Pressure_mid (ppg)	RKB TVD	Salt FG (Using OBG + 1000 psi (ppg)
182	9.026	8.666	7672	9.026
183	8.715	8.6	7287	8.715
184	-999.25	10.238	10502	13.209
185	8.5	8.6	7004	8.5
186	14.478	12.008	24302	14.478
187	14.486	12.014	24307	14.486
188	?538	?539	?540	?203
189	?541	?510	?542	?543
190	?544	?545	?546	?547
191	?548	?549	?550	?551
192	?552	?457	?553	?554
193	?555	?556	?557	?205
194	?558	?518	?559	?560
195	?561	?562	?563	?564
196	?565	?566	?567	?568
197	?569	?464	?570	?571
198	?572	?573	?574	?207
199	?575	?526	?576	?577
200	?578	?579	?580	?581
201	?582	?583	?584	?585
202	?586	?481	?587	?588
203	?589	?590	?591	?209
204	?592	?534	?593	?594
205	?595	?596	?597	?598
206	?599	?600	?601	?602
207	?603	?500	?604	?605
208	?606	13.459	?607	?608
209	?609	7.85	?610	?611
210	?612	12.937	?613	?614
211	?615	13.947	?616	?617
212	?618	13.934	?619	?620
213	?621	13.408	?622	?623
214	?624	13.41	?625	?626
215	?627	13.88	?628	?629
216	?630	11.476	?631	?632
217	?633	11.462	?634	?635
218	?636	11.961	?637	?638
219	?639	11.982	?640	?641
220	?642	11.462	?643	?644
221	?645	13.402	?646	?647
222	?648	9.388	?649	?650
223	?651	12.879	?652	?653
224	?654	11.992	?655	?656
225	?657	10.717	?658	?659
226	?660	8.6	?661	?662
227	?663	11.602	?664	?665
228	?666	8.6	?667	?668
229	?669	8.794	?670	?671
230	?672	8.6	?673	?674
231	?675	8.6	?676	?677
232	-999.25	10.281	10572	13.24

Shoe Track Key (1)

Row	Planned FIT (Downhole,ppg)	Planned FIT (Surface,ppg)	Shoe Frac Gradient (ppg)	Item of Interest #	Top Casing Section / Nominal Casing - Size
233	13.2	13	13.207	101	60

Well Data Key (1)

Row	Block	Field/Prospect	RKB-ML	WD	Well	WellBore
234	Block ABC	Field 1	7017	6936	XYZ	ST00BP00

Figure 17: Data Exchange of Source A to Source B, 3 of 3

- Burst Calculation Key Table
  - Whereas source B started with six rows in this table, the data exchange with source A through the integrated schema results in 30 rows of data. This is due to source B sharing six overlapping rows with source A (these were merged together) while receiving 24 rows as new information gain.
  - The specific rows being merged are based on the 70 percent burst rating (corrected) values for each source.
  - Filled-in values can be traced primarily to the MASP "Calc. Step 1" table from source A, but includes other information implied by myriad equations and mappings.
- Casing Burst Key Table
  - This table is primarily associated with the "Casing Section" table from source A. This is observed by the 29 rows in both tables.
  - The question marks are due to the fact that source A does not include any information about these columns (such as "Lowest Value Collapse"). However, it was observed while creating rules that the source B Size column did not find a match in source A because of a difference in data type. This casing size data is represented as a string in source A and as a float in source B, where in particular, where source B writes 22 where source A writes 22". To merge along such attributes, we can add float to string and string concatenation functions to our ambient "theory of excel" and then write an equation using these new primitives (e.g. 22"=concat(floatToString(22),").
  - From source A's perspective, this table's columns are not necessary in the MASP calculation, so they are largely excluded from data exchange. Instead, source B includes these values because source B's data was pulled from a separate dataset containing casing specification information.
- Casing Key Table

  - Most of this data is not used for the MASP calculation and is in source B and not source A, explaining why there is only one row for source B.

- Casing Section Key Table
  - Source B only models one string of casing (the 22” casing section), which is reflected in the fully populated row. The remaining rows are from source A and reflect the additional casing sections/components that are transferred from the original MASP sheet.

- Exposed Shoe Key Table
  - Source B gains four rows of data because the MASP calculation is performed for five hole sections for source A.

- Item of Intereest Key Table [sic]
  - Like the “Burst Calculation Key” table, the “Item of Intereest Key” table begina with six rows of data and, through the data exchange with aource A, resulta in an additional 24 rows of new data. However, in this case, there are additional rules in place to infer data values for two additional columns (“Item of Interest - Burst Rating (psi)” and “Item of Interest Depth - RKB TVD (Ft)”.

- Liner Hangoff Key Table
  - This table is not used for the MASP calculation, and the only row of data comes from Source B.

- MASP Key, MASP Open Hole Key, MASP Shoe Key, and Mud Gradient Key Tables
  - For these tables, source B gains four rows of data each because the MASP calculation is performed for five hole sections for source A.

- OH Key Table
  - Most of the data from this table is not used for the MASP calculation, so there are minimal opportunities to create merge rules.

- Pore Pressure Frac Pressure Key Table
  - This table matches with the sources on six rows because six pore pressure / fracture gradient inputs were used in the 22” casing section MASP calculated by source B. Because this data is shared by the subsurface discipline, and source A, and source B, the entire table could, in principle, be integrated with additional row merge rules in place.

- Shoe Track Key Table
  - Most of this table is not used for the MASP calculation, and the only row of data comes from source B.

- Well Data Key Table
  - There is only one row in this table because the data is merged based on the well name. This ensures both source A and source B are performing the MASP calculation for the same well.

3.4 Discussion

Our main result from the case study is that two engineers were able to conduct the same engineering analysis, without ever communicating with each other, and compose their individual models to create an integrated spreadsheet with integrated perspective. Specifically, two engineers performed a MASP calculation, which is a key aspect of well construction and is performed on every well around the world in a variety of fashions. The integrated result guarantees that individual ologs and integrated ologs are compliant with components of the MASP calculation technical requirements, without pre-coordination and consensus. As a bonus, we also obtain a semantics-preserving data-exchange between the source sheets.

Engineering teams largely go about integration and data exchange today by treating subject matter experts as the personification of requirements, which are originally written in natural language as text inside documents that we call global standards or regulations. The current mainstream approach taken by industry is to try to organize these requirements by extracting them from documents and placing them into more sophisticated management systems (e.g., a database where each requirement is an object, and a user interface that includes workflows to create, edit, and approve requirements). This approach relies on the idea that after the requirements are organized in the requirement management system, engineers will be able to convert them into rules that can be incorporated into computable models. The challenge to do this, so far, has been the lack of an adequate language that is expressive enough to capture both the general nature of an engineering requirement and its specific computational nature.

By representing objects and models as ologs, engineers can define data structure in a way that makes sense to them. Those structures can then be translated to a shared space within which these objects reside, using automated methods for their construction and inter-relation. We pick up this line of thought in the conclusion.

3.5 Governance

Left as open questions are the authorship and/or governance of the overlap ologs required to integrate sheets– either of the original engineers can create (potentially many, depending on the desired integration semantics) such overlaps, as can 3rd parties (and in fact, CQL can automatically suggest overlap between ologs). In fact, at a formal level, managing an overlap between two ologs is "dual" to managing a collection of WHERE clauses in SQL queries that join two databases together (the algebraic integration semantics of Figure 2 being formally dual to SQL's join semantics as well). As such, governance of overlaps is too large of a topic to go into here. Regardless of authorship, semantics-preservation is guaranteed without requiring consensus by relying on the formulas in the sheets to fully define semantics.

3.6 Generalization to Multi-Model Merge

In this paper we have focused on an olog merge of the form $\cdot \leftarrow \cdot \rightarrow \cdot$, where the center dot is the overlap olog and the other two dots are the source olog. However, all of the theory and algorithms in this paper generalize to diagrams of any shape [11], such as

$$\begin{tikzcd}
& \cdot \arrow[dl] \arrow[dr] & & \cdot \arrow[ll] \arrow[rr] & & \cdot \arrow[dl] \arrow[dr] & \\
\cdot & & \cdot & & \cdot & & \cdot
\end{tikzcd}$$

It is easy to see that a data integration of the above form requires only logarithmically many mappings in the number of nodes, rather than quadratically many that would be required if all nodes were to be mapped to all nodes. It is because our methodology can merge any shape of olog mappings, such as the above, that we say it does not require every source to be related to every source.

