Appendix I: Exemplar Power System Engineering Optimisation Assignment Interaction with Large Language Model

This Appendix includes the prompts supplied to ChatGPT for the exemplar assignment questions included in Figure 1 of the letter. This exercise was carried out using ChatGPT Plus in July 2023, with access to the Wolfram plug-in enabled.

Question (a), Question (b)

The interaction with the LLM for solving question (a) and question (b) may be viewed here: https://bit.ly/46HC4rH.

Prompt

Solve the following optimisation problem given the objective function and constraint set as follows, where x and y are real numbers:

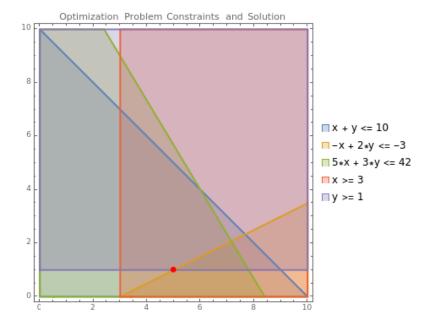
```
\underbrace{\text{minimise}}_{x,y}4x+7y
x+y\leq10
-x+2y\leq-3
5x+3y\leq42
x\geq3
y\geq1
```

Produce a graph showing x on the x-axis and y on the y-axis and showing that the solution obtained is in fact optimal. In the plots, include the constraints. Also report the value of the objective function at the optimal solution.

Response

The optimal solution to the given optimization problem is x = 5 and y = 1. The value of the objective function at this optimal solution is 27.

Now, let's plot the constraints and the optimal solution on a graph.



The red point represents the optimal solution (x = 5, y = 1). As you can see, this point lies within the feasible region defined by the constraints, indicating that it is indeed an optimal solution. The value of the objective function at this point is 27.

Question (c)

The interaction with the LLM for solving question (c) may be viewed here: https://bit.ly/3NNOibl.

Prompt

A power system consists of nine generators (G1, G2, G3, G4, G5, G6, G7, G8, G9). Each generator has a maximum and minimum output. Each generator may be assumed to be online and operational. Each generator has a cost curve of the form $C_i = a_i * P_i ^2 + b_i * P_i + c_i$, where P_i is the output of generator i. These parameters are detailed below. It is required to determine the generator outputs which minimises cost when demand is 1495.2 MW.

|Unit|P_min,i|P_max,i|a_i|b_i|c_i| |G1|2.4|12|0.025|25.5|24.4| |G2|4|20|0.012|37.6|117.8| |G3|0|50|0|0.5|0| |G4|15.2|76|0.009|13.3|81.1| |G5|25|100|0.006|8|217.9| |G6|54.24|155|0.005|10.7|142.7| |G7|68.95|197|0.003|23|259.1| |G8|140|350|0.002|10.9|177.1| |G9|100|400|0.0021|17.5|311.9|

First, I would like you to formulate this as an optimisation problem with an objective function and constraints - remember to consider both equality and inequality constraints.

Then I would like you to form the corresponding Lagrangian function of the optimisation problem and describe how it may be used to find the optimal solution.

Finally, I would like you to write GAMS code which may be used to determine the optimal dispatch of the generators.

Response

This is a classic economic dispatch problem in power systems. The goal is to minimize the total cost of generation while satisfying the demand and the operational limits of each generator.

Formulation of the optimization problem

The objective function is the total cost of generation, which is the sum of the cost of each generator:

$$\min_{P_i} \sum_{i=1}^{9} C_i = \sum_{i=1}^{9} (a_i * P_i^2 + b_i * P_i + c_i)$$

The constraints are:

• The total generation must equal the demand:

$$\sum_{i=1}^{9} P_i = 1495.2$$

• Each generator must operate within its limits:

$$P_{\min,i} \leq P_i \leq P_{\max,i}$$

for
$$i = 1, 2, ..., 9$$

Formulation of the Lagrangian function

The Lagrangian function is formed by incorporating the constraints into the objective function using Lagrange multipliers. For this problem, we have one equality constraint (power balance) and nine inequality constraints (generator limits). The Lagrangian function is:

$$L(P_i, \lambda, \mu_i, \nu_i) = \sum_{i=1}^{9} (a_i * P_i^2 + b_i * P_i + c_i) + \lambda (\sum_{i=1}^{9} P_i - 1495.2) - \sum_{i=1}^{9} \mu_i (P_i - P_{\min,i}) + \sum_{i=1}^{9} \nu_i (P_i - P_{\max,i})$$

where P_i is the power output of the generator i, λ is the Lagrange multiplier associated with the power balance constraint, μ_i is and v_i are the Lagrange multipliers associated with the lower and upper limits of generator i, respectively.

The optimal solution is found by setting the derivative of the Lagrangian function with respect to P_i , almbda, μ_i and v_i to zero and solving the resulting system of equations. This gives the optimal power output of each generator and the optimal values of the Lagrange multipliers.

GAMS code

Here is a simple GAMS code for this problem.

```
Set i / G1*G9 /;
Alias (i,j);
Parameters
    a(i) /G1 0.025, G2 0.012, G3 0, G4 0.009, G5 0.006, G6 0.005, G7
    0.003, G8 0.002, G9 0.00211/
    b(i) /G1 25.5, G2 37.6, G3 0.5, G4 13.3, G5 8, G6 10.7, G7 23, G8
    10.9, G9 7.5/
    c(i) /G1 24.4, G2 117.8, G3 0, G4 81.1, G5 217.9, G6 142.7, G7 259.1,
    G8 177.1, G9 311.9/
    Pmin(i) /G1 2.4, G2 4, G3 0, G4 15.2, G5 25, G6 54.24, G7 68.95, G8
    140, G9 100/
    Pmax(i) /G1 12, G2 20, G3 50, G4 76, G5 100, G6 155, G7 197, G8 350,
    G9 400/
    demand /1495.2/;
Variables
    P(i)
    cost;
Positive Variable P;
Equations
    obj
    power_balance
    lower limit(i)
    upper_limit(i);
obj..
    cost = e = sum(i, a(i) *P(i) * (P(i)) + b(i) *P(i) + c(i));
power_balance..
    sum(i, P(i)) = e = demand;
lower_limit(i)..
    P(i) = q = Pmin(i);
upper_limit(i)..
    P(i) = l = Pmax(i);
Model dispatch /all/;
Solve dispatch using NLP minimizing cost;
```

This GAMS code defines the parameters and variables, sets up the objective function and constraints, and solves the problem using a nonlinear programming (NLP) solver. The solution will give the optimal power output of each generator that minimizes the total cost while satisfying the demand and the generator limits.