Macrofinance Project - Bahrain

FIN-406

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1 Introduction

First order conditions are given by:

$$
\begin{aligned}
c_t &: c_t^{-\frac{1}{\sigma}} - \lambda_t = 0, \\
b_t &: \frac{\beta \lambda_{t+1}}{\Pi_{t+1}} - \frac{\lambda_t}{i_t + 1} = 0, \\
N_t &: -\nu N_t^{\frac{1}{\varphi}} - \lambda_t w_t (\tau_t - 1) = 0, \\
\lambda_t &: Pr_t - c_t + \frac{b_{t-1}}{\Pi_t} - \frac{b_t}{i_t + 1} - N_t w_t (\tau_t - 1) = 0.
\end{aligned}
$$

Question 1

a) The coefficient measuring emissions per unit of output for Bahrain is given by $\psi = 0.4$.

b) Total carbon dioxide is equal to 419.12 ppm in October. This yields $M = 41.912$.

Question 2

The value of variables at the steady state in levels can be found in Table 1.

i	0.010
Π	1.000
c	0.132
z	0.088
λ	7.574
y	0.220
n	0.853
w	0.164
mc	0.909
Pr	0.036
τ	0.314
$\mathcal{I}$	0.754
M	41.912
Z^*	1.000

Table 1: Steady state variables

Question 3

The impulse response of all variables after a 20 p.p. increase in Z^* relative to its steady-state level can be seen in Figure 1.

