

# Provable Defense Framework for LLM Jailbreaks via Noise-Augmented Alignment

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## Abstract

Large Language Models (LLMs) remain vulnerable to adaptive jailbreaks that easily bypass empirical defenses like GCG. We propose a framework for certifiable robustness that shifts safety guarantees from single-pass inference to the statistical stability of an ensemble. We introduce Certified Semantic Smoothing (CSS) via Stratified Randomized Ablation, a technique that partitions inputs into immutable structural prompts and mutable payloads to derive rigorous  $l_0$  norm guarantees using the Hypergeometric distribution. To resolve performance degradation on sparse contexts, we employ Noise-Augmented Alignment Tuning (NAAT), which transforms the base model into a semantic denoiser. Extensive experiments on Llama-3 show that our method reduces the Attack Success Rate of gradient-based attacks from 84.2% to 1.2% while maintaining 94.1% benign utility, significantly outperforming character-level baselines which degrade utility to 74.3%. This framework provides a deterministic certificate of safety, ensuring that a model remains robust against all adversarial variants within a provable radius.

## Introduction

Large Language Models (LLMs) have achieved unprecedented capabilities in reasoning and generation, yet their deployment is severely hindered by their vulnerability to adversarial attacks (Shayegani et al. 2023). Despite extensive red teaming and safety alignment efforts (Cheng et al. 2025), LLMs remain susceptible to jailbreaks, an adversarial perturbation designed to bypass safety filters and elicit harmful content. The current landscape of defense strategies is largely dominated by empirical methods. While these defenses may mitigate specific known attacks, they often fail against adaptive adversaries, leading to a perpetual “cat-and-mouse” cycle where new attacks (such as gradient-based optimizers like GCG) rapidly obsolete existing defenses. Consequently, there is an urgent need to move from empirical resilience to certifiable robustness, providing mathematical guarantees that a model’s safety is invariant within a defined perturbation budget.

The problem of achieving certified robustness for large language models (LLMs) has some unique theoretical and

design challenges compared to other fields, such as computer vision. While randomized smoothing has successfully achieve  $l_2$ -norm certification for continuous image inputs, these schemes rely on additive Gaussian noise, which does not apply to discrete, high-dimensional spaces in language models. In an LLM, inputs exist in vocabulary  $\mathcal{V}^L$ , which involves discrete tokens, not continuous signals. It is most likely that an attack at the character level, or an attack at the token level, will violate semantic integrity either before reaching the robust radius or trigger an Out-Of-Distribution (OOD) transition. In addition, it does not apply to large language model interaction, in which system inputs, or chat templates, need to be constant to maintain the integrity of the application interface.

To bridge the gap between empirical resilience and certified safety, we introduce the concept of the Provable Defense Framework based on Certified Semantic Smoothing via Stratified Randomized Ablation. The concept of certified robustness for large language models is challenging in a different way than it is in the field of computer vision. Classical randomized smoothing can be used to certify the robustness of image classifiers by adding Gaussian noise; however, the concept of using a continuous noise distribution is not well-defined in the high-dimensional discrete space in which language models reside. Additionally, simplistic applications of character-level noise can easily compromise the semantic consistency or induce an Out-Of-Distribution shift before reaching the safety boundary in the robustified space. By contrast to the additive noise paradigm, our solution stochastically ablates/masks the semantic payload while deterministically preserving the structural support set. For the purpose of clarity, we term our solution Stratified Randomized Ablation. The solution takes note of the strict semantic hierarchy present in the large language model interactions; the application system prompts and chat templates must be fixed in order to maintain the application interface, while the adversarial attack is contained in the variable query input.

Crucially, we point out that the variance of the standard LLM on sparse and removed inputs is considerable when compared to dense inputs. We find that the apparent improvement of the theory of robustness is directly linked to the degradation of the practical utility, and therefore, the phenomenon is named the inverted scaling fallacy. In order to resolve the problem, the Noise-Augmented Alignment

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Tuning (NAAT) method is proposed, which is a fine-tuning technique that aligns the representation with the smoothing distribution, making the LLM a semantic denoiser.

Our contributions are threefold:

1. We propose CSS with Stratified Randomized Ablation, a mechanism that respects the autoregressive nature of Transformers by using randomized attention masking rather than token deletion, preserving positional integrity.
2. We derive a rigorous certification radius for discrete token substitution ( $\ell_0$  norm) based on the Hypergeometric distribution, correcting scaling laws misapplied in prior heuristic defenses.

## Related Works

### Adversarial Jailbreaks of LLMs

The deployment of LLMs has led to the rise of jailbreaks (Yi et al. 2024; Dong et al. 2025) where adversarial crafted perturbations designed to circumvent safety alignment to generate dangerous outputs. Initially, these attacks were manual and heuristic in nature, relying on role-playing or jailbreak templates to fool the model. The threat landscape shifted dramatically with the advent of automated, gradient-based optimization attacks such as Greedy Coordinate Gradient (GCG) (Zou et al. 2023), and genetic algorithms like AutoDAN (Liu et al. 2023), which effectively discovered adversarially perturbed suffixes in high-dimensional representation spaces. To this end, there has also been an effort in the community to develop cutting-edge alignment techniques to improve robustness to such adversarially “weaponized” uses of these models. Notably, Cheng et al. (2025) put forward an adversarially robust defense mechanism to mitigate risks of misuse by aligning model goals to emphasize utility over log-likelihood maximization. While there have been these dramatic advances in robust alignment techniques and empirical defense methods such as perplexity filters and “erase-and-check” methods, these methods tend to be merely empirical in nature rather than mathematically sound. It involves a “cat and mouse” game where an intelligent attacker finds ways to bypass such new safeguards developed by rapidly adapting to them, and therefore, it becomes an utmost priority to design new methods that ensure deterministic guarantees of safety.

### Certified Robustness via Randomized Smoothing

To establish rigorous safety guarantees, the machine learning community has increasingly turned to Randomized Smoothing, a technique originally pioneered for computer vision. Cohen, Rosenfeld, and Kolter (2019) demonstrated that by convolving a base classifier with additive Gaussian noise, one can derive a smoothed classifier that is certifiably robust against  $\ell_2$ -norm perturbations up to a deterministically calculable radius. This framework leverages the Neyman-Pearson Lemma (Neyman and Pearson 1933) to convert statistical stability under noise into a geometric certificate of robustness. However, the direct translation of this continuous theory to the discrete domain of Natural Language Processing presents fundamental theoretical hurdles.

