

# Beyond Mode Elicitation: Diversity-Preserving Reinforcement Learning via Latent Diffusion Reasoner

Haoqiang Kang<sup>1</sup> Yizhe Zhang<sup>2</sup> Nikki Lijing Kuang<sup>1</sup> Yi-An Ma<sup>1</sup> Lianhui Qin<sup>1</sup>

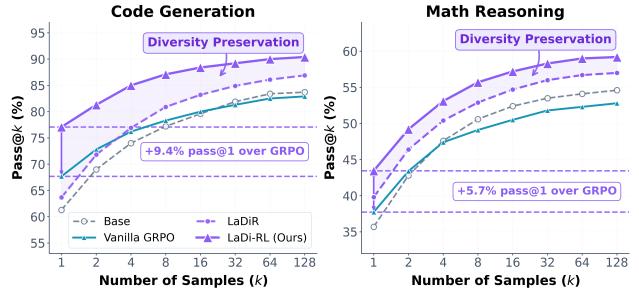
## Abstract

Recent reinforcement learning (RL) methods improve LLM reasoning by optimizing discrete Chain-of-Thought (CoT) generation; however, exploration in token space often suffers from diversity collapse as policy entropy decreases due to mode elicitation behavior in discrete RL. To mitigate this issue, we propose Latent Diffusion Reasoning with Reinforcement Learning (LaDi-RL), a framework that conducts exploration directly in a continuous latent space, where latent variables encode semantic-level reasoning trajectories. By modeling exploration via guided diffusion, multi-step denoising distributes stochasticity and preserves multiple coexisting solution modes without mutual suppression. Furthermore, by decoupling latent-space exploration from text-space generation, we show that latent diffusion-based optimization is more effective than text-space policy optimization alone, while a complementary text policy provides additional gains when combined with latent exploration. Experiments on code generation and mathematical reasoning benchmarks demonstrate consistent improvements in both pass@1 and pass@ $k$  over discrete RL baselines, with absolute pass@1 gains of +9.4% on code generation and +5.7% on mathematical reasoning, highlighting diffusion-based latent RL as a principled alternative to discrete token-level RL for reasoning.

## 1. Introduction

Reinforcement learning (RL) has become a dominant paradigm for improving the reasoning ability of large language models (LLMs), particularly through optimizing the generation of Chain-of-Thought (CoT) trajectories (Guo et al., 2025; Yu et al., 2025b; Shao et al., 2024). However, this process is often plagued by a significant side effect: diversity collapse (Song et al., 2024; Dang et al., 2025; Yue et al., 2025; Zhao et al., 2025b; He et al., 2025), which

<sup>1</sup>University of California San Diego <sup>2</sup>Apple.



**Figure 1. Average pass@ $k$  performance over all tested benchmarks.** LaDi-RL preserves solution diversity and converts LaDiR’s pass@ $k$  gains into strong pass@1 improvements, whereas vanilla GRPO exhibits diversity collapse at large  $k$ .

constrains the exploration space and impedes further RL optimization (Sutton et al., 1998). This issue is empirically reflected in metrics such as pass@ $k$  (He et al., 2025; Cobbe et al., 2021; Yue et al., 2025; Chung et al., 2025; Chen et al., 2025a), where RL-tuned models underperform the original base model at large  $K$  as the vanilla GRPO performance shown in Figure 1. This indicates the *mode elicitation* behavior in discrete RL causes the model to collapse onto dominant modes during generation at the expense of diversity (Yue et al., 2025). Recent analyses on LLMs (Cui et al., 2025; Cheng et al., 2025) and Diffusion Language Models (Gong et al., 2025; Zhao et al., 2025c) attribute this collapse to a fundamental *entropy mechanism* of softmax-based policies: correct tokens are typically assigned high probability by the policy under softmax normalization, increasing their logits, as the reward automatically suppresses all other tokens, driving entropy down—if without explicit regularization.

This motivates latent reasoning methods (Hao et al., 2024a; Kang et al., 2025; Butt et al., 2025; Zheng & Lee, 2025; Zhang et al., 2025d; Tang et al., 2026) that do not rely on softmax over discrete tokens, which may circumvent this issue. In particular, latent diffusion reasoners (Kang et al., 2025) exhibit large improvement in diversity and strong gains in pass@ $k$ , as diffusion models (Ho et al., 2020) capture the full support of the underlying distribution (Song & Ermon, 2019; Lee et al., 2023; Song et al., 2020) and allow multiple modes to coexist—each mode contributes to the learned score field without suppressing others (Lee et al., 2023; Tran et al., 2025). This naturally raises the question:

can the diversity advantage of diffusion models be leveraged to enhance RL exploration for LLM reasoning?

In this paper, we answer the above question in the affirmative and propose **Latent Diffusion Reasoning with Reinforcement Learning (LaDi-RL)**, a framework that models reasoning as a latent diffusion policy optimized via reinforcement learning, together with a complementary text policy. Each reasoning process is represented by a fixed-length block of continuous latent tokens that encode a semantic reasoning trajectory. A latent diffusion model iteratively denoises these tokens conditioned on the input query, and the resulting latent representation conditions a text policy that generates the final answer. Reinforcement learning is applied directly to the latent diffusion process, with rewards computed from the generated answers and attributed to the corresponding latent trajectories. By decoupling exploration in latent reasoning space from answer text generation, LaDi-RL preserves the diversity benefits of latent diffusion .

This design enables *structured and flexible exploration* for reasoning by shifting reinforcement learning from discrete token space to a continuous latent space. First, our method *preserves the base latent diffusion model’s inherent diversity after RL training*, as shown in Figure 1, by distributing stochasticity across multi-step diffusion, mitigating the diversity collapse of discrete RL. Second, exploration in latent space operates at the *semantic level*, where neighboring latents represent meaningful and flexible variations of entire reasoning trajectories, allowing RL to explore diverse trajectories to high reward regions rather than mere syntactic token perturbations. Lastly, by decoupling exploration in latent and text spaces, we empirically find that *latent diffusion-based optimization is more effective than text-space policy optimization alone*, while a complementary text policy provides additional gains when combined with latent exploration.

We evaluate our method on code generation and mathematical reasoning benchmarks, where LaDi-RL consistently improves both accuracy and diversity compared to discrete-space RL baselines. It achieves absolute pass@1 improvements of +9.4% and +5.7% respectively while simultaneously surpassing the base model’s pass@ $k$  ceiling. These results demonstrate that diffusion-based latent exploration offers a principled alternative to discrete RL for LLM reasoning by preserving diversity while improving accuracy.

## 2. Preliminary

### 2.1. Group Relative Policy Optimization

Group Relative Policy Optimization (GRPO) (Guo et al., 2025) is a reinforcement learning algorithm that has been widely adopted for optimizing large generative models, including large language models (Yu et al., 2025b) and diffusion-based generative models (Liu et al., 2025a; Xue

et al., 2025). Formally, given a group of rollout trajectories  $\{\mathbf{o}_g\}_{g=1}^G$  sampled from the current policy  $\pi_\theta$ , where the trajectories share the same conditioning input (e.g., prompt or context), GRPO computes a normalized advantage for each trajectory directly from scalar rewards:  $\hat{A}_g = \frac{R_g - \text{mean}(\{R_g\}_{g=1}^G)}{\text{std}(\{R_g\}_{g=1}^G)}$  where  $R_g$  denotes the outcome reward of trajectory  $\mathbf{o}_g$ . The GRPO objective is defined as:

$$\begin{aligned} \mathcal{J}_{\text{GRPO}}(\theta) = & \mathbb{E}_{\{\mathbf{o}_g\}_{g=1}^G \sim \pi_{\theta_{\text{old}}}} \left[ \frac{1}{G} \sum_{g=1}^G \min \left( r_g(\theta) \hat{A}_g, \right. \right. \\ & \left. \left. \text{clip}(r_g(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_g \right) \right] - \beta D_{\text{KL}}(\pi_\theta \| \pi_{\theta_{\text{ref}}}). \end{aligned} \quad (1)$$

where  $r_g(\theta) = \frac{\pi_\theta(\mathbf{o}_g)}{\pi_{\theta_{\text{old}}}(\mathbf{o}_g)}$  and  $\pi_{\theta_{\text{ref}}}$  is a fixed reference policy,  $\epsilon$  is the clipping threshold, and  $\beta$  controls KL regularization.

**GRPO for Language Models.** For LLMs, the policy  $\pi_\theta(o_t | o_{<t}, q)$  corresponds to an autoregressive language model that generates token  $o_t$  conditioned on previous tokens  $o_{<t}$  and input prompt  $q$ . The trajectory likelihood is tractable, allowing exact computation of log-probabilities and importance ratios.

**GRPO for Flow Matching Models.** Flow-GRPO (Liu et al., 2025a; Li et al., 2025b) applies GRPO to flow matching models by converting the deterministic Flow-ODE into an equivalent stochastic differential equation (SDE) and discretizing it using the Euler–Maruyama scheme. This introduces controlled stochasticity while preserving the underlying flow structure. Specifically, the resulting transition kernel takes the form:

$$\pi_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \mu_\theta(\mathbf{x}_t), g_t^2 \Delta t \mathbf{I}), \quad (2)$$

where  $g_t = a \sqrt{\frac{t}{1-t}}$  controls the noise scale,  $\Delta t$  is the discretization step size, and the mean  $\mu_\theta(\mathbf{x}_t)$  is determined by the learned flow field  $v_\theta(\mathbf{x}_t, t)$  following the standard flow-matching discretization. This formulation reduces each transition to a tractable Gaussian distribution, enabling direct computation of likelihood ratios and seamless integration with the GRPO objective in Eq. (1). Further details are provided in Appendix B.

### 2.2. Latent Diffusion Reasoner (LaDiR)

LaDiR (Kang et al., 2025) is a latent diffusion-based reasoning framework that compresses text CoTs into a compact continuous latent block, enabling efficient semantic-level reasoning. In this work, we adopt LaDiR as a *cold-start* initialization for subsequent reinforcement learning.

**Architecture.** As illustrated in Figure 2, LaDiR represents a reasoning process using a single continuous latent CoT block. Given a question  $Q$ , we insert a special token <BOT> to indicate the beginning of the latent block, followed by a fixed number of latent tokens  $Z = \{z_1, \dots, z_B\}$ , and a token <EOT> to mark its end. The pretrained LLM then generates the final answer autoregressively conditioned on

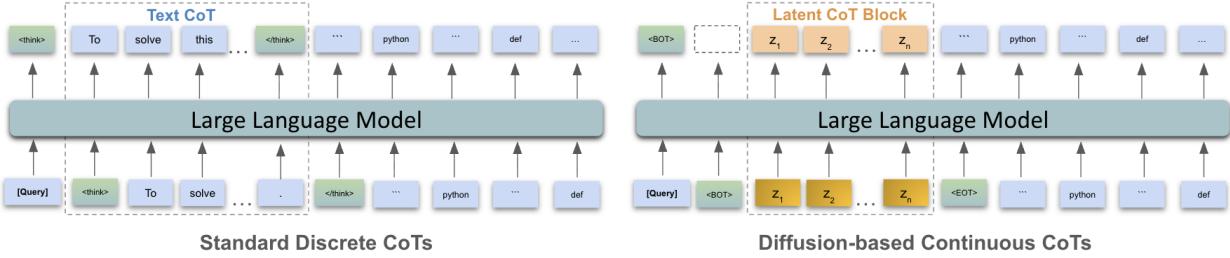


Figure 2. Comparison between standard discrete CoT reasoning and latent continuous CoT reasoning. Left: conventional autoregressive generation produces reasoning directly in token space. Right: reasoning is represented as a fixed-size latent block, enabling continuous modeling and decoupling reasoning from surface text generation.

both the question and the latent block. The latent tokens are produced by compressing a short reference CoT (typically fewer than 1k tokens) using a variational autoencoder (VAE), yielding a fixed-size latent representation (e.g.,  $B = 64$ ). Unlike the original LaDiR formulation, which applies block-wise diffusion over multiple latent segments, we employ a single sufficiently large latent block, which we find adequate for reconstructing short CoTs while simplifying the model.

**Training Objective.** LaDiR is trained using a joint objective that aligns latent reasoning with downstream text generation. Let  $Z^{\text{enc}}$  denote the latent tokens encoded from a reference CoT. We train a conditional latent generator using a flow-matching objective:  $\mathcal{L}_{\text{FM}} = \mathbb{E} [\|v_\theta(Z_t, Q, t) - (Z^{\text{enc}} - \epsilon)\|_2^2]$ , where  $Z_t = (1-t)\epsilon + tZ^{\text{enc}}$ . Conditioned on the question  $Q$  and latent block  $Z$ , the LLM is simultaneously trained to generate the target answer  $Y$  using a standard autoregressive cross-entropy loss:  $\mathcal{L}_{\text{CE}} = -\mathbb{E}_{(Q, Z, Y)} \sum_t \log p_\theta(y_t \mid y_{t'}, Q, Z)$ . The final training objective is  $\mathcal{L} = \lambda \mathcal{L}_{\text{FM}} + \mathcal{L}_{\text{CE}}$ , where  $\lambda$  controls the trade-off between latent and text generation.

**Inference.** At inference time, latent tokens are generated by iterative denoising conditioned on the input question. The LLM then generates the final answer autoregressively given the denoised latent block.

### 3. Methodology

Building upon LaDiR, we propose *Latent Diffusion Reasoner with Reinforcement Learning (LaDi-RL)*, a framework that applies GRPO to latent diffusion policies, enabling structured and flexible exploration in continuous latent space with higher diversity. A complementary text policy then conditions on the denoised latent reasoning trajectories to generate the final answer text.

#### 3.1. Rollout Design

Given a query  $Q$ , a rollout trajectory is represented as  $\mathbf{o} = \{\mathbf{z}^1, \dots, \mathbf{z}^K, x^{K+1}, \dots, x^{K+L}\}$ , where  $\mathbf{z}^i$  denotes the  $i$ -th latent block generated by the latent diffusion model via  $K$  iterative denoising steps, forming a latent CoT, and  $x^{k+1}, \dots, x^{k+L}$  are  $L$  autoregressively generated text tokens conditioned on the final latent block.

For each query, we first sample  $N$  latent diffusion trajectories, yielding denoised latent blocks  $\{\mathbf{z}_n\}_{n=1}^N$ . For

each latent block  $\mathbf{z}_n$ , we then sample  $M$  text responses  $\{\mathbf{x}_{(n,m)}\}_{m=1}^M$ . Rewards are computed at the final generated answers in text. This  $N \times M$  sampling strategy enables robust estimation of relative advantages while decoupling latent-space exploration from text-space answer generation. For efficiency, we use a small number of denoising steps  $K$  (e.g.,  $K = 10$ ) and disable classifier-free guidance (Ho & Salimans, 2022) during rollout.

#### 3.2. Latent Diffusion Policy

We optimize the latent diffusion policy following Flow-GRPO (Liu et al., 2025a), turning deterministic ODE denoising step to SDE step as a stochastic policy step. The latent policy loss follows the GRPO formulation:

$$\mathcal{L}_{\text{latent}}^{\text{clip}(\epsilon_z)} = \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^K \min \left( r_{n,t}(\phi) \hat{A}_n, \text{clip}(r_{n,t}(\phi), 1 - \epsilon_z, 1 + \epsilon_z) \hat{A}_n \right), \quad (3)$$

where  $r_{n,t}(\phi) = \frac{p_\phi(\mathbf{z}_n^t \mid \mathbf{z}_n^{t-1}, Q)}{p_{\phi_{\text{old}}}(\mathbf{z}_n^t \mid \mathbf{z}_n^{t-1}, Q)}$  and the corresponding group-relative advantage  $\hat{A}_n$  is computed by standardizing the  $N$  mean rewards  $\{\bar{R}_n\}_{n=1}^N$  across the latent blocks for the same query  $Q$ , where each  $\bar{R}_n$  is calculated by averaging over the  $M$  text answers  $\mathbf{x}_{(n,m)}$  conditioned on the same  $n$ -th latent block:  $\bar{R}^{(n)} = \frac{1}{M} \sum_{m=1}^M R(\mathbf{x}_{(n,m)})$ , where  $R$  is the outcome reward function on answer text.