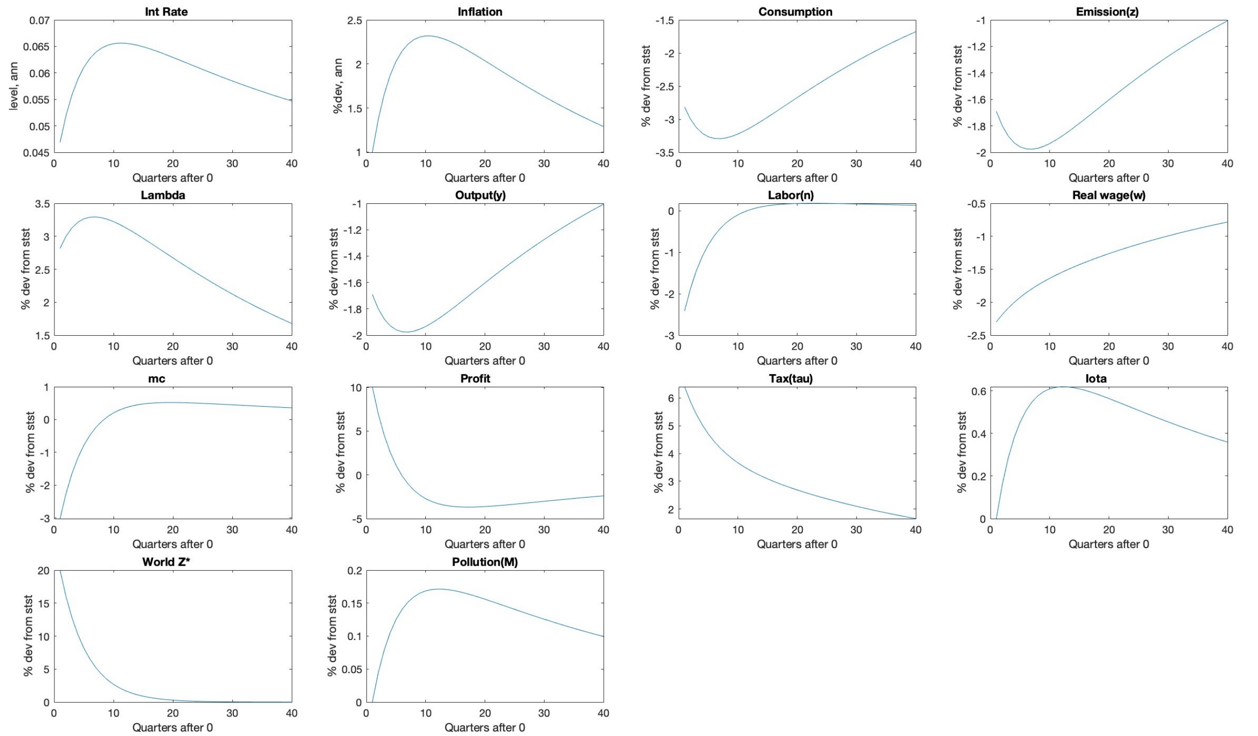

Figure 1: Impulse response of the variables after a 20 p.p. increase in Z^*.

Given the shock in world pollution, the stock of pollution M accumulates and thus increases over time after the initial shock, because the shock in Z^* is positive (9). This in turn leads to an increase in $\mathcal{I}$ (10) meaning that productivity is gradually reduced (higher $\mathcal{I}$ means lower productivity), directly affecting output negatively (3). Because output goes down, emissions emitted by the firms (Z) also go down (11), and so do wages (due to lower productivity) which in turn reduces n, or hours worked, as well as consumption c. In addition, lower wages result in lower marginal costs mc. Taxes τ must then go up to clear the government budget (6) (because wages and hours worked go down). Because hours worked decrease, profits of the firms Pr go up (less wages to pay) (4). From the Taylor rule (7), we have that due to the decrease in output Y, inflation Π will increase (or, equivalently, π). This is due to the fact that although output is below full level (which would make the interest rate go down), the coefficient ϕ_π is higher than ϕ_y, meaning that the central bank is more leniant towards deviations in output rather than inflation, so it will keep interest rates higher to reduce inflation rather than increase output. Because inflation goes up, interest rates increase.

Question 4

Setting the price of carbon emissions equal to 0.75 (compared to the previous 0.5), the impulse responses of all the variables in the model are shown in Figure 2.