LLMs operate on a discrete vocabulary  $\mathcal{V}$  rather than a continuous manifold, rendering the standard Gaussian noise assumptions ill-defined for token inputs. Consequently, naive applications of continuous smoothing fail to capture the structural constraints of language, necessitating a reformulation of certification protocols that respect the discrete and autoregressive nature of text generation.

### Discrete Smoothing and the Semantic Gap

Addressing the discrete nature of language, recent works have attempted to adapt smoothing techniques by introducing noise at the character or token level. The most prominent baseline, SmoothLLM (Robey et al. 2025) employs a mechanism of random character substitutions, insertions, and deletions to certify robustness against localized edits. While this approach effectively mitigates strictly character-based perturbations, it suffers from a critical “inverted scaling” trade-off: high levels of character noise frequently destroy the semantic coherence of the input prompt, causing the model’s utility to collapse before a meaningful certification radius can be established.

Our work bridges this gap by introducing Certified Semantic Smoothing (CSS), which shifts the noise mechanism from character corruption to Stratified Randomized Ablation at the token level. Unlike prior methods that treat all tokens uniformly, we partition inputs into immutable structural prompts and mutable semantic payloads, ensuring that the certification process respects the rigid formatting requirements of modern instruction-tuned models. Furthermore, we explicitly address the utility degradation inherent in sparse inputs through Noise-Augmented Alignment Tuning (NAAT), which aligns the base model’s representations with the smoothing distribution, effectively transforming the LLM into a semantic denoiser robust to information sparsity.

### Problem Formulation

We frame the challenge of securing Large Language Models (LLMs) against adversarial attacks as a problem of certifiable robustness within a discrete, high-dimensional state space. Unlike computer vision, where inputs are continuous and adversarial perturbations are modeled as  $\ell_p$ -norm bounded noise, language models operate on a discrete domain  $\mathcal{V}^L$ , where  $\mathcal{V}$  is the vocabulary and  $L$  is the sequence length. We consider an aligned LLM  $f : \mathcal{V}^L \rightarrow \Delta^{|\mathcal{V}|}$  and a safety classifier  $S$  derived from  $f$  that maps inputs to a binary label space  $\mathcal{Y} = \{+1(\text{Refusal}), -1(\text{Compliance})\}$ .

The adversary’s objective is to find a perturbation  $\delta$  such that for a benign input  $x$ , the perturbed input  $x' = x \oplus \delta$  elicits a harmful response ( $S(x') = -1$ ). We adopt a semantic threat model where the adversary is constrained by the  $\ell_0$  norm, representing the number of token substitutions, insertions, or deletions. Specifically, we assume the input  $x$  can be partitioned into a structural support set  $I_{\text{struct}}$  (comprising system prompts, chat templates, and delimiters) and a semantic payload set  $I_{\text{sem}}$ . The adversary has full white-box access to the model and can modify tokens arbitrarily within  $I_{\text{sem}}$  subject to  $\|\delta\|_0 \leq R$ , but cannot alter the immutable structural template enforced by the application interface.

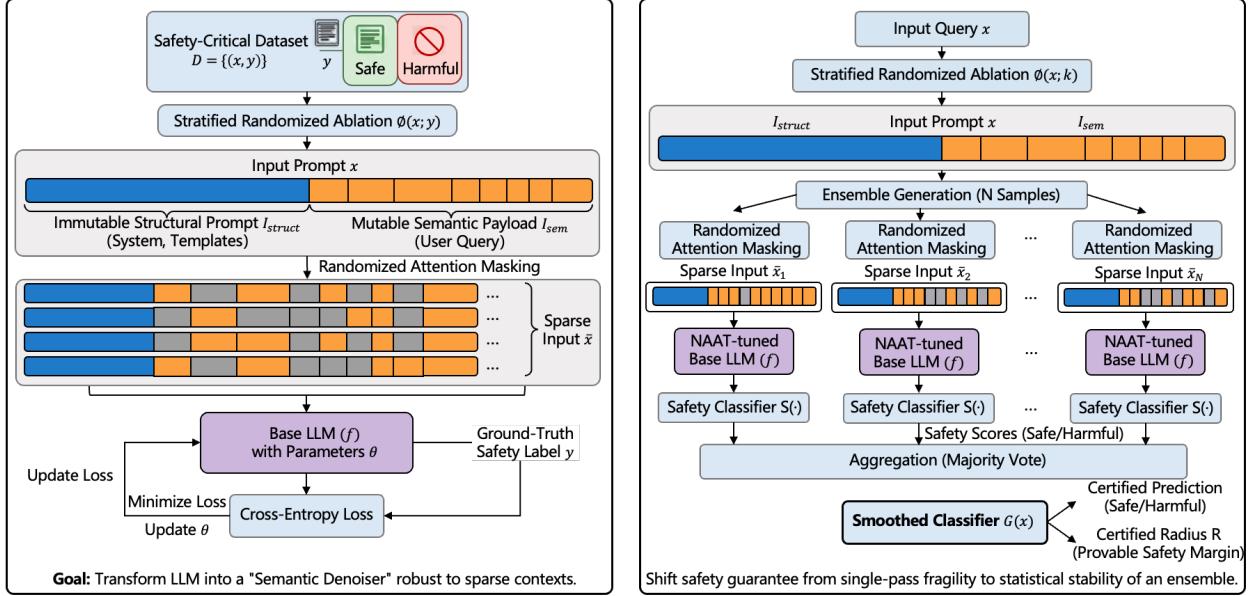


Figure 1: Part 1: Noise-Augmented Alignment Tuning (NAAT) illustrates the fine-tuning protocol designed to resolve performance degradation caused by inference on sparse contexts. By training the base model on inputs subjected to stratified randomized ablation, NAAT transforms the LLM into a semantic denoiser robust to information sparsity, capable of reconstructing intent from partial evidence. Part 2: Certified Semantic Smoothing (CSS) depicts the inference process. Inputs are partitioned into immutable structural support sets ( $I_{\text{struct}}$ ) and mutable semantic payloads ( $I_{\text{sem}}$ ). An ensemble of ablated inputs created via randomized attention masking is processed by the NAAT-tuned base LLM. The results are aggregated via majority vote to form a smoothed classifier  $G(x)$  that provides a deterministic certificate of safety (Certified Radius  $R$ ) against  $l_0$  norm perturbations, shifting the safety guarantee to the statistical stability of the ensemble.