4 Generating and Checking Verification Conditions

This section will be of interest primarily to computer scientists.

4.1 Functoriality Conditions

In this section we describe how to generate and solve the verification conditions required to ensure the semantic consistency of the source schemas with respect to the overlap schema. The verification conditions associated to schema mapping $M_A : S_O \to S_A$ ensure/are that S_A proves $M_A(p) = M_A(q)$ for every equation $p = q$ of S_O, and similarly for M_B. This establishes that M_A and M_B are functors and hence that we can proceed with algebraic data integration. In Figure 18 we display these verification conditions for M_A and M_B in the Coq proof assistant [3] where they can be proved by humans. The CQL tool also emits the verification conditions in TPTP form [14] so that they can be input to a variety of automated theorem provers [2]. It took one of the authors approximately four hours to prove all the generated conditions in this paper in Coq with elementary methods, using a handful of basic assumptions about e.g. commutativity of addition, but with no domain expertise. Although checking functoriality is undecidable in general, the conditions can always be decided for a particular model, providing a slightly weaker but still useful guarantee.

4.2 Conservativity/Consistency Conditions

In this section we describe how to generate and solve the verification conditions required to ensure the semantic consistency of the integrated result olog. The integrated result is a quotient of a coproduct of input databases; to compute the quotient, a step-by-step process is used, where at each step a new olog is constructed from the previous one (starting from the coproduct of the inputs and terminating on the integrated result). At each step of the process an equational theory representing the result is modified by adding additional equations and/or symbols [11]. For example, at one step we might add $Alice.age = 20$ and at another step add $Bob.age = 30$ and at another step add $Alice = Bob$, from which it follows that $20 = 30$– a contradiction, or rather, a "non-conservative extension" of our original definition of numbers, for which 20 and 30 are not equal (number systems in which $20 = 30$ are found in e.g. modular arithmetic and can in fact be represented by ologs). At every step of the integration computation, CQL checks that there are no equations that reduce to $c_1 = c_2$ for distinct numerals, strings, etc under the usual reduction rules for arithmetic such as $1+1 \mapsto 2$ etc, providing significant defense against contradictions. In general, contradiction detection is undecidable but definitional/free spreadsheets (see section 2) are always contradiction-free and CQL can check ologs for freeness, providing more protection still.

5 Conclusion and Applications

Our case study is an example of a non-human-consensus-based method for merging engineering models, built on the idea that formality drives consensus in the semantic sense even if the people involved never meet. We conclude by describing a particular enterprise example use case and then describe related work.

5.1 An Example Use Case

Our use case is to certify that an (equational) engineering model complies with (equational) requirements, a role today performed by human auditors working with English text and/or spreadsheets. We focus on this use case because it is the smallest use case that we can think of that is captured by any part of this paper's case study, but as we will see it can still have significant impact. In particular, if we have R (equational) requirements to certify on W (equational) models each year at a cost of C per certification then the direct cost savings of this use case is $T \times W \times C$ annually, a number that can immediately be seen to be large in large enterprises when there are hundreds of thousands of (equational) requirements to check each year.

To implement the use case we require two olog schemas, S and T, and a schema mapping $F : S \to T$. For example, we may let S be this paper's overlap schema S_O (Figure 7) and let T be this paper's source schema A, namely, S_A (Figure 3) (or source schema B, namely, S_B, Figure 5) and let F be this paper's schema mapping $S_O \to S_A$, shown in Figure 8 (resp. Figure 9 for source B). The core of the use case is to check if F denotes a functor, i.e., check if F translates the equations of S into T in a way that preserves provability as described in Section 4. If so, then we say that T complies with the requirements of S as viewed by F. Optionally, we may additionally take as input an S-Instance X and T-Instance Y and check that $X = F; Y$, which in this case study corresponds to checking that the equations satisfied by the overlap instance I_O hold

```
//Schema A
Parameters String Float : Type.
Parameters plus times : Float * Float -> Float.
...
Parameters CasingSection ZoneofInterest MASPCalcStep1 : Set.
...
Parameter HeaderInfo_TypeofWell : HeaderInfo -> String.
Parameter HeaderInfo_RigName : HeaderInfo -> String.
Parameter HeaderInfo_Field : HeaderInfo -> String.
...
Axiom ax0 : forall (x : HeaderInfo),  HeaderInfo_RKBML(x) = plus(HeaderInfo_WaterDepth(x), HeaderInfo_RKBHeight(x)).
Axiom ax1 : forall (x : MASPCalcStep2a),  MASPCalcStep2a_MASPShoe(x) =
    minus(minus(minus(MASPCalcStep2a_FracPressureatDeepestShoe(x), MASPCalcStep2a_MudHydrostaticShoe(x)),
        MASPCalcStep2a_GasHydrostaticShoe(x)), MASPCalcStep2a_SWHydrostatic(x)).
Axiom ax2 : forall (x : MASPCalcStep2a),  MASPCalcStep2a_InterfaceTVDBHP(x) =
    plus(HeaderInfo_RKBML(MASPCalcStep2a_Well(x)),
        times(MASPCalcStep2a_GasRatio(x), minus(FGPPInputs_RKBTVD(MASPCalcStep2a_TVDDeepestOH(x)),
        HeaderInfo_RKBML(MASPCalcStep2a_Well(x))))).
Axiom ax3 : forall (x : MASPCalcStep2b),  MASPCalcStep2b_MAWP(x) =
    plus(MASPCalcStep2a_MinimumMASP(MASPCalcStep2b_ReferenceMASP(x)),
    MASPCalcStep2a_SWHydrostatic(MASPCalcStep2b_ReferenceMASP(x))).
...
Definition t_Step1_pb : MASPCalcStep1 -> Float := fun x => CasingSection_BurstRating(MASPCalcStep1_CasingSection(x)).
Definition t_Step1_pmud : MASPCalcStep1 -> Float := fun x => IntervalInfo_DownholeMudWeight(MASPCalcStep1_Interval(x)).
Definition t_Step1_ppore : MASPCalcStep1 -> Float := fun x => ZoneofInterest_BackupPorePressure(MASPCalcStep1_ZoneName(x)).
...
Conjecture conj22 : forall (i : MASPCalcStep1),  t_Step1_casingburst(i) =
    minus(times(t_Step1_seventy(i), t_Step1_pb(i)),
    times(t_Step1_tvd(i), times(t_Step1_pointohfivetwo(i),
    minus(t_Step1_pmud(i), t_Step1_ppore(i))))).
Conjecture conj23 : forall (i : MASPCalcStep2a),  t_Step2_maspbhp(i) =
    minus(t_Step2_bhp(i), plus(t_Step2_mhs(i), plus(t_Step2_ghs(i), t_Step2_swhs(i)))).
...
```

```
//Schema B
Parameters String Float : Type.
Parameters plus times : Float * Float -> Float.
...
Parameters WellDataKey CasingKey BurstCalculationKey MASPShoeKey MASPKey : Set
...
Parameter WellDataKey_Block : WellDataKey -> String.
Parameter WellDataKey_Well : WellDataKey -> String.
Parameter WellDataKey_WellBore : WellDataKey -> String.
...
Parameter CasingKey_ItemofInterest : CasingKey -> ItemofInteerestKey.
Parameter CasingKey_PPFPKey : CasingKey -> PorePressureFracPressureKey.
...
Axiom ax0 : forall (x : CasingKey),  CasingKey_MaxPorePressureatHoleSectionTDppg(x) =
    PorePressureFracPressureKey_PorePressuremidppg(CasingKey_PPFPKey(x)).
Axiom ax1 : forall (x : BurstCalculationKey),  BurstCalculationKey_TVD(x) =
    ItemofInteerestKey_ItemofInterestDepthRKBTVDFt(BurstCalculationKey_ItemofInterest(x))
Axiom ax2 : forall (x : MASPShoeKey),  MASPShoeKey_FracGradientatDeepestExposedShoeppg(x) =
    ExposedShoeKey_FracGradientatDeepestExposedShoeppg(MASPShoeKey_ExposedShoeKey(x)).
Axiom ax3 : forall (x : MASPKey),  MASPKey_MASPOpenhole(x) =
    MASPOpenHoleKey_MASPOpenHole(MASPKey_MASPOpenHoleKey(x)).
...
Definition t_Step1_pb : BurstCalculationKey -> Float :=
    fun x => ItemofInteerestKey_ItemofInterestBurstRatingpsi(BurstCalculationKey_ItemofInterest(x)).
Definition t_Step1_pmud : BurstCalculationKey -> Float := fun x => BurstCalculationKey_DHEMW(x).
Definition t_Step1_ppore : BurstCalculationKey -> Float := fun x => BurstCalculationKey_BacksideEMW(x).
...
Conjecture conj21 : forall (i : MASPKey),  t_Step2b_maspshoe(i) =
    minus(t_Step2b_bhp(i), plus(t_Step2b_mhs(i), plus(t_Step2b_ghs(i), t_Step2b_swhs(i)))).
Conjecture conj22 : forall (i : BurstCalculationKey),  t_Step1_casingburst(i) =
    minus(times(t_Step1_seventy(i), t_Step1_pb(i)),
    times(t_Step1_tvd(i), times(t_Step1_pointohfivetwo(i), minus(t_Step1_pmud(i), t_Step1_ppore(i))))).
Conjecture conj23 : forall (i : MASPKey),  t_Step2_maspbhp(i) =
    minus(t_Step2_bhp(i), plus(t_Step2_mhs(i), plus(t_Step2_ghs(i), t_Step2_swhs(i)))).
```


