Similarity-based Selection

Similarity Selection

Relational selection is based on a boolean condition *C*

- evaluate *C* for each tuple *t*
- if *C(t)* is true, add *t* to result set
- if *C(t)* is false, *t* is not part of solution
- result is a set of tuples $\{ t_1, t_2, ..., t_n \}$ all of which satisfy *C*

Uses for relational selection:

- precise matching on structured data

... Similarity Selection

Similarity selection is used in contexts where

- cannot define a precise matching condition
- *can* define a measure *d* of "distance" between tuples
- *d=0* is an exact match, *d>0* is less accurate match
- result is a list of pairs $[(t_1,d_1), (t_2,d_2), ..., (t_n,d_n)]$ (ordered by d_i)

Uses for similarity matching:

- text or multimedia (image/music) retrieval
- ranked queries in conventional databases

Similarity-based Retrieval

Similarity-based retrieval typically works as follows:

- query is given as a *query object q* (e.g. sample image)
- system finds objects that are *like q* (i.e. small distance)

The system can measure distance between any object and *q* ...

How to restrict solution set to only the "most similar" objects:

- *threshold* d_{max} (only objects *t* such that $dist(t,q) \le d_{max}$)
- *count k* (*k* closest objects (*k* nearest neighbours))

... Similarity-based Retrieval

Tuple structure for storing such data typically contains

- `id` to uniquely identify object (e.g. PostgreSQL `oid`)
- metadata (e.g. artist, title, genre, date taken, ...)
- `value` of object itself (e.g. PostgreSQL BLOB or `bytea`)

Properties of typical distance functions (on objects *x,y,z*)

- $dist(x,y) \ge 0$, $dist(x,x) = 0$, $dist(x,y) = dist(y,x)$
- $dist(x,z) < dist(x,y) + dist(y,z)$ (triangle inequality)
- often require substantial computational effort

... Similarity-based Retrieval

Naive approach to similarity-based retrieval

```
q = ...    // query object
dmax = ... // dmax > 0  =>  using threshold
knn = ...  // knn > 0   =>  using nearest-neighbours
Dists = [] // empty list
foreach tuple t in R {
    d = dist(t.val, q)
    insert (t.oid,d) into Dists  // sorted on d
}
n = 0;  Results = []
foreach (i,d) in Dists {
    if (dmax > 0 && d > dmax) break;
    if (knn > 0 && ++n > knn) break;
    insert (i,d) into Results  // sorted on d
}
return Results;
```

Cost = read all *r* feature vectors + compute *distance()* for each

---

... Similarity-based Retrieval

For some applications, *Cost(dist(x,y))* is comparable to T_r

⇒ computing `dist(t.val,q)` for every tuple `t` is infeasible.

To improve this aspect:

- compute feature vector which captures "critical" object properties
- store feature vectors "in parallel" with objects (cf. signatures)
- compute distance using feature vectors (not objects)

i.e. replace $dist(t,t_q)$ by $dist'(vec(t),vec(t_q))$ in previous algorithm.

Further optimisation: dimension-reduction to make vectors smaller

---

... Similarity-based Retrieval

Content of feature vectors depends on application ...

- image ... colour histogram (e.g. 100's of values/dimensions)
- music ... loudness/pitch/tone (e.g. 100's of values/dimensions)
- text ... term frequencies (e.g. 1000's of values/dimensions)

Typically use multiple features, concatenated into single vector.

Feature vectors represent points in a very high-dimensional space.

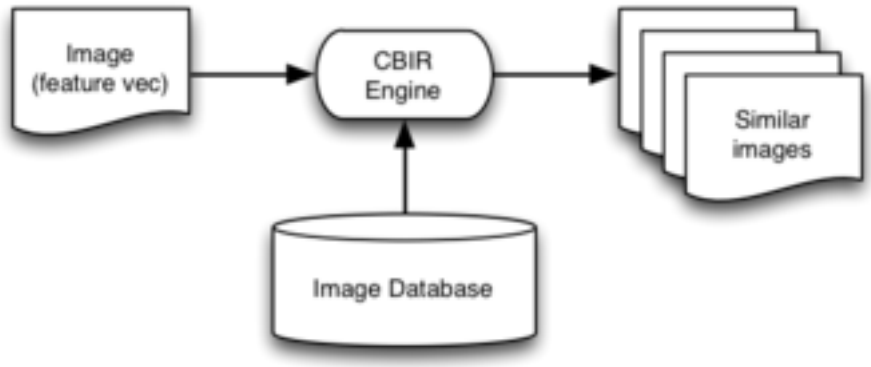
Query: feature vector representing one point in vh-dim space.