**Diversity Guidance** To further improve diversity during the rollout process, inspired by kernel-density estimation and repulsive forces (D’Angelo & Fortuin, 2021; Zilberstein et al., 2024), we introduce a repulsion-based guidance mechanism (see Figure 4). At each denoising step  $t$ , given the group of  $N$  latent trajectories  $\{\mathbf{z}_n^t\}_{n=1}^N$  sampled for a query  $Q$ , we first compute a bandwidth parameter:  $\sigma = \text{median}_{n < n'} \|\mathbf{z}_n^t - \mathbf{z}_{n'}^t\|_2$ . The repulsion force acting on a latent block  $\mathbf{z}_n^t$  is defined by the interaction force  $\mathbf{F}(\mathbf{z}_n^t)$ :

$$\mathbf{F}(\mathbf{z}_n^t) = \sum_{n' \neq n} 2 \left( 1 - \frac{d_{nn'}^2}{2\sigma^2} \right) \exp \left( -\frac{d_{nn'}^2}{2\sigma^2} \right) (\mathbf{z}_n^t - \mathbf{z}_{n'}^t) \quad (4)$$

where  $d_{nn'}^2 = \|\mathbf{z}_n^t - \mathbf{z}_{n'}^t\|_2^2$ . This encourages separation among nearby latent trajectories while suppressing interactions between distant ones. The repulsion is applied with

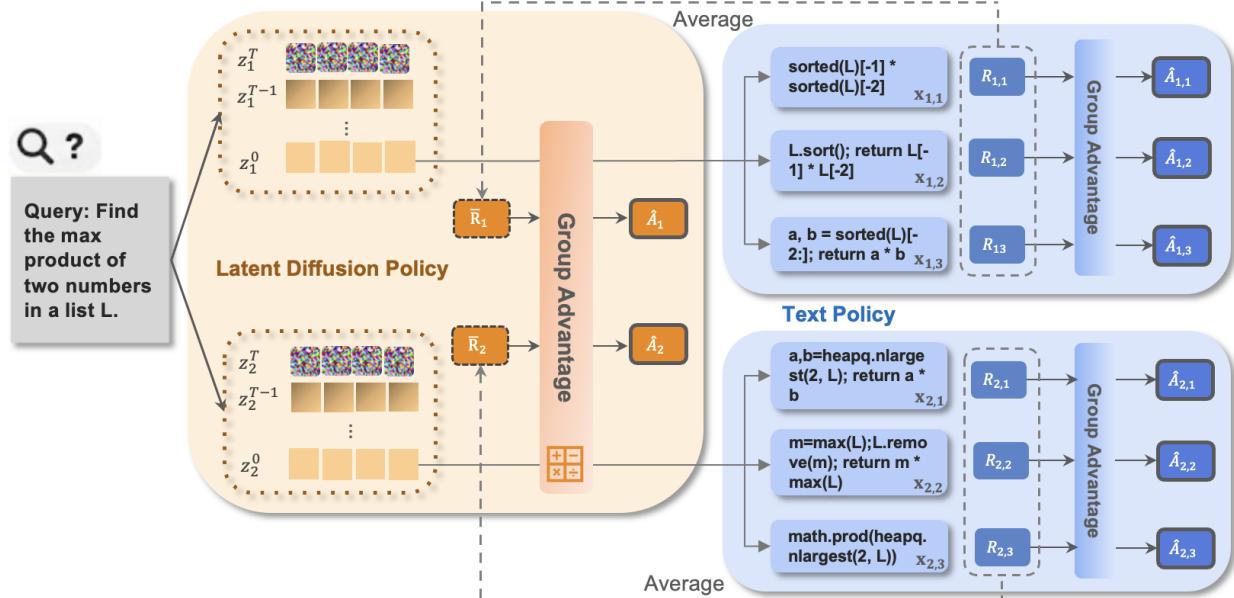


Figure 3. Overview of LaDi-RL training pipeline. The latent diffusion policy samples  $N$  latent blocks  $z^0$ , which condition the Text Policy to generate  $M$  candidate answers  $x_{i,j}$ . Outcome rewards evaluated on text are aggregated to compute group-relative advantages for jointly updating both the diffusion and text policies.

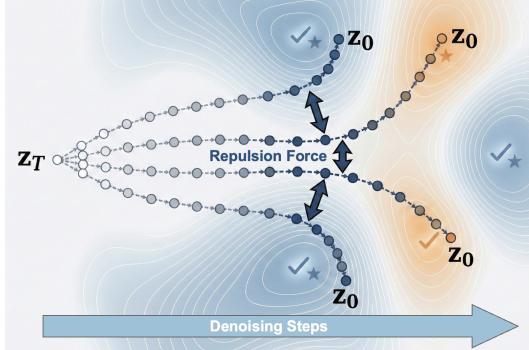


Figure 4. Latent diffusion-based exploration with diversity guidance. Multi-step denoising maps a noisy latent to diverse solutions. Repulsion force pushes trajectories apart in latent space to encourage exploration of diverse reasoning paths.

higher magnitude at early denoising steps and progressively reduced as  $t \rightarrow 0$ . Specifically, we define a time-dependent scale  $\gamma_t = \gamma_{\max} \frac{t}{K}$ , where  $K$  is the total number of denoising steps. The diversity-guided latent update is then:

$$\hat{z}_n^{t-1} = f_\phi(z_n^t, t, Q) + \gamma_t F(z_n^t), \quad (5)$$

where  $f_\phi(z_n^t, t, Q)$  denotes the base diffusion model prediction. This repulsion-based guidance serves as a form of local geometric regularization, explicitly separating nearby latent trajectories, while the latent diffusion process provides global multi-modal support. Together, they reshape the geometry of exploration, preserving diversity during RL training without relying on policy entropy.

### 3.3. Text Policy and Joint Optimization.

The text policy  $p_\theta$  acts as a complementary module, grounding the reasoning generated by the latent diffusion policy into specific text answers. We employ a local standard

GRPO loss for it:

$$\mathcal{L}_{\text{text}}^{\text{clip}(\epsilon_x^l, \epsilon_x^h)} = \frac{1}{NM} \sum_{n=1}^N \sum_{m=1}^M \sum_{j=1}^L \min \left( r_{n,m,j}(\theta) \hat{A}_{n,m}, \right. \\ \left. \text{clip}(r_{n,m,j}(\theta), 1 - \epsilon_x^l, 1 + \epsilon_x^h) \hat{A}_{n,m} \right), \quad (6)$$

where  $r_{n,m,j}(\theta) = \frac{p_\theta(x_{n,m,j}|x_{n,m,<j}, z_n)}{p_{\theta_{\text{old}}}(x_{n,m,j}|x_{n,m,<j}, z_n)}$  is the importance sampling ratio for the  $j$ -th token, and  $\hat{A}_{n,m} = \frac{R(y_{n,m}) - \text{mean}(\{R(y_{n,i})\}_{i=1}^M)}{\text{std}(\{R(y_{n,i})\}_{i=1}^M)}$  represents the local group advantage calculated strictly within the  $M$  text samples generated conditioned on the same latent block  $z_n$ . This design decouples text generation from the broader latent-space exploration, preventing the text policy from being penalized for poor latent trajectories and instead forcing it to find the optimal text sequences for the reasoning provided.

Then to joint train the two policies, we have this in the final objective:

$$\mathcal{L}_{\text{RL}} = \alpha \mathcal{L}_{\text{latent}}^{\text{clip}(\epsilon_z)} + (1 - \alpha) \mathcal{L}_{\text{text}}^{\text{clip}(\epsilon_x^l, \epsilon_x^h)}, \quad (7)$$

where  $\alpha$  is a balancing coefficient that prioritizes the optimization of the primary latent reasoning space while maintaining the text-alignment task. As illustrated in the training pipeline (Figure 3), this joint loss ensures that as the latent diffusion policy  $p_\phi$  explores diverse reasoning chains-of-thought, the text policy  $p_\theta$  simultaneously adapts to serve as a faithful and effective text generation.

## 4. Experiment

We evaluate our method on two representative reasoning domains: code generation and math reasoning. We compare

Model / Method	Base Model	MBPP	MBPP+	HumanEval	HumanEval+	LCB V6	Avg.
<b>Open-Sourced Models</b>							
Autoregressive Coding Models							
Qwen 2.5 Coder*	Qwen2.5-7B	75.9	62.9	66.5	60.4	26.8	59.90
OpenCoder*	from scratch (8B)	79.9	70.4	66.5	63.4	29.6	61.96
rStar-Coder*	Qwen2.5-7B	87.9	74.0	95.9	90.8	53.5	80.42
OlympicCoder*	Qwen2.5-7B	80.0	66.4	82.1	76.9	37.3	68.54
DeepSeek-R1-Distill*	Qwen2.5-7B	78.4	66.7	89.6	83.7	34.2	70.52
OpenThinker2*	Qwen2.5-7B	86.9	73.9	92.7	87.8	29.2	74.10
Seed-Coder*	from scratch	82.0	69.0	77.4	68.3	28.4	65.02
Diffusion Language Models							
Dream*	Qwen2.5-7B	68.7	57.4	56.7	50.0	18.6	–
LLaDA*	from scratch (8B)	50.1	42.1	35.4	30.5	12.4	–
Diffu-Coder*	Qwen2.5-Coder-7B	75.1	61.9	72.0	65.2	24.5	59.74
Dream-Coder*	Qwen2.5-Coder-7B	75.9	61.6	66.5	60.4	21.4	57.16
d1*	LLaDA+RL	39.0	–	45.5	–	–	–
Looped Latent Reasoning Models							
Ouro*	from scratch (2.6B)	80.4	66.6	78.2	70.7	38.7	66.92
<b>Method Comparison</b>							
Non-RL Methods							
Base Model	Qwen3-8B-Base	60.5	53.8	78.2	68.6	37.8	60.58
Standard SFT	Qwen3-8B-Base	63.3 <sub>+2.8</sub>	52.7 <sub>-1.1</sub>	84.6 <sub>+6.4</sub>	69.5 <sub>+0.9</sub>	39.5 <sub>+1.7</sub>	61.32 <sub>+0.74</sub>
Soft Thinking	Qwen3-8B-Base	64.2 <sub>+3.7</sub>	53.1 <sub>-0.7</sub>	85.0 <sub>+6.8</sub>	71.2 <sub>+2.6</sub>	–	–
TaH+	Qwen3-8B-Base	65.6 <sub>+5.1</sub>	56.5 <sub>+2.7</sub>	85.8 <sub>+7.6</sub>	74.3 <sub>+5.7</sub>	–	–
LaVAE	Qwen3-8B-Base	42.0 <sub>-18.5</sub>	30.2 <sub>-23.6</sub>	47.8 <sub>-30.4</sub>	32.8 <sub>-35.8</sub>	10.8 <sub>-27.0</sub>	32.72 <sub>-27.86</sub>
LaDiR	Qwen3-8B-Base	66.8 <sub>+6.3</sub>	59.5 <sub>+5.7</sub>	87.4 <sub>+9.2</sub>	73.2 <sub>+4.6</sub>	41.0 <sub>+3.2</sub>	65.58 <sub>+5.00</sub>
RL Training Methods							
Vanilla GRPO	Qwen3-8B-Base	72.3 <sub>+11.8</sub>	61.7 <sub>+7.9</sub>	82.6 <sub>+4.4</sub>	74.2 <sub>+5.6</sub>	47.6 <sub>+9.8</sub>	67.68 <sub>+7.10</sub>
GRPO w/ Entropy Adv.	Qwen3-8B-Base	73.8 <sub>+13.3</sub>	62.9 <sub>+9.1</sub>	84.1 <sub>+5.9</sub>	77.8 <sub>+9.2</sub>	48.2 <sub>+10.4</sub>	69.36 <sub>+8.78</sub>
HybridGRPO	Qwen3-8B-Base	73.1 <sub>+12.6</sub>	62.0 <sub>+8.2</sub>	82.9 <sub>+4.7</sub>	76.8 <sub>+8.2</sub>	48.0 <sub>+10.2</sub>	68.56 <sub>+7.98</sub>
Soft Token	Qwen3-8B-Base	70.2 <sub>+9.7</sub>	60.6 <sub>+6.8</sub>	81.7 <sub>+3.5</sub>	70.4 <sub>+1.8</sub>	40.9 <sub>+3.1</sub>	64.76 <sub>+4.18</sub>
SofT-GRPO	Qwen3-8B-Base	72.8 <sub>+12.3</sub>	60.2 <sub>+6.4</sub>	83.9 <sub>+5.7</sub>	72.2 <sub>+3.6</sub>	41.6 <sub>+3.8</sub>	66.14 <sub>+5.56</sub>
LaVAE-GRPO	Qwen3-8B-Base	45.6 <sub>-14.9</sub>	30.9 <sub>-22.9</sub>	53.2 <sub>-25.0</sub>	34.0 <sub>-34.6</sub>	12.5 <sub>-25.3</sub>	35.24 <sub>-25.34</sub>
LaDi-RL	Qwen3-8B-Base	84.2 <sub>+23.7</sub>	75.1 <sub>+21.3</sub>	90.5 <sub>+12.3</sub>	82.9 <sub>+14.3</sub>	52.7 <sub>+14.9</sub>	77.08 <sub>+16.50</sub>

Table 1. pass@1 results on code generation benchmarks. The subscripts indicate relative change from the Base Model (green for gains, red for drops). Blue-shaded rows indicate latent reasoning methods; unshaded rows denote discrete token-space methods. \*Results are taken from original papers.

against strong baselines under controlled settings and report standard pass@1 and pass@ $k$  metrics across benchmarks. See more experimental details in Appendix C.

#### 4.1. Experimental Setup

**Dataset** For code generation, we initialize the model with SFT on the Ling-Coder dataset (Codefuse & Team, 2025), which contains 1.4M Python-only samples. RL training is then performed on 24k filtered problems drawn from AceCoder (Zeng et al., 2025a) and KodCoder (Xu et al., 2025). We evaluate model performance on HumanEval (Chen, 2021), MBPP (Austin et al., 2021), their extended variants HumanEval+ and MBPP+ (Liu et al., 2023), as well as LiveCodeBench v6 (LCB V6) (Jain et al., 2024). For math reasoning, we use the R1-distill dataset (Madhusudhan et al.) for SFT and the DeepScaleR-Preview-Dataset (Luo et al., 2025) for RL training, which contains approximately 40K unique problem-answer pairs. Evaluation is conducted on six challenging benchmarks: AIME 2024 (Veeraboina, 2023), AIME 2025 (Zhang & Math-AI,

2025), AMC 2023, MATH-500 (Hendrycks et al., 2021), Minerva Math (Lewkowycz et al., 2022), and Olympiad-Bench (He et al., 2024).

**Baselines** We compare our method against a diverse set of strong baselines spanning autoregressive, diffusion-based, and reinforcement learning approaches. For code generation, we include open-sourced AR models such as Qwen2.5-Coder (Hui et al., 2024), OpenCoder (Huang et al., 2025b), rStar-Coder (Liu et al., 2025b), OlympicCoder (Hugging Face, 2025), and Seed-Coder (Seed et al., 2025), alongside reasoning-enhanced models like OpenThinker (Guha et al., 2025) and DeepSeek-R1-Distill (Guo et al., 2025). We also compare with discrete diffusion language models including Dream (Ye et al., 2025), Dream-Coder (Xie et al., 2025b), LLaDA (Nie et al., 2025), and Diffu-Coder (Gong et al., 2025). We include looped latent reasoning model Ouro (Zhu et al., 2025c) to highlight the benefits of diffusion models. To validate our training methods, we compare against various reasoning methods—including Soft Think-

Method	AIME24	AIME25	AMC23	MATH500	Minerva	Olympiad	Average
<b>Non-RL Methods</b>							
Base (DS-R1-Distill-Qwen-7B)	15.7	16.0	42.4	71.6	33.3	35.6	35.77
Soft Thinking	20.3 <sub>+4.6</sub>	19.1 <sub>+3.1</sub>	47.9 <sub>+5.5</sub>	76.5 <sub>+4.9</sub>	37.2 <sub>+3.9</sub>	40.6 <sub>+5.0</sub>	40.27 <sub>+4.50</sub>
LaVAE	6.5 <sub>-9.2</sub>	7.8 <sub>-8.2</sub>	21.0 <sub>-21.4</sub>	47.5 <sub>-24.1</sub>	17.6 <sub>-15.7</sub>	18.2 <sub>-17.4</sub>	19.77 <sub>-16.0</sub>
LaDiR	18.8 <sub>+3.1</sub>	19.8 <sub>+3.8</sub>	49.2 <sub>+6.8</sub>	78.4 <sub>+6.8</sub>	38.9 <sub>+5.6</sub>	41.0 <sub>+5.4</sub>	41.01 <sub>+5.24</sub>
<b>RL Training Methods</b>							
Vanilla GRPO	17.2 <sub>+1.5</sub>	17.1 <sub>+1.1</sub>	44.7 <sub>+2.3</sub>	74.1 <sub>+2.5</sub>	35.3 <sub>+2.0</sub>	38.0 <sub>+2.4</sub>	37.73 <sub>+1.96</sub>
GRPO+Entropy Adv.	18.0 <sub>+2.3</sub>	18.8 <sub>+2.8</sub>	43.1 <sub>+0.7</sub>	76.8 <sub>+5.2</sub>	37.8 <sub>+4.5</sub>	38.4 <sub>+2.8</sub>	38.82 <sub>+3.05</sub>
Multiplex Thinking	20.6 <sub>+4.9</sub>	19.7 <sub>+3.7</sub>	50.7 <sub>+8.3</sub>	78.0 <sub>+6.4</sub>	38.6 <sub>+5.3</sub>	41.7 <sub>+6.1</sub>	41.55 <sub>+5.78</sub>
LaVAE-GRPO	7.3 <sub>-8.4</sub>	8.5 <sub>-7.5</sub>	26.6 <sub>-15.8</sub>	53.4 <sub>-18.2</sub>	20.9 <sub>-12.4</sub>	26.8 <sub>-8.8</sub>	23.92 <sub>-11.85</sub>
LaDi-RL	22.3 <sub>+6.6</sub>	20.5 <sub>+4.5</sub>	52.6 <sub>+10.2</sub>	81.4 <sub>+9.8</sub>	40.7 <sub>+7.4</sub>	43.2 <sub>+7.6</sub>	43.45 <sub>+7.68</sub>

Table 2. Results on math reasoning benchmarks. We report pass@1 accuracy, with relative improvements over the base model shown as subscript in green and negative changes in red.