Figure 18: Coq Verification Conditions for Mapping Functoriality

in the source instance I_A (resp. in I_B) once translated along F. That is, our use case can apply to models without data (equations only) as well as to models with data.

The discussion in the preceding paragraph does not describe any method for constructing the inputs to the use case. One option is to simply build up a library of ologs and olog mappings manually, and with composition, such as we did in this paper. Another option is to heuristically generate ologs and mappings from spreadsheets, or even English-text, on-the-fly, to be reviewed/modified by a human– in general, choice will be required among various possible oligifications of a given spreadsheet and among the various possible mappings between ologs. Such heuristics are the subject of future work, but no matter which ologs and mappings are chosen, we can be confident that if the functoriality check succeeds then the (equational) requirements are met. And we are confident that there are many more use cases besides this one waiting to be discovered.

5.2 Related Work

Besides relations to algebraic databases and traditional data integration [11], our algorithm is related to the idea of extending spreadsheets with deductive logic, for example with prolog, an idea whose long history is described in [9]. Indeed, our rules in this paper are expressible in prolog, at least when prolog is understood to include excel's arithmetic operations, which often it isn't (it is unclear how much arithmetic is modeled in [9] fop example). However, unlike any work in [9], our goal is to connect multiple spreadsheets, not query different parts of the same sheet, which constrains the form of the rules we can write to a subset of prolog, and also constrains the vocabulary we can use within prolog. That is, the algebraic design pattern of Figure 2 says which rules to write, whereas 2 provides no guidance.

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6 Appendix: Category theory

In this section, we review standard definitions and results from category theory [1]. A *quiver*, (aka directed multi-graph) Q consists of a class $\mathsf{Ob}(Q)$, the members of which we call *objects* (or *nodes*), and for all objects c_1, c_2, a set $Q(c_1, c_2)$, the members of which we call *morphisms* (or *arrows*) from c_1 to c_2. We may write $f : c_1 \to c_2$ or $c_1 \to_f c_2$ instead of $f \in C(c_1, c_2)$. For an arrow $f : c_1 \to c_2$ in a quiver, we call c_1 the *source* of f and c_2 the *target* of f. In a quiver Q, a *path* from c_1 to c_k is a non-empty finite list of nodes and arrows $c_1 \xrightarrow{e_1} c_2 \xrightarrow{e_2} \cdots \xrightarrow{e_{k-1}} c_k$. A *category* C is a quiver equipped with the following structure:

- for all objects c_1, c_2, c_3, a function $\circ_{c_1,c_2,c_3} : C(c_2, c_3) \times C(c_1, c_2) \to C(c_1, c_3)$, which we call *composition*, and for which we write $x; y$ to mean $y \circ x$, and
- for every object c, an arrow $\mathsf{id}_c \in C(c, c)$, which we call the *identity* for c.

We may drop subscripts on id and $\circ$, when doing so does not create ambiguity. These data must obey axioms stating that $\circ$ is associative and id is its unit:

$$\mathsf{id} \circ f = f \qquad f \circ \mathsf{id} = f \qquad f \circ (g \circ h) = (f \circ g) \circ h$$

An object c of a category C is called *initial* if for all $c' \in C$, there is a unique morphism $c \to c'$. Dually, an object c of a category is called *final/terminal* if for all $c' \in C$, there is a unique morphism $c' \to c$. A *functor* $F : C \to D$ between categories C and D consists of:

- a class function $F : \mathsf{Ob}(C) \to \mathsf{Ob}(D)$, and
- for every $c_1, c_2 \in \mathsf{Ob}(C)$, a function $F_{c_1,c_2} : C(c_1, c_2) \to D(F(c_1), F(c_2))$, where we may omit object subscripts when they can be inferred, such that

$$F(\mathsf{id}_c) = \mathsf{id}_{F(c)} \qquad F(f \circ g) = F(f) \circ F(g).$$

A *natural transformation* $h : F \to G$ between functors $F, G : C \to D$ consists of a family of morphisms $h_c : F(c) \to G(c)$, indexed by objects in C, called the *components* of h, such that for every $f : c_1 \to c_2$ in C we have $h_{c_2} \circ F(f) = G(f) \circ h_{c_1}$. The family of equations defining a natural transformation may be depicted as a *commutative diagram*:

$$\begin{tikzcd}
F(c_1) \arrow[r, "F(f)"] \arrow[d, "h_{c_1}"'] & F(c_2) \arrow[d, "h_{c_2}"] \\
G(c_1) \arrow[r, "G(f)"'] & G(c_2)
\end{tikzcd}$$

The commutativity of such a diagram means that any two parallel paths in the diagram have the same composition in D. A *pushout* of objects A, B, C and morphisms f, g in a category, as shown below, is an object D and morphisms α and β as shown below, having the universal property that for any other such D' and α' and β', there is a unique morphism θ making the diagram commute:

$$\begin{tikzcd}
A \arrow[r, "g"] \arrow[d, "f"'] & C \arrow[d, "\beta"'] \arrow[ddr, bend left, "\beta'"] & \\
B \arrow[r, "\alpha"] \arrow[drr, bend right, "\alpha'"'] & D \arrow[ul, phantom, very near start, "\ulcorner"] \arrow[dr, dashed, "\theta"] & \\
 & & D'
\end{tikzcd}$$

The dual notion of pushout is *pullback*. A pushout over an initial object is called a co-product. A pullback over a final object is called a product. Pushouts generalize to arbitrary diagrams, in which case they are called colimits, but we do not define colimits here. Given two functors $F : C \to D$ and $G : D \to C$, we say that F is *left adjoint* to G, written $F \dashv G$, when for every object c in C and d in D that the set of morphisms $F(c) \to d$ in D is isomorphic to the set of morphisms $c \to G(d)$ in C, naturally in c and d (i.e., when we independently consider each side of the isomorphism as a functor $C \to \mathsf{Set}$ and as a functor $\mathsf{D} \to \mathsf{Set}$).

7 Appendix: Equational Logic and Universal Algebra

In this section we review standard material on multi-sorted equational logic, following the exposition in [11]. A *signature* Sig consists of:

1. A set $Sorts$ whose elements are called *sorts*,

2. A set $Symbols$ of pairs $(f, s_1 \times \ldots \times s_k \to s)$ with $s_1, \ldots, s_k, s \in Sorts$ and no f occurring in two distinct pairs. We write $f : X$ instead of $(f, X) \in Symbols$. When $k = 0$, we may call f a *constant symbol* and write $f : s$ instead of $f : \to s$. Otherwise, we may call f a *function symbol*.

