Assignment 5 –

NUMB11/FMNN01: Numerical Linear Algebra

C. Führer/P. Birken/R. Klöfkorn , Numerical Analysis

The purpose of these exercises is to experiment with algorithms for finding eigenvalues.

Task 1

Consider the matrix

$$A = \begin{pmatrix} 5 & 0 & 0 & -1 \\ 1 & 0 & -1 & 1 \\ -1.5 & 1 & -2 & 1 \\ -1 & 1 & 3 & -3 \end{pmatrix}$$

and the matrix $A(p)$ with the same diagonal elements as A and all other elements being scaled by $p \in [0, 1]$, i.e. pA_{ij}. Note, $A(0)$ is a diagonal matrix. Compute the eigenvalues of $A(p)$ with `eig` and plot them in the complex plane. Vary the parameter p so that you trace in the plot their dependency on p. Mark in the plot the diagonal elements of A and also the eigenvalues of A with fat circles. Mark in the same plot the Gerschgorin discs and explain by your figure Gerschgorin's theorm. If you like programming you can make a film of your result. In that case we will use it in class (if we get your permission).

Task 2

Implement the QR method with Rayleigh shifts and deflations (Alg. 28.2). Test it on several symmetric matrices and compare the result with `eig`. Test it even on a symmetric orthogonal matrix. Apply it on many random symmetric matrices and make a statement about the average number of iterations you need to get the eigenvalues with a relative error less than 10^{-8}. Note a matrix given by

$$A := \mathrm{rand}(n, n); \quad A := \frac{1}{2}(A + A^{\mathrm{T}})$$

is a random symmetric matrix.

Task 3

Solve Exercise 25.1 in the book. Please note the hint given there. Explain how the statement $\text{rank}(A - \lambda \text{I}) \geq m - 1 \ \forall \ \lambda \in \mathbb{C}$ is related to the statement you should prove.

Task 4

This task is a chalenge! Study the description of the bisection algorithm on pp. 227-229 of the course book. Implement the method and test it on matrices which are in tridiangular form. You might take a general symmetric matrix and transform it to Hessenberg form with the Python command `scfipy.linalg.hessenberg` or the matlab command `hessenb`.