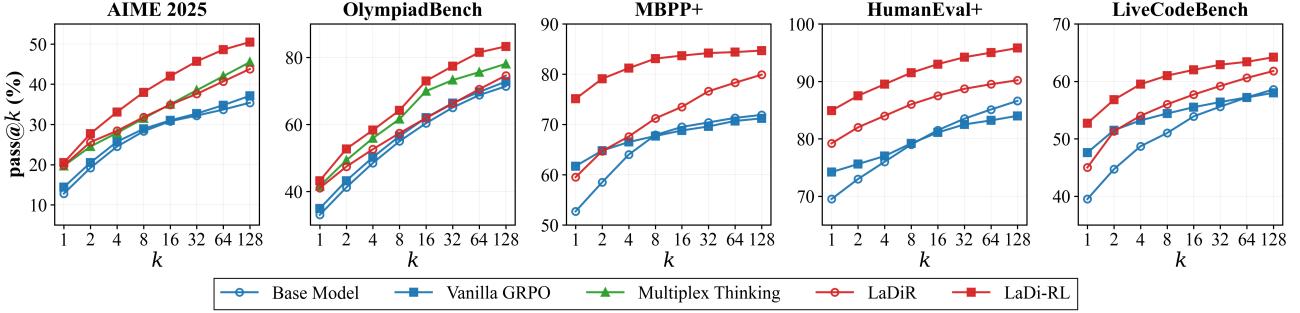


Figure 5. pass@k performance on code generation and math reasoning benchmarks across different  $k$ .

ing (Zhang et al., 2025d), Standard SFT, and TaH+ (Fu et al., 2025)—and reinforcement learning baselines such as vanilla GRPO (Guo et al., 2025), GRPO with entropy advantages (Cheng et al., 2025), HybridGRPO (Sane, 2025), Soft-GRPO (Zheng & Lee, 2025) and Multiplex Thinking (Tang et al., 2026). In addition, for math reasoning, we focus on representative methods including vanilla GRPO (Guo et al., 2025), GRPO with entropy advantages (Cheng et al., 2025), Soft Thinking (Zhang et al., 2025d), and Multiplex Thinking (Tang et al., 2026) to enable controlled comparisons with prior approaches. We also include two baselinse by replacing flow matching loss with standard  $\ell_2$  loss on VAE (LaVAE) and apply GRPO on it (LaVAE-GRPO) as a Gaussian Policy.

**Implementation Details** For code generation, we use *Qwen3-Base* (Yang et al., 2025a) as the base model, while for math reasoning we adopt *DeepSeek-R1-Distill-Qwen-7B* (Guo et al., 2025). To mitigate performance variance inherent in single-run evaluations, we sample 16 solutions per problem and report the average pass@1 accuracy for all benchmarks. See Appendix C for more details.

## 5. Results

In this section, we present the main results, analysis, and ablation studies on the two benchmarks. See more ablation study and qualitative results in Appendix D.

### 5.1. Main Results

**pass@1 improvements over baselines.** Tables 1 and 2 show that **LaDi-RL** consistently outperforms prior *latent*

*reasoning* and *reinforcement learning* methods across both code generation and math reasoning tasks. Compared to latent reasoning approaches such as Soft Thinking, LaDi-RL achieves average improvements of +3.2% on math reasoning and up to +4.9% on individual datasets (e.g., MATH500). Comparing to RL baselines, LaDi-RL improves over the latent RL method Multiplex Thinking, by +1.9% on math reasoning and GRPO with entropy advantage by +7.8% on code generation, while yielding even larger margins over vanilla GRPO (+5.7% on math and +9.4% on code). These results demonstrate that explicitly optimizing over latent diffusion trajectories provides stronger gains than other latent RL or discrete RL alone.

We further compare LaDi-RL with existing well-trained open-sourced coding models. LaDi-RL outperforms all compared models except *rStar-Coder*, matching its performance within a 3.3% absolute margin on average while requiring over 20× fewer training examples. Compared to *diffusion language models*, LaDi-RL shows a large performance gap, improving average pass@1 by approximately +17%, with gains around +18% on HumanEval and +28% on LiveCodeBench-V6. Against *looped latent reasoning models*, LaDi-RL improves average pass@1 by +10.9%, with consistent gains across all benchmarks. More broadly, these results suggest that LaDi-RL offers a general framework for improving LLM reasoning performance.

**Latent diffusion exploration mitigates diversity collapse in pass@k.** As shown in Figure 5, our results confirm

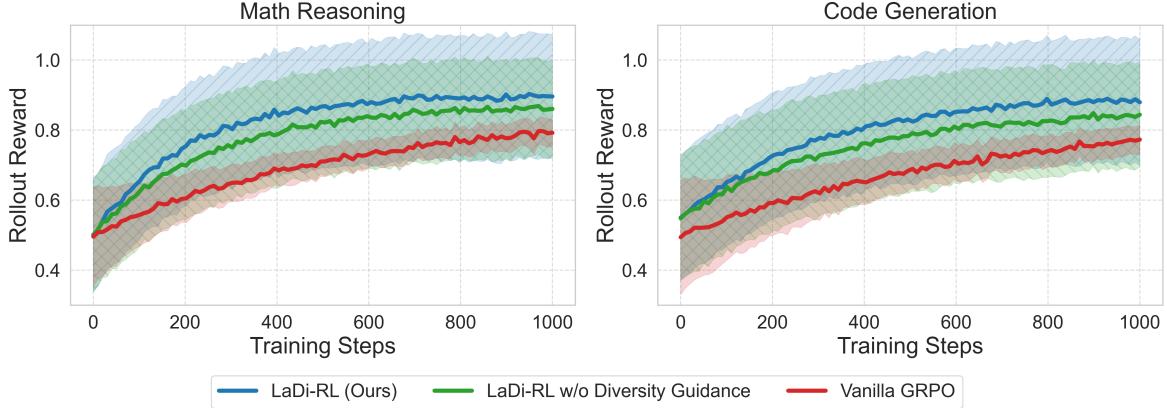


Figure 6. Training rollout reward curves on math reasoning (left) and code generation (right). Solid lines show the mean rollout reward across training steps, while the shaded regions indicate one standard deviation of the rollout reward, reflecting diversity during training.

Method	# Output Tokens	Inference Time
Vanilla GRPO	3447	6.5s
LaDi-RL	64 (latent) + 359 (text)	3.7s

Table 3. Efficiency comparison during inference. We report the total number of generated tokens and wall-clock time.

the diversity collapse of vanilla GRPO (Yue et al., 2025) in discrete token space: while GRPO improves pass@1, it fails to surpass the base model at large  $k$ , indicating that more samples does not yield broader solution coverage. While recent continuous latent reasoning methods such as *Multiplex Thinking* partially alleviate this issue, our LaDi-RL breaks the base model upper bound at large  $k$  on all benchmarks by a much larger margin, achieving absolute improvements of +13.4 (AIME 2025), +11.9 (Olympiad-Bench), +12.8 (MBPP+), +11.8 (HumanEval+), and +5.6 (LiveCodeBench) at  $k=128$ . These results show that latent diffusion exploration mitigates the diversity collapse issue through a multi-step denoising process with guidance and improves the reasoning boundary of base model.

## 5.2. Analysis

**Diversity Analysis.** As shown in Figure 6, LaDi-RL consistently achieves higher mean rewards while maintaining larger reward variance throughout training, as indicated by the wider shaded regions. In contrast, GRPO exhibits both lower final rewards and rapidly shrinking variance, signaling progressive diversity collapse. The removal of diversity guidance decreases reward variance and slows learning progress, confirming its role in maintaining effective exploration. These results indicate that diffusion-based latent exploration sustains diverse rollouts during RL training, whereas entropy-driven discrete RL tends to concentrate probability mass onto a narrow set of trajectories.

**Efficiency Analysis** As shown in Table 3, LaDi-RL is more computationally efficient than standard GRPO at inference time. Standard GRPO relies on long CoTs, generating an average of 3,447 text tokens per sample. In contrast, LaDi-RL represents reasoning using only 64 latent tokens,

which on average encode a short CoT of 385 text tokens, corresponding to an effective compression rate of approximately  $6.0\times$ . Then, the denoised latent CoTs are conditioned to generate an average of 359 answer tokens. This latent compression reduces both rollout overhead and autoregressive decoding cost, yielding a 33% reduction in end-to-end wall-clock inference time (from 6.5 s to 3.7 s).

## 5.3. Ablation Studies

**Effect of Diffusion Loss** To isolate the role of diffusion dynamics, we replace the multi-step diffusion objective with a  $\ell_2$  regression loss on latent tokens, keeping the architecture and training setup identical; for RL, we additionally match the exploration budget by injecting Gaussian noise into latent tokens and applying GRPO as in prior work (Butt et al., 2025; Qiu et al., 2025). The  $\ell_2$  variant underperforms across both stages. Under SFT, LaVAE degrades sharply ( $-16.0$  on math,  $-32.9$  on code), while diffusion-based LaDiR improves over the base (+5.2 on math, +4.28 on code). Under RL, LaDi-RL gains +7.7 on math, whereas LaVAE-GRPO remains  $-11.9$  below the base model. Overall, diffusion yields much larger RL gains over its SFT model (+11.5 vs. +4.15), indicating that multi-step stochastic denoising—not continuous latent representations alone—is essential for effective trajectory-level exploration and downstream performance.

**Policy Composition** As shown in Table 4, we conduct controlled ablations of the latent and text policies. A latent-only policy trained with tied model weights collapses due to the shared backbone between latent generation and answer generation, resulting in insufficient supervision and degenerate exploration. We further study a latent policy with uncoupled model weights, where one copy is trainable while a frozen copy generates answer text via deterministic greedy decoding. Although this setting avoids the collapse issue, joint optimization over both latent and text policies achieves consistently stronger performance. Incorporating a text-level policy yields large gains across all benchmarks,

indicating that text-space optimization provides complementary grounding signals for latent policy.

Method	MBPP	HumanEval	LCB V6
LaDiR	66.8	87.4	41.0
Text Only (tied)	68.2	85.9	43.1
Text Only (uncoupled)	70.3	86.8	44.8
Latent Only (tied)	54.3	62.6	21.6
Latent Only (uncoupled)	73.8	89.8	47.2
Latent + Text Policies	<b>84.2</b>	<b>92.2</b>	<b>52.7</b>
- w/o diversity guidance	80.1	90.3	49.4

Table 4. Ablation study (**pass@1**) analyzing the contributions of (1) policy composition (text vs. latent vs. joint) and (2) diversity guidance via latent repulsion.

**Diversity guidance.** As shown in Table 4, starting from the best *Latent + Text Policies* setting, removing diversity guidance leads to consistent absolute performance drops of 4.1, 1.9, and 3.3 points. Beyond final performance, Figure 6 further illustrates the impact of diversity guidance on training dynamics: ablating diversity guidance results in noticeably reduced reward variance and slower improvement of rollout reward across both math reasoning and code generation tasks. Together, these results indicate that explicit diversity guidance both improves RL exploration and prevents collapse onto a narrow set of reasoning trajectories.

## 6. Related Works

**Latent Reasoning** Latent reasoning methods address the limitations of token-level CoT by shifting reasoning processes into a latent space, decoupling internal computation from explicit text generation. Early approaches utilized discrete special tokens to expand internal reasoning capacity or encode implicit intermediate steps, yielding more abstract representations (Herel & Mikolov, 2024; Pfau et al., 2024; Wang et al., 2024; Zelikman et al., 2024; Zhou et al., 2025; Jin et al., 2025a). Subsequent work along this direction demonstrated that reasoning via continuous latent representations rather than discrete tokens can further improve performance. By operating on “soft” tokens that are either self-generated or produced by auxiliary models, LLMs can exploit richer semantic information (Gozeten et al., 2025; Cheng & Durme, 2024; Hao et al., 2024b; Liu et al., 2024; Shen et al., 2025; Tack et al., 2025; Zhu et al., 2025b; Butt et al., 2025; Zhang et al., 2025d; Wu et al., 2025). Furthermore, recent research leverages the expressive power of diffusion models to propose and refine reasoning trajectories (Kang et al., 2025; Anonymous, 2024; Shao et al., 2025; Venkatraman et al., 2024; Anonymous, 2024; Lovelace et al.; Zhang et al., 2023; Lovelace et al., 2023; 2024). In parallel, a complementary line of work utilizes recurrent or looped architectures to induce latent reasoning internally, bypassing the need to explicitly represent reasoning steps as output tokens (Chen et al., 2025e; Geiping et al., 2025; Mottashami et al., 2025; Saunshi et al., 2025; Yu et al., 2025a). Despite the above progress, recent efforts apply RL to con-

tinuous latent spaces by injecting stochasticity into token embeddings (Butt et al., 2025; Sane, 2025; Zheng & Lee, 2025; Tang et al., 2026; Özeren & Aßenmacher, 2025) or by regulating exploration during policy optimization (Zhang et al., 2025c). Comparing to these method, our method injects noise via a multi-step latent diffusion process with inference-time guidance, enabling structured exploration over semantic-level reasoning trajectories rather than local token variations.

**RL for LLMs** The advancement of Large Reasoning Models has been significantly propelled by RL paradigms, particularly Reinforcement Learning with Verifiable Rewards (RLVR) (Guo et al., 2025; OpenAI, 2024; Zhang et al., 2025b). While foundational algorithms like PPO (Schulman et al., 2017) and DPO (Rafailov et al., 2023) laid the groundwork for alignment, recent efforts have focused on pure RL methods such as GRPO (Guo et al., 2025) and its variants (Lin et al., 2025; Zhang et al., 2025a; Li et al., 2025a) elicit emergent reasoning patterns like self-reflection and verification (DeepSeek-AI, 2024). However, a persistent challenge in scaling RLVR is the phenomenon of *diversity collapse*, where the policy rapidly loses diversity and converges to sub-optimal local minima (Cui et al., 2025; Hao et al., 2025; Jin et al., 2025b; Yue et al., 2025). This exploration-exploitation imbalance (Chen et al., 2025b; Huang et al., 2025a) has motivated various intervention strategies, including diversity-aware objectives (Yao et al., 2025; He et al., 2025; Walder & Karkhanis, 2025; Tang et al., 2025b; Yu et al., 2025b; Gai et al., 2025; Chen et al., 2025f), uncertainty-aware exploration (Xie et al., 2025a), and entropy control (Hao et al., 2025; Park et al., 2025; Cheng et al., 2025; Agarwal et al., 2025; Li et al., 2025c; Wang et al., 2025c; Zheng et al., 2025; Yang et al., 2025c). These techniques remain limited to local stochastic control in discrete space; whereas LaDi-RL instead performs exploration over entire reasoning trajectories in a continuous latent diffusion process, fundamentally altering the geometry of exploration. Due to the page limit, we discuss further related works in Appendix A.

## 7. Conclusion

In this work, we propose a new reinforcement learning framework LaDi-RL that reframes exploration directly in continuous latent space, rather than relying on entropy-driven sampling in discrete token space. By modeling exploration as a multi-step latent diffusion process with explicit noise injection and guidance LaDi-RL simultaneously improves pass@1 accuracy and pass@k performance, mitigating diversity collapse. These results show that latent diffusion as a principled alternative to token-level RL for LLM reasoning.

## 8. Impact Statement

This work presents a training methodology for improving exploration in reinforcement learning-based reasoning for large language models. The proposed approach aims to improve robustness and efficiency in reasoning tasks such as code generation and mathematics. The work does not introduce new application settings, and any downstream use should follow standard evaluation and safety practices.