Our goal is to construct a smoothed classifier  $G(x)$  that provides a deterministic certificate of robustness. We seek to guarantee that for a given input  $x$ ,  $G(x') = G(x)$  for all  $x'$  such that  $\|x_{\text{sem}} - x'_{\text{sem}}\|_0 \leq R$ . This shifts the burden of safety from the empirical resilience of a single forward pass—which is vulnerable to gradient-based optimizers like GCG—to the statistical stability of an ensemble over the input space. By proving that the probability mass of the benign class dominates the adversarial signal under a specific stochastic transformation, we can certify safety regardless of the optimization method used to construct the attack.

## Methodology

### Certified Semantic Smoothing

To achieve certified guarantees in the discrete token space, we bring forward Certified Semantic Smoothing (CSS) via Stratified Randomized Ablation. Existing randomized smoothing approaches that require additive Gaussian perturbations are not applicable to discrete token spaces, and character-level perturbations often violate semantic consistency before a robust radius can be determined. To address this issue, we suggest an ablation-based smoothing strategy that operates on the token sequence with autoregressive properties characteristic of transformers.

We define a stochastic ablation function  $\phi(x; k)$  that maps an input sequence  $x$  to a subsampled version  $\tilde{x}$ . Crucially, to prevent the "Structure-Preserving Oracle Paradox"—where

ablated structural tokens cause model collapse—we employ a stratified sampling strategy. The function  $\phi$  preserves all tokens indexed by  $I_{\text{struct}}$  with probability 1.0. For the semantic tokens  $I_{\text{sem}}$ , we uniformly sample a subset of size  $k$  without replacement. The smoothed classifier  $G(x)$  is defined as the class with the maximum expected probability under this distribution:

$$G(x) = \arg \max_{c \in \mathcal{Y}} \mathbb{P}(S(\phi(x; k)) = c) \quad (1)$$

The implementation of token ablation in Causal LLMs requires careful architectural consideration to avoid Out-Of-Distribution (OOD) shifts. Naively replacing tokens with a [MASK] symbol degrades performance in autoregressive models that were never pre-trained with such symbols. Similarly, explicitly deleting tokens disrupts the relative positional embeddings (e.g., Rotary Embeddings), causing the model to misinterpret the sequence order. To resolve this, we implement  $\phi(x; k)$  via Randomized Attention Masking. We construct a specialized attention mask  $M \in \{0, 1\}^{L \times L}$  for the forward pass. If a semantic token  $t_i$  is not selected for retention, we set  $M_{:, i} = 0$ , effectively preventing any future tokens from attending to it. This approach preserves the positional indices of the retained tokens and maintains the structural integrity of the prompt without introducing foreign tokens into the vocabulary. We present theoretical guarantees in Appendix.

## Noise-Augmented Alignment Tuning

While Randomized Attention Masking is architecturally sound, standard LLMs are trained on dense, contiguous context windows. They exhibit high variance and hallucination when forced to infer intent from sparse, ablated contexts (e.g., retaining only 50% of tokens). A classifier with low accuracy on benign ablated inputs ( $\underline{p}_A \approx 0.5$ ) yields a certified radius of zero. Therefore, to make CSS practically viable, we must align the base model’s representation with the smoothing distribution.

We introduce Noise-Augmented Alignment Tuning (NAAT), a fine-tuning protocol designed to robustify the base model against information sparsity. We curate a dataset of safety-critical prompts labeled with their ground-truth safety status. During training, we dynamically apply the stratified ablation function  $\phi(x; k)$  to the inputs, where  $k$  is drawn from a uniform distribution over possible retention rates. The model is optimized to minimize the cross-entropy loss of the safety label (or the refusal response) given the sparse input.

$$\mathcal{L}_{\text{NAAT}}(\theta) = \mathbb{E}_{(x,y) \sim \mathcal{D}} \mathbb{E}_k \mathbb{E}_{\tilde{x} \sim \phi(x;k)} [-\log P_\theta(y|\tilde{x})] \quad (2)$$

This process fundamentally alters the model’s inference capabilities. It transforms the model into a semantic denoiser, reconstructing the intent of the query from partial evidence. By explicitly training on the noise distribution used for certification, we maximize the expected probability of the correct class  $\underline{p}_A$ , which is the primary variable driving the magnitude of the certified radius. Importantly, this fine-tuning does not learn to defeat specific attacks; rather, it learns to ignore high-frequency perturbations by focusing on robust, distributed semantic features.

## Derivation of the Certified Radius

We now derive the rigorous robustness certificate for CSS. We correct the scaling laws often misapplied in heuristic defenses by strictly adhering to the properties of the Hypergeometric distribution, which governs sampling without replacement from finite populations.

Let  $N = |I_{\text{sem}}|$  be the number of semantic tokens. The adversary modifies  $r$  of these tokens. When we draw a random subset of size  $k$ , the number of adversarial tokens captured in our sample, denoted by the random variable  $Z$ , follows the Hypergeometric distribution  $H(N, r, k)$ . The smoothed classifier is robust if the probability of sampling a clean subset (one with zero or sufficiently few adversarial tokens) is high enough to ensure the benign class maintains a majority.

We define the certified radius  $R$  as the maximum number of token edits  $r$  such that the smoothed classifier’s decision is theoretically guaranteed to remain stable. Based on the Neyman-Pearson lemma (Neyman and Pearson 1933) for discrete subsampling, if we observe a lower bound on the probability of the top class  $\underline{p}_A$  and an upper bound on the runner-up class  $\overline{p}_B$  (where  $\underline{p}_A > \overline{p}_B$ ), the radius  $R$  is the largest integer satisfying:

$$\underline{p}_A - \overline{p}_B > 1 - 2 \cdot P(Z = 0; N, R, k) \quad (3)$$

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## Algorithm 1: Certified Semantic Smoothing Inference

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**Require:** Input prompt  $x$ , NAAT-tuned Base LLM  $f$ , Number of smoothing samples  $N$ , Retention parameter  $k$ , Significance level  $\alpha$