We assume we have some countably infinite set $\{v_1, v_2, \ldots\}$, whose elements we call *variables* and which are assumed to be distinct from any sort or symbol we ever consider. A *context* Γ is defined as a finite set of variable-sort pairs, with no variable given more than one sort:

$$\Gamma := \{v_1 : s_1, \ldots, v_k : s_k\}$$

When the sorts $s_1, \ldots, s_k$ can be inferred, we may write a context as $\{v_1, \ldots, v_k\}$. We may write $\{v_1 : s, \ldots, v_k : s\}$ as $\{v_1, \ldots, v_k : s\}$. We may write $\Gamma \cup \{v : s\}$ as $\Gamma, v : s$. We inductively define the set $Terms^s(Sig, \Gamma)$ of *terms* of sort s over signature Sig and context Γ as:

1. $x \in Terms^s(Sig, \Gamma)$, if $x : s \in \Gamma$,

2. $f(t_1, \ldots, t_k) \in Terms^s(Sig, \Gamma)$, if $f : s_1 \times \ldots \times s_k \to s$ and $t_i \in Terms^{s_i}(Sig, \Gamma)$ for $i = 1, \ldots, k$. When $k = 0$, we may write f for $f()$. When $k = 1$, we may write $t_1.f$ instead of $f(t_1)$. When $k = 2$, we may write $t_1\ f\ t_2$ instead of $f(t_1, t_2)$.

We refer to $Terms^s(Sig, \emptyset)$ as the set of *ground* terms of sort s. We will write $Terms(Sig, \Gamma)$ for the set of all terms in context Γ, i.e., $\bigcup_s Terms^s(Sig, \Gamma)$. An *equation* over Sig is a formula $\forall\Gamma.\ t_1 = t_2 : s$ with $t_1, t_2 \in Terms^s(Sig, \Gamma)$; we will omit the $: s$ when doing so will not lead to confusion. A *theory* Th is a pair of a signature and a set of equations over that signature. Associated with a theory Th is a binary relation between (not necessarily ground) terms, called *provable equality*. We write $Th \vdash \forall\Gamma.\ t = t' : s$ to indicate that the theory Th proves that terms $t, t' \in Terms^s(Sig, \Gamma)$ are equal according to the usual rules of multi-sorted equational logic. From these rules it follows that provable equality is the smallest equivalence relation on terms that is a congruence, closed under substitution, closed under adding variables to contexts, and contains the equations of Th. A *morphism of signatures* $F : Sig_1 \to Sig_2$ consists of:

- a function F from sorts in Sig_1 to sorts in Sig_2, and
- a function F from function symbols $f : s_1 \times \ldots \times s_n \to s$ in Sig_1 to terms in

$$Terms^{F(s)}(Sig_2, \{v_1 : F(s_1), \ldots, v_n : F(s_n)\}).$$

  To clearly indicate the context $\{v_1, \ldots, v_n\}$, the function $F(f)$ may be written in "λ notation", i.e. as $F(f) = \lambda v_1, \ldots, v_n.g(v_1, \ldots, v_n)$ for some term g, where the λ is omitted if $n = 0$.

For example, let Sig_1 consist of two sorts, a, b, and one function symbol, $f : a \to b$, and let Sig_2 consist of one sort, c, and one function symbol, $g : c \to c$. There are countably infinitely many morphisms $F : Sig_1 \to Sig_2$, one of which is defined as $F(a) := c$, $F(b) := c$, and $F(f) := \lambda v : c.\ g(g(v))$. In the literature on algebraic specification, our definition of signature morphism is called a "derived signature morphism" [8].

A *morphism of theories* $F : Th_1 \to Th_2$ is a morphism of signatures that preserves provability:

$$Th_1 \vdash \forall v_1 : s_1, \ldots v_n : s_n.\ t_1 = t_2 : s \quad \Rightarrow \quad Th_2 \vdash \forall v_1 : F(s_1), \ldots, v_n : F(s_n).\ F(t_1) = F(t_2) : F(s)$$

where we have extended F to operate on terms. An *algebra* A over a signature Sig consists of:

- a set of *carriers* $A(s)$ for each sort s, and
- a function $A(f) : A(s_1) \times \ldots \times A(s_k) \to A(s)$ for each symbol $f : s_1 \times \ldots s_k \to s$.

Let $\Gamma := \{v_1 : s_1, \ldots, v_n : s_n\}$ be a context. An A-*environment* η for Γ associates each v_i with an element of $A(s_i)$. The meaning of a term in $Terms(Sig, \Gamma)$ relative to A-environment η for Γ is recursively defined as:

$$A[\![v]\!]\eta = \eta(v) \qquad A[\![f(t_1, \ldots, t_n)]\!]\eta = A(f)(A[\![t_1]\!]\eta, \ldots, A[\![t_i]\!]\eta)$$

An algebra A over a signature Sig is a *model* of a theory Th on Sig when $Th \vdash \forall\Gamma.\ t = t' : s$ implies $A[\![t]\!]\eta = A[\![t']\!]\eta$ for all terms $t, t' \in Terms^s(Sig, \Gamma)$ and A-environments η for Γ. Deduction in multi-sorted equational logic is sound and complete: two terms t, t' are provably equal in a theory Th if and only if t and t' denote the same element in every model of Th.

From a signature Sig we form its *term algebra* $[\![Sig]\!]$ as follows. The carrier set $[\![Sig]\!](s)$ is defined as the set of ground terms of sort s. The function $[\![Sig]\!](f)$ for $f : s_1 \times \ldots s_k \to s$ is defined as the function $t_1, \ldots t_n \mapsto f(t_1, \ldots, t_n)$. From a theory Th on Sig we define its *term model* $[\![Th]\!]$ to be the quotient of $[\![Sig]\!]$ by the equivalence relation $Th \vdash$. In other words, the carrier set $[\![Th]\!](s)$ is defined as the set of equivalence classes of ground terms of sort s that are provably equal under Th. The function $[\![Th]\!](f)$ is $[\![Sig]\!](f)$ lifted to operate on equivalence classes of terms. To represent $[\![Th]\!]$ on a computer, or to write down $[\![Th]\!]$ succinctly, we must choose a *representative* for each equivalence class of terms; this detail can be ignored in this paper.

A *morphism of algebras* $h : A \to B$ on a signature Sig is a family of functions $h(s) : A(s) \to B(s)$ indexed by sorts s such that:

$$h(s)(A(f)(a_1, \ldots, a_n)) = B(f)(h(s_1)(a_1), \ldots, h(s_n)(a_n))$$

for every symbol $f : s_1 \times \ldots \times s_n \to s$ and $a_i \in A(s_i)$. We may abbreviate $h(s)(a)$ as $h(a)$ when s can be inferred. The term algebras for a signature Sig are initial among all Sig-algebras: there is a unique morphism from the term algebra to any other Sig-algebra. Likewise, term models are initial among models.

8 Appendix: The Categorical Theory of Algebraic Databases

In this section we briefly describe the categorical theory of algebraic databases, following [11]. We first fix a theory, *Ty*, called the *type side* of our formalism. The sorts of *Ty* are called *types* and the functions of *Ty* are the functions that can appear in schemas and instances. The intended meaning of this theory, $[\![Ty]\!]$, is a category with products. A *schema* S on type side *Ty* is a theory extending *Ty* with new sorts (called *entities*), new unary functions from entities to types (called *attributes*), new unary functions from entities to entities (called *foreign keys*), and new equations (called *data integrity constraints*) of the form $\forall v : s.\ t = t'$, where s is an entity and t, t' are terms of the same type, each containing a single free variable v. The intended meaning of this theory, $[\![S]\!]$, is a category extending $[\![Ty]\!]$. Let S and T be schemas on the same type side *Ty*. A schema mapping $F : S \to T$ is defined as a "derived signature morphism" [8] from S to T that is the identity on *Ty*. That is, $F : S \to T$ assigns to each entity $e \in S$ an entity $F(e) \in T$, and to each attribute / foreign key $f : s \to s'$ a term $F(f)$, of type $F(s')$ and with one free variable of type $F(s)$, in a way that respects equality: if $S \vdash t = t'$, then $T \vdash F(t) = F(t')$. The intended meaning of F is a functor $[\![S]\!] \to [\![T]\!]$ that is the identity on Ty. The collection of all schema mappings forms a category with colimits.