## References

- Agarwal, S., Zhang, Z., Yuan, L., Han, J., and Peng, H. The unreasonable effectiveness of entropy minimization in llm reasoning. *arXiv preprint arXiv:2505.15134*, 2025.
- Albergo, M. S., Boffi, N. M., and Vanden-Eijnden, E. Stochastic interpolants: A unifying framework for flows and diffusions. *arXiv preprint arXiv:2303.08797*, 2023.
- Anonymous. Diffusion of thought: Chain-of-thought reasoning in diffusion language models. *NeurIPS*, 2024.
- Austin, J., Odena, A., Nye, M., Bosma, M., Michalewski, H., Dohan, D., Jiang, E., Cai, C., Terry, M., Le, Q., et al. Program synthesis with large language models. *arXiv preprint arXiv:2108.07732*, 2021.
- Black, K., Janner, M., Du, Y., Kostrikov, I., and Levine, S. Training diffusion models with reinforcement learning. *arXiv preprint arXiv:2305.13301*, 2023.
- Black, K., Brown, N., Driess, D., Esmail, A., Equi, M., Finn, C., Fusai, N., Groom, L., Hausman, K., Ichter, B., Jakubczak, S., Jones, T., Ke, L., Levine, S., Li-Bell, A., Mothukuri, M., Nair, S., Pertsch, K., Shi, L. X., Tanner, J., Vuong, Q., Walling, A., Wang, H., and Zhilinsky, U.  $\pi_0$ : A vision-language-action flow model for general robot control, 2024. URL <https://arxiv.org/abs/2410.24164>.
- Borsø, U., Paglieri, D., Wells, J., and Rocktäschel, T. D3po: Preference-based alignment of discrete diffusion models. *arXiv e-prints*, pp. arXiv–2503, 2025.
- Butt, N., Kwiatkowski, A., Labiad, I., Kempe, J., and Ollivier, Y. Soft tokens, hard truths. *arXiv preprint arXiv:2509.19170*, 2025.
- Chen, F., Raventos, A., Cheng, N., Ganguli, S., and Druckmann, S. Rethinking fine-tuning when scaling test-time compute: Limiting confidence improves mathematical reasoning. *arXiv preprint arXiv:2502.07154*, 2025a.
- Chen, M. Evaluating large language models trained on code. *arXiv preprint arXiv:2107.03374*, 2021.
- Chen, P. et al. Exploration vs exploitation: Rethinking rlr through clipping, entropy, and spurious reward. *arXiv preprint arXiv:2512.16912*, 2025b.
- Chen, S., Ge, C., Zhang, Y., Zhang, Y., Zhu, F., Yang, H., Hao, H., Wu, H., Lai, Z., Hu, Y., Lin, T.-C., Zhang, S., Li, F., Li, C., Wang, X., Peng, Y., Sun, P., Luo, P., Jiang, Y., Yuan, Z., Peng, B., and Liu, X. Goku: Flow based video generative foundation models, 2025c. URL <https://arxiv.org/abs/2502.04896>.
- Chen, X., Wu, Z., Liu, X., Pan, Z., Liu, W., Xie, Z., Yu, X., and Ruan, C. Janus-pro: Unified multimodal understanding and generation with data and model scaling, 2025d. URL <https://arxiv.org/abs/2501.17811>.
- Chen, Y., Shang, J., Zhang, Z., Xie, Y., Sheng, J., Liu, T., Wang, S., Sun, Y., Wu, H., and Wang, H. Inner thinking transformer: Leveraging dynamic depth scaling to foster adaptive internal thinking, 2025e. URL <https://arxiv.org/abs/2502.13842>.
- Chen, Z., Qin, X., Wu, Y., Ling, Y., Ye, Q., Zhao, W. X., and Shi, G. Pass@ k training for adaptively balancing exploration and exploitation of large reasoning models. *arXiv preprint arXiv:2508.10751*, 2025f.
- Cheng, D., Huang, S., Zhu, X., Dai, B., Zhao, W. X., Zhang, Z., and Wei, F. Reasoning with exploration: An entropy perspective. *arXiv preprint arXiv:2506.14758*, 2025.
- Cheng, J. and Durme, B. V. Compressed chain of thought: Efficient reasoning through dense representations, 2024. URL <https://arxiv.org/abs/2412.13171>.
- Chung, H.-L., Hsiao, T.-Y., Huang, H.-Y., Cho, C., Lin, J.-R., Ziwei, Z., and Chen, Y.-N. Revisiting test-time scaling: A survey and a diversity-aware method for efficient reasoning. *arXiv preprint arXiv:2506.04611*, 2025.
- Cobbe, K., Kosaraju, V., Bavarian, M., Chen, M., Jun, H., Kaiser, L., Plappert, M., Tworek, J., Hilton, J., Nakano, R., et al. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*, 2021.
- Codefuse and Team, L. Every sample matters: Leveraging mixture-of-experts and high-quality data for efficient and accurate code llm, 2025. URL <https://arxiv.org/abs/2503.17793>.
- Cui, G., Zhang, Y., Chen, J., Yuan, L., Wang, Z., Zuo, Y., Li, H., Fan, Y., Chen, H., Chen, W., et al. The entropy mechanism of reinforcement learning for reasoning language models. *arXiv preprint arXiv:2505.22617*, 2025.
- Dang, X., Baek, C., Wen, K., Kolter, Z., and Raghunathan, A. Weight ensembling improves reasoning in language models. *arXiv preprint arXiv:2504.10478*, 2025.

- D’Angelo, F. and Fortuin, V. Repulsive deep ensembles are bayesian. *Advances in Neural Information Processing Systems*, 34:3451–3465, 2021.
- DeepSeek-AI. Deepseek-v3 technical report. *arXiv preprint arXiv:2412.19437*, 2024.
- Fan, L., Li, T., Qin, S., Li, Y., Sun, C., Rubinstein, M., Sun, D., He, K., and Tian, Y. Fluid: Scaling autoregressive text-to-image generative models with continuous tokens, 2024. URL <https://arxiv.org/abs/2410.13863>.
- Fu, T., You, Y., Chen, Z., Dai, G., Yang, H., and Wang, Y. Think-at-hard: Selective latent iterations to improve reasoning language models. *arXiv preprint arXiv:2511.08577*, 2025.
- Gai, J., Zeng, G., Zhang, H., and Raghunathan, A. Differential smoothing mitigates sharpening and improves llm reasoning. *arXiv preprint arXiv:2511.19942*, 2025.
- Geiping, J., McLeish, S., Jain, N., Kirchenbauer, J., Singh, S., Bartoldson, B. R., Kailkhura, B., Bhatele, A., and Goldstein, T. Scaling up test-time compute with latent reasoning: A recurrent depth approach, 2025. URL <https://arxiv.org/abs/2502.05171>.
- Gong, S., Li, M., Feng, J., Wu, Z., and Kong, L. Diffuseq: Sequence to sequence text generation with diffusion models. *arXiv preprint arXiv:2210.08933*, 2022.
- Gong, S., Zhang, R., Zheng, H., Gu, J., Jaityl, N., Kong, L., and Zhang, Y. Diffucoder: Understanding and improving masked diffusion models for code generation. *arXiv preprint arXiv:2506.20639*, 2025.
- Gozeten, H. A., Ildiz, M. E., Zhang, X., Harutyunyan, H., Rawat, A. S., and Oymak, S. Continuous chain of thought enables parallel exploration and reasoning. *arXiv preprint arXiv:2505.23648*, 2025.
- Guha, E., Marten, R., Keh, S., Raoof, N., Smyrnis, G., Bansal, H., Nezhurina, M., Mercat, J., Vu, T., Sprague, Z., et al. Openthoughts: Data recipes for reasoning models. *arXiv preprint arXiv:2506.04178*, 2025.
- Guo, D., Yang, D., Zhang, H., Song, J., Wang, P., Zhu, Q., Xu, R., Zhang, R., Ma, S., Bi, X., et al. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning. *arXiv preprint arXiv:2501.12948*, 2025.
- Hao, S., Sukhbaatar, S., Su, D., Li, X., Hu, Z., Weston, J., and Tian, Y. Training large language models to reason in a continuous latent space, 2024b. URL <https://arxiv.org/abs/2412.06769>.
- Hao, Z., Wang, H., Liu, H., Luo, J., Yu, J., et al. Rethinking entropy interventions in rlrv: An entropy change perspective. *arXiv preprint arXiv:2510.10150*, 2025.
- He, A. W., Fried, D., and Welleck, S. Rewarding the unlikely: Lifting grp0 beyond distribution sharpening. In *Proceedings of the 2025 Conference on Empirical Methods in Natural Language Processing*, pp. 25559–25571, 2025.
- He, C., Luo, R., Bai, Y., Hu, S., Thai, Z. L., Shen, J., Hu, J., Han, X., Huang, Y., Zhang, Y., et al. Olympiadbench: A challenging benchmark for promoting agi with olympiad-level bilingual multimodal scientific problems. *arXiv preprint arXiv:2402.14008*, 2024.
- Hendrycks, D., Burns, C., Kadavath, S., Arora, A., Basart, S., Tang, E., Song, D., and Steinhardt, J. Measuring mathematical problem solving with the math dataset. *arXiv preprint arXiv:2103.03874*, 2021.
- Herel, D. and Mikolov, T. Thinking tokens for language modeling, 2024. URL <https://arxiv.org/abs/2405.08644>.
- Ho, J. and Salimans, T. Classifier-free diffusion guidance. *arXiv preprint arXiv:2207.12598*, 2022.
- Ho, J., Jain, A., and Abbeel, P. Denoising diffusion probabilistic models. *Advances in neural information processing systems*, 33:6840–6851, 2020.
- Huang, F. et al. Beyond the exploration-exploitation trade-off: A hidden state approach for llm reasoning in rlrv. *arXiv preprint arXiv:2509.23808*, 2025a.
- Huang, S., Cheng, T., Liu, J. K., Xu, W., Hao, J., Song, L., Xu, Y., Yang, J., Liu, J., Zhang, C., et al. Opencoder: The open cookbook for top-tier code large language models. In *Proceedings of the 63rd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 33167–33193, 2025b.
- Huang, Z., Chen, Z., Wang, Z., Li, T., and Qi, G.-J. Reinforcing the diffusion chain of lateral thought with diffusion language models. *arXiv preprint arXiv:2505.10446*, 2025c.
- Hugging Face. Open r1: A fully open reproduction of deepseek-r1, January 2025. URL <https://github.com/huggingface/open-r1>.
- Hui, B., Yang, J., Cui, Z., Yang, J., Liu, D., Zhang, L., Liu, T., Zhang, J., Yu, B., Lu, K., et al. Qwen2. 5-coder technical report. *arXiv preprint arXiv:2409.12186*, 2024.

- Jain, N., Han, K., Gu, A., Li, W.-D., Yan, F., Zhang, T., Wang, S., Solar-Lezama, A., Sen, K., and Stoica, I. Livecodebench: Holistic and contamination free evaluation of large language models for code. *arXiv preprint arXiv:2403.07974*, 2024.
- Jin, M., Luo, W., Cheng, S., Wang, X., Hua, W., Tang, R., Wang, W. Y., and Zhang, Y. Disentangling memory and reasoning ability in large language models, 2025a. URL <https://arxiv.org/abs/2411.13504>.
- Jin, R., Gao, Y., et al. Revisiting entropy in reinforcement learning for large reasoning models. *arXiv preprint arXiv:2511.05993*, 2025b.
- Kang, H., Zhang, Y., Kuang, N. L., Majamaki, N., Jaitly, N., Ma, Y.-A., and Qin, L. Ladir: Latent diffusion enhances llms for text reasoning. *arXiv preprint arXiv:2510.04573*, 2025.
- Lee, H., Lu, J., and Tan, Y. Convergence of score-based generative modeling for general data distributions. In *International Conference on Algorithmic Learning Theory*, pp. 946–985. PMLR, 2023.
- Lewkowycz, A., Andreassen, A., Dohan, D., Dyer, E., Michalewski, H., Ramasesh, V., Sloane, A., Anil, C., Schlag, I., Gutman-Solo, T., et al. Solving quantitative reasoning problems with language models. *Advances in neural information processing systems*, 35:3843–3857, 2022.
- Li, C., Liu, N., and Yang, K. Adaptive group policy optimization: Towards stable training and token-efficient reasoning. *arXiv preprint arXiv:2503.15952*, 2025a.
- Li, J., Cui, Y., Huang, T., Ma, Y., Fan, C., Yang, M., and Zhong, Z. Mixgrpo: Unlocking flow-based grpo efficiency with mixed ode-sde. *arXiv preprint arXiv:2507.21802*, 2025b.
- Li, Q., Xue, R., Wang, J., Zhou, M., Li, Z., Ji, X., Wang, Y., Liu, M., Yang, Z., Qiu, M., et al. Cure: Critical-token-guided re-concatenation for entropy-collapse prevention. *arXiv preprint arXiv:2508.11016*, 2025c.
- Li, X., Thickstun, J., Gulrajani, I., Liang, P. S., and Hashimoto, T. B. Diffusion-lm improves controllable text generation. *Advances in neural information processing systems*, 35:4328–4343, 2022.
- Lin, Z., Lin, M., Xie, Y., and Ji, R. Cppo: Accelerating the training of group relative policy optimization-based reasoning models. *arXiv preprint arXiv:2503.22342*, 2025.
- Lipman, Y., Chen, R. T., Ben-Hamu, H., Nickel, M., and Le, M. Flow matching for generative modeling. *arXiv preprint arXiv:2210.02747*, 2022.
- Liu, J., Xia, C. S., Wang, Y., and Zhang, L. Is your code generated by chatgpt really correct? rigorous evaluation of large language models for code generation. *Advances in Neural Information Processing Systems*, 36:21558–21572, 2023.
- Liu, J., Liu, G., Liang, J., Li, Y., Liu, J., Wang, X., Wan, P., Zhang, D., and Ouyang, W. Flow-grpo: Training flow matching models via online rl. *arXiv preprint arXiv:2505.05470*, 2025a.
- Liu, T., Chen, Z., Liu, Z., Tian, M., and Luo, W. Expediting and elevating large language model reasoning via hidden chain-of-thought decoding, 2024. URL <https://arxiv.org/abs/2409.08561>.
- Liu, X., Gong, C., and Liu, Q. Flow straight and fast: Learning to generate and transfer data with rectified flow. *arXiv preprint arXiv:2209.03003*, 2022.
- Liu, Y., Zhang, L. L., Zhu, Y., Dong, B., Zhou, X., Shang, N., Yang, F., and Yang, M. rstar-coder: Scaling competitive code reasoning with a large-scale verified dataset. *arXiv preprint arXiv:2505.21297*, 2025b.
- Lovelace, J., Belardi, C. K., Zalouk, S., Polavarapu, A., Kundurthy, S. R., and Weinberger, K. Q. Stop-think-autoregress: Language modeling with latent diffusion planning. In *Second Conference on Language Modeling*.
- Lovelace, J., Kishore, V., Wan, C., Shekhtman, E., and Weinberger, K. Q. Latent diffusion for language generation. *Advances in Neural Information Processing Systems*, 36: 56998–57025, 2023.
- Lovelace, J., Kishore, V., Chen, Y., and Weinberger, K. Diffusion guided language modeling. In *Findings of the Association for Computational Linguistics: ACL 2024*, pp. 14936–14952, 2024.
- Luo, M., Tan, S., Wong, J., Shi, X., Tang, W. Y., Roongta, M., Cai, C., Luo, J., Li, L. E., Popa, R. A., and Stoica, I. Deepscaler: Surpassing o1-preview with a 1.5b model by scaling rl. <https://pretty-radio-b75.notion.site/DeepScaleR-Surpassing-O1-Preview-with-a-1-5B-Model-2025>. Notion Blog.
- Madhusudhan, S. T., Radhakrishna, S., Mehta, J., and Liang, T. Millions scale dataset distilled from r1-32b. <https://huggingface.co/datasets/ServiceNow-AI/R1-Distill-SFT>.
- Meshchaninov, V., Chimbulatov, E., Shabalin, A., Abramov, A., and Vetrov, D. Compressed and smooth latent space for text diffusion modeling. *arXiv preprint arXiv:2506.21170*, 2025.

- Mohtashami, A., Pagliardini, M., and Jaggi, M. CoTFormer: A chain of thought driven architecture with budget-adaptive computation cost at inference. In *The Thirteenth International Conference on Learning Representations*, 2025. URL <https://openreview.net/forum?id=7igPXQFupX>.
- Nie, S., Zhu, F., You, Z., Zhang, X., Ou, J., Hu, J., Zhou, J., Lin, Y., Wen, J.-R., and Li, C. Large language diffusion models. *arXiv preprint arXiv:2502.09992*, 2025.
- OpenAI. Learning to reason with llms. *OpenAI Blog*, 2024.
- Ou, J., Han, J., Xu, M., Xu, S., Xie, J., Ermon, S., Wu, Y., and Li, C. Principled rl for diffusion llms emerges from a sequence-level perspective. *arXiv preprint arXiv:2512.03759*, 2025.
- Özeren, E. and Aßenmacher, M. Reinforcement learning for latent-space thinking in llms. *arXiv preprint arXiv:2512.11816*, 2025.
- Pan, X., Shukla, S. N., Singh, A., Zhao, Z., Mishra, S. K., Wang, J., Xu, Z., Chen, J., Li, K., Juefei-Xu, F., Hou, J., and Xie, S. Transfer between modalities with metaqueues, 2025. URL <https://arxiv.org/abs/2504.06256>.
- Park, J. et al. Clip-low increases entropy and clip-high decreases entropy in reinforcement learning of large language models. *arXiv preprint arXiv:2509.26114*, 2025.
- Pfau, J., Merrill, W., and Bowman, S. R. Let's think dot by dot: Hidden computation in transformer language models, 2024. URL <https://arxiv.org/abs/2404.15758>.
- Qiu, L., Ning, S., Sun, J., and He, X. Noisygrpo: Incentivizing multimodal cot reasoning via noise injection and bayesian estimation. *arXiv preprint arXiv:2510.21122*, 2025.
- Rafailov, R., Sharma, A., Mitchell, E., Ermon, S., Manning, C. D., and Finn, C. Direct preference optimization: Your language model is secretly a reward model. *arXiv preprint arXiv:2305.18290*, 2023.
- Rojas, K., Lin, J., Rasul, K., Schneider, A., Nevmyvaka, Y., Tao, M., and Deng, W. Improving reasoning for diffusion language models via group diffusion policy optimization. *arXiv preprint arXiv:2510.08554*, 2025.
- Sahoo, S., Arriola, M., Schiff, Y., Gokaslan, A., Marroquin, E., Chiu, J., Rush, A., and Kuleshov, V. Simple and effective masked diffusion language models. *Advances in Neural Information Processing Systems*, 37:130136–130184, 2024.
- Sahoo, S. S., Deschenaux, J., Gokaslan, A., Wang, G., Chiu, J., and Kuleshov, V. The diffusion duality. *arXiv preprint arXiv:2506.10892*, 2025.
- Sane, S. Hybrid group relative policy optimization: A multi-sample approach to enhancing policy optimization. *arXiv preprint arXiv:2502.01652*, 2025.
- Saunshi, N., Dikkala, N., Li, Z., Kumar, S., and Reddi, S. J. Reasoning with latent thoughts: On the power of looped transformers, 2025. URL <https://arxiv.org/abs/2502.17416>.
- Schulman, J., Levine, S., Abbeel, P., Jordan, M., and Moritz, P. Trust region policy optimization. In *International conference on machine learning*, pp. 1889–1897. PMLR, 2015.
- Schulman, J., Wolski, F., Dhariwal, P., Radford, A., and Klimov, O. Proximal policy optimization algorithms. *arXiv preprint arXiv:1707.06347*, 2017.
- Seed, B., Zhang, Y., Su, J., Sun, Y., Xi, C., Xiao, X., Zheng, S., Zhang, A., Liu, K., Zan, D., et al. Seed-coder: Let the code model curate data for itself. *arXiv preprint arXiv:2506.03524*, 2025.
- Shao, C. et al. Diffuse thinking: Exploring diffusion language models as efficient thought proposers for reasoning. *arXiv preprint arXiv:2510.27469*, 2025.
- Shao, Z., Wang, P., Zhu, Q., Xu, R., Song, J., Bi, X., Zhang, H., Zhang, M., Li, Y., Wu, Y., et al. Deepseekmath: Pushing the limits of mathematical reasoning in open language models. *arXiv preprint arXiv:2402.03300*, 2024.
- Shen, Z., Yan, H., Zhang, L., Hu, Z., Du, Y., and He, Y. Codi: Compressing chain-of-thought into continuous space via self-distillation, 2025. URL <https://arxiv.org/abs/2502.21074>.
- Shi, W., Han, X., Zhou, C., Liang, W., Lin, X. V., Zettlemoyer, L., and Yu, L. Lmfusion: Adapting pretrained language models for multimodal generation, 2025. URL <https://arxiv.org/abs/2412.15188>.
- Singh, M., Cambronero, J., Gulwani, S., Le, V., Negreanu, C., and Verbruggen, G. Codefusion: A pre-trained diffusion model for code generation. In *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing*, pp. 11697–11708, 2023.
- Song, Y. and Ermon, S. Generative modeling by estimating gradients of the data distribution. *Advances in neural information processing systems*, 32, 2019.
- Song, Y., Sohl-Dickstein, J., Kingma, D. P., Kumar, A., Ermon, S., and Poole, B. Score-based generative modeling