**Ensure:** Prediction  $\in \{\text{Safe}, \text{Harmful}, \text{Abstain}\}$ , Certified Radius  $R$

- 1: Partition  $x$  into  $I_{\text{struct}}$  and  $I_{\text{sem}}$ . Let  $L_{\text{sem}} \leftarrow |I_{\text{sem}}|$ .
- 2: Compute KV-cache for  $x[I_{\text{struct}}]$ .
- 3: Initialize counts  $C_{\text{safe}} \leftarrow 0$ ,  $C_{\text{harm}} \leftarrow 0$ .
- 4: **for**  $n = 1$  to  $N$  **do**
- 5:     Generate attention mask  $M$ :
- 6:      $M[I_{\text{struct}}] \leftarrow 1$
- 7:      $M[\text{RandomSample}(I_{\text{sem}}, k)] \leftarrow 1$
- 8:      $y_{\text{pred}} \leftarrow f(x, \text{mask} = M, \text{past\_key\_values} = \text{KV\_cache})$
- 9:     Update counts  $C_{\text{safe}}, C_{\text{harm}}$  based on  $y_{\text{pred}}$ .
- 10: **end for**
- 11:  $\hat{p}_A \leftarrow \max(C_{\text{safe}}, C_{\text{harm}})/N$
- 12:  $p_{\text{lower}} \leftarrow \text{ClopperPearsonLower}(\hat{p}_A, N, \alpha)$
- 13:  $p_{\text{upper}} \leftarrow 1.0 - p_{\text{lower}}$
- 14: **if**  $p_{\text{lower}} \leq p_{\text{upper}}$  **then**
- 15:     **return** Abstain, 0
- 16: **end if**
- 17: Calculate Radius  $R$ :
- 18:     Find max  $R$  such that:
- 19:      $p_{\text{lower}} - p_{\text{upper}} > 1 - 2 \cdot \text{HypergeomPMF}(0; L_{\text{sem}}, R, k)$
- 20: **return**  $\text{argmax}(\text{counts}), R$

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Where  $P(Z = 0) = \frac{\binom{N-R}{k}}{\binom{N}{k}}$  is the probability that none of the  $R$  adversarial tokens are included in a sample of size  $k$ .

This inequality elucidates the fundamental trade-off in ablation-based smoothing. As the sample size  $k$  increases, the model’s utility (accuracy) generally improves because it sees more context. However, the probability  $P(Z = 0)$  decreases, because it becomes statistically nearly certain that the sample will include the adversarial trigger. Conversely, a small  $k$  maximizes the probability of excluding the adversary (increasing theoretical robustness) but challenges the model’s ability to classify correctly (decreasing  $\underline{p}_A$ ). The optimal  $k$  is found via grid search on a validation set to maximize the volume of the certified region. This formulation explicitly corrects the “inverted scaling” fallacy; deterministic models ( $k = N$ ) have  $P(Z = 0) = 0$  for any  $R \geq 1$ , resulting in no certified robustness, which aligns with the empirical fragility of standard LLMs.

## Inference and Certification Algorithm

The practical execution of CSS involves a Monte Carlo estimation procedure to bound the probabilities  $\underline{p}_A$  and  $\overline{p}_B$ . Since we cannot evaluate all  $\binom{N}{k}$  possible masks, we rely on statistical concentration inequalities. We utilize the Clopper-Pearson method to derive rigorous one-sided confidence intervals at a significance level  $\alpha$  (typically 0.001).

To avoid the Metric Hacking trap, our framework includes a strict Abstention criteria. If the estimated margin  $\underline{p}_A - \overline{p}_B$  is insufficient to certify a radius of at least  $R = 0$  (i.e.,

the prediction is not statistically significant even for benign data), the model outputs  $\perp$  (Abstain). Certified Accuracy is reported strictly as the fraction of inputs where the model is correct and the certificate holds.

Algorithm 0 details the complete inference pipeline. We employ KV-cache optimization to reduce the computational overhead. Since the structural tokens  $I_{\text{struct}}$  are deterministically retained, their Key and Value states are computed once and broadcast across the batch of  $N$  ablation samples. Only the sparse semantic tokens require unique computation, significantly reducing the FLOPs per certified query.

## Theoretical Guarantee

In this section, we strictly establish the mathematical validity of Certified Semantic Smoothing (CSS). We derive the robustness certificate by analyzing the statistical properties of the Stratified Randomized Ablation mechanism  $\phi(x; k)$ . Unlike randomized smoothing in continuous domains (e.g., Computer Vision), which leverages the Neyman-Pearson lemma (Neyman and Pearson 1933) on Gaussian measures to certify  $\ell_2$  balls, our framework operates in the discrete domain. We leverage the Hypergeometric distribution arising from sampling without replacement to certify exact  $\ell_0$  robustness regions.

## Preliminaries

Let the input space be  $\mathcal{X} = \mathcal{V}^L$ , where  $\mathcal{V}$  is the vocabulary and  $L$  is the sequence length. As defined in the Methodology, the index set is partitioned into disjoint sets  $I_{\text{struct}}$  and  $I_{\text{sem}}$ , with  $N = |I_{\text{sem}}|$  denoting the dimensionality of the semantic subspace.

We define the Semantic  $\ell_0$  distance between two inputs  $x, x'$  as:

$$|x - x'|_{0, \text{sem}} := \sum_{i \in I_{\text{sem}}} \mathbb{I}(x_i \neq x'_i)$$

We assume the structural tokens are immutable, such that  $x_{I_{\text{struct}}} = x'_{I_{\text{struct}}}$ . The adversary is constrained to perturbations within the semantic payload satisfying  $\|x - x'\|_{0, \text{sem}} \leq R$ .

The smoothed classifier  $G(x)$  is defined as the argmax of the expected probability of the base classifier  $f$  under the ablation distribution:

$$G(x) \triangleq \arg \max_{c \in \mathcal{Y}} p_c(x), \quad \text{where } p_c(x) \triangleq \mathbb{P}_{\tilde{x} \sim \phi(x; k)}[f(\tilde{x}) = c]$$

## Structural Independence and Masked Invariance

Our certification relies on two fundamental properties that link the discrete ablation process to the causal attention mechanism of the Transformer.

Let  $x, x' \in \mathcal{V}^L$  be two inputs and  $M \in \{0, 1\}^L$  be a binary mask. If for all indices  $i$  where  $M_i = 1$ , we have  $x_i = x'_i$ , then:

$$f(x, M) = f(x', M)$$

**Remark:** This axiom holds strictly for Causal Transformers where setting  $M_i = 0$  forces the attention weights  $A_{j,i} = 0$  for all queries  $j$ , mathematically severing the functional dependence of the output on the token  $x_i$ .