Answer: list of objects "near to" query object in this space.

Example: Content-based Image Retrieval

User supplies a description or sample of desired image (features).

System returns a ranked list of "matching" images from database.



... Example: Content-based Image Retrieval

At the SQL level, this might appear as ...

```
create view Sunset as
select image from MyPhotos
where  title = 'Pittwater Sunset'
       and taken = '2009-01-01';

create view SimilarSunsets as
select title, image
from   MyPhotos
where  (image ~~ (select * from Sunset)) < 0.05
order  by (image ~~ (select * from Sunset));
```

where the ~~ operator measures distance between images.

... Example: Content-based Image Retrieval

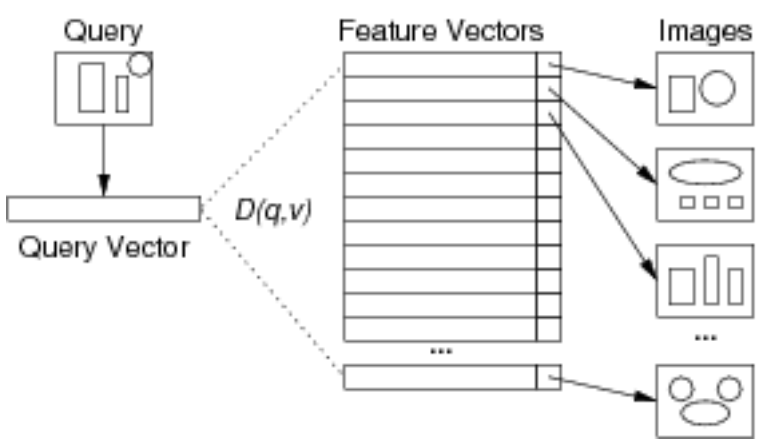
Implementing content-based retrieval requires ...

- a collection of "pertinent" image features
  - e.g. colour, texture, shape, keywords, ...
- some way of describing/representing image features
  - typically via a vector of numeric values
- a distance/similarity measure based on features
  - e.g. Euclidean distance between two vectors

$$dist(x,y) = \sqrt{((x_1-y_1)^2 + (x_2-y_2)^2 + \ldots (x_n-y_n)^2)}$$

... Example: Content-based Image Retrieval

Data structures for naive similarity-retrieval:



... Example: Content-based Image Retrieval

Insertion of an image into the database:

- use image processing algorithms to compute feature vector
- insert image (either into file system or as a BLOB in the DBMS)
- insert tuple *(img,vec)* associating image and feature vector

Insertion cost:

- image processing (relatively expensive, $\cong T_r$)
- copy image in file sys OR insertion of image as BLOB
- insertion of *(img,vec)* tuple ($1_r + 1_w$)

... Example: Content-based Image Retrieval

Inputs to content-based similarity-retrieval:

- a database of r objects ($obj_1, obj_2, ..., obj_r$) plus associated ...
- $r \times n$-dimensional feature vectors ($v_{obj_1}, v_{obj_2}, ..., v_{obj_r}$)
- a query image q with associated n-dimensional vector (v_q)
- a distance measure $D(v_i,v_j) : [0..1)$ ($D=0 \rightarrow v_i=v_j$)

Outputs from content-based similarity-retrieval:

- a list of the k nearest objects in the database [a_1, a_2, ... a_k]
- ordered by distance $D(v_{a_1},v_q) \leq D(v_{a_2},v_q) \leq ... \leq D(v_{a_k},v_q)$

Approaches to *k*NN Retrieval

Partition-based

- use auxiliary data structure to identify candidates
- space-partitioning methods: Grid file, k-d-B-tree, quadtree
- data-partitioning methods: R-tree, X-tree, SS-tree, TV-tree, ...
- unfortunately, such methods "fail" when #dims > 10..20

Approximation-based

- use approximating data structure to identify candidates
- signatures: VA-files
- projections: iDistance, LSH, MedRank, CurveIX, Pyramid

... Approaches to *k*NN Retrieval

Above approaches mostly try to reduce number of objects considered.