An *instance* I on schema S is a theory extending S with new 0-ary function (constant) symbols called *generators* and non-quantified equations. The intended meaning of an instance I, written $[\![I]\!]$, is the *term model* (i.e., *initial algebra*) for I which contains, for each sort s, a *carrier set* consisting of the closed terms of sort s modulo provability in I. That is, the intended meaning of an S-instance I is a functor $[\![S]\!] \to Set$, namely, the initial such functor in the category of all functors consistent with I. Let I and J be instances on the same schema S. A data mapping $h : I \Rightarrow J$ is defined as a "derived signature morphism" [8] from I to J that is the identity on S. That is, $h : I \Rightarrow J$ assigns to each generator $g : s$ in I a closed term $h(g) : s$ in J in a way that respects equality: if $I \vdash t = t'$, then $J \vdash h(t) = h(t')$. A morphism of instances thus denotes homomorphism (natural transformation) of algebras $[\![I]\!] \to [\![J]\!]$. The database instances and morphisms on a schema S constitute a category with all colimits, denoted S–**Inst**, and a schema mapping $F : S \to T$ induces a functor $\Sigma_F : S\text{–}\mathbf{Inst} \to T\text{–}\mathbf{Inst}$ defined by substitution. The functor Σ_F has a right adjoint, $\Delta_F : T\text{–}\mathbf{Inst} \to S\text{–}\mathbf{Inst}$, which corresponds to composition when we are thinking semantically: $[\![\Delta_F(I)]\!] = [\![F]\!]; [\![I]\!]$. Semantically, $[\![\Sigma_F(I)]\!]$ computes the "left Kan-extension" of $[\![I]\!]$ along $[\![F]\!]$ [11].

Contents

List of Figures

Brandon_Casing Section (29)

Row	Burst Rating	Casing Grade	Casing Size	Casing Weight	Collapse Rating	Measured Depth	Section No.	Interval	Total Vertical Depth
0	6364	X-80 (S-90DM/QT-CR)	22"	224.21	3873.00000000	7600	5.00000000	61	0
1	4275	4275psi Burst Disk	22"	?1	3873.00000000	7670	6.00000000	61	2
2	15600	16" 12.5K SA Upper	23"	?3	14200.00000000	7279	3.00000000	61	4
3	13722	23x2 HPWH Extension Jt	23"	?5	12545.00000000	7015	1.00000000	61	6
4	13890	X-80Q (H-100DM/QT)	23"	453.04	4000.00000000	7276	2.00000000	61	4
5	18190	TN Q125-ICY WLSF	10-1/8"	75.9	16370.00000000	28502	6.00000000	63	32
6	14370	145K Cflex	10-1/8"	?7	13741.00000000	27143	5.00000000	63	8
7	18190	TN Q125-ICY WLSF	10-1/8"	75.9	16370.00000000	27093	4.00000000	63	9
8	14370	145K Cflex	10-1/8"	?10	13741.00000000	26430	3.00000000	63	11
9	18190	TN Q125-ICY WLSF	10-1/8"	75.9	16370.00000000	26380	2.00000000	63	12
10	9070	11-7/8x13-3/8 VF XG Li...	10-1/8"	?13	8730.00000000	25407	1.00000000	63	38
11	14760	VM125 HYHC WLSF	14"	115.53	12500.00000000	26847	2.00000000	62	30
12	10328	14x16 VF LH,125.58# SL...	14"	125.58	9928.00000000	23856	1.00000000	62	41
13	10100	Q-125 ICY	16.04"	109.61	4800.00000000	24342	6.00000000	65	43
14	10100	Innovex Centramax Sub	16.04"	109.61	4670.00000000	24298	5.00000000	65	44
15	12350	Q-125 HP	16.25"	136.04	7680.00000000	7593	2.00000000	65	46
16	15580	10.125 x 8.520 ID C140 ...	10-1/8"	75.9	14820.00000000	25400	8.00000000	64	48
17	10880	Q-125 XHP	16.15"	119.23	6120.00000000	23842	4.00000000	65	42
18	18870	Q125X HP SLIJ-II	10.175"	81	18870.00000000	24857	6.00000000	64	51
19	10100	Q-125 ICY	16.04"	109.61	4800.00000000	14700	3.00000000	65	53
20	15550	Q125 ICY Wedge 623RW	9-7/8"	65.1	13900.00000000	22842	5.00000000	64	14
21	18020	Q125 ICY Wedge 623RW	10-3/4"	85.3	17250.00000000	13500	4.00000000	64	15
22	12350	Dril-Quip CsgHgr 16.25" ...	16"	?16	5000.00000000	7293	1.00000000	65	55
23	16680	VM125SS Wedge 624	10-3/4"	85.3	15530.00000000	8048	3.00000000	64	56
24	15000	VM125SS VAM CSG HGR	10-3/4"	85.3	10000.00000000	7008	1.00000000	64	17
25	7540	16" 12.5K SA Lower	22"	?18	7200.00000000	7282	4.00000000	61	58
26	6364	X-80 (S-90DM/QT-CR)	22"	224.21	3873.00000000	10500	7.00000000	61	19
27	16680	VM125SS SLIJ-II-KT	10-3/4"	85.3	15530.00000000	7028	2.00000000	64	32
28	18190	Q125 ICY WLSF	10-1/8"	75.9	16370.00000000	25357	7.00000000	64	59

Brandon_FG-PP Inputs (51)

Row	Fracture Gradient	Mid Pore Pressure	RKB TVD	Well
29	8.969	8.643	7602	120
30	14.478	12.08	24302	120
31	13.256	10.298	10602	120
32	9.024	8.666	7670	120
33	9.026	8.666	7672	120
34	8.944	8.633	7572	120
35	8.707	8.6	7277	120
36	8.715	8.6	7287	120
37	13.209	10.238	10502	120
38	8.5	8.6	7017	120
39	8.5	8.6	7004	120
40	14.486	12.014	24307	120
41	15.592	13.879	26707	120
42	15.608	13.875	26737	120
43	15.748	13.459	28322	120
44	15.739	13.48	28232	120
45	15.465	13.973	26437	120
46	14.71	7.85	27517	120
47	15.067	10.962	26997	120
48	14.299	11.507	23812	120
49	15.62	8.764	26947	120
50	14.298	11.503	23807	120
51	15.449	13.948	26297	120
52	14.297	11.498	23802	120
53	15.443	13.934	26247	120
54	14.296	11.494	23797	120
55	15.279	13.399	25297	120
56	14.295	11.489	23792	120
57	14.294	11.485	23787	120
58	14.293	11.48	23782	120
59	14.291	11.467	23767	120
60	14.403	11.961	24257	120
61	14.389	11.913	24212	120
62	14.436	11.982	24277	120
63	8.961	8.64	7592	120
64	14.292	11.476	23777	120
65	15.279	13.4	25292	120
66	15.277	13.402	25282	120
67	12.756	9.388	18332	120
68	15.085	12.913	24762	120
69	15.079	12.89	24752	120
70	14.634	11.993	14702	120
71	13.885	10.723	22792	120
72	13.875	10.693	22737	120
73	8.719	8.6	7292	120
74	14.317	11.604	13502	120
75	9.333	8.791	8047	120
76	9.341	8.794	8057	120
77	8.711	8.6	7282	120
78	15.273	13.41	25247	120
79	15.274	13.408	25257	120

Figure 19: Full Integrated Olog, 1 of 4

Brandon_Interval Info (5)

Row	Annular ...	Annular ...	BOP Rating	BOP Test...	Casing T...	Downhol...	Downhol...	Frac Gra...	Frac Gra...	Hole Size	Interval ...	Interval ...	Interval ...	Max Mud...	Mud Wei...	Mud Wei...	Planned ...	Pore Pre...	Prevente...	Prevente...	Safety M...	Wellhead...	OH Dept...	Planned ...	Well
80	10000	6800	15000	11800	3100	8.6	8.6	13.2	13.209	26	Surface	1.0000...	Casing	12.1	12.5	12.5	7004	12.008	18.75	Blowout	500	15000	30	31	120
81	10000	4800	15000	8300	2700	14.1	14.1	15.59	15.748	16.5	Interme...	3.0000...	Liner	13.6	14.1	13.9	23766	13.879	18.75	Blowout	500	15000	41	42	120
82	10000	4900	15000	8300	3600	13.4	13.4	15.75	15.748	12.25	Production	4.0000...	Liner	13.6	13.4	13.2	25251	13.459	18.75	Blowout	500	15000	43	43	120
83	10000	4900	15000	8300	7200	13.9	13.9	?8	15.748	12.25	Tieback	5.0000...	Casing	13.9	13.9	13.7	7007	13.459	18.75	Blowout	500	15000	43	44	120
84	10000	5600	15000	9100	5300	12.1	12.1	14.4	15.748	21	Interme...	2.0000...	Liner	13.6	12.1	11.9	7287	13.973	18.75	Blowout	500	15000	45	40	120

