Algorithms for Public Transit Graphs

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December 25, 2023

Abstract

1 Introduction

These algorithms will be used to construct a public transit system as a graph. We will use the WGS84 coordinate system and represent the Earth as an oblate spheroid. The difficulty in these problems comes from the issue of the Earth's irregular shape.

2 Point-Geodesic Problem

The problem states: Given a Geodesic, defined by the shortest path between two points on a sphere's surface, and a random point on the sphere's surface, find the closest point on that line to that random point.

2.1 Solution in Euclidean Space

Take Points A and B, and parametrize $\overline{AB}$ to create a function of a vector, $f(t)$. Derive $f(t)$ to find the critical point, t_c. The closest point will be $f(t_c)$.

2.2 Kyler's Solution

Take the Geodesic's endpoints (P_1, P_2), along with the random point, R_1, and construct $\triangle P_1P_2R_1$, using Vincenty's formulae to construct the edges. Then, take the midpoint of $\overline{P_1P_2}$, and construct a new line using this new midpoint, M_1, and R_1. Throw out whichever line is longer $\overline{R_1P_1}$ or $\overline{R_1P_2}$, and replace it with $\overline{R_1M_1}$. Repeat this process until $\overline{P_1P_2}$ is within the specified tolerance. This algorithm is $O(\log_2(n))$, where n is equal to the original length of $\overline{P_1P_2}$. It takes $\log_2(n)$ steps to reach the specified tolerance.

2.3 Final Solution

We just ended up using Baselga and Martínez-Llario's Solution on this problem [1], with Karney's improvements. [2]

References

[1] S. Baselga and J. C. Martínez-Llario, "Intersection and point-to-line solutions for geodesics on the ellipsoid," *Studia Geophysica et Geodaetica*, vol. 62, no. 3, pp. 353–363, July 2018. [Online]. Available: https://doi.org/10.1007/s11200-017-1020-z

[2] C. F. F. Karney, "Geodesic intersections," 2023. [Online]. Available: https://doi.org/10.48550/arXiv.2308.00495