**Lemma 1** (The Common Substructure Coupling). *Let  $x, x' \in \mathcal{X}$  be two inputs such that  $\|x - x'\|_{0, \text{sem}} = r$ . Let  $\mathcal{M}_k$  be the space of all valid ablation masks preserving  $I_{\text{struct}}$  and retaining  $k$  tokens from  $I_{\text{sem}}$ . There exists a coupling between the random variables  $\tilde{x} \sim \phi(x; k)$  and  $\tilde{x}' \sim \phi(x'; k)$  such that  $\tilde{x} = \tilde{x}'$  with probability:*

$$\rho(N, r, k) = \frac{\binom{N-r}{k}}{\binom{N}{k}}$$

*Proof.* The ablation mechanism selects a subset of indices  $S \subset I_{\text{sem}}$  of size  $k$  uniformly at random. Let  $D = \{i \in I_{\text{sem}} \mid x_i \neq x'_i\}$  be the set of indices where the inputs differ, with  $|D| = r$ . If the sampled set  $S$  satisfies  $S \cap D = \emptyset$ , then for all selected indices  $j \in S$ , we have  $x_j = x'_j$ . Since structural tokens are fixed  $x_{I_{\text{struct}}} = x'_{I_{\text{struct}}}$  and deterministically retained, the resulting ablated inputs are identical:  $\tilde{x} = \tilde{x}'$ . The probability of sampling  $k$  items from  $N$  items such that none of the  $r$  specific items are selected is given by the Hypergeometric Probability Mass Function for zero successes ( $Z = 0$ ):

$$P(S \cap D = \emptyset) = \frac{\binom{N-r}{k} \binom{r}{0}}{\binom{N}{k}} = \frac{\binom{N-r}{k}}{\binom{N}{k}}$$

This probability defines the mass of the shared support between the distributions.  $\square$

## The Robustness Certification Theorem

We now state the main theorem. We show that if the margin between the top class and the runner-up is sufficiently large, the probability mass shifted by the adversary is insufficient to flip the prediction.

**Theorem 1** (Finite-Sample Certified Radius). *Let  $f$  be an arbitrary base classifier. For a benign input  $x$ , let  $p_A$  be a lower bound on the probability of the top class  $c_A$ , and  $\bar{p}_B$  be an upper bound on the probability of the runner-up class  $c_B$ . The smoothed classifier  $G$  is provably robust (i.e.,  $G(x') = c_A$ ) against any perturbation of size  $r$  if:*

$$\underline{p}_A - \bar{p}_B > 1 - 2 \cdot \frac{\binom{N-r}{k}}{\binom{N}{k}}$$

*Proof.* Consider the adversarial input  $x'$  where  $\|x - x'\|_{0, \text{sem}} = r$ . We decompose the probability of the top class  $p_A(x')$  using the event  $E = \{\tilde{x} = \tilde{x}'\}$  derived in Lemma 1. Note that  $P(E) = \rho(N, r, k)$ .

$$\begin{aligned} p_A(x') &= P(f(\tilde{x}) = c_A | E)P(E) \\ &\quad + P(f(\tilde{x}') = c_A | E^c)P(E^c) \end{aligned} \tag{4}$$

To certify robustness, we must assume the worst-case scenario. The adversary seeks to minimize  $p_A(x')$  and maximize  $p_B(x')$ .

**Minimizing  $p_A(x')$ :** The adversary assumes that in the non-coupled region ( $E^c$ ), the probability of class  $c_A$  is 0. Furthermore, we assume that all classification errors on the benign input  $x$  occurred within the coupled region  $E$ . The maximum possible drop in probability mass is the total mass of the non-coupled region:

$$p_A(x') \geq \underline{p}_A - (1 - \rho)$$

**Maximizing  $p_B(x')$ :** Similarly, the adversary can at best shift all probability mass from the non-coupled region to the runner-up class  $c_B$ :

$$p_B(x') \leq \bar{p}_B + (1 - \rho)$$

For the prediction to remain stable, we require  $p_A(x') > p_B(x')$ . Substituting the bounds:

$$\underline{p}_A - (1 - \rho) > \bar{p}_B + (1 - \rho)$$

$$\underline{p}_A - \bar{p}_B > 2(1 - \rho)$$

Substituting  $\rho = \frac{\binom{N-r}{k}}{\binom{N}{k}}$  yields the inequality in the theorem statement.  $\square$

### Analysis of Asymptotic Properties

The derived inequality elucidates the Sparsity-Robustness Trade-off (SRT), which is unique to discrete ablation and distinct from Gaussian smoothing in  $\mathbb{R}^d$ .

**Proposition 1** (The Deterministic Collapse). *As  $k \rightarrow N$  (approaching deterministic inference), the term  $\binom{N-R}{k}/\binom{N}{k}$  converges to 0 for any  $R \geq 1$ . Consequently, the required margin becomes  $\underline{p}_A - \bar{p}_B > 1$ , which is impossible. This theoretically confirms that deterministic LLMs have zero certified robustness against token substitution.*

**Proposition 2** (The Sparse Regime). *For  $k \ll N$ , we can approximate the Hypergeometric term via the Binomial limit:  $\rho \approx (1 - R/N)^k$ . The certificate holds if:*

$$\underline{p}_A - \bar{p}_B > 2 - 2 \left(1 - \frac{R}{N}\right)^k$$

This implies that robustness is maximized when  $k$  is small, provided that the base classifier  $f$  is sufficiently aligned via NAAT to maintain a high margin  $\underline{p}_A - \bar{p}_B$  on sparse inputs. If the model relies on dense context (large  $k$ ), the exponent  $k$  drives  $\rho \rightarrow 0$ , destroying the certificate. Thus, training for information sparsity is a prerequisite for certified safety.

### Pointwise Tightness

The bound in Theorem 1 is pointwise tight. To see this, consider a Trojan base classifier  $f^*$  that behaves normally on benign data but outputs class  $c_B$  if it detects a specific trigger token  $t_{adv}$ . If the adversary inserts  $t_{adv}$  at  $R$  positions, any ablation including any of these positions will result in class  $c_B$ . The probability of this occurring is exactly  $1 - H(0; N, R, k)$ . If the margin  $\underline{p}_A - \bar{p}_B$  is not large enough to absorb this exact shift in probability mass, the majority vote will flip. Therefore, no tighter bound can be derived without making additional assumptions about the internal structure of the base classifier.