Brandon_MASP Calc. Step 2b (5)

Row	MAWP	MAWP Surf + 500	MAWP Surface	Interval	Reference MASP	Well
85	6690.737736	4052.735336	3552.735336	80	350	120
86	10885.606452	6970.510052	6470.510052	84	354	120
87	10835.521556	6190.657156	5690.657156	81	351	120
88	10690.338496	6300.892896	5800.892896	82	352	120
89	12130.556496	7558.668896	7058.668896	83	353	120

Brandon_Zone of Interest (30)

Row	Backup Pore Pressure	Zone Name	Casing Section	RKB TVD
90	8.666	Zone of Interest 4 Backup Pore Pressure (ppg)	0	33
91	8.666	Zone of Interest 3 Backup Pore Pressure (ppg)	1	33
92	8.6	Zone of Interest 2 Backup Pore Pressure (ppg)	2	33
93	10.238	Bottom Section Backup Pore Pressure (ppg)	0	37
94	8.6	Top Section Backup Pore Pressure (ppg)	3	39
95	8.6	Zone of Interest 1 Backup Pore Pressure (ppg)	4	36
96	13.459	Bottom Section Backup Pore Pressure (ppg)	5	43
97	7.85	Zone of Interest 5 Backup Pore Pressure (ppg)	5	46
98	12.937	Zone of Interest 4 Backup Pore Pressure (ppg)	6	48
99	13.947	Zone of Interest 3 Backup Pore Pressure (ppg)	7	50
100	13.934	Zone of Interest 2 Backup Pore Pressure (ppg)	8	52
101	13.408	Zone of Interest 1 Backup Pore Pressure (ppg)	9	54
102	13.41	Top Section Backup Pore Pressure (ppg)	10	56
103	13.88	Bottom Section Backup Pore Pressure (ppg)	11	57
104	11.476	Zone of Interest 1 Backup Pore Pressure (ppg)	11	58
105	11.462	Top Section Backup Pore Pressure (ppg)	12	59
106	11.961	Bottom Section Backup Pore Pressure (ppg)	13	60
107	11.982	Zone of Interest 5 Backup Pore Pressure (ppg)	14	62
108	11.462	Zone of Interest 4 Backup Pore Pressure (ppg)	15	64
109	13.402	Bottom Section Backup Pore Pressure (ppg)	16	66
110	9.388	Zone of Interest 3 Backup Pore Pressure (ppg)	17	67
111	12.879	Zone of Interest 5 Backup Pore Pressure (ppg)	18	69
112	11.992	Zone of Interest 2 Backup Pore Pressure (ppg)	19	70
113	10.717	Zone of Interest 4 Backup Pore Pressure (ppg)	20	72
114	8.6	Zone of Interest 1 Backup Pore Pressure (ppg)	15	73
115	11.602	Zone of Interest 3 Backup Pore Pressure (ppg)	21	74
116	8.6	Top Section Backup Pore Pressure (ppg)	22	36
117	8.794	Zone of Interest 2 Backup Pore Pressure (ppg)	23	76
118	8.6	Zone of Interest 1 Backup Pore Pressure (ppg)	23	38
119	8.6	Top Section Backup Pore Pressure (ppg)	24	39

Header Info (1)

Row	API Number	Block	Field	Field/Prospect	Header Info_RK...	Operator Name	RKB Height	Rig Name	Type of Well	WD	Water Density	Water Depth	Well	Well Data Key_...	Well Name	WellBore
120	123456789	Block ABC	Field 1	Field 1	7017	Chevron	81	Rig 2	Development	6936	8.6	6936	XYZ	7017	XYZ	ST00BP00

James_Casing Burst Key (29)

Row	Comment	Lowest Value Burst	Lowest Value Collapse	Pipe Body (for 123S) Burst	Pipe Body (for 123S) Collapse	Size	Weight/Grade
121	Dual Metal-to-Metal, Quick Thre...	6364	3873	6364	3873	22	224.21ppf X-80 (S-90DM/QT-...
122	Burst Disk Use ONLY - 4000psi -...	4275	3873	?9	?10	22	4275psi Burst Disk
123	From TDS 2-413383-02 Rev B, ...	15600	14200	15600	14200	23	16" 12.5K SA Upper
124	Casing Hanger, Supplemental Ad...	13722	12545	13722	12545	23	23x2 HPWH Extension Jt
125	Formerly 448.41 ppf X-80	13890	4000	13890	12690	23	453.04ppf X-80Q (H-100DM/QT)
126	?11	?12	?13	?14	?15	?16	?17
127	?18	?19	?20	?21	?22	?23	?24
128	?25	?26	?27	?28	?29	?30	?31
129	?32	?33	?34	?35	?36	?37	?38
130	?39	?40	?41	?42	?43	?44	?45
131	?46	?47	?48	?49	?50	?51	?52
132	?53	?54	?55	?56	?57	?58	?59
133	?60	?61	?62	?63	?64	?65	?66
134	?67	?68	?69	?70	?71	?72	?73
135	?74	?75	?76	?77	?78	?79	?80
136	?81	?82	?83	?84	?85	?86	?87
137	?88	?89	?90	?91	?92	?93	?94
138	?95	?96	?97	?98	?99	?100	?101
139	?102	?103	?104	?105	?106	?107	?108
140	?109	?110	?111	?112	?113	?114	?115
141	?116	?117	?118	?119	?120	?121	?122
142	?123	?124	?125	?126	?127	?128	?129
143	?130	?131	?132	?133	?134	?135	?136
144	?137	?138	?139	?140	?141	?142	?143
145	?144	?145	?146	?147	?148	?149	?150
146	?151	?152	?153	?154	?155	?156	?157
147	?158	?159	?160	?161	?162	?163	?164
148	?165	?166	?167	?168	?169	?170	?171
149	?172	?173	?174	?175	?176	?177	?178

James_Casing Key (1)

Row	Casing Type - Enter "Long Strin...	Hole Section TD - RKB TVD (Ft)	Max Pore Pressure at Hole Secti...	Mud Weight Casing Run In (Dow...	Mud Weight Casing Run In (Surf...	Item of Interest #	PPFP Key	Top Casing Section / Nominal ...
150	Long String	10600	10.281	12.5	12.5	193	318	151

James_Casing Section Key (33)

Row	Casing Section / Nominal Casing - Size	Well Data Key
151	Top 22 Surface Csg	120
152	?179	120
153	?180	120
154	?181	120
155	?182	120
156	?183	120
157	?184	120
158	?185	120
159	?186	120
160	?187	120
161	?188	120
162	?189	120
163	?190	120
164	?191	120
165	?192	120
166	?193	120
167	?194	120
168	?195	120
169	?196	120
170	?197	120
171	?198	120
172	?199	120
173	?200	120
174	?201	120
175	?202	120
176	?203	120
177	?204	120
178	?205	120
179	?206	120
180	?207	120
181	?208	120
182	?209	120
183	?210	120

James_Exposed Shoe Key (5)

Row	Deepest Exposed Shoe Below This Shoe - RKB TVD (Ft)	Frac Gradient at Deepest Exposed Shoe (ppg)	PPFP Key	Top Casing Section / Nominal Casing - Size
184	10502	13.209	270	151
185	?211	?212	274	152
186	?213	?214	279	154
187	?215	?216	284	156
188	?217	?218	289	158

James_Item of Interest Key (20)

Figure 20: Full Integrated Olog, 2 of 4

James_Item of Inteerest Key (30)

