Lund University, Numerical Analysis **Spring Term 2, 2020**
Philipp Birken

Numerical Analysis: Basic Course

Assignment 5

Problem 1
Assume a computer where numbers are stored as 6 bit binary numbers in the form

$$float(x) = \pm 1.b_1b_0 \cdot 2^{-c_2c_1c_0}.$$

Hereby, $1.b_1b_0$ and $c_2c_1c_0$ are binary numbers. Complete the following table:

x	float(x)		rel. error
	binary	decimal	
0.1	$1.10 \cdot 2^{-100}$	0.09375	0.0625
0.2			
0.3			
0.4			
0.5			

Problem 2
Compute the best possible constants for the equivalence of the 1- and ∞-norm, as well as the 1- and 2-norm in $\mathbb{R}^n$:

$$c_{1,\infty}\|\mathbf{x}\|_1 \leq \|\mathbf{x}\|_\infty \leq c_{\infty,1}\|\mathbf{x}\|_1, \qquad c_{1,2}\|\mathbf{x}\|_1 \leq \|\mathbf{x}\|_2 \leq c_{2,1}\|\mathbf{x}\|_1.$$

Best possible means that the inequalities hold sharp, meaning that for each of the four inequalities, there is a vector $\mathbf{x} \in \mathbb{R}^n$ such that this inequality holds with equality. Provide such a vector for each inequality.

Problem 3
Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & \alpha & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

with $\alpha \in \mathbb{R}$.

a) Compute $\|\mathbf{A}\|_2$ as a function of α.

b) Compute the condition number of $\mathbf{A}$ in $\|\cdot\|_\infty$ as a function of α for $\alpha \neq 0$.

c) Make a sketch of this function.

Problem 4
Consider the two dimensional system of ordinary differential equations

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 0 & -100 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

with unknown function $\mathbf{y}(t) = (y_1(t), y_2(t))^T$ and initial values $\mathbf{y}(t_0) = (1,1)^T$ for $t \in [0,1]$.

- What is the exact solution?
- Implement the explicit and the implicit Euler method for the above problem using a constant time step size h.
- Plot the exact solution and the numerical solutions for both methods for different choices of h.
- What do you see?

Now that you have your code, you can play around a bit. Consider the predator-prey model (also called Lotka-Volterra equation)

$$\begin{aligned} \dot{x} &= ax - bxy, \\ \dot{y} &= cxy - dy, \end{aligned}$$

with positive constants a, b, c and d. x and y denote the number of rabbits and foxes over time. Use the explicit Euler method to solve this and choose initial values, parameters, time steps and end times. Can you make the rabbits win? How much of your code did you rewrite, how much did you reuse?

Return: **Friday, May 8th**