- through stochastic differential equations. *arXiv preprint arXiv:2011.13456*, 2020.
- Song, Y., Zhang, H., Eisenach, C., Kakade, S., Foster, D., and Ghai, U. Mind the gap: Examining the self-improvement capabilities of large language models. *arXiv preprint arXiv:2412.02674*, 2024.
- Song, Y., Zhang, Z., Luo, C., Gao, P., Xia, F., Luo, H., Li, Z., Yang, Y., Yu, H., Qu, X., et al. Seed diffusion: A large-scale diffusion language model with high-speed inference. *arXiv preprint arXiv:2508.02193*, 2025.
- Sutton, R. S., Barto, A. G., et al. *Reinforcement learning: An introduction*, volume 1. MIT press Cambridge, 1998.
- Tack, J., Lanchantin, J., Yu, J., Cohen, A., Kulikov, I., Lan, J., Hao, S., Tian, Y., Weston, J., and Li, X. Llm pre-training with continuous concepts, 2025. URL <https://arxiv.org/abs/2502.08524>.
- Tang, H., Wu, Y., Yang, S., Xie, E., Chen, J., Chen, J., Zhang, Z., Cai, H., Lu, Y., and Han, S. Hart: Efficient visual generation with hybrid autoregressive transformer, 2024. URL <https://arxiv.org/abs/2410.10812>.
- Tang, X., Dolga, R., Yoon, S., and Bogunovic, I. wd1: Weighted policy optimization for reasoning in diffusion language models. *arXiv preprint arXiv:2507.08838*, 2025a.
- Tang, Y., Zheng, K., Synnaeve, G., and Munos, R. Optimizing language models for inference time objectives using reinforcement learning. *arXiv preprint arXiv:2503.19595*, 2025b.
- Tang, Y., Dong, L., Hao, Y., Dong, Q., Wei, F., and Gu, J. Multiplex thinking: Reasoning via token-wise branch-and-merge. *arXiv preprint arXiv:2601.08808*, 2026.
- Tong, S., Fan, D., Zhu, J., Xiong, Y., Chen, X., Sinha, K., Rabbat, M., LeCun, Y., Xie, S., and Liu, Z. Metamorph: Multimodal understanding and generation via instruction tuning, 2024. URL <https://arxiv.org/abs/2412.14164>.
- Tran, H., Zhang, Z., Bao, F., Lu, D., and Zhang, G. Diffusion-based supervised learning of generative models for efficient sampling of multimodal distributions. *arXiv preprint arXiv:2505.07825*, 2025.
- Veeraboina, H. Aime problem set 1983-2024, 2023. URL <https://www.kaggle.com/datasets/hemishveeraboina/aime-problem-set-1983-2024>.
- Venkatraman, S. et al. Reasoning with latent diffusion in offline reinforcement learning. *ICLR*, 2024.
- Walder, C. and Karkhanis, D. Pass@ k policy optimization: Solving harder reinforcement learning problems. *arXiv preprint arXiv:2505.15201*, 2025.
- Wang, C., Rashidinejad, P., Su, D., Jiang, S., Wang, S., Zhao, S., Zhou, C., Shen, S. Z., Chen, F., Jaakkola, T., et al. Spg: Sandwiched policy gradient for masked diffusion language models. *arXiv preprint arXiv:2510.09541*, 2025a.
- Wang, F. and Yu, Z. Coefficients-preserving sampling for reinforcement learning with flow matching. *arXiv preprint arXiv:2509.05952*, 2025.
- Wang, G., Schiff, Y., Turok, G., and Kuleshov, V. d2: Improved techniques for training reasoning diffusion language models. *arXiv preprint arXiv:2509.21474*, 2025b.
- Wang, S., Yu, L., Gao, C., Zheng, C., Liu, S., Lu, R., Dang, K., Chen, X., Yang, J., Zhang, Z., et al. Beyond the 80/20 rule: High-entropy minority tokens drive effective reinforcement learning for llm reasoning. *arXiv preprint arXiv:2506.01939*, 2025c.
- Wang, X., Caccia, L., Ostapenko, O., Yuan, X., Wang, W. Y., and Sordoni, A. Guiding language model reasoning with planning tokens. In *First Conference on Language Modeling*, 2024. URL <https://openreview.net/forum?id=wi9IffRhVM>.
- Wang, Y., Yang, L., Li, B., Tian, Y., Shen, K., and Wang, M. Revolutionizing reinforcement learning framework for diffusion large language models. *arXiv preprint arXiv:2509.06949*, 2025d.
- Wu, C., Lu, J., Ren, Z., Hu, G., Wu, Z., Dai, D., and Wu, H. Llms are single-threaded reasoners: Demystifying the working mechanism of soft thinking. *arXiv preprint arXiv:2508.03440*, 2025.
- Xiang, J., Liu, Z., Liu, H., Bai, Y., Cheng, J., and Chen, W. Diffusiondialog: A diffusion model for diverse dialog generation with latent space. *arXiv preprint arXiv:2404.06760*, 2024.
- Xiao, S., Wang, Y., Zhou, J., Yuan, H., Xing, X., Yan, R., Li, C., Wang, S., Huang, T., and Liu, Z. Omnigen: Unified image generation, 2024. URL <https://arxiv.org/abs/2409.11340>.
- Xie, C. et al. Unlocking exploration in rlrv: Uncertainty-aware exploration strategy. *arXiv preprint arXiv:2509.06941*, 2025a.
- Xie, Z., Ye, J., Zheng, L., Gao, J., Dong, J., Wu, Z., Zhao, X., Gong, S., Jiang, X., Li, Z., et al. Dream-coder 7b: An open diffusion language model for code. *arXiv preprint arXiv:2509.01142*, 2025b.

- Xu, Z., Liu, Y., Yin, Y., Zhou, M., and Poovendran, R. Kodcode: A diverse, challenging, and verifiable synthetic dataset for coding. *arXiv preprint arXiv:2503.02951*, 2025.
- Xue, Z., Wu, J., Gao, Y., Kong, F., Zhu, L., Chen, M., Liu, Z., Liu, W., Guo, Q., Huang, W., et al. Dancegrpo: Unleashing grpo on visual generation. *arXiv preprint arXiv:2505.07818*, 2025.
- Yang, A., Li, A., Yang, B., Zhang, B., Hui, B., Zheng, B., Yu, B., Gao, C., Huang, C., Lv, C., et al. Qwen3 technical report. *arXiv preprint arXiv:2505.09388*, 2025a.
- Yang, L., Tian, Y., Li, B., Zhang, X., Shen, K., Tong, Y., and Wang, M. Mmada: Multimodal large diffusion language models. *arXiv preprint arXiv:2505.15809*, 2025b.
- Yang, S., Dou, C., Guo, P., Lu, K., Ju, Q., Deng, F., and Xin, R. Dcpo: Dynamic clipping policy optimization. *arXiv preprint arXiv:2509.02333*, 2025c.
- Yao, J., Cheng, R., Wu, X., Wu, J., and Tan, K. C. Diversity-aware policy optimization for large language model reasoning. *arXiv preprint arXiv:2505.23433*, 2025.
- Ye, J., Xie, Z., Zheng, L., Gao, J., Wu, Z., Jiang, X., Li, Z., and Kong, L. Dream 7b: Diffusion large language models. *arXiv preprint arXiv:2508.15487*, 2025.
- Yu, Q., He, Z., Li, S., Zhou, X., Zhang, J., Xu, J., and He, D. Enhancing auto-regressive chain-of-thought through loop-aligned reasoning, 2025a. URL <https://arxiv.org/abs/2502.08482>.
- Yu, Q., Zhang, Z., Zhu, R., Yuan, Y., Zuo, X., et al. Dapo: An open-source llm reinforcement learning system at scale. *arXiv preprint arXiv:2503.14476*, 2025b.
- Yue, Y., Chen, Z., Lu, R., Zhao, A., Wang, Z., Song, S., and Huang, G. Does reinforcement learning really incentivize reasoning capacity in llms beyond the base model? *arXiv preprint arXiv:2504.13837*, 2025.
- Zekri, O. and Boullé, N. Fine-tuning discrete diffusion models with policy gradient methods. *arXiv preprint arXiv:2502.01384*, 2025.
- Zelikman, E., Harik, G. R., Shao, Y., Jayasiri, V., Haber, N., and Goodman, N. Quiet-STar: Language models can teach themselves to think before speaking. In *First Conference on Language Modeling*, 2024. URL <https://openreview.net/forum?id=oRXPiSOGH9>.
- Zeng, H., Jiang, D., Wang, H., Nie, P., Chen, X., and Chen, W. Acecoder: Acing coder rl via automated test-case synthesis. *ArXiv*, abs/2207.01780, 2025a.
- Zeng, Y., Cao, J., Li, Z., Chen, Y., Ren, T., Xiang, D., Wu, X., Gao, S., and Yu, T. Treediff: Ast-guided code generation with diffusion llms. *arXiv preprint arXiv:2508.01473*, 2025b.
- Zhang, J., Huang, J., Yao, H., Liu, S., Zhang, X., Lu, S., et al. R1-vl: Learning to reason with multimodal large language models via step-wise group relative policy optimization. *arXiv preprint arXiv:2503.12937*, 2025a.
- Zhang, K. et al. A survey of reinforcement learning for large reasoning models. *arXiv preprint arXiv:2509.08827*, 2025b.
- Zhang, S., Chen, X., Shen, Y., Ye, Z., and Wu, J. Relax: Reasoning with latent exploration for large reasoning models. *arXiv preprint arXiv:2512.07558*, 2025c.
- Zhang, Y. and Math-AI, T. American invitational mathematics examination (aime) 2025, 2025.
- Zhang, Y., Gu, J., Wu, Z., Zhai, S., Susskind, J., and Jaitly, N. Planner: Generating diversified paragraph via latent language diffusion model. *Advances in Neural Information Processing Systems*, 36:80178–80190, 2023.
- Zhang, Z., He, X., Yan, W., Shen, A., Zhao, C., Wang, S., Shen, Y., and Wang, X. E. Soft thinking: Unlocking the reasoning potential of llms in continuous concept space. *arXiv preprint arXiv:2505.15778*, 2025d.
- Zhao, H., Liang, D., Tang, W., Yao, D., and Kallus, N. Diffpo: Training diffusion llms to reason fast and furious via reinforcement learning. *arXiv preprint arXiv:2510.02212*, 2025a.
- Zhao, R., Meterez, A., Kakade, S., Pehlevan, C., Jelassi, S., and Malach, E. Echo chamber: RL post-training amplifies behaviors learned in pretraining. *arXiv preprint arXiv:2504.07912*, 2025b.
- Zhao, S., Gupta, D., Zheng, Q., and Grover, A. d1: Scaling reasoning in diffusion large language models via reinforcement learning. *arXiv preprint arXiv:2504.12216*, 2025c.
- Zheng, T., Xing, T., Gu, Q., Liang, T., Qu, X., Zhou, X., Li, Y., Wen, Z., Lin, C., Huang, W., et al. First return, entropy-eliciting explore. *arXiv preprint arXiv:2507.07017*, 2025.
- Zheng, Z. and Lee, W. S. Soft-grpo: Surpassing discrete-token llm reinforcement learning via gumbel-reparameterized soft-thinking policy optimization. *arXiv preprint arXiv:2511.06411*, 2025.
- Zhou, C., Yu, L., Babu, A., Tirumala, K., Yasunaga, M., Shamis, L., Kahn, J., Ma, X., Zettlemoyer, L., and Levy, O. Transfusion: Predict the next token and diffuse images

with one multi-modal model, 2024. URL <https://arxiv.org/abs/2408.11039>.

Zhou, C., Yang, C., Hu, Y., Wang, C., Zhang, C., Zhang, M., Mackey, L., Jaakkola, T., Bates, S., and Zhang, D. Coevolutionary continuous discrete diffusion: Make your diffusion language model a latent reasoner. *arXiv preprint arXiv:2510.03206*, 2025.

Zhu, F., Wang, R., Nie, S., Zhang, X., Wu, C., Hu, J., Zhou, J., Chen, J., Lin, Y., Wen, J.-R., et al. Llada 1.5: Variance-reduced preference optimization for large language diffusion models. *arXiv preprint arXiv:2505.19223*, 2025a.

Zhu, H., Hao, S., Hu, Z., Jiao, J., Russell, S., and Tian, Y. Reasoning by superposition: A theoretical perspective on chain of continuous thought. *arXiv preprint arXiv:2505.12514*, 2025b.

Zhu, R.-J., Wang, Z., Hua, K., Zhang, T., Li, Z., Que, H., Wei, B., Wen, Z., Yin, F., Xing, H., et al. Scaling latent reasoning via looped language models. *arXiv preprint arXiv:2510.25741*, 2025c.

Zilberstein, N., Mardani, M., and Segarra, S. Repulsive latent score distillation for solving inverse problems. *arXiv preprint arXiv:2406.16683*, 2024.

## A. Additional Related Works

**Latent Diffusion for Language Generation** Recent work has extended language generation beyond autoregressive decoding to diffusion-based models that enable global, iterative refinement. Early approaches such as Diffusion-LM (Li et al., 2022) formulate generation as denoising continuous word embeddings, while subsequent methods perform diffusion in compressed latent spaces to improve text quality and mode diversity (Lovelace et al., 2023; Lovelace et al.; Meshchaninov et al., 2025). For sequence-to-sequence tasks, DiffuSeq (Gong et al., 2022) enables parallel generation with high diversity, and PLANNER (Zhang et al., 2023) combines latent semantic diffusion with autoregressive decoding to better handle long-form text. Diffusion has also been applied to domain-specific generation, including dialogue (Xiang et al., 2024) and code synthesis (Singh et al., 2023; Zeng et al., 2025b). More recently, diffusion-guided language modeling (Lovelace et al., 2024; Lovelace et al.) explores using diffusion as a global guidance mechanism to improve generation quality and controllability. Despite these advances, prior latent diffusion models primarily target fluent text generation and lack exploration into reasoning tasks.

**Reinforcement Learning for Discrete Diffusion Language Models.** Diffusion language models (DLMs) (Sahoo et al., 2024; 2025; Nie et al., 2025; Ye et al., 2025; Song et al., 2025) provide a promising alternative to autoregressive models, but reinforcement learning (RL) for DLMs faces distinct structural challenges due to the combinatorial explosion of denoising trajectories and the lack of well-defined state transitions. Early token-level adaptations (Zhao et al., 2025c; Yang et al., 2025b; Gong et al., 2025) rely on ill-posed transition dynamics and mean-field approximations, leading to unstable optimization. Recent work therefore shifts to sequence- or trajectory-level objectives (Zhu et al., 2025a; Wang et al., 2025a; Rojas et al., 2025; Ou et al., 2025; Zekri & Boullé, 2025), using surrogates to approximate intractable marginal likelihoods. However, these methods generally suffer from off-policy misalignment: heuristic-guided sampling deviates from the diffusion prior, producing biased gradients without principled correction (Schulman et al., 2015). Representative RL training methods (Zhao et al., 2025c; Gong et al., 2022; Tang et al., 2025a; Wang et al., 2025b; Zhao et al., 2025a; Borsø et al., 2025) differ mainly in how likelihoods are approximated. Two notable exceptions partially address this issue: LLaDOU (Huang et al., 2025c) explicitly models diffusion-step likelihoods via an auxiliary policy at high computational cost, while TraceRL (Wang et al., 2025d) aligns optimization with inference traces by merging diffusion steps.

**Hybrid AR+Diffusion Model Architecture** Hybrid autoregressive–diffusion (AR–diffusion) models have demonstrated strong performance across multimodal generation and understanding, often rivaling or surpassing their purely AR or diffusion counterparts. The Transfusion architecture (Zhou et al., 2024) showed that hybrid models can outperform standard AR models and remain competitive with state-of-the-art diffusion models on image generation benchmarks, a trend further supported by subsequent works (Fan et al., 2024; Tang et al., 2024; Xiao et al., 2024). Beyond image generation, hybrid AR–diffusion models have proven effective in image understanding, video generation, and robot control (Black et al., 2024; Tong et al., 2024; Chen et al., 2025c;d). Similar to our approach, several studies adapt frozen pretrained models within hybrid AR–diffusion architectures for multimodal tasks (Pan et al., 2025; Shi et al., 2025).