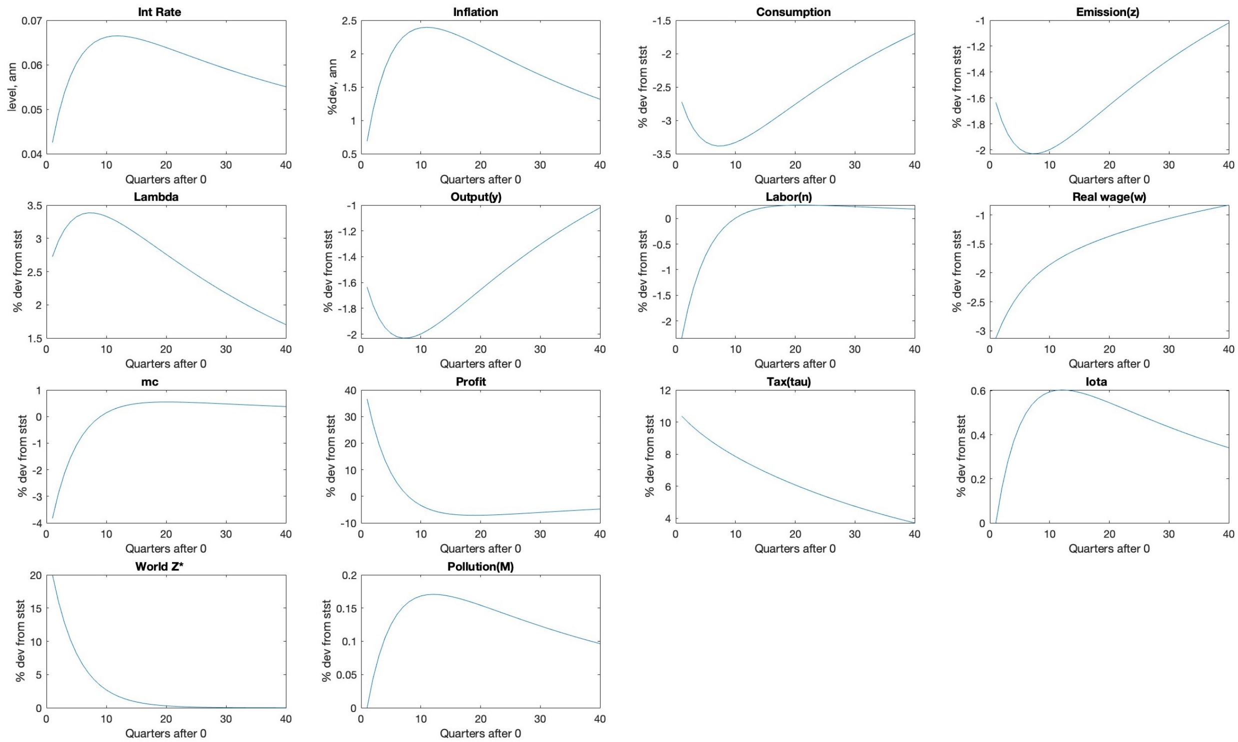

Figure 2: Impulse responses when $p_Z = 0.75$.

In this case, the higher carbon tax affects firms much more than it does individual agents: although most variables change with respect to the benchmark case, consumption, for example, is only very slightly different. However, because the carbon tax is now higher, the effect on firms is clear: to maximize their profits, firms lower labor so wages are lower than in the benchmark case, and thus the marginal cost is also lower. The profit is higher than in the benchmark case, due to the fact that although the carbon permits are higher, the reduction in wages has a more significant effect. Finally, to clear the budget, the government also has to increase labor tax more than in the benchmark case.

Question 5

Setting the price of emitting emissions equal to 0.5 again and increasing the government spending to 0.5Y, the the impulse responses of all the variables in the model are in figure 3.