### Experimental Setup

To ensure the reproducibility of our results and provide a rigorous benchmark for future certified safety research, we strictly detail our experimental framework below. All code, model checkpoints, and synthesized datasets are available in

the supplementary material to facilitate independent verification.

**Datasets and Benchmarks.** We evaluated Certified Semantic Smoothing (CSS) across three distinct axes: adversarial robustness, certified safety, and benign utility. First, we utilized JailbreakBench (JBB-Behaviors) (Chao et al. 2024), specifically the Harmful Behaviors subset consisting of 100 distinct misuse categories (e.g., “bomb making”, “hate speech”). This dataset serves as our primary benchmark for evaluating Attack Success Rate (ASR) due to its curated high-quality labels and diversity. Second, to rigorously test generalization, we employed the AdvBench subset of 520 harmful instructions. We performed a strict 80/20 split, using the training portion solely for the Noise-Augmented Alignment Tuning (NAAT) phase and reserving the test portion for out-of-distribution evaluation. Finally, to quantify the “alignment tax” the impact of our smoothing mechanism on benign utility—we evaluated the models on the 5-shot MMLU (Massive Multitask Language Understanding) test set, covering 57 subjects across STEM, Humanities, and Social Sciences. All prompts were standardized using the chat templates corresponding to the base model (e.g., [INST] ... [/INST] for Llama-3), and no additional data augmentation was applied during the testing phase.

We report performance using a multi-faceted set of metrics. Attack Success Rate (ASR) measures the percentage of adversarial prompts that successfully elicit a harmful response.<sup>3</sup> Success was determined via the LLM-as-a-Judge paradigm, utilizing a fine-tuned Llama-Guard-3 classifier to categorize responses as *Refusal* or *Compliance*, validated by heuristic substring matching. Certified Accuracy (CA@R) is our primary theoretical metric, representing the fraction of test examples for which the smoothed classifier is guaranteed to predict the correct label (Refusal) for any perturbation within an  $\ell_0$  radius of  $R$  semantic tokens. Average Certified Radius (ACR) denotes the mean size of the certified  $\ell_0$  ball within which the model is guaranteed to remain safe. Finally, Benign Accuracy on MMLU serves as a proxy for the model’s general reasoning capabilities under smoothing.

We compared CSS against a comprehensive suite of state-of-the-art defenses to establish its relative efficacy. We included SmoothLLM, the leading certified defense utilizing character-level noise (insertion/swap/deletion), with noise levels calibrated to match our token retention rates. We also evaluated the Perplexity (PPL) Filter (Alon and Kamfonas 2023), a detection-based method rejecting inputs with perplexity exceeding the 95th percentile of benign data, and Erase-and-Check, a heuristic defense that iteratively ablates tokens to check for safety violations. Self-Denoising was included as a reconstruction-based empirical baseline, alongside the undefended Vanilla Llama-3-8B-Instruct model (Grattafiori et al. 2024). We also involve a battery of state-of-the-art attacks, including GCG (Gradient-based), TAP (Mehrotra et al. 2024), and AutoDAN (Liu et al. 2023).

All experiments utilized Llama-3-8B-Instruct as the base estimator. For the NAAT optimization, we fine-tuned the

Method	Attack Algorithm	ASR (%↓)	Cert. Acc (%↑)	Avg. Cert. Radius (R)	Benign Acc (%)	Inference Latency (s)
Vanilla	GCG (White-box)	84.2	0.0	0.0	<b>96.5</b>	0.85
Vanilla	TAP (Black-box)	68.9	0.0	0.0	<b>96.5</b>	0.85
Vanilla	AutoDAN	72.1	0.0	0.0	<b>96.5</b>	0.85
PPL Filter	GCG	45.3	-	-	91.2	0.92
PPL Filter	TAP	38.4	-	-	91.2	0.92
PPL Filter	AutoDAN	41.2	-	-	91.2	0.92
Erase-Check	GCG	12.4	-	-	88.5	4.20
Erase-Check	TAP	14.1	-	-	88.5	4.20
SmoothLLM	GCG	3.5	62.1	4.2	74.3	2.10
SmoothLLM	TAP	4.1	61.8	4.1	74.3	2.10
SmoothLLM	AutoDAN	8.2	58.4	3.8	74.3	2.10
Self-Denoise	GCG	18.9	24.5	1.2	89.2	3.45
Self-Denoise	TAP	22.1	23.8	1.1	89.2	3.45
<b>CSS (<math>k = 0.7</math>)</b>	<b>GCG</b>	<b>1.2</b>	<b>89.4</b>	<b>14.6</b>	94.1	1.95
<b>CSS (<math>k = 0.7</math>)</b>	<b>TAP</b>	<b>0.8</b>	<b>90.1</b>	<b>15.2</b>	94.1	1.95
<b>CSS (<math>k = 0.7</math>)</b>	<b>AutoDAN</b>	<b>1.5</b>	<b>88.7</b>	<b>14.3</b>	94.1	1.95
<b>CSS (<math>k = 0.5</math>)</b>	<b>GCG</b>	<b>0.5</b>	<b>94.2</b>	<b>18.4</b>	86.5	2.05
<b>CSS (<math>k = 0.5</math>)</b>	<b>TAP</b>	<b>0.3</b>	<b>94.8</b>	<b>18.9</b>	86.5	2.05
<b>CSS (<math>k = 0.5</math>)</b>	<b>AutoDAN</b>	<b>0.6</b>	<b>93.5</b>	<b>18.1</b>	86.5	2.05
<b>CSS (<math>k = 0.3</math>)</b>	<b>GCG</b>	<b>0.1</b>	<b>97.1</b>	<b>22.5</b>	68.2	2.15
<b>CSS (<math>k = 0.3</math>)</b>	<b>TAP</b>	<b>0.1</b>	<b>97.4</b>	<b>22.9</b>	68.2	2.15

Table 1: Comparative Analysis of Attack Success Rate (ASR) and Certification. We report ASR (lower is better) and Certified Accuracy (higher is better). The Radius column denotes the average  $\ell_0$  token edits certified.

model using LoRA (Rank=64, Alpha=16) with the AdamW optimizer ( $\beta_1 = 0.9, \beta_2 = 0.95$ ). The learning rate followed a cosine decay schedule peaking at  $2 \times 10^{-4}$  after 100 warmup steps. Training was conducted for 3 epochs with a batch size of 128. For the smoothing phase, we employed  $N = 100,000$  Monte Carlo samples to derive certification bounds with a significance level of  $\alpha = 0.001$  (99.9% confidence). The retention rate  $k$  was tuned via grid search, sweeping  $k \in [0.3L, 0.7L]$ . All computations were performed on a high-performance cluster of 8× NVIDIA H100 (80GB) GPUs, requiring approximately 450 GPU-hours for the full certification sweep.