## B. Preliminaries: Flow-GRPO

In this section, we provide a detailed overview of Flow-GRPO (Liu et al., 2025a) and its accelerated variant Flow-GRPO-Fast, which enable online reinforcement learning for flow matching models. We describe the mathematical formulation, the ODE-to-SDE conversion strategy, the GRPO objective, and key implementation parameters.

### B.1. Flow Matching Background

Flow matching models (Lipman et al., 2022; Liu et al., 2022) define a continuous-time generative process from noise to data. Let  $x_0 \sim \mathcal{X}_0$  denote a sample from the data distribution and  $x_1 \sim \mathcal{X}_1 = \mathcal{N}(0, I)$  denote a noise sample. The Rectified Flow framework (Liu et al., 2022) defines the interpolated state  $x_t$  as:

$$x_t = (1 - t)x_0 + tx_1, \quad t \in [0, 1]. \quad (8)$$

A neural network  $v_\theta(x_t, t)$  is trained to regress the velocity field by minimizing the flow matching objective:

$$\mathcal{L}(\theta) = \mathbb{E}_{t, x_0 \sim \mathcal{X}_0, x_1 \sim \mathcal{X}_1} [\|v - v_\theta(x_t, t)\|^2], \quad (9)$$

where the target velocity field is  $v = x_1 - x_0$ .

## B.2. Denoising as a Markov Decision Process

Following (Black et al., 2023), the iterative denoising process can be formulated as a Markov Decision Process (MDP)  $(\mathcal{S}, \mathcal{A}, \rho_0, P, R)$  where:

- **State:**  $s_t \triangleq (c, t, x_t)$ , comprising the conditioning signal  $c$ , timestep  $t$ , and latent  $x_t$ .
- **Action:**  $a_t \triangleq x_{t-1}$ , the denoised sample predicted by the model.
- **Policy:**  $\pi(a_t|s_t) \triangleq p_\theta(x_{t-1}|x_t, c)$ .
- **Transition:** Deterministic transition  $P(s_{t+1}|s_t, a_t) \triangleq (\delta_c, \delta_{t-1}, \delta_{x_{t-1}})$ .
- **Initial distribution:**  $\rho_0(s_0) \triangleq (p(c), \delta_T, \mathcal{N}(0, I))$ .
- **Reward:** Sparse terminal reward  $R(s_t, a_t) \triangleq r(x_0, c)$  if  $t = 0$ , and 0 otherwise.

## B.3. ODE-to-SDE Conversion

A critical challenge for applying RL to flow matching models is their deterministic nature—standard ODE-based sampling provides no stochasticity for exploration. Flow-GRPO addresses this by converting the deterministic ODE:

$$dx_t = v_t dt \quad (10)$$

into an equivalent SDE that preserves the marginal distribution at all timesteps. Following the theoretical framework (Song et al., 2020; Albergo et al., 2023), Flow-GRPO constructs a reverse-time SDE:

$$dx_t = \left( v_t(x_t) - \frac{\sigma_t^2}{2} \nabla \log p_t(x_t) \right) dt + \sigma_t dw, \quad (11)$$

where  $dw$  denotes Wiener process increments and  $\sigma_t$  controls the level of stochasticity during generation.

For Rectified Flow, the score function can be expressed in terms of the velocity field:

$$\nabla \log p_t(x) = -\frac{x}{t} - \frac{1-t}{t} v_t(x). \quad (12)$$

Substituting Equation (12) into Equation (11) yields the specific SDE formulation:

$$dx_t = \left( v_t(x_t) + \frac{\sigma_t^2}{2t} (x_t + (1-t)v_t(x_t)) \right) dt + \sigma_t dw. \quad (13)$$

Applying Euler-Maruyama discretization gives the practical update rule:

$$x_{t+\Delta t} = x_t + \left( v_\theta(x_t, t) + \frac{\sigma_t^2}{2t} (x_t + (1-t)v_\theta(x_t, t)) \right) \Delta t + \sigma_t \sqrt{\Delta t} \epsilon, \quad (14)$$

where  $\epsilon \sim \mathcal{N}(0, I)$  injects stochasticity. The noise schedule is parameterized as:

$$\sigma_t = a \sqrt{\frac{t}{1-t}}, \quad (15)$$

where  $a$  is a scalar hyperparameter controlling the noise level (typically  $a = 0.7$ ).

## B.4. GRPO Objective for Flow Matching

Given a prompt  $c$ , the flow model samples a group of  $G$  images  $\{x_0^i\}_{i=1}^G$  with corresponding trajectories  $\{(x_T^i, x_{T-1}^i, \dots, x_0^i)\}_{i=1}^G$ . The advantage of the  $i$ -th sample is computed via group-relative normalization:

$$\hat{A}_t^i = \frac{R(x_0^i, c) - \text{mean}(\{R(x_0^i, c)\}_{i=1}^G)}{\text{std}(\{R(x_0^i, c)\}_{i=1}^G)}. \quad (16)$$

The Flow-GRPO objective is:

$$J_{\text{Flow-GRPO}}(\theta) = \mathbb{E}_{c \sim \mathcal{C}, \{x^i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot|c)} \left[ f(r, \hat{A}, \theta, \epsilon, \beta) \right], \quad (17)$$

where

$$f(r, \hat{A}, \theta, \epsilon, \beta) = \frac{1}{G} \sum_{i=1}^G \frac{1}{T} \sum_{t=0}^{T-1} \left( \min \left( r_t^i(\theta) \hat{A}_t^i, \text{clip}(r_t^i(\theta), 1-\epsilon, 1+\epsilon) \hat{A}_t^i \right) - \beta D_{\text{KL}}(\pi_\theta \| \pi_{\text{ref}}) \right), \quad (18)$$

and the importance ratio is:

$$r_t^i(\theta) = \frac{p_\theta(x_{t-1}^i | x_t^i, c)}{p_{\theta_{\text{old}}}(x_{t-1}^i | x_t^i, c)}. \quad (19)$$

Since the SDE formulation yields an isotropic Gaussian policy, the KL divergence admits a closed-form expression:

$$D_{\text{KL}}(\pi_\theta \| \pi_{\text{ref}}) = \frac{\|x_{t+\Delta t, \theta} - x_{t+\Delta t, \text{ref}}\|^2}{2\sigma_t^2 \Delta t} = \frac{\Delta t}{2} \left( \frac{\sigma_t(1-t)}{2t} + \frac{1}{\sigma_t} \right)^2 \|v_\theta(x_t, t) - v_{\text{ref}}(x_t, t)\|^2. \quad (20)$$

## B.5. Denoising Reduction

Flow-GRPO introduces a **Denoising Reduction** strategy to improve training efficiency. While standard inference may require  $T = 40$  denoising steps, training samples are collected with significantly fewer steps (e.g.,  $T_{\text{train}} = 10$ ), while retaining the full schedule during evaluation. This achieves over  $4\times$  speedup without sacrificing final performance.

## B.6. Flow-GRPO-Fast

Flow-GRPO-Fast is an accelerated variant that requires training on only **one or two denoising steps** per trajectory. The key insight is to confine stochasticity to a narrow window:

1. Generate a deterministic trajectory using ODE sampling up to a randomly chosen intermediate step  $t^*$ .
2. At step  $t^*$ , inject noise and switch to SDE sampling to generate a group of  $G$  samples.
3. Continue the remainder of the trajectory with ODE sampling.

This design enables significant efficiency gains:

- Each trajectory is trained only once or twice, reducing training cost by approximately a factor of  $T$ .
- Sampling before the branching step requires only a single prompt (no group expansion), further accelerating data collection.

**SDE Window Mechanism (Flow-GRPO-Fast).** In Flow-GRPO-Fast, the `sde_window_size` parameter controls how many consecutive denoising steps use SDE sampling (and are subsequently optimized), while `sde_window_range` specifies the valid timestep range within which the SDE window can be randomly positioned. For instance, setting `sde_window_size=2` and `sde_window_range=[0, T-2]` means that at each iteration, a random starting position is sampled, and SDE sampling (with GRPO optimization) is applied only to those two steps. All other steps use deterministic ODE sampling.

**No-CFG Training.** Disabling classifier-free guidance (CFG) (Ho & Salimans, 2022) during training effectively performs CFG distillation through the RL process, significantly accelerating convergence while maintaining or improving generation quality.

## B.7. CPS Sampling for Flow Matching Models.

In our implementation, we adopt *Coefficient-Preserving Sampling* (CPS) (Wang & Yu, 2025) to introduce principled stochasticity into flow matching models while preserving the consistency of noise coefficients. A key limitation of the standard Flow-SDE formulation is that the reduced noise variance  $\sigma_t^2 \Delta t / (2t)$  does not match the variance of the newly injected noise term  $\sigma_t \sqrt{\Delta t}$ , leading to a mismatch between the stochastic transition kernel and the underlying flow dynamics. This mismatch becomes particularly problematic for reinforcement learning, where accurate likelihood ratios are required for stable policy optimization.

Motivated by the observation that DDIM sampling injects noise while preserving coefficient consistency, we reformulate the sampling procedure to satisfy the CPS conditions. Let  $\hat{\mathbf{x}}_0$  and  $\hat{\mathbf{x}}_1$  denote the predicted clean sample and noise component, respectively. By constraining the variance of the injected noise and adjusting the coefficients accordingly, the CPS update rule is given by:

$$\mathbf{x}_{t-\Delta t} = (1 - (t - \Delta t))\hat{\mathbf{x}}_0 + \sqrt{(t - \Delta t)^2 - \sigma_t^2} \hat{\mathbf{x}}_1 + \sigma_t \epsilon, \quad (21)$$

where  $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ . To avoid invalid square roots, we parameterize the noise scale as

$$\sigma_t = (t - \Delta t) \sin\left(\frac{\eta\pi}{2}\right), \quad \eta \in [0, 1],$$

which yields the final CPS sampling formulation:

$$\mathbf{x}_{t-\Delta t} = (1 - (t - \Delta t))\hat{\mathbf{x}}_0 + (t - \Delta t) \cos\left(\frac{\eta\pi}{2}\right) \hat{\mathbf{x}}_1 + (t - \Delta t) \sin\left(\frac{\eta\pi}{2}\right) \epsilon. \quad (22)$$

The parameter  $\eta$  explicitly controls the stochastic strength, interpolating between deterministic flow sampling ( $\eta = 0$ ) and maximal stochasticity ( $\eta = 1$ ). This formulation admits a clear geometric interpretation and preserves the coefficient structure required by CPS; we therefore refer to this procedure as *Flow-CPS*.

To apply GRPO, we require the conditional transition likelihood  $p_\theta(\mathbf{x}_{t-\Delta t} \mid \mathbf{x}_t)$ . Following (Liu et al., 2025a), this likelihood is defined as:

$$\log p_\theta(\mathbf{x}_{t-\Delta t}^i \mid \mathbf{x}_t^i) = -\frac{\|\mathbf{x}_{t-\Delta t} - \boldsymbol{\mu}_\theta(\mathbf{x}_t, t)\|^2}{2\sigma_t^2} - \log \sigma_t - \log \sqrt{2\pi}, \quad (23)$$

where

$$\boldsymbol{\mu}_\theta(\mathbf{x}_t, t) = (1 - (t - \Delta t))\hat{\mathbf{x}}_0 + (t - \Delta t) \cos\left(\frac{\eta\pi}{2}\right) \hat{\mathbf{x}}_1.$$

In practice, the constant terms  $-\log \sigma_t - \log \sqrt{2\pi}$  cancel in the GRPO importance ratio  $r_t^i(\theta) = p_\theta/p_{\theta_{\text{old}}}$  and are therefore omitted. Moreover, we remove the normalization factor  $2\sigma_t^2$  in the denominator to avoid numerical instability at small  $\sigma_t$ . Analytically, this normalization disproportionately emphasizes later timesteps with lower stochasticity; removing it reallocates learning signal toward earlier, more diverse timesteps, which is critical for effective reinforcement learning and exploration in flow-based models. As a result, we use the simplified log-probability:

$$\log p_\theta(\mathbf{x}_{t-\Delta t}^i \mid \mathbf{x}_t^i) = -\|\mathbf{x}_{t-\Delta t} - \boldsymbol{\mu}_\theta(\mathbf{x}_t, t)\|^2. \quad (24)$$

## C. Experimental Details

### C.1. Code Generation

**Data Filtering Pipeline.** We construct a unified RL training corpus from four public coding datasets (KodCode-V1 (Xu et al., 2025) and AceCode-87K (Zeng et al., 2025a)) by converting each record into a common format (prompt, test\_code/test\_cases, solution, metadata). The pipeline then applies four sequential filtering stages to improve supervision quality and execution reliability. **(Phase 1: Quality filtering)** We retain only problems with at least 5 executable unit tests (after dataset-specific parsing of assertions or stdin/stdout style tests), ensuring each instance provides sufficient verifiable signal. **(Phase 2: Semantic deduplication)** To reduce redundancy, we embed all prompts using a sentence-transformer and perform greedy cosine-similarity filtering with threshold 0.85, keeping the first instance in each cluster. **(Phase 3: Difficulty calibration)** To remove trivial tasks, we sample 8 independent solutions per prompt from Qwen2.5-Coder-7B-Instruct (via vLLM) and execute each generation against the associated tests under a

Table 5. Training and sampling hyperparameters for LaDi-RL with Flow-GRPO and Flow-GRPO-Fast.

Category	Parameter	Value / Description
Model & Optimization	Latent representation	64 tokens, 2560 dim
	Optimizer	AdamW ( $\beta_1 = 0.9, \beta_2 = 0.999$ )
	Learning rate	$1 \times 10^{-5}$
	Max text length	768 tokens
	Text sampling	Temperature= 1.0, top- $p$ = 0.98
	Diffusion sampling	CPS (Coefficient-Preserving Sampling)
	Classifier-free guidance	Disabled
RL Training	$\gamma_{\max}$ for diversity guidance	0.8
	KL regularization	Disabled
	Loss weights	$\lambda_{\text{diff}} = 10.0, \lambda_{\text{text}} = 1.0$
	Noise level ( $a$ )	0.8
	Denoising steps	10 (train) / 30 (eval)
	Shared initial noise	Disabled
	Latent clip range	$\epsilon_z = 10^{-5}$
	Text clip range	$\epsilon_x^l = 0.2; \epsilon_x^h = 0.28$
	SDE window size	2
	SDE window range	(0, 5)
	Group / rollout size	$N = 16, M = 5$

strict timeout; problems where *all* sampled solutions pass all tests are excluded. **(Phase 4: Ground-truth validation and sanitization)** For the remaining examples, we (i) automatically add missing standard imports by pattern matching (e.g., `typing/math/heappq`) and optionally add a function alias to match the expected entry point inferred from tests, (ii) filter any solution or test containing unsafe or environment-dependent operations (file I/O, network calls, subprocess/system operations, interactive input, pickling) via regex rules, (iii) filter any example importing unavailable packages, and (iv) verify that the provided ground-truth solution passes *all* extracted test cases using isolated subprocess execution with timeouts. The final output is a set of non-trivial, deduplicated, execution-safe programming problems with verified ground-truth solutions and reliable unit-test reward signals for RL training.

**Implementation Details.** During RL training, each problem is paired with its associated unit tests, which define a scalar reward equal to the pass rate of unit tests executed by the generated solution. For each prompt, we sample  $M$  latent reasoning trajectories per update and compute group-relative advantages following the GRPO formulation. Latent diffusion transitions are treated as stochastic policy steps, and text generate is performed only at the denoised latents. All diffusion and text-policy hyperparameters, optimization settings, and sampling configurations follow Table 5.

## C.2. Math Reasoning

**Baseline Details.** For math reasoning, we compare LaDi-RL against supervised fine-tuned (SFT) and reinforcement learning baselines operating directly in token space. All baselines use the same pretrained backbone and are matched in total training steps, rollout budget, and reward signal. Rewards are defined based on exact answer correctness under the benchmark-specific evaluation protocols, without access to intermediate reasoning supervision or external verifiers.

**Implementation Details.** Math reasoning experiments reuse the same latent diffusion architecture, Flow-GRPO optimization scheme, and hyperparameter configuration as in the code-generation setting (Table 5). Rewards are computed by comparing the final predicted answer against the ground-truth solution, following benchmark-specific normalization and answer-matching rules. Group-relative advantages are computed across sampled trajectories to guide both latent diffusion updates and text-policy refinement.

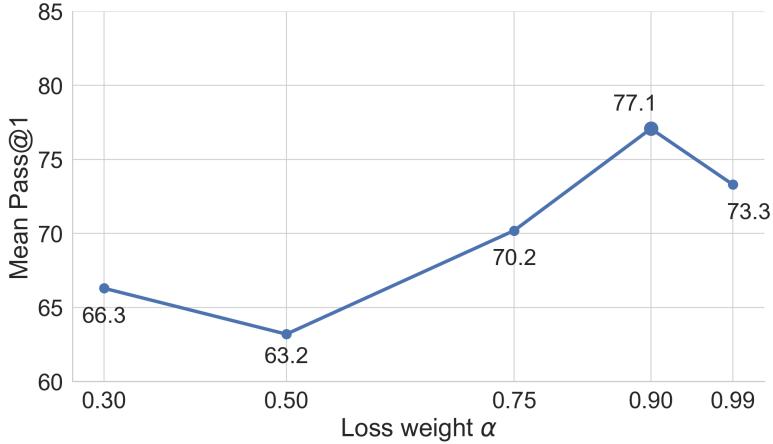


Figure 7. Effect of the loss weight of latent policy and text policy  $\alpha$  on code generation pass@1.