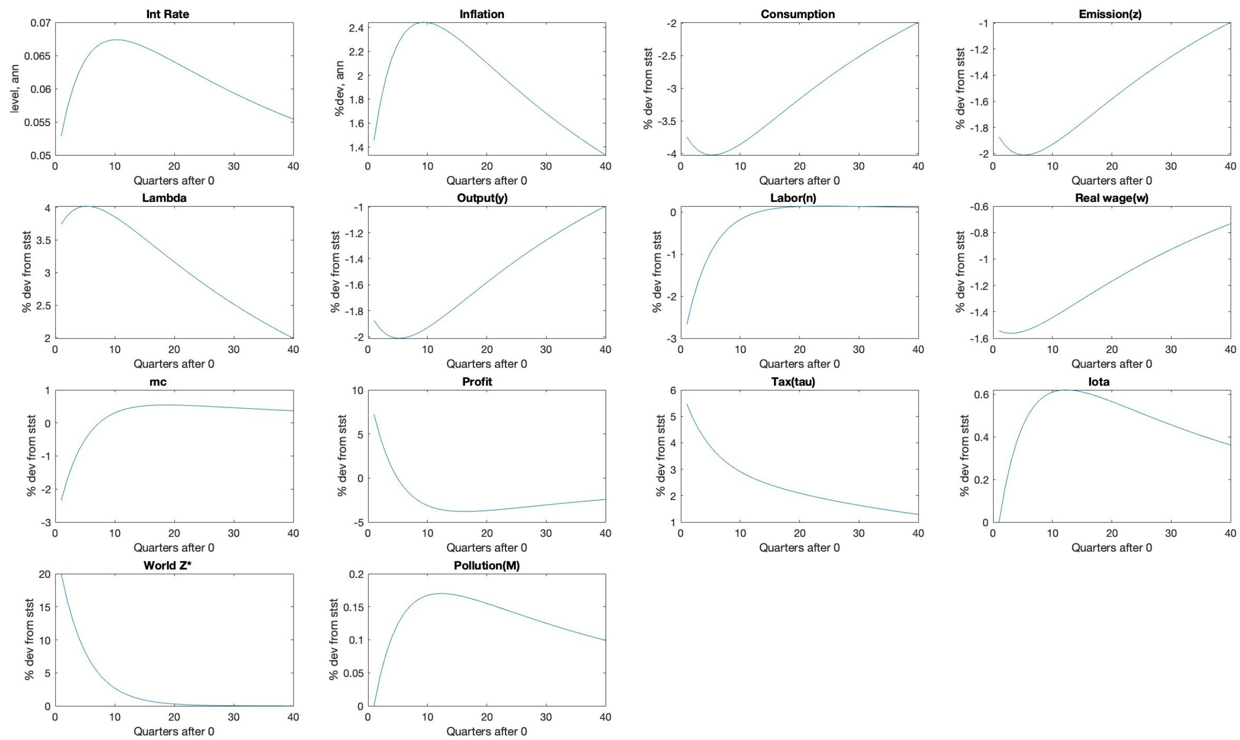

Figure 3: Impulse responses when $p_Z = 0.5$, $G = 0.5Y$.

With respect to the benchmark, we only some variables change significantly: for example real wage now has a smaller response: in fact, the increase in government spending, when total emissions (Z) and labor (n) don't change, help real wage to have a smaller negative response than the benchmark (6). Despite this, consumption has a higher negative value with respect to the benchmark. Output also starts at a slightly higher negative value than it does in the benchmark case , however, because inflation experiences a much higher shock than in the benchmark case and that the central bank considers deviations in inflation more important than ones in output, the interest rate starts at a higher level in the aims to reduce this inflation. Thanks to the governments budget constraint (6), the increase in government spending helps to absorb the shock in the increase in world pollution would cause in real wage and labor, compared to the benchmark. Therefore taxes τ increase less since it is not needed to compensate the smaller decrease in these variables.

Question 6

The shock in Z^* increases the stock of pollution (M) and M changes the value of the other variables only through its effect on $\mathcal{I}$. Here our hypotheses, $\iota_1 = \iota_2 = 0$, imply that $\mathcal{I}$ is now constant, and hence the stock of pollution M has no effect anymore (such as a worsening in workers' health) on the different economic variables. This implies that all the economic variables stay at the steady state. The responses of the variables can be seen in Figure 4.