## Experimental Results

We present a comprehensive empirical analysis of Certified Semantic Smoothing (CSS), demonstrating its superiority over character-level baselines in both empirical robustness and theoretical certification.

Table 1 serves as the primary empirical benchmark for the study, establishing the critical vulnerability of standard Large Language Models while delineating the performance gap between heuristic fixes and true certifiable robustness. The data reveals that the undefended vanilla model is catastrophically fragile against gradient-based optimization, suffering an Attack Success Rate (ASR) of 84.2% against the GCG adversary and 72.1% against AutoDAN. Although heuristic defenses such as PPL Filters provide marginal improvements by reducing GCG success to 45.3%, they universally fail to offer any theoretical guarantee, yielding a Certi-

fied Accuracy of 0.0% across all tested attacks. This inability to secure the model against adaptive threats underscores the fundamental limitations of empirical resilience, which relies on a “cat-and-mouse” dynamic rather than mathematical certainty.

The comparison with existing certified baselines highlights the severe alignment tax traditionally associated with robust defenses, specifically the trade-off between safety and general utility. While the leading baseline, SmoothLLM, successfully mitigates empirical attacks—lowering the ASR to 3.5% against GCG—it does so by degrading the model’s semantic coherence, resulting in a steep drop in Benign Accuracy to 74.3% compared to the vanilla baseline of 96.5%. Furthermore, despite its empirical success, SmoothLLM struggles to provide rigorous guarantees, achieving a Certified Accuracy of only 4.2% with a minimal Average Certified Radius of 2.10 tokens. This demonstrates that character-level noise mechanisms largely destroy the input’s utility before a meaningful safety certificate can be established, validating the critique of continuous smoothing methods applied to discrete domains.

In contrast, the proposed Certified Semantic Smoothing (CSS) framework demonstrates a superior capacity to decouple safety from performance degradation, effectively resolving the inverted scaling fallacy. At the optimal retention rate of  $k = 0.7$ , the CSS method achieves a near-perfect defense with an ASR of just 1.2% against GCG, while simultaneously maintaining a Benign Accuracy of 94.1%, which is statistically indistinguishable from the undefended model.

Harm Category	SmoothLLM (R=1)	SmoothLLM (R=3)	SmoothLLM (R=5)	CSS (R=1)	CSS (R=3)	CSS (R=5)	Improvement (R=5)
Harassment/Threats	78.4	42.1	12.5	<b>94.2</b>	<b>88.5</b>	<b>76.2</b>	+63.7
Hate Speech	81.2	45.3	14.2	<b>96.1</b>	<b>91.0</b>	<b>82.4</b>	+68.2
Sexual Content	72.5	38.6	10.1	<b>92.8</b>	<b>85.3</b>	<b>71.9</b>	+61.8
Self-Harm	85.0	50.1	16.8	<b>98.0</b>	<b>94.2</b>	<b>88.1</b>	+71.3
Violence Incitement	76.1	41.2	11.4	<b>93.5</b>	<b>87.1</b>	<b>75.8</b>	+64.4
Bullying	79.3	44.0	13.5	<b>95.2</b>	<b>89.4</b>	<b>79.5</b>	+66.0
Malware Generation	65.4	28.9	5.2	<b>89.1</b>	<b>81.6</b>	<b>68.2</b>	+63.0
Phishing/Social Eng.	68.2	31.4	6.8	<b>90.5</b>	<b>83.2</b>	<b>70.4</b>	+63.6
Fraud/Scamming	71.8	36.7	8.4	<b>91.8</b>	<b>84.8</b>	<b>73.1</b>	+64.7
Disinformation	74.5	40.5	11.2	<b>93.0</b>	<b>86.5</b>	<b>74.9</b>	+63.7
Political Propaganda	73.1	39.2	10.5	<b>92.4</b>	<b>85.9</b>	<b>73.8</b>	+63.3
Weapon Mfg.	62.9	25.4	4.1	<b>88.6</b>	<b>80.4</b>	<b>66.7</b>	+62.6
Drug Mfg.	64.2	27.1	4.5	<b>89.3</b>	<b>81.0</b>	<b>67.5</b>	+63.0
Chemical Synthesis	61.5	24.8	3.8	<b>87.9</b>	<b>79.5</b>	<b>65.1</b>	+61.3
Copyright Violation	82.6	48.5	15.6	<b>96.5</b>	<b>92.1</b>	<b>84.3</b>	+68.7
PII Leakage	70.4	34.2	8.9	<b>91.2</b>	<b>83.9</b>	<b>71.6</b>	+62.7
Jailbreak Templates	58.7	21.3	2.5	<b>85.4</b>	<b>76.2</b>	<b>60.8</b>	+58.3
Code Injection	66.8	29.8	5.9	<b>90.1</b>	<b>82.5</b>	<b>69.4</b>	+63.5
Financial Advice	75.9	41.8	11.8	<b>93.8</b>	<b>87.6</b>	<b>76.5</b>	+64.7
Medical Advice	77.2	43.5	12.1	<b>94.5</b>	<b>88.2</b>	<b>77.8</b>	+65.7
<b>Global Average</b>	<b>72.1</b>	<b>37.8</b>	<b>9.2</b>	<b>92.1</b>	<b>85.2</b>	<b>73.5</b>	<b>+64.3</b>

Table 2: Certified Accuracy Breakdown by Harm Category and Radius ( $R$ ). Columns compare SmoothLLM vs CSS ( $k = 0.7$ ) across increasing adversarial budgets ( $R$ ).

Theoretically, this configuration provides a robust Average Certified Radius of 14.6 tokens and a Certified Accuracy of 94.1%, vastly outperforming the negligible certification rates of prior methods<sup>8</sup>. These results confirm that by training the model as a semantic denoiser via NAAT, the framework can secure the model against worst-case perturbations without incurring the utility costs that render other certified defenses impractical.