Other optimisations to make *k*NN retrieval faster

- reduce I/O by reducing size of vectors (compression, *d*-reduction)
- reduce I/O by placing "similar" tuples together (clustering)
- reduce I/O by remembering previous pages (caching)
- reduce cpu by making distance computation faster

VA Files

VA (Signature) Files

Vector **A**pproximation (VA) file developed by Weber and Blott.

- targetted at very high-dimensional feature vectors
- abandons the idea of search complexity better than *O(n)*

Why give up on sub-linear complexity?

- analysis shows that *all* methods degenerate to this as *d* increases
- absolute upper bound on *d* before linear scan is best *d = 610*
- in practice, observe that most methods degenerate for $10 \le d \le 40$

Note: *d* = number of dimensions

... VA (Signature) Files

Uses a signature file "parallel" to the main feature vector file

... VA (Signature) Files

VA signatures have properties:

- (much) more compact than feature vectors
- provide approximation to information in vectors

Approach to querying:

- perform filtering by fast scan of signatures, then
- compute expensive *D* only on (hopefully) small set of candidates

---

VA-File Signatures

Computing VA signatures:

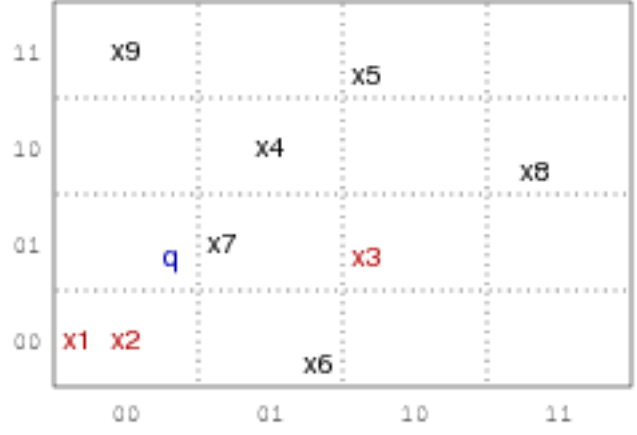
- partition space into small number of regions on each dimension
- use region values in object signatures (coarse-grained view of data)

Implemented by taking *m* high-order bits from each feature vector element.

Consider a *3-d* RGB feature vector, with 0..255 for each RGB value:

- feature vector: *v = (255,128,0) = (**11**111111,**10**000000,**00**00000)*
- partition each dimension into 4 regions (⇒ *m=2* bits per *d*)
- VA signature for this vector: *va(v) = (11,10,00)*

... VA-File Signatures



v1 = (00000011,00001111) va1 = 0000 *d = 2*

v2 = (00001111,00001111) va2 = 0000 *m = 2*

v3 = (10000011,01000011) va3 = 1001

Insertion with VA-Files

Given: a feature vector v_{obj} for a new object.

Insertion is extremely simple.

```
sig = signature(v_obj)
append v_obj to feature vector file
append sig to signature file
```

Storage overhead determined by *m* (e.g. 3/32).

Query with VA-Files

Input: query vector `vq`

```
results = [];  maxD = infinity;
for (i = 0; i < r; i++) {
   // fast distance calculation
   dist = minDistance(region[va[i]], vq)
   if  (#results < k  ||  dist < maxD)  {
      dist = distance(v[i],vq)
      if (#results < k  ||  dist < maxD)  {
         insert (tid[i],dist) into results
         // sorted(results) && length(results) <= k
         maxD = largest distance in results
      }
   }
}
```

Result is a list f *tid*s of *k* database objects nearest to query.