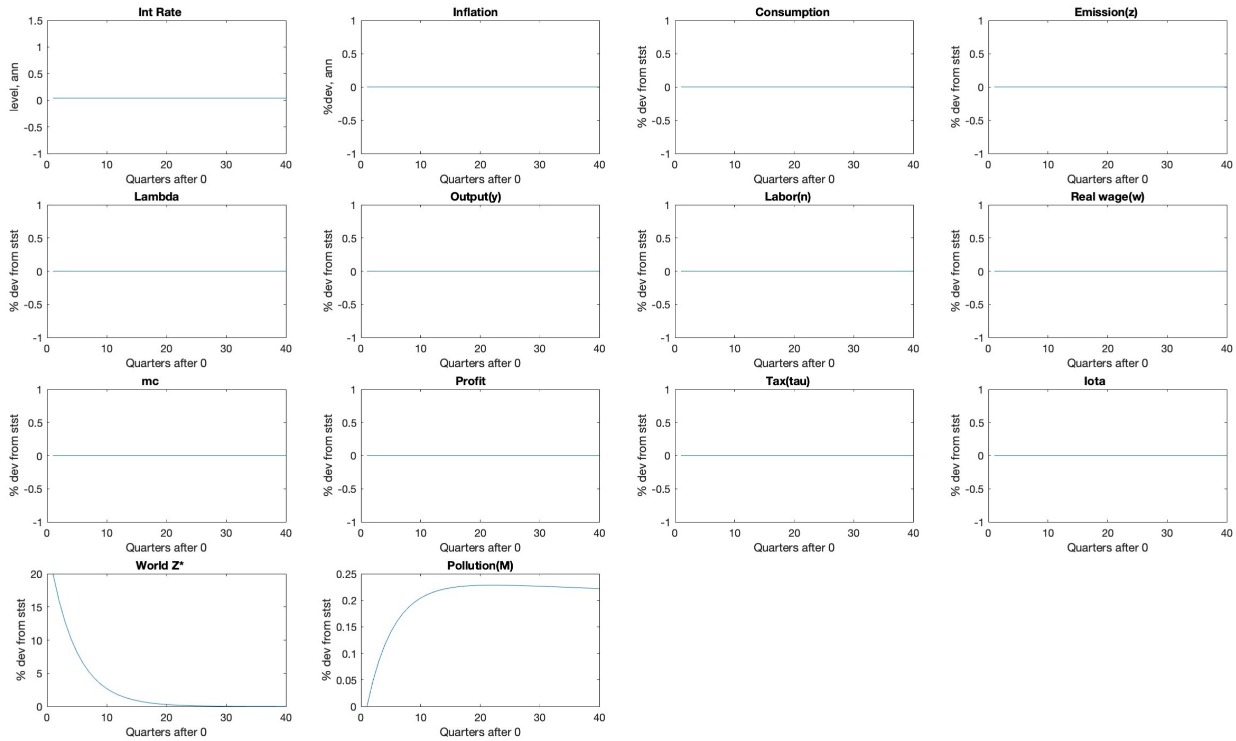

Figure 4: Impulse response of the variables after a 20 p.p. increase in Z^* and $\mathcal{I} = \iota_0$.

Question 7

Figure 5 shows the impulse response of the different variables after a 20 p.p. increase in world emissions and a new tighter monetary policy, i.e. when firms pollute more, interest rates go higher, which make them produce less.