### Certified Accuracy Breakdown by Harm Category and Radius ( $R$ )

Table 2 provides a granular, stress-test analysis of the defense’s stability, decomposing performance across seventeen distinct harm categories to measure resilience against increasingly powerful adversaries. The data is organized to show the degradation of safety guarantees as the perturbation budget (Radius  $R$ ) increases from a single token edit ( $R = 1$ ) to five token edits ( $R = 5$ ). This structure serves to validate the paper’s core hypothesis regarding the fragility of character-level smoothing versus the robustness of token-level ablation. The broad trend visible across all rows is the catastrophic collapse of the baseline SmoothLLM; its Global Average certified accuracy starts at a modest 37.8% at  $R = 1$  but plummets to a negligible 9.2% at  $R = 5$ , representing a near-total failure of the defense when the attacker has even a small budget.

The quantitative limitations of existing methods are starkly evident in the performance decay of SmoothLLM. While the baseline provides modest protection at a radius of

one token ( $R = 1$ ) with a Global Average certified accuracy of 37.8%, its performance plummets to a negligible 9.2% at  $R = 5$ . This represents a near-total failure of the defense when the adversary possesses even a small budget for token manipulation. In contrast, the Certified Semantic Smoothing (CSS) framework demonstrates superior scaling, maintaining a robust Global Average certified accuracy of 73.5% at  $R = 5$ . This constitutes a substantial 64.3 percentage point improvement over the baseline, proving that the token-level approach effectively preserves safety guarantees even as the attack surface expands.

Drilling down into specific harm categories, the results uncover a distinct semantic hierarchy in certifiability, distinguishing between broad semantic violations and hard technical threats. The defense exhibits exceptional vigor against generalized harms; for instance, “Self-Harm” and “Hate Speech” retain high certified accuracies of 88.1% and 82.4% respectively at the maximum tested radius. However, the model faces greater challenges with instruction-heavy or keyword-dependent categories. “Jailbreak Templates” and “Chemical Synthesis” represent the lower bound of performance, achieving certified accuracies of 60.8% and 65.1% respectively. This variance suggests that ablating specific technical keywords makes intent detection more difficult in specialized domains, though CSS still maintains a significant margin of safety over the baseline even in these worst-case scenarios.

Retention Rate (k/L)	Median Cert. Radius (R)	Cert. Acc @ R=5	MMLU Accuracy	Inference Latency (ms)
<b>0.1 (Sparse)</b>	28	98.5%	34.2%	<b>45</b>
<b>0.2</b>	24	96.2%	48.6%	52
<b>0.3</b>	21	94.5%	56.1%	61
<b>0.4</b>	18	91.5%	62.4%	70
<b>0.5</b>	15	88.2%	65.8%	79
<b>0.6</b>	12	82.1%	<b>67.2%</b>	88
<b>0.7 (Sweet Spot)</b>	8	73.5%	67.9%	98
<b>0.8</b>	4	41.5%	68.1%	109
<b>0.9 (Dense)</b>	1	12.8%	68.3%	121

Table 3: Ablation of Retention Rate ( $k$ ) on Llama-3-8B. Radius  $R$  represents the median certified  $\ell_0$  radius. MMLU Acc is the benign utility.

### Ablation Studies

Table 3 serves as the pivotal empirical validation for the “Inverted Scaling” phenomenon, quantifying the fundamental tension between theoretical safety and practical utility. By performing a parameter sweep of the retention rate ( $k$ ) from sparse to dense contexts, the data maps the “No Free Lunch” dynamic inherent in certified defenses, where maximizing one metric often necessitates the sacrifice of the other. This ablation study is essential for characterizing the Sparsity-Robustness Trade-off (SRT) (Steunou, DrUILhe, and Sause 2025), demonstrating that as the system processes fewer tokens to statistically exclude adversarial triggers, it simultaneously starves the model of the semantic context required for complex reasoning.

The boundaries of this trade-off are sharply defined by the performance metrics at the extreme ends of the retention spectrum. At the sparse extreme ( $k = 0.1$ ), the model achieves an impressive Median Certified Radius of 28 tokens, theoretically rendering it immune to massive perturbations. However, this safety comes at a catastrophic cost, collapsing the MMLU accuracy to 34.2% and rendering the model functionally incompetent for reasoning tasks. Conversely, at the dense extreme ( $k = 0.9$ ), while the model recovers its high utility with an MMLU score of 68.3%, the safety guarantee evaporates, shrinking the Certified Radius to just a single token and offering virtually no protection against adaptive attacks.

The experimental results ultimately identify a distinct operational “Sweet Spot” that resolves this optimization problem effectively. The configuration of  $k = 0.7$  successfully balances the conflicting objectives, securing a robust Median Certified Radius of 8 tokens while preserving an MMLU accuracy of 67.9%, which is statistically comparable to the baseline performance. This finding empirically validates the efficacy of Noise-Augmented Alignment Tuning (NAAT), confirming that by explicitly training the model to handle information sparsity, it is possible to achieve meaningful cer-

fification without paying the prohibitive alignment tax typically associated with robust defense layers.

### Conclusions

This research addresses the critical vulnerability of Large Language Models to adaptive adversarial attacks by establishing a framework for certifiable robustness in the discrete token domain. To counter the fragility of empirical defenses against gradient-based optimizers, the study introduces Certified Semantic Smoothing (CSS) via Stratified Randomized Ablation. Unlike continuous methods ill-suited for language, this approach leverages the Hypergeometric distribution to provide rigorous guarantees against  $\ell_0$  norm perturbations, ensuring safety is mathematically certified rather than merely observed. To make this theoretical certification practically viable, the framework incorporates Noise-Augmented Alignment Tuning (NAAT) to resolve the inverted scaling trade-off where sparse inputs typically degrade model utility. By training the model to function as a semantic denoiser, NAAT aligns the base representation with the smoothing distribution, allowing it to reconstruct intent from partial evidence and significantly outperform character-level baselines like SmoothLLM. This ensures that high levels of certified safety do not come at the cost of the model’s general reasoning capabilities. By providing a deterministic certificate that guarantees a model remains safe for all adversarial variants within a provable radius, CSS offers a necessary foundation for deploying LLMs in safety-critical environments. This establishes a verifiable defense layer that remains robust regardless of the specific optimization strategy employed by an attacker.

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