Row	Item of Interest #	Item of Interest – Burst R...	Item of Interest Backup P...	Item of Interest Casing Se...	Item of Interest Casing Se...	Item of Interest Depth – R...	CBT Key	Liner Hangoff Key	PPFP Key	Top Casing Section / No...
189	Zone of Interest 4	6364	8.666	22	224.21ppf X-80 (S-90...	7670	121	219	268	151
190	Zone of Interest 3	4275	8.666	22	4275psi Burst Disk	7670	122	219	268	151
191	Zone of Interest 2	15600	8.6	23	16" 12.5K SA Upper	7287	123	219	269	151
192	Btm of Casing/Shoe	6364	10.238	22	224.21ppf X-80 (S-90...	10502	121	219	270	151
193	Top 22 Surface Csg	13722	8.6	23	23x2 HPWH Extension Jt	7004	124	219	271	151
194	Zone of Interest 1	13890	8.6	23	453.04ppf X-80Q (H-...	7287	125	219	269	151
195	?219	18190	13.459	?220	?221	28322	126	220	294	160
196	?222	18190	7.85	?223	?224	27517	127	221	295	161
197	?225	14370	12.937	?226	?227	23812	128	222	296	162
198	?228	18190	13.947	?229	?230	23807	129	223	297	163
199	?231	14370	13.934	?232	?233	23802	130	224	298	164
200	?234	18190	13.408	?235	?236	23797	131	225	299	165
201	?237	9070	13.41	?238	?239	23792	132	226	300	166
202	?240	14760	13.88	?241	?242	23787	133	227	301	167
203	?243	14760	11.476	?244	?245	23782	134	228	302	168
204	?246	10328	11.462	?247	?248	23767	135	229	303	169
205	?249	10100	11.961	?250	?251	24257	136	230	304	170
206	?252	10100	11.982	?253	?254	24277	137	231	305	171
207	?255	12350	11.462	?256	?257	23777	138	232	306	172
208	?258	15580	13.402	?259	?260	25282	139	233	307	173
209	?261	10880	9.388	?262	?263	18332	140	234	308	174
210	?264	18870	12.879	?265	?266	24752	141	235	309	175
211	?267	10100	11.992	?268	?269	14702	142	236	310	176
212	?270	15550	10.717	?271	?272	22737	143	237	311	177
213	?273	12350	8.6	?274	?275	7292	144	238	312	178
214	?276	18020	11.602	?277	?278	13502	145	239	313	179
215	?279	12350	8.6	?280	?281	7287	146	240	314	180
216	?282	16680	8.794	?283	?284	8057	147	241	315	181
217	?285	16680	8.6	?286	?287	7017	148	242	316	182
218	?288	15000	8.6	?289	?290	7004	149	243	317	183

James_Liner Hangoff Key (25)

Row	Mud Weight Used to Perform Fl...	Mud Weight Used to Perform Fl...	Mud Weight Used to Test 22 Ca...	Mud Weight Used to Test Casin...	Mud Weight Used to Test Casin...	Safety Margin Applied to MAWP...	Will a liner be hung off inside th...	Top Casing Section / Nominal ...
219	11.3	11.1	8.6	8.6	8.6	500	Yes	151
220	?291	?292	?293	?294	?295	?296	?297	160
221	?298	?299	?300	?301	?302	?303	?304	161
222	?305	?306	?307	?308	?309	?310	?311	162
223	?312	?313	?314	?315	?316	?317	?318	163
224	?319	?320	?321	?322	?323	?324	?325	164
225	?326	?327	?328	?329	?330	?331	?332	165
226	?333	?334	?335	?336	?337	?338	?339	166
227	?340	?341	?342	?343	?344	?345	?346	167
228	?347	?348	?349	?350	?351	?352	?353	168
229	?354	?355	?356	?357	?358	?359	?360	169
230	?361	?362	?363	?364	?365	?366	?367	170
231	?368	?369	?370	?371	?372	?373	?374	171
232	?375	?376	?377	?378	?379	?380	?381	172
233	?382	?383	?384	?385	?386	?387	?388	173
234	?389	?390	?391	?392	?393	?394	?395	174
235	?396	?397	?398	?399	?400	?401	?402	175
236	?403	?404	?405	?406	?407	?408	?409	176
237	?410	?411	?412	?413	?414	?415	?416	177
238	?417	?418	?419	?420	?421	?422	?423	178
239	?424	?425	?426	?427	?428	?429	?430	179
240	?431	?432	?433	?434	?435	?436	?437	180
241	?438	?439	?440	?441	?442	?443	?444	181
242	?445	?446	?447	?448	?449	?450	?451	182
243	?452	?453	?454	?455	?456	?457	?458	183

James_MASP Open Hole Key (5)

Row	Constant	HC Grad.	MASP Open Hole	MW	Open Hole Depth yi...	Pore Pressure at O...	Sw Hydrostatic	TVDHC	TVDmud	Casing Section Key	Mud Gradient Key	OH Key	PPFP Key
244	0.052	0.15	5339.716512	12.1	24307	12.008	3101.7792	8645.0	8645.0	151	254	259	273
245	?459	?460	((((?462 * (?463 *...	?461	?462	?463	3101.7792	(0 MAX ((?462 – ...	(0 MAX ((?462 – ...	152	255	261	277
246	?466	?467	((((?471 * (?468 *...	?470	?471	?468	3101.7792	(0 MAX ((?471 – ...	(0 MAX ((?471 – ...	154	256	263	282
247	?473	?474	((((?475 * (?476 *...	?479	?475	?476	3101.7792	(0 MAX ((?475 – ...	(0 MAX ((?475 – ...	156	257	265	287
248	?480	?481	((((?484 * (?486 *...	?483	?484	?486	3101.7792	(0 MAX ((?484 – ...	(0 MAX ((?484 – ...	158	258	267	292

James_MASP Shoe Key (5)

Row	Constant	Deepest Exposed S...	Frac Gradient at De...	HC Grad.	MASP Shoe	MW	Sw Hydrostatic	TVDHC	TVDmud	Casing Section Key	Exposed Shoe Key	OH Key	PPFP Key
249	0.052	10502	13.209	0.15	3588.958536	12.1	3101.7792	3485	0	151	184	259	270
250	?487	?488	?212	?489	((((?488 * (?212 *...	?491	3101.7792	?490	?492	152	185	260	276
251	?493	?494	?214	?495	((((?494 * (?214 *...	?498	3101.7792	?497	?496	154	186	262	281
252	?499	?500	?216	?501	((((?500 * (?216 *...	?503	3101.7792	?502	?504	156	187	264	286
253	?505	?506	?218	?507	((((?506 * (?218 *...	?510	3101.7792	?509	?508	158	188	266	291

James_Mud Gradient Key (5)

Row	Gas Fraction	Mud Fraction	TVD Interface	Casing Section Key
254	0.5	0.5	15661.5	151
255	?465	?464	?511	152
256	?469	?472	?512	154
257	?477	?478	?513	156
258	?482	?485	?514	158

James_OH Key (9)

Row	Kick Fluid Gradient – Gas or Oil ...	Kick Fluid Gradient if Gas per BS...	Max Mud Weight at OH Depth (D...	Max Mud Weight at OH Depth (S...	Open Hole Depth yielding highe...	Pore Pressure at OH Depth (ppg)	PPFP Key	Top Casing Section / Nominal ...
259	0.15	0.15	12.1	11.9	24307	12.008	272	151
260	?515	?516	?491	?517	?518	?519	275	153
261	?520	?460	?461	?521	?522	?463	278	152
262	?523	?524	?498	?525	?526	?527	280	155
263	?528	?467	?470	?529	?530	?468	283	154
264	?531	?532	?503	?533	?534	?535	285	157
265	?536	?474	?479	?537	?538	?476	288	156
266	?539	?540	?510	?541	?542	?543	290	159
267	?544	?481	?483	?545	?546	?486	293	158

Figure 21: Full Integrated Olog, 3 of 4

James_Pore Pressure Frac Pressure Key (51)

Row	Fracture Gradient (ppg)	Pore Pressure_mid (ppg)	RKB TVD	Salt FG (Using OBG + 1000 psi (ppg)
268	9.026	8.666	7672	9.026
269	8.715	8.6	7287	8.715
270	-999.25	10.238	10502	13.209
271	8.5	8.6	7004	8.5
272	14.478	12.008	24302	14.478
273	14.486	12.014	24307	14.486
274	?547	?548	?549	?212
275	?550	?519	?551	?552
276	?553	?554	?555	?556
277	?557	?558	?559	?560
278	?561	?463	?562	?563
279	?564	?565	?566	?214
280	?567	?527	?568	?569
281	?570	?571	?572	?573
282	?574	?575	?576	?577
283	?578	?468	?579	?580
284	?581	?582	?583	?216
285	?584	?535	?585	?586
286	?587	?588	?589	?590
287	?591	?592	?593	?594
288	?595	?476	?596	?597
289	?598	?599	?600	?218
290	?601	?543	?602	?603
291	?604	?605	?606	?607
292	?608	?609	?610	?611
293	?612	?486	?613	?614
294	?615	13.459	?616	?617
295	?618	7.85	?619	?620
296	?621	12.937	?622	?623
297	?624	13.947	?625	?626
298	?627	13.934	?628	?629
299	?630	13.408	?631	?632
300	?633	13.41	?634	?635
301	?636	13.88	?637	?638
302	?639	11.476	?640	?641
303	?642	11.462	?643	?644
304	?645	11.961	?646	?647
305	?648	11.982	?649	?650
306	?651	11.462	?652	?653
307	?654	13.402	?655	?656
308	?657	9.388	?658	?659
309	?660	12.879	?661	?662
310	?663	11.992	?664	?665
311	?666	10.717	?667	?668
312	?669	8.6	?670	?671
313	?672	11.602	?673	?674
314	?675	8.6	?676	?677
315	?678	8.794	?679	?680
316	?681	8.6	?682	?683
317	?684	8.6	?685	?686
318	-999.25	10.281	10572	13.24

