Lund University, Numerical Analysis **Spring Term 2, 2020**
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Numerical Analysis: Basic Course

Assignment 2

Problem 1

Let $f \in \mathcal{C}^4(\mathbb{R})$ be a real valued function, $t_0 < t_1 < t_2$ real numbers and p a polynomial of degree ≤ 3, which satisfies $p(t_j) = f(t_j)$, $j = 0, 1, 2$ and $p'(t_1) = f'(t_1)$. An interpolation problem where both requirements on the function values and its derivatives are given is called a Hermite interpolation problem.

a) Prove that for $t \in [t_0, t_2]$:

$$|f(t) - p(t)| \leq \frac{1}{24} \max_{\xi \in [t_0, t_2]} |f^{(4)}(\xi)||(t - t_0)(t - t_1)^2(t - t_2)|.$$

Hint: Employ the auxiliary function

$$F(t) = r(t) - \frac{f(x) - p(x)}{(x - t_0)(x - t_1)^2(x - t_2)}(t - t_0)(t - t_1)^2(t - t_2).$$

b) Is there a function $f \in \mathcal{C}^4(\mathbb{R})$, $f \notin \mathcal{P}_3$ and a corresponding polynomial $p \in \mathcal{P}_3$ and a $t \in (t_0, t_2)$, $t \neq t_1$, for which you have equality in the above inequality?

Problem 2

Write a program coeff=cubspline(xint,yint), which takes as input the x- and y-values of $m + 1$ interpolation points and returns a $m \times 4$ coefficient matrix of the natural spline which interpolates the data. The i-th row of this matrix contains the coefficients a_i, b_i, c_i, d_i of the cubic subpolynomial s_i of the spline. The program may be written for equidistant node-points x_i (h constant.)

Then write a program yval=cubsplineval(coeff,xint,xval), which evaluates the spline at xval. Test both programs on the test problems from the last assignment.

Problem 3

We will now discuss a wheel profile function, that is based the standard rail wheel profile S1002, which is defined sectionwise by polynomials up to degree 7. The profile and its sections are shown in Fig. 1. The polynomials are defined by

$$\begin{array}{llcl}
\text{Section A:} & F(s) & = & a_A - b_A s \\
\text{Section B:} & F(s) & = & a_B - b_B s + c_B s^2 - d_B s^3 + e_B s^4 - f_B s^5 + g_B s^6 - h_B s^7 + i_B s^8 \\
\text{Section C:} & F(s) & = & -a_C - b_C s - c_C s^2 - d_C s^3 - e_C s^4 - f_C s^5 - g_C s^6 - h_C s^7 \\
\text{Section D:} & F(s) & = & a_D - \sqrt{b_D^2 - (s + c_D)^2} \\
\text{Section E:} & F(s) & = & -a_E - b_E s \\
\text{Section F:} & F(s) & = & a_F + \sqrt{b_F^2 - (s + c_F)^2} \\
\text{Section G:} & F(s) & = & a_G + \sqrt{b_G^2 - (s + c_G)^2} \\
\text{Section H:} & F(s) & = & a_H + \sqrt{b_H^2 - (s + c_H)^2}
\end{array}$$

and

	A	B	C	D
a	1.364323640	0.0	$4.320221063\,10^{+3}$	16.446
b	0.066666667	$3.358537058\,10^{-2}$	$1.038384026\,10^{+3}$	13.
c	–	$1.565681624\,10^{-3}$	$1.065501873\,10^{+2}$	26.210665
d	–	$2.810427944\,10^{-5}$	$6.051367875\,10^{+0}$	–
e	–	$5.844240864\,10^{-8}$	$2.054332446\,10^{-1}$	–
f	–	$1.562379023\,10^{-8}$	$4.169739389\,10^{-3}$	–
g	–	$5.309217349\,10^{-15}$	$4.687195829\,10^{-5}$	–
h	–	$5.957839843\,10^{-12}$	$2.252755540\,10^{-7}$	–
i	–	$2.646656573\,10^{-13}$	–	–
$\xi_{\min}$	32.15796	-26	-35	-38.426669071
$\xi_{\max}$	60	32.15796	-26	-35

	E	F	G	H
a	93.576667419	8.834924130	16.	9.519259302
b	2.747477419	20	12.	20.5
c	–	58.558326413	55.	49.5
$\xi_{\min}$	-39.764473993	-49.662510381	-62.764705882	-70.0
$\xi_{\max}$	-38.426669071	-39.764473993	-49.662510381	-62.764705882

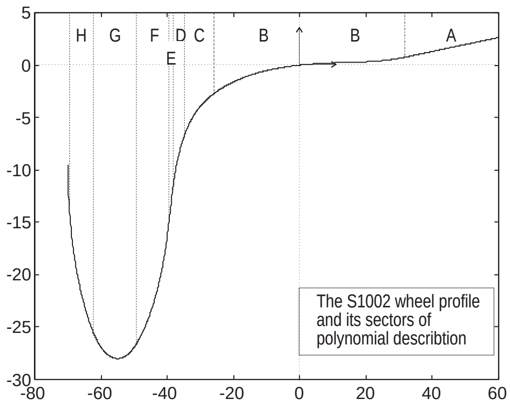

Figure 1.

Describe this wheel profile by means of a natural cubic spline. For this end, download the file `s1002.py`, which contains the above description of the s1002 wheel standard and generate from this data, which you then use to generate an interpolating spline with your programs from task 2. Plot the resulting curve.

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