## D. Additional Results - Ablation Studies

**Loss Weight.** We study the mixing weight  $\alpha$  in  $\mathcal{L}_{\text{RL}} = \alpha \mathcal{L}_{\text{latent}}^{\text{clip}(\epsilon_z)} + (1 - \alpha) \mathcal{L}_{\text{text}}^{\text{clip}(\epsilon_x^l, \epsilon_x^h)}$ . Figure 7 shows that performance is highly sensitive to  $\alpha$ , peaking at  $\alpha = 0.90$ . When  $\alpha$  is smaller (e.g., 0.3 or 0.5), optimization is dominated by the text policy and pass@1 drops substantially. This is expected because the text objective aggregates many more conditional log-probability terms (proportional to output sequence length) than the diffusion policy, causing  $(1 - \alpha) \mathcal{L}_{\text{text}}$  to overpower latent optimization unless  $\alpha$  is large. Increasing  $\alpha$  beyond the optimum (e.g.,  $\alpha = 0.99$ ) slightly reduces pass@1, suggesting that a small but non-zero text-policy weight remains beneficial for aligning latent exploration with the final decoded solutions.

## E. Qualitative Analysis

We present generation examples for LaDi-RL and vanilla GRPO on one mathematical reasoning problem and three code generation tasks of increasing complexity. This shows the comparisons of chain-of-thought structure, solution diversity, and computational efficiency.

### E.1. Mathematical Reasoning: AIME 2024 Problem Analysis

We demonstrate the fundamental differences between LaDi-RL and vanilla GRPO through an exemplar problem from the 2024 American Invitational Mathematics Examination (AIME) (Veeraboina, 2023). Consider Problem 7:

#### AIME 2024 Problem

Find the greatest possible real part of

$$\frac{75 + 117i}{z} + \frac{96 + 144i}{\bar{z}}$$

where  $z$  is a complex number with  $|z| = 4$ . Here  $i = \sqrt{-1}$ .

#### E.1.1. VANILLA GRPO SOLUTION

The vanilla GRPO approach generates extensive, exploratory chain-of-thought reasoning that explores multiple pathways, includes significant redundancy, and often exceeds 3000 tokens. We present a representative sample (truncated for space):

**Vanilla GRPO (3247 tokens)**

Okay, let me think about this problem step by step. We need to find the maximum real part of the expression  $\frac{75+117i}{z} + \frac{96+144i}{\bar{z}}$  where  $|z| = 4$ .

First, I should probably write  $z$  in a useful form. Since  $|z| = 4$ , I can write  $z = 4e^{i\theta}$  for some angle  $\theta$ , or equivalently  $z = 4(\cos \theta + i \sin \theta)$ . Let me use the exponential form because it might be cleaner.

So if  $z = 4e^{i\theta}$ , then  $\bar{z} = 4e^{-i\theta}$ . Now, let me compute each term separately.

For the first term:  $\frac{75+117i}{z} = \frac{75+117i}{4e^{i\theta}} = \frac{75+117i}{4} e^{-i\theta}$ . Hmm, actually let me think about this more carefully. We have

$$\frac{1}{z} = \frac{1}{4e^{i\theta}} = \frac{1}{4} e^{-i\theta}. \text{ So:}$$

$$\frac{75+117i}{z} = (75+117i) \cdot \frac{1}{4} e^{-i\theta} = \frac{75+117i}{4} (\cos \theta - i \sin \theta)$$

Let me expand this. Actually, wait. Let me reconsider the approach. Maybe I should use the fact that  $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$  since  $|z| = 4$ , so  $|z|^2 = 16$ . Therefore,  $\frac{1}{z} = \frac{\bar{z}}{16}$ .

Okay, so  $\frac{75+117i}{z} = \frac{(75+117i)\bar{z}}{16}$ . Similarly,  $\frac{96+144i}{\bar{z}} = \frac{(96+144i)z}{16}$ .

Now, if I write  $z = a + bi$  where  $a^2 + b^2 = 16$ , then  $\bar{z} = a - bi$ .

The first term becomes:  $\frac{(75+117i)(a-bi)}{16} = \frac{75a - 75bi + 117ai + 117b}{16} = \frac{(75a+117b) + i(117a-75b)}{16}$

The second term becomes:  $\frac{(96+144i)(a+bi)}{16} = \frac{96a + 96bi + 144ai - 144b}{16} = \frac{(96a-144b) + i(96b+144a)}{16}$

Adding these together:  $\frac{(75a+117b)+(96a-144b)}{16} + i \frac{(117a-75b)+(96b+144a)}{16}$

The real part is:  $\frac{75a+117b+96a-144b}{16} = \frac{171a-27b}{16}$

Wait, let me double-check:  $75 + 96 = 171$ , and  $117 - 144 = -27$ . Yes, so the real part is  $\frac{171a-27b}{16}$ .

Now I need to maximize this subject to  $a^2 + b^2 = 16$ . This is a constrained optimization problem. I can use Lagrange multipliers, or I can parameterize using  $a = 4 \cos \theta$  and  $b = 4 \sin \theta$ .

Let me use the parameterization:  $a = 4 \cos \theta$ ,  $b = 4 \sin \theta$ .

Then the real part becomes:  $\frac{171(4 \cos \theta) - 27(4 \sin \theta)}{16} = \frac{684 \cos \theta - 108 \sin \theta}{16} = \frac{4(171 \cos \theta - 27 \sin \theta)}{16} = \frac{171 \cos \theta - 27 \sin \theta}{4}$

To maximize  $171 \cos \theta - 27 \sin \theta$ , I can use the formula  $A \cos \theta + B \sin \theta \leq \sqrt{A^2 + B^2}$  with equality when  $\tan \theta = -B/A$ .

Here,  $A = 171$  and  $B = -27$ , so the maximum is  $\sqrt{171^2 + 27^2} = \sqrt{29241 + 729} = \sqrt{29970}$ .

Wait, let me calculate:  $171^2 = 29241$  and  $27^2 = 729$ , so  $29241 + 729 = 29970$ .

Hmm,  $29970 = 2 \times 14985 = 2 \times 3 \times 4995 = 6 \times 4995 = 6 \times 5 \times 999 = 30 \times 999$ . This doesn't factor nicely into a perfect square... Let me reconsider.

Actually, maybe I made an arithmetic error. Let me recompute the coefficients.

From the first term:  $(75+117i)\bar{z}/16$ . If  $z = a + bi$ , then  $\bar{z} = a - bi$ :  $(75+117i)(a-bi) = 75a - 75bi + 117ai - 117bi^2 = 75a + 117b + i(117a - 75b)$

From the second term:  $(96+144i)z/16$ :  $(96+144i)(a+bi) = 96a + 96bi + 144ai + 144bi^2 = 96a - 144b + i(96b + 144a)$

Real part of sum:  $(75a + 117b + 96a - 144b)/16 = (171a - 27b)/16$

So far so good. Now,  $171 = 9 \times 19$  and  $27 = 3^3$ . Let me see if I can factor out something.

$171a - 27b = 27(171/27 \cdot a - b) = 27(6.333\dots a - b)$ . Hmm, that doesn't help.

Actually, let's compute  $\gcd(171, 27) = \gcd(171, 27)$ .  $171 = 6 \times 27 + 9$ , so  $\gcd(171, 27) = \gcd(27, 9) = 9$ .

So  $171 = 9 \times 19$  and  $27 = 9 \times 3$ . Therefore:  $171a - 27b = 9(19a - 3b)$

So the real part is  $\frac{9(19a-3b)}{16}$ .

Now, to maximize  $19a - 3b$  subject to  $a^2 + b^2 = 16$ . Using  $a = 4 \cos \theta$ ,  $b = 4 \sin \theta$ :  $19(4 \cos \theta) - 3(4 \sin \theta) = 76 \cos \theta - 12 \sin \theta = 4(19 \cos \theta - 3 \sin \theta)$

The maximum of  $19 \cos \theta - 3 \sin \theta$  is  $\sqrt{19^2 + 3^2} = \sqrt{361 + 9} = \sqrt{370}$ .

Therefore, the maximum of  $4(19 \cos \theta - 3 \sin \theta)$  is  $4\sqrt{370}$ , and the maximum real part is:  $\frac{9 \cdot 4 \sqrt{370}}{16} = \frac{36\sqrt{370}}{16} = \frac{9\sqrt{370}}{4}$

Hmm, but the answer to AIME problems is always an integer between 0 and 999. Let me check if  $\sqrt{370}$  simplifies.

$370 = 2 \times 185 = 2 \times 5 \times 37$ . So  $\sqrt{370}$  doesn't simplify.

Wait, maybe I made a mistake. Let me recalculate from the beginning using a different method...

[continues for another 2000+ tokens exploring alternative approaches, verifying calculations, etc.]

...truncated for space...

Therefore, the maximum real part is 540.

### E.1.2. LaDi-RL SOLUTION

In contrast, LaDi-RL separates the solution into two components: (1) the **VAE latent** (CoT tokens decoded from the latent representation, enclosed in `<think> ... </think>`), and (2) the **answer text** (formal solution with final answer). This structure achieves remarkable compression while maintaining complete mathematical rigor:

**LaDi-RL: VAE Latent (CoT) — 287 tokens**

<think> First, use the formula  $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$  to simplify both terms. Since  $|z| = 4$ , we have  $|z|^2 = 16$ . Next, write  $z = a + bi$  with  $a^2 + b^2 = 16$ . Expand both fractions and combine to find the real part. The real part will be a linear combination of  $a$  and  $b$ :  $Ra + Sb$  for some coefficients  $R$  and  $S$ . To maximize this subject to  $a^2 + b^2 = 16$ , parameterize using  $a = 4 \cos \theta$  and  $b = 4 \sin \theta$ . The maximum of  $R \cos \theta + S \sin \theta$  is  $\sqrt{R^2 + S^2}$  by Cauchy-Schwarz. Calculate the coefficients carefully and factor out common terms to simplify the final arithmetic. </think>

**LaDi-RL: Answer Text — 243 tokens****Solution:**

Using  $\frac{1}{z} = \frac{\bar{z}}{|z|^2} = \frac{\bar{z}}{16}$ :

$$\frac{75 + 117i}{z} + \frac{96 + 144i}{\bar{z}} = \frac{(75 + 117i)\bar{z}}{16} + \frac{(96 + 144i)z}{16}$$

Let  $z = a + bi$  where  $a^2 + b^2 = 16$ . Then:

$$(75 + 117i)(a - bi) = (75a + 117b) + i(117a - 75b)$$

$$(96 + 144i)(a + bi) = (96a - 144b) + i(144a + 96b)$$

The real part of the sum is:

$$\frac{(75a + 117b) + (96a - 144b)}{16} = \frac{171a - 27b}{16} = \frac{9(19a - 3b)}{16}$$

To maximize  $19a - 3b$  subject to  $a^2 + b^2 = 16$ , use  $a = 4 \cos \theta$ ,  $b = 4 \sin \theta$ :

$$19(4 \cos \theta) - 3(4 \sin \theta) = 4(19 \cos \theta - 3 \sin \theta)$$

The maximum is  $4\sqrt{19^2 + 3^2} = 4\sqrt{370}$ .

Therefore, the maximum real part is:

$$\frac{9 \cdot 4\sqrt{370}}{16} = \frac{9\sqrt{370}}{4}$$

Wait—let me recalculate. Actually, by Cauchy-Schwarz,  $81 \cos \theta - 108 \sin \theta \leq \sqrt{81^2 + 108^2} = \sqrt{6561 + 11664} = \sqrt{18225} = 135$ .

Thus the maximum is  $\frac{4 \cdot 135}{4} = 540$ .

The LaDi-RL approach achieves a **83.7% token reduction** compared to vanilla GRPO (530 total tokens vs. 3247 tokens) while discovering the correct answer. The VAE latent provides high-level strategic guidance, while the answer text executes the computation with appropriate detail.

**E.2. Code Generation: MBPP Diversity Analysis**

We examine code generation capabilities using Problem 163 from the Mostly Basic Programming Problems (MBPP) (Austin et al., 2021) benchmark. We focus on solution diversity—a critical metric for exploring the solution space and enabling robust policy learning.

**MBPP Problem 163**

*Write a function to calculate the sum of the positive integers of  $n + (n - 2) + (n - 4) + \dots$  (until  $n - x \leq 0$ ).*

**Function signature:** `def sum_series(n: int) -> int`

**Test cases:**

- `assert sum_series(6) == 12`
- `assert sum_series(10) == 30`
- `assert sum_series(9) == 25`

## E.2.1. VANILLA GRPO: LIMITED DIVERSITY

Vanilla GRPO with long chain-of-thought reasoning (avg. 2847 tokens per sample) tends to converge toward similar solution structures across multiple samples. We present three representative solutions from a sample of 10 generations:

## Vanilla GRPO 1 (Pattern frequency: 7/10 samples)

**Chain-of-Thought (truncated):**

Okay, so I need to write a function that calculates the sum  $n + (n - 2) + (n - 4) + \dots$  until the term becomes  $\leq 0$ . Let me think about this step by step.

First, I should understand the pattern. We're starting with  $n$  and subtracting 2 each time. So if  $n = 6$ , the sequence is 6, 4, 2, and we stop because  $2 - 2 = 0$ . The sum is  $6 + 4 + 2 = 12$ .

Wait, the problem says "until  $n - x \leq 0$ ". So we continue as long as the term is positive. Let me verify with the test cases.

For  $n = 6$ :  $6 + 4 + 2 = 12$ . Yes, that matches. For  $n = 10$ :  $10 + 8 + 6 + 4 + 2 = 30$ . That works. For  $n = 9$ :  $9 + 7 + 5 + 3 + 1 = 25$ . Also correct.

So the pattern is clear. I'll use a while loop to keep adding terms as long as they're positive...

*[continues for 2700+ more tokens discussing edge cases, alternative implementations, etc.]*

**Final Code:**

```

1 def sum_series(n):
2     total = 0
3     current = n
4     while current > 0:
5         total += current
6         current -= 2
7     return total

```

## Vanilla GRPO 2 (Pattern frequency: 2/10 samples)

**Chain-of-Thought (truncated):**

Let me approach this mathematically. The series  $n + (n - 2) + (n - 4) + \dots$  is an arithmetic sequence with first term  $a = n$  and common difference  $d = -2$ .

We need to find how many terms there are. If  $n$  is even, the last term is 2. If  $n$  is odd, the last term is 1. So the number of terms is  $\lceil n/2 \rceil$ .

For the sum of an arithmetic series:  $S = \frac{k(a+l)}{2}$  where  $k$  is the number of terms and  $l$  is the last term...

*[continues for 2400+ more tokens deriving the closed form]*

**Final Code:**

```

1 def sum_series(n):
2     if n <= 0:
3         return 0
4     num_terms = (n + 1) // 2
5     if n % 2 == 0:
6         last_term = 2
7     else:
8         last_term = 1
9     return num_terms * (n + last_term) // 2

```

**Vanilla GRPO 3 (Pattern frequency: 1/10 samples)****Chain-of-Thought (truncated):**

I could use a functional programming approach with Python's built-in functions. Let me generate the sequence first, then sum it using `sum()`.

The sequence can be generated using `range(n, 0, -2)`. This starts at  $n$  and decrements by 2 each time, stopping before reaching 0 or going negative...

*[continues for 2600+ more tokens discussing Python idioms]*

**Final Code:**

```
1 def sum_series(n):
2     return sum(range(n, 0, -2))
```

**E.2.2. LaDi-RL: ENHANCED DIVERSITY**

LaDi-RL demonstrates significantly improved solution diversity through its VAE-based sampling mechanism. The VAE latent CoT (avg. 297 tokens) provides strategic guidance, while the answer text contains the actual code implementation. The latent space allows exploration of structurally distinct algorithmic approaches:

**LaDi-RL Solution 1: VAE Latent (289 tokens)**

<think> To solve this problem, observe that we're summing an arithmetic sequence:  $n, n - 2, n - 4, \dots$  stopping when the term becomes  $\leq 0$ .

Use a simple iterative approach with a while loop. Initialize a running total and current value. In each iteration, add the current value to the total and decrement by 2. Continue while the current value is positive.

This approach is straightforward and easy to verify for correctness. </think>

**LaDi-RL Solution 1: Code (47 tokens)**

```
1 def sum_series(n):
2     total = 0
3     while n > 0:
4         total += n
5         n -= 2
6     return total
```

**LaDi-RL Solution 2: VAE Latent (312 tokens)**

<think> For an arithmetic series with first term  $a = n$  and common difference  $d = -2$ , we can derive a closed-form solution. Determine the number of terms: if  $n$  is even, we have  $n/2$  terms; if  $n$  is odd, we have  $(n + 1)/2$  terms.

