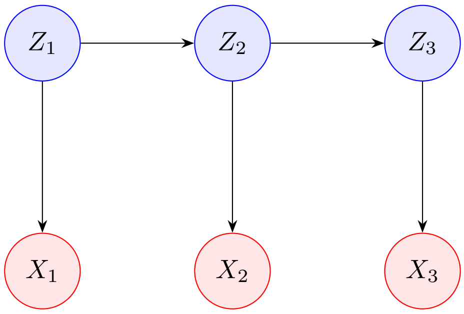
1 Inference

1. Consider the following graphical probability model:



   - Write down the corresponding factor graph
   - Apply the sum-product algorithm to compute the marginal probability $p(x_2)$.

2. Compute the E-step and M-step for Gaussian mixture model in detail.

3. Using mean field method to approximate the marginal probability $p_\theta(X)$ of Gaussian mixture model. The variational distribution takes the product form:

$$q(Z, \pi, \mu_k, \Sigma_k) = q(Z)q(\pi)\prod_{k=1}^{K} q(\mu_k, \Sigma_k).$$

4. Write a simple Python code for rejection sampling.

5. Consider a Markov chain with transition matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

   Let $\pi = (\pi_1, \pi_2, \pi_3)$ be the stationary distribution, find the equations that π should satisfy.

6. Write Python codes for the following MCMC methods:
   (a) Basic MCMC
   (b) Hamiltonian MC
   (c) Langevin dynamics

   Use your code to do the sampling for general two-dimensional Gaussian distributions, compare the results for three methods (the acceptance rates, etc).

7. Prove: the Gibbs sampling method satisfies the detailed balance condition.