... Query with VA-Files

VA signatures allow fast elimination of objects by region

- if nearest point in region containing object is further than maxD, ignore object

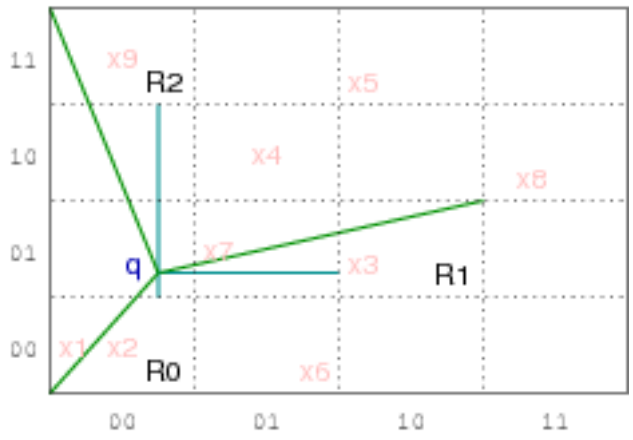
Given query vector *q*, data vector *v*, can quickly compute:

- *lower(q,v)* = distance between *q* and nearest point in region *R[v]*
- *upper(q,v)* = distance between *q* and furthest point in region *R[v]*

Thus, instead of *r* expensive *D* computations, we require

- *r* cheap computations of *(lower(q,v), upper(q,v))*
- small number of real *distance* computations
  (how many are actually required depends on effectiveness of filtering)

... Query with VA-Files



Marks on each dimension *i* are stored as array $R_i[0], R_i[1], .. R_i[2^m]$.

... Query with VA-Files

Distance bound formula:

```
lower(q,v) = √∑_{i=1}^d low_i^2
   where
      low_i = q_i - R_i[v_i+1],                  if v_i < q_i
            = 0,                                 if v_i = q_i
            = R_i[v_i] - q_i,                    if v_i > q_i

upper(q,v) = √∑_{i=1}^d upp_i^2
   where
      upp_i = q_i - R_i[v_i],                    if v_i < q_i
            = max(q_i-R_i[v_i], R_i[v_i+1]-q_i), if v_i = q_i
            = R_i[v_i+1] - q_i,                  if v_i > q_i
```

Cost of VA File Searching

Performing VA search requires:

- read *r* VA signatures
- compute $lower(q,v_j)$ for each of *r* signatures
  (could potentially cache region-based distance results in look-up table)
- compute real distance, *D*, *rf* times, where *f* is filtering factor
  (expect an *f* value somewhere near 0.001 under ideal conditions)

A problem with VA files:

- works best with uniform data distribution (many image DBs are not)
- observed filtering levels are 0.01-0.1 rather than 0.001

Can fix this problem using uneven region sizes (but needs more storage)

... Cost of VA File Searching

Example: $d=200$, $m=3$, $r=10^6$, $B=4096$, $T_D \cong 0.001 T_r$

- size of tuples *(tid,vec)* = 4 bytes + 400 bytes = 404 bytes
- number of tuples per page $\lfloor 4096/404 \rfloor = 10$
- total number of feature vector pages = $\lceil 10^6/10 \rceil = 100000$

Cost (without VA file) = $100000T_r + 10^6 T_D \cong 101000T_r$

- size of VA signature = 200×3 bits = 75 bytes
- number sigs in VA file page = $\lfloor 4096/75 \rfloor = 54$
- total number of VA file pages = $\lceil 10^6/54 \rceil = 18518$
- with $f=0.001$, fetch only 1000 feature vectors

Cost (with VA file) = $18518T_r + 1000T_r + 1000T_D \cong 19519T_r$

Improved VA Search Algorithm

An improved query algorithm that guarantees minimal *D* computations:

```
vaq = vasignature(vq)
results = [];  maxD = infinity;  pending = [];
for (i = 0; i < r; i++) {
   lowD = lower(vq,region[va[i]])
   uppD = upper(vq,region[va[i]])
   if  (#results < k  ||  dist < uppD)  {
      sortedInsert (i,uppD) into results
      heapInsert (i,lowD) into pending
   }
}
results = [];  heapRemove (i,lowD) from pending
while (lowD < maxD) {
   dist = distance(v[i],vq)
   if (#results < k  ||  dist < maxD) {
      sortedInsert (i,dist) into results
      // sorted(results) && length(results) <= k
      maxD = largest distance in results
   }
   heapRemove (i,lowD) from pending
}
```