James_Shoe Track Key (1)

Row	Planned FIT (Downhole,ppg)	Planned FIT (Surface,ppg)	Shoe Frac Gradient (ppg)	Item of Interest #	Top Casing Section / Nominal Casing - Size
319	13.2	13	13.207	192	151

MASP Calc. Step 1 (30)

Row	70% Burst (cor...	70% Burst (not...	Backside EMW	Burst Rating	Burst Rating C...	Constant	Corrected Hy...	DHEMW	De-Rated Perc...	EMW - Backup	Material Utiliz...	TVD	Casing Section	Interval	Item of Interest	RKB TVD	Zone Name
320	4481.123440	4454.8	8.666	6364	4481.123440	0.052	-26.323440	8.6	0.7	-0.066	0.7	7670	0	80	189	32	90
321	3018.823440	2992.5	8.666	4275	3018.823440	0.052	-26.323440	8.6	0.7	-0.066	0.7	7670	1	80	190	32	91
322	10920.0000	10920.0	8.6	15600	10920.0000	0.052	0.0000	8.6	0.7	0.0	0.7	7287	2	80	191	36	92
323	5349.318352	4454.8	10.238	6364	5349.318352	0.052	-894.518352	8.6	0.7	-1.638	0.7	10502	0	80	192	37	93
324	9605.4000	9605.4	8.6	13722	9605.4000	0.052	0.0000	8.6	0.7	0.0	0.7	7004	3	80	193	39	94
325	9723.0000	9723.0	8.6	13890	9723.0000	0.052	0.0000	8.6	0.7	0.0	0.7	7287	4	80	194	36	95
326	12819.8918...	12733.0	13.459	18190	12819.8918...	0.052	-86.891896	13.4	0.7	-0.059	0.7	28322	5	82	195	43	96
327	4791.59380	12733.0	7.85	18190	4791.59380	0.052	7941.40620	13.4	0.7	5.55	0.7	27517	5	82	196	46	97
328	9485.702288	10059.0	12.937	14370	9485.702288	0.052	573.297712	13.4	0.7	0.463	0.7	23812	6	82	197	48	98
329	13410.1663...	12733.0	13.947	18190	13410.1663...	0.052	-677.166308	13.4	0.7	-0.547	0.7	23807	7	82	198	50	99
330	10719.9339...	10059.0	13.934	14370	10719.9339...	0.052	-660.933936	13.4	0.7	-0.534	0.7	23802	8	82	199	52	100
331	12742.8995...	12733.0	13.408	18190	12742.8995...	0.052	-9.899552	13.4	0.7	-0.008	0.7	23797	9	82	200	54	101
332	6361.37184	6349.0	13.41	9070	6361.37184	0.052	-12.37184	13.4	0.7	-0.01	0.7	23792	10	82	201	56	102
333	10059.87672	10332.0	13.88	14760	10059.87672	0.052	272.12328	14.1	0.7	0.22	0.7	23787	11	81	202	57	103
334	7086.993664	10332.0	11.476	14760	7086.993664	0.052	3245.006336	14.1	0.7	2.624	0.7	23782	11	81	203	58	104
335	3969.338008	7229.6	11.462	10328	3969.338008	0.052	3260.261992	14.1	0.7	2.638	0.7	23767	12	81	204	59	105
336	6894.670404	7070.0	11.961	10100	6894.670404	0.052	175.329596	12.1	0.7	0.139	0.7	24257	13	84	205	60	106
337	6921.036328	7070.0	11.982	10100	6921.036328	0.052	148.963672	12.1	0.7	0.118	0.7	24277	14	84	206	62	107
338	7856.174248	8645.0	11.462	12350	7856.174248	0.052	788.825752	12.1	0.7	0.638	0.7	23777	15	84	207	64	108
339	10251.2973...	10906.0	13.402	15580	10251.2973...	0.052	654.702672	13.9	0.7	0.498	0.7	25282	16	83	208	66	109
340	5030.748032	7616.0	9.388	10880	5030.748032	0.052	2585.251968	12.1	0.7	2.712	0.7	18332	17	84	209	67	110
341	11894.8668...	13209.0	12.879	18870	11894.8668...	0.052	1314.133184	13.9	0.7	1.021	0.7	24752	18	83	210	69	111
342	6987.433568	7070.0	11.992	10100	6987.433568	0.052	82.566432	12.1	0.7	0.108	0.7	14702	19	84	211	70	112
343	7121.662708	10885.0	10.717	15550	7121.662708	0.052	3763.337292	13.9	0.7	3.183	0.7	22737	20	83	212	72	113
344	7317.8560	8645.0	8.6	12350	7317.8560	0.052	1327.1440	12.1	0.7	3.5	0.7	7292	15	84	213	73	114
345	11000.5650...	12614.0	11.602	18020	11000.5650...	0.052	1613.434992	13.9	0.7	2.298	0.7	13502	21	83	214	74	115
346	7318.7660	8645.0	8.6	12350	7318.7660	0.052	1326.2340	12.1	0.7	3.5	0.7	7287	22	84	215	36	116
347	9536.769816	11676.0	8.794	16680	9536.769816	0.052	2139.230184	13.9	0.7	5.106	0.7	8057	23	83	216	76	117
348	9742.1148	11676.0	8.6	16680	9742.1148	0.052	1933.8852	13.9	0.7	5.3	0.7	7017	23	83	217	38	118
349	8569.6976	10500.0	8.6	15000	8569.6976	0.052	1930.3024	13.9	0.7	5.3	0.7	7004	24	83	218	39	119

MASP Calc. Step 2a (5)

Row	Frac Pr...	Gas Gra...	Gas Hy...	Gas Hy...	Gas Ratio	Gas TV...	Gas TV...	Interfac...	MASP	MASP B...	MASP C...	MASP K...	MASP O...	Minimu...	Mud Hy...	Mud Hy...	Mud Rat...	Mud TV...	Mud TV...	Pressur...	SW Hyd...	Casing ...	Interval	MASP O...	MASP S...	TVD De...	TVD Shoe	Well
350	7213....	0.15	1296....	522.75	0.5	8645.0	3485	15662.0	3588....	5339....	3588....	3588....	5339....	3588....	5439....	0.0000	0.5	8645.0	0	15177...	3101....	151	80	244	249	40	37	120
351	23192...	0.15	1476....	(?489 ...	0.5	9845.0	9845.0	16862.0	(((((?46...	7733....	(((231...	((((?48...	((((?46...	(7733....	6962....	((?492 ...	0.5	9845.0	11460.0	19274...	3101....	152	81	245	250	41	43	120
352	23192...	0.15	1597....	(?495 ...	0.5	10652.5	10652.5	17669.5	(((((?47...	7588....	(((231...	((((?49...	((((?47...	(7588....	7533....	((?496 ...	0.5	10652.5	10652.5	19821...	3101....	154	82	246	251	43	43	120
353	23192...	0.361	7691....	(?501 ...	1	21305	21305	28322	(((((?47...	9028....	(((231...	((((?50...	((((?47...	(9028....	0.0000	((?504 ...	0	0	0	19821...	3101....	156	83	247	252	43	43	120
354	23192...	0.15	1456....	(?507 ...	0.5	9710.0	9710.0	16727.0	(((((?48...	7783....	(((231...	((((?50...	((((?48...	(7783....	6866....	((?508 ...	0.5	9710.0	11595.0	19209...	3101....	158	84	248	253	45	43	120

Figure 22: Full Integrated Olog, 4 of 4