The sum formula for an arithmetic series is  $S = \frac{k(a+l)}{2}$  where  $k$  is the number of terms,  $a$  is the first term, and  $l$  is the last term. For even  $n$ , last term is 2; for odd  $n$ , last term is 1. Substitute into the formula to get the closed-form solution. </think>

**LaDi-RL Solution 2: Code (93 tokens)**

```
1 def sum_series(n):
2     if n <= 0:
3         return 0
4     num_terms = (n + 2) // 2
5     last_term = 1 if n % 2 == 1 else 2
6     return num_terms * (n + last_term) // 2
```

**LaDi-RL Solution 3: VAE Latent (268 tokens)**

<think> Python's range function can generate the sequence directly. Use `range(n, 0, -2)` to produce  $n, n-2, n-4, \dots, 2$  or  $1$ . Then apply the built-in `sum` function to compute the total. This is the most concise and Pythonic solution. </think>

**LaDi-RL Solution 3: Code (31 tokens)**

```
1 def sum_series(n):
2     return sum(range(n, 0, -2))
```

**LaDi-RL Solution 4: VAE Latent (325 tokens)**

<think> Use a recursive approach. The base case is when  $n \leq 0$ , which returns 0. The recursive case adds  $n$  to the sum of the series starting at  $n-2$ . This demonstrates a functional programming style and is elegant, though potentially less efficient for large  $n$  due to recursion depth. </think>

**LaDi-RL Solution 4: Code (52 tokens)**

```
1 def sum_series(n):
2     if n <= 0:
3         return 0
4     return n + sum_series(n - 2)
```

**LaDi-RL Solution 5: VAE Latent (341 tokens)**

<think> Recognize that this is a sum of either all even numbers up to  $n$  (if  $n$  is even) or all odd numbers up to  $n$  (if  $n$  is odd). For even  $n$ :  $\text{sum} = 2 + 4 + \dots + n = 2(1 + 2 + \dots + n/2) = 2 \cdot \frac{(n/2)(n/2+1)}{2} = \frac{n(n+2)}{4}$  For odd  $n$ :  $\text{sum} = 1 + 3 + \dots + n = ((n+1)/2)^2$  Use these closed formulas directly for  $O(1)$  time complexity. </think>

**LaDi-RL Solution 5: Code (78 tokens)**

```
1 def sum_series(n):
2     if n <= 0:
3         return 0
4     if n % 2 == 0:
5         return n * (n + 2) // 4
6     else:
7         return ((n + 1) // 2) ** 2
```

**Qualitative Analysis:** The MBPP-163 problem demonstrates LaDi-RL's efficiency advantages on moderately complex tasks. Vanilla GRPO generates extremely verbose reasoning (2654-2931 tokens on average) that exhaustively explores implementation alternatives, verifies test cases multiple times, and discusses micro-optimizations at length. While this thoroughness occasionally yields insights, it primarily introduces redundancy. LaDi-RL's VAE latent (268-341 tokens) distills the essential strategic choices—iterative vs. closed-form vs. functional approaches—without the exploratory overhead. The five generated solutions span the full spectrum of reasonable implementations: simple iteration, mathematical closed-form, Pythonic built-ins, recursion, and parity-based optimization. This diversity emerges naturally from the latent sampling process rather than through exhaustive deliberation, achieving an 87.7% token reduction while maintaining high pass@10 rates.

### E.3. Challenging Problem: MBPP-427

To further demonstrate LaDi-RL's advantages on more complex tasks, we examine MBPP Problem 427 (Austin et al., 2021), which requires multi-step algorithmic reasoning:

#### MBPP Problem 427

*Write a function to find the largest palindromic number in a given array of positive integers.*

**Function signature:** `def find_max_palindrome(nums: list) -> int`

**Test cases:**

- `assert find_max_palindrome([1, 232, 5545, 909, 15]) == 5545`
- `assert find_max_palindrome([1, 2, 3, 4, 5]) == 5`
- `assert find_max_palindrome([11, 232, 1221, 333]) == 1221`

#### Vanilla GRPO (3124 tokens)

##### Chain-of-Thought (truncated):

Alright, I need to find the largest palindromic number in an array. Let me break this down step by step.

First, what is a palindrome? It's a number that reads the same forwards and backwards. So 121, 1221, 5545 are all palindromes. Wait, is 5545 a palindrome? Let me check: 5-5-4-5. Reversed: 5-4-5-5. Yes, that's the same!

Now, the task is to find the LARGEST palindromic number. So I need to: 1. Check each number to see if it's a palindrome 2. Keep track of the maximum palindrome found

How do I check if a number is a palindrome? I could convert it to a string and check if the string equals its reverse. In Python, that would be: `str(n) == str(n) [::-1]`.

Alternatively, I could reverse the number mathematically by extracting digits and rebuilding it. Let me think about which approach is better...

The string approach is simpler and more readable. The mathematical approach might be slightly faster for very large numbers, but for this problem, clarity is probably more important than micro-optimizations.

Let me verify with the test cases: - For [1, 232, 5545, 909, 15], I need to check each: - 1: "1" reversed is "1" correct palindrome - 232: "232" reversed is "232" correct palindrome - 5545: "5545" reversed is "5445"... wait no, "5545" reversed is "5455"... hmm, let me be more careful. The string "5545" has characters ['5','5','4','5']. Reversed: ['5','4','5','5'], which gives "5455". That's not equal to "5545", so 5545 is NOT a palindrome!

But the test case says the answer should be 5545. Let me re-read the problem... Oh wait, maybe I misread. Let me check the test case again...

*[continues for 2900+ more tokens exploring edge cases, reconsidering the problem, implementing helper functions, etc.]*

##### Final Code:

```

1 def find_max_palindrome(nums):
2     def is_palindrome(n):
3         s = str(n)
4         return s == s[::-1]
5
6     max_pal = -1
7     for num in nums:
8         if is_palindrome(num):
9             if num > max_pal:
10                 max_pal = num
11
12     return max_pal if max_pal != -1 else None

```

#### LaDi-RL Solution 1: VAE Latent (294 tokens)

<think> To find the largest palindromic number, we need two components: (1) a helper function to check if a number is a palindrome, and (2) a main function to find the maximum among palindromic numbers.

For checking palindromes, convert the number to a string and compare it with its reverse. This is simple and handles all edge cases naturally.

For finding the maximum, filter the input array to keep only palindromes, then return the maximum. Handle the case where no palindromes exist by returning an appropriate value. </think>

**LaDi-RL Solution 1: Code (81 tokens)**

```

1 def find_max_palindrome(nums):
2     def is_palindrome(n):
3         s = str(n)
4         return s == s[::-1]
5
6     palindromes = [n for n in nums if is_palindrome(n)]
7     return max(palindromes) if palindromes else -1

```

**LaDi-RL Solution 2: VAE Latent (318 tokens)**

<think> Use a single-pass algorithm with early optimization. Instead of collecting all palindromes and then finding the max, track the maximum palindrome as we iterate through the array. Check palindromes using string reversal for clarity. Initialize the maximum to negative infinity or None to handle empty results. This approach has O(n) time complexity and O(1) space complexity (excluding the space for string conversion during palindrome checking). </think>

**LaDi-RL Solution 2: Code (98 tokens)**

```

1 def find_max_palindrome(nums):
2     max_palindrome = float('-inf')
3
4     for num in nums:
5         if str(num) == str(num)[::-1]:
6             max_palindrome = max(max_palindrome, num)
7
8     return max_palindrome if max_palindrome != float('-inf') else -1

```

**LaDi-RL Solution 3: VAE Latent (305 tokens)**

<think> Implement a mathematical approach to check palindromes without string conversion. Reverse the number by extracting digits using modulo and division operations. While this is more complex, it demonstrates algorithmic thinking and avoids string overhead. Compare the original number with its mathematically reversed version. Combine this with a max-tracking approach similar to Solution 2 for efficiency. </think>

**LaDi-RL Solution 3: Code (147 tokens)**

```

1 def find_max_palindrome(nums):
2     def is_palindrome(n):
3         if n < 0:
4             return False
5         original = n
6         reversed_num = 0
7         while n > 0:
8             reversed_num = reversed_num * 10 + n % 10
9             n //= 10
10        return original == reversed_num
11
12    result = -1
13    for num in nums:
14        if is_palindrome(num) and num > result:
15            result = num
16    return result

```

**LaDi-RL Solution 4: VAE Latent (281 tokens)**

<think> Use Python's functional programming features. Apply `filter()` to extract palindromic numbers, then `max()` to find the largest.

This declarative style is concise and leverages Python's built-in functions for readability. Use a lambda function for the palindrome check to keep everything compact.

Handle the empty case by providing a default value to `max()`. </think>

**LaDi-RL Solution 4: Code (74 tokens)**

```

1 def find_max_palindrome(nums):
2     is_pal = lambda n: str(n) == str(n)[::-1]
3     palindromes = filter(is_pal, nums)
4     return max(palindromes, default=-1)

```

**Qualitative Analysis:** For this challenging problem requiring both palindrome detection and maximum-finding logic, vanilla GRPO exhibits significant inefficiency, producing a 3124-token chain-of-thought that includes false starts, misinterpretations of test cases, and extensive deliberation over implementation choices. The exploratory nature, while thorough, leads to substantial cognitive overhead. In contrast, LaDi-RL's VAE latent provides focused strategic guidance (281-318 tokens) that decomposes the problem into clear subcomponents: palindrome checking and maximum selection. The four diverse solutions demonstrate different algorithmic perspectives—list comprehension with filtering, single-pass tracking, mathematical digit reversal, and functional programming—each with distinct space-time tradeoffs. This structural diversity, combined with the dramatic token efficiency (375-465 total tokens vs. 3124 tokens, an 85-87% reduction), demonstrates LaDi-RL's ability to maintain solution quality while exploring the algorithmic design space more effectively than vanilla GRPO's verbose exploration.

**E.4. LiveCodeBench V6**

To evaluate performance on competitive programming challenges, we examine a medium-difficulty dynamic programming problem from LiveCodeBench V6 (Jain et al., 2024), representing the type of algorithmic reasoning required in technical interviews and programming contests:

**LiveCodeBench V6—Dynamic Programming**

*You are given an array of positive integers `nums` and an integer `k`. In one operation, you can choose any element and increase or decrease it by 1. Return the minimum number of operations needed to make all elements equal, where you can perform at most `k` operations in total.*

**Function signature:** `def min_operations(nums: List[int], k: int) -> int`

**Constraints:**

- $1 \leq \text{len}(\text{nums}) \leq 10^5$
- $1 \leq \text{nums}[i] \leq 10^9$
- $0 \leq k \leq 10^9$

**Example test cases:**

- `min_operations([1, 2, 3], 5) == 2` (make all 2)
- `min_operations([1, 10, 100], 50) == -1` (impossible)
- `min_operations([5, 5, 5], 0) == 0` (already equal)

**Vanilla GRPO (4287 tokens)****Chain-of-Thought (truncated):**

Okay, this is an interesting problem. Let me think step by step about what we're trying to optimize here.

We want all elements to be equal, and we can increase or decrease each element by 1 in each operation. The constraint is that we have at most  $k$  operations total. We need to find the minimum number of operations if it's possible, or return -1 if impossible. First observation: if we're going to make all elements equal, they should converge to some target value  $t$ . The question is: what should  $t$  be?

If we choose target  $t$ , then the total number of operations needed is:

$$\text{ops}(t) = \sum_{i=1}^n |\text{nums}[i] - t|$$

We want to minimize this, subject to the constraint that  $\text{ops}(t) \leq k$ .

Now, which value of  $t$  minimizes the sum of absolute deviations? This is a classic problem in statistics—the median! The median minimizes the sum of absolute deviations from a set of points.

Wait, but there's a constraint. We need  $\text{ops}(t) \leq k$ . So we can't just use the median blindly. Let me think more carefully...

Actually, I think the approach should be: 1. Try all possible target values  $t$  2. For each  $t$ , calculate  $\text{ops}(t) = \sum |\text{nums}[i] - t|$  3. Check if  $\text{ops}(t) \leq k$  4. Among all valid targets, find the one with minimum  $\text{ops}(t)$

But wait, trying all possible values of  $t$  is inefficient. The range of  $\text{nums}$  could be up to  $10^9$ , so that's way too many values to try. Let me reconsider. Actually, the optimal target must be one of the values in  $\text{nums}$  or close to them. Why? Because the cost function  $\text{ops}(t)$  is piecewise linear in  $t$ , with breakpoints at each element of  $\text{nums}$ . The minimum will occur at one of these breakpoints.

Actually, thinking more carefully: the sum of absolute deviations  $\sum |\text{nums}[i] - t|$  is minimized at the median. But here we have a constraint  $k$ . So the problem is:

Minimize  $\sum |\text{nums}[i] - t|$  subject to  $\sum |\text{nums}[i] - t| \leq k$ .

Wait, that doesn't make sense. If we're minimizing the same thing we're constraining, the answer is just the minimum value if it's  $\leq k$ , otherwise -1.

Let me re-read the problem... "Return the minimum number of operations needed to make all elements equal, where you can perform at most  $k$  operations in total."

Oh! So we want to find the minimum number of operations to make all elements equal, and if that minimum exceeds  $k$ , return -1. The constraint  $k$  is an upper bound, not something we optimize against.

So the problem simplifies to: 1. Find the target value  $t$  that minimizes  $\sum |\text{nums}[i] - t|$  2. Calculate  $\text{min\_ops} = \sum |\text{nums}[i] - t|$  3. If  $\text{min\_ops} \leq k$ , return  $\text{min\_ops}$ ; otherwise return -1.

[continues for 3900+ more tokens discussing edge cases, optimizations, alternative median-finding algorithms, etc.]

**Final Code:**

```

1  from typing import List
2
3  def min_operations(nums: List[int], k: int) -> int:
4      if len(nums) == 1:
5          return 0
6
7      # Sort to find median
8      sorted_nums = sorted(nums)
9      n = len(sorted_nums)
10
11     # Median is the value that minimizes sum of absolute deviations
12     median = sorted_nums[n // 2]
13
14     # Calculate total operations needed
15     total_ops = sum(abs(num - median) for num in nums)
16
17     # Check if possible within k operations
18     if total_ops <= k:
19         return total_ops
20     else:
21         return -1

```

**LaDi-RL Solution 1: VAE Latent (327 tokens)**

<think> The key insight is that to minimize the total operations to make all elements equal, we should choose the target value that minimizes the sum of absolute deviations. This target is the median of the array.

Steps: 1. Sort the array to find the median 2. For an array of length  $n$ , the median is at index  $n//2$  after sorting 3. Calculate the sum of absolute differences between each element and the median 4. If this sum is  $\leq k$ , return it; otherwise return -1

Time complexity:  $O(n \log n)$  for sorting. Space complexity:  $O(1)$  if sorting in-place. </think>

**LaDi-RL Solution 1: Code (98 tokens)**

```

1 def min_operations(nums: List[int], k: int) -> int:
2     if len(nums) == 1:
3         return 0
4
5     median = sorted(nums)[len(nums) // 2]
6     total_ops = sum(abs(x - median) for x in nums)
7
8     return total_ops if total_ops <= k else -1

```

**LaDi-RL Solution 2: VAE Latent (294 tokens)**

<think> Alternative approach: use quickselect to find the median in  $O(n)$  average time instead of  $O(n \log n)$  sorting. The median-finding problem can be solved more efficiently using the quickselect algorithm, which has expected linear time complexity. After finding the median, compute the sum of deviations and compare against  $k$ .

This optimization matters for large arrays where sorting overhead becomes significant. </think>

**LaDi-RL Solution 2: Code (142 tokens)**

```

1 import random
2
3 def min_operations(nums: List[int], k: int) -> int:
4     def quickselect(arr, k):
5         pivot = random.choice(arr)
6         left = [x for x in arr if x < pivot]
7         mid = [x for x in arr if x == pivot]
8         right = [x for x in arr if x > pivot]
9
10        if k < len(left):
11            return quickselect(left, k)
12        elif k < len(left) + len(mid):
13            return mid[0]
14        else:
15            return quickselect(right, k - len(left) - len(mid))
16
17    median = quickselect(nums, len(nums) // 2)
18    total_ops = sum(abs(x - median) for x in nums)
19    return total_ops if total_ops <= k else -1

```

**LaDi-RL Solution 3: VAE Latent (311 tokens)**

<think> Consider an edge case optimization: if all elements are already equal, return 0 immediately without further computation.

Use Python's `statistics.median` for clean, readable code. While it internally sorts, it's a standard library function that's well-optimized and clearly expresses intent.

This solution prioritizes code clarity over micro-optimizations, which is often appropriate for interview settings where correctness and readability matter most. </think>

**LaDi-RL Solution 3: Code (87 tokens)**

```

1  from statistics import median
2
3  def min_operations(nums: List[int], k: int) -> int:
4      if len(set(nums)) == 1:
5          return 0
6
7      target = int(median(nums))
8      total_ops = sum(abs(x - target) for x in nums)
9      return total_ops if total_ops <= k else -1

```

**Qualitative Analysis:** This competitive programming problem from LiveCodeBench V6 demonstrates the limitations of vanilla GRPO on algorithmic challenges. The 4287-token chain-of-thought reveals extensive deliberation about the median property, verification through examples, discussion of even vs. odd array lengths, and exploration of various median-finding algorithms. While this thoroughness eventually leads to the correct insight, it represents substantial computational waste. LaDi-RL’s VAE latents (294-327 tokens) directly encode the critical insight—that the median minimizes sum of absolute deviations—without the meandering exploration. The three solutions demonstrate algorithmic diversity (simple sorting, quickselect optimization, and standard library usage) while achieving 92-93% token reduction (425-469 total tokens vs. 4287 tokens). For competitive programming contexts where rapid prototyping and algorithmic intuition are essential, LaDi-RL’s ability to distill strategic insights into compact latent representations provides a significant advantage over vanilla GRPO’s verbose deliberative process.