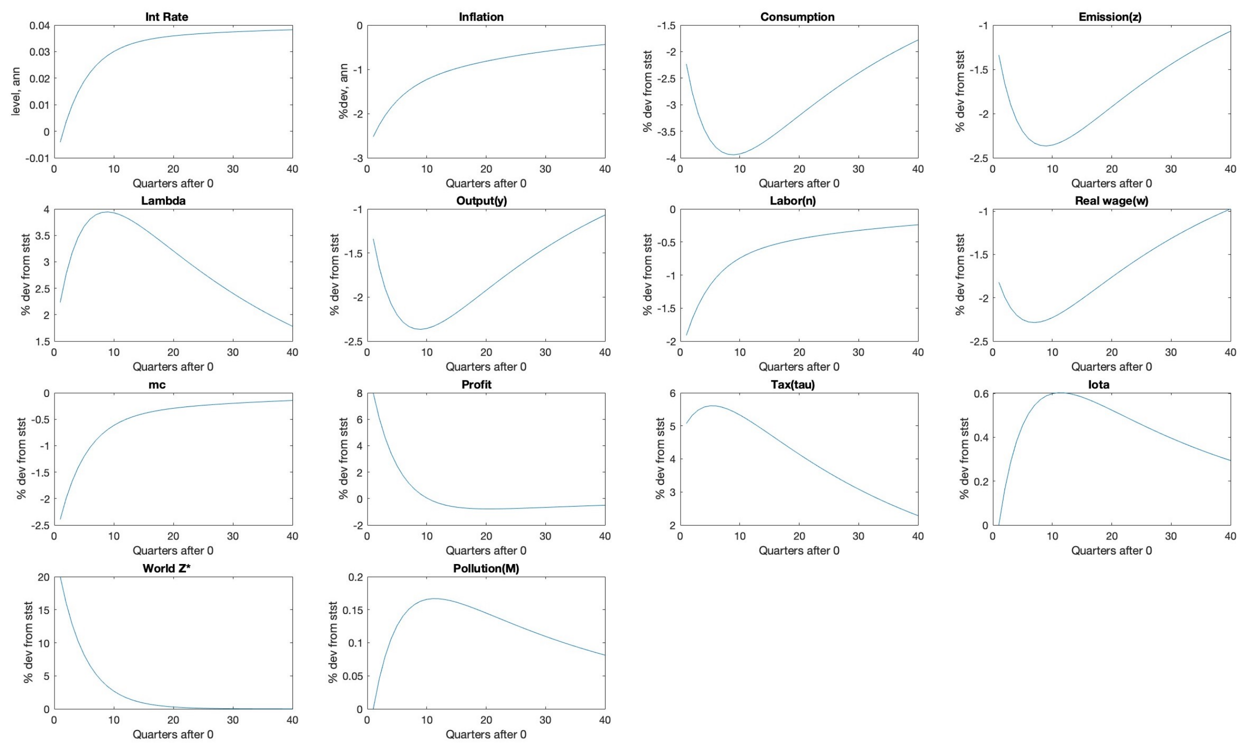

Figure 5: Impulse response of the variables after a 20 p.p. increase with new monetary policy.

Now the interest rate is more sensitive to deviation from the steady state of the accumulation of pollution because $\phi_m = 3 > \phi_\pi, \phi_y$. In the Taylor rule that considers the deviation from steady state air pollution, the interest rate would go up as the deviation is larger. Thus, because M stays above 0% deviation from steady state, the interest rate stays above 0 throughout that same time and does not decrease with the fact that inflation is negative at that time. Initially the air pollution is not highly different from its steady state value and due to the fact that inflation is lower than the steady state value the interest rate starts low; then because the deviation from steady state, air pollution gets larger and pushes the interest rate up, even though inflation is still lower than its steady state value. At the beginning consumption is higher than the benchmark case because the interest rate is lower. At the time of the shock, the emissions Z are less deviated from their steady state value than for the benchmark case. This leads to lower profits than the benchmark case because the firms have to pay for more pollution permits in this case, lowering their profits. So labor is less reduced in this case as well as wages.

Equations

Utility:

$$\max_{C_t,N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{N_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right). \tag{1}$$

Budget constraint of representative agent:

$$C_t + \frac{b_t}{1+i_t} = (1-\tau_t)\, w_t N_t + \frac{b_{t-1}}{\Pi_t} + Pr_t. \tag{2}$$

Technology of firm i:

$$Y_t(i) = (1 - \mathcal{I}(M_t))\, N_t(i)^{1-\alpha}, \quad \alpha \in (0,1). \tag{3}$$

Profits:

$$\max_{N_t(i)} Pr_t \equiv Y_t(i) - w_t N_t(i) - p_Z Z_t(i). \tag{4}$$

Law of motion of inflation in the economy:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \widehat{mc} c_t. \tag{5}$$

Government budget constraints:

$$G + \frac{b_{t-1}}{\Pi_t} = \tau_t w_t N_t + \frac{b_t}{1+i_t} + p_Z Z_t. \tag{6}$$

Taylor rule:

$$1 + i_t = (1+i)\Pi_t^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}, \quad \phi_\pi > 1, \phi_y \geq 0. \tag{7}$$

Taylor rule in log terms (Question 7):

$$i_t = -\ln\beta + \phi_\pi \pi_t + \phi_y (y_t - y) + \phi_m (m_t - m). \tag{8}$$

Stock of pollution:

$$M_t = (1-\delta_M)\, M_{t-1} + Z_t + 0.1 \times Z_t^*. \tag{9}$$

Reduction in productivity according to the stock of pollution:

$$\mathcal{I}(M_t) = \iota_0 + \iota_1 M_t + \iota_2 M_t^2. \tag{10}$$

Emissions emitted by the firms are proportional to output:

$$Z_t(i) = \psi Y_t(i). \tag{11}$$

Rest of world gas emissions:

$$Z_{t+1}^* = \rho Z_t^* + \xi \varepsilon_{t+1}. \tag{12}$$