Curve-based Similarity Search

Curve-based Searching

The strategy when using curve-based indexing is as follows:

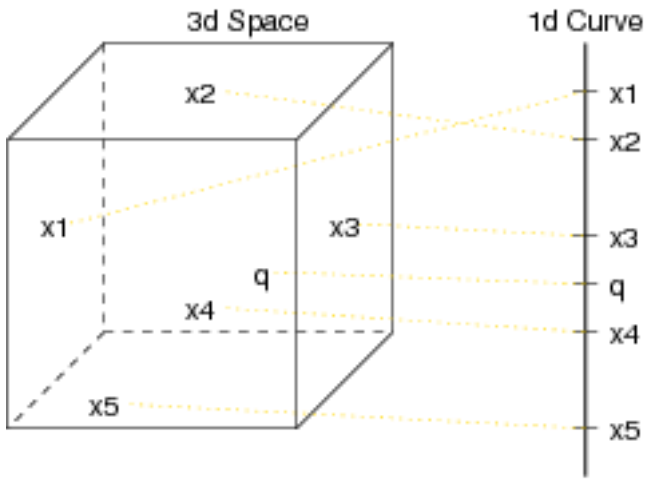
- dealing with many dimensions ($d > 10$) is too difficult
- so, project the feature space onto one dimension (line/curve)
- this gives a linear ordering of the objects
- can then perform search over this linear ordering
- using existing efficient 1d access methods (e.g. B-trees)

Indexing based on curves needs to ...

- minimise information loss to reduce missing answers
- at the same time, maximise filtering effectiveness

... Curve-based Searching

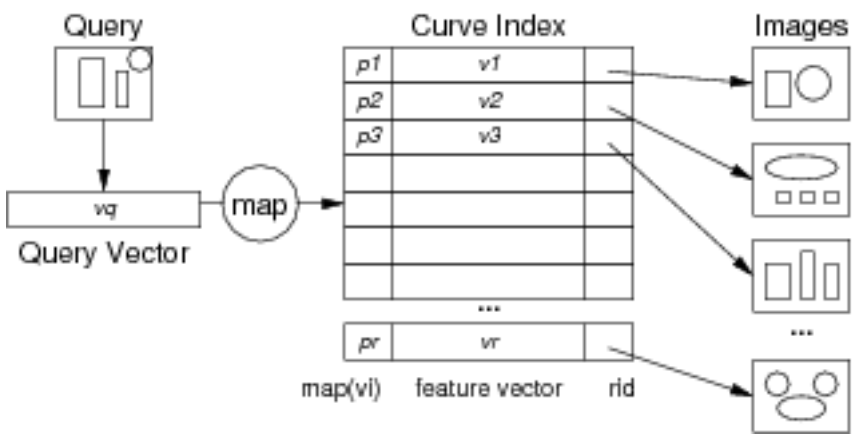
Mapping from 3-d to 1-d



Note: impossible to preserve all NN-in-space as NN-on-curve.

Curve Index File Organisation

Data structures for curve-based searching:



Insertion with Curve Index

For each image inserted into the database:

- determine a feature vector *v* for the image
- determine a *tid* for the image (file name?)
- map the feature vector onto the curve *p = map(v)*
- *p* is the primary key for the curve index file
- insert *(p,v,tid)* into the curve index file using e.g. B-tree

---

Searching with Curve Index

Overview of curve-based searching algorithm:

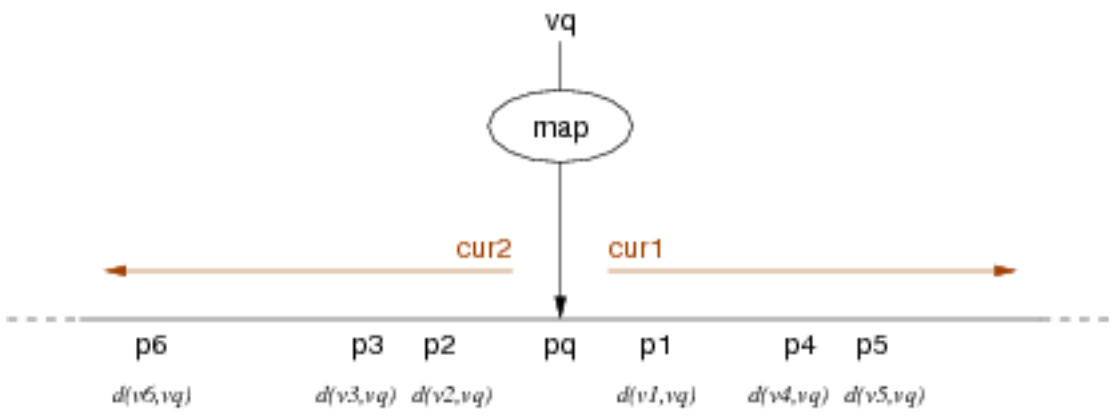
```
Input: query feature vector v_q

p = map(v_q)
cur_1 = cur_2 = lookup(p)
while (not enough answers) {
    (pt,vec,tid) = next(cur_1)
    remember tid if D(vec,v_q) small enough
    (pt,vec,tid) = prev(cur_2)
    remember tid if D(vec,v_q) small enough
}
```

Output: collected *tid*s of similar objects

... Searching with Curve Index

How curve is scanned to find potential near-neighbours:



... Searching with Curve Index

What kind of curves are required to make this approach viable?

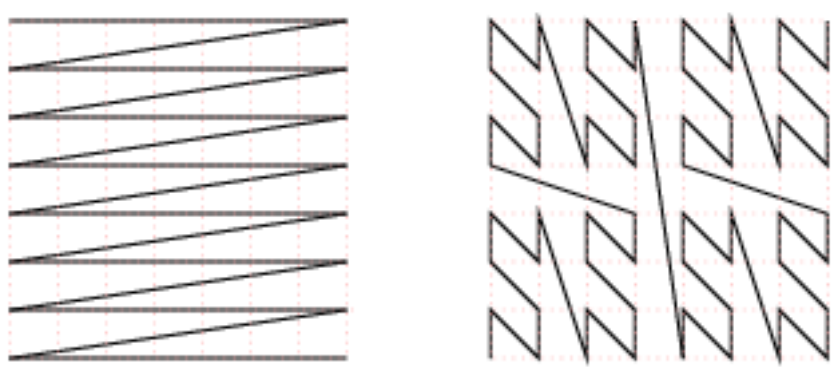
- must pass through every data point exactly once
- must pass through every "point" in the underlying space exactly once

Possible candidate curves:

- space-filling curves (e.g. Hilbert curve, Peano curve, Gray ordering)

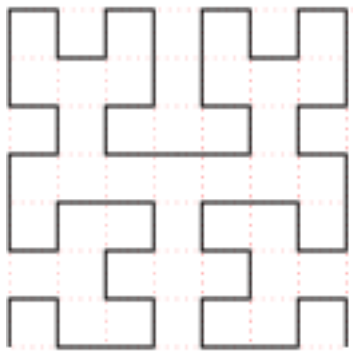
Experiments have shown that the Hilbert curve is the most suitable for data access.

2d Space-filling Curves

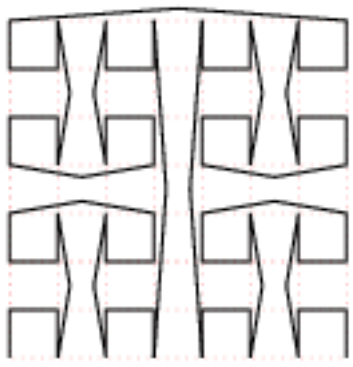


Row–wise enumeration Peano curve

... 2d Space-filling Curves



Hilbert curve



Gray ordering

The Curvelx Scheme

What we would like from the above curve mapping:

- object close to the query on the curve ⇒ close in feature space
- object close to the query in feature space ⇒ close on the curve

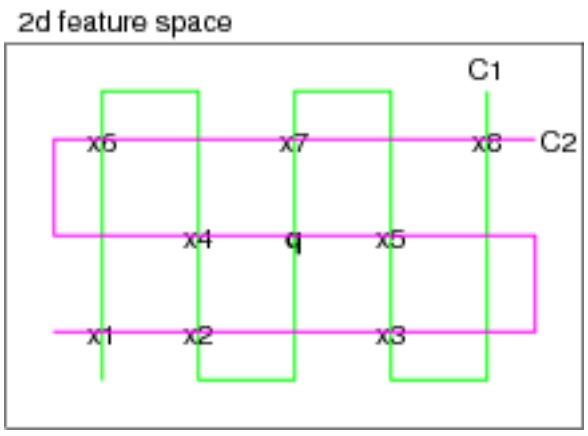
With a single Hilbert curve (although not some other curves)

- $map(v) \cong map(v_q) \;\Rightarrow\; D(v,v_q) \cong 0$ is generally true
  (in other words, we usually find similar objects near the query on the curve)
- $D(v,v_q) \cong 0 \;\Rightarrow\; map(v) \cong map(v_q)$ is sometimes false
  (in other words, we sometimes fail to find objects that are similar to the query)

One curve is not enough, so use several "complementary" curves.

... The Curvelx Scheme

Example curveix scheme with 2 curves on a 2*d* space:



... The Curvelx Scheme

The basic idea:

- project *d*-dimensional vector onto 1-d space-filling curve
- collect set of curve-neighbours (should contain some *k*NN)
- repeat above for several curves with different paths through space
- union of neighbour sets from all curves gives candidates
- retrieve and compute distance only for candidates

We want candidate set to contain most/all of *k*NN ...

- how many curves do we need?
- how many neighbours do we examine on each curve?

Curve Mapping Functions

Each C_i has the following properties:

- maps *d*-dimensional vectors onto points on a line i.e. $[0,1)^d \rightarrow [0,1)$
- mapping is one-to-one and defines an order on $[0,1)^d$
- maps onto a space-filling Hilbert curve

Points within a small region of the curve are *likely* to have been mapped from vectors that are close in *d*-space.

How to generate multiple C_is from a single vector:

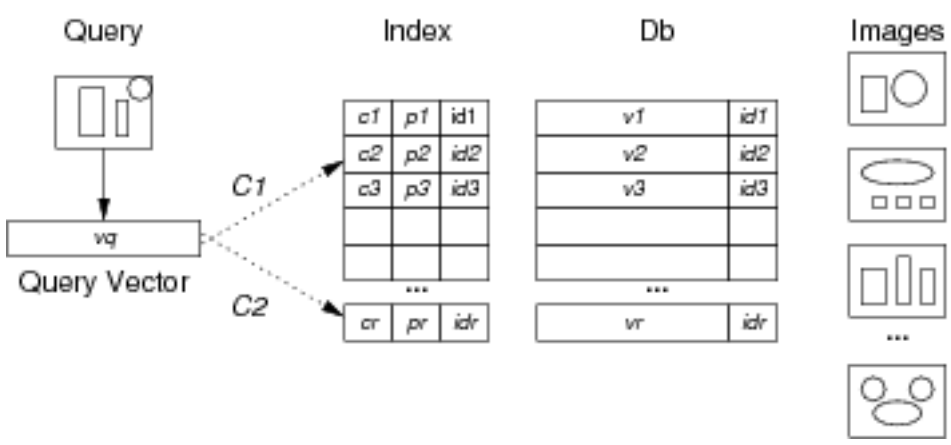
- reorder elements in vector, translate individual elements

Data Structures for CurveIx

Derived from data structures for the QBIC system:

C_i	a mapping function for each of the *m* different space-filling curves (*i=1..m*)
Db	a database of *r d*-dimensional feature vectors; each entry is a pair `(imageId, vector)` where `imageId` forms a primary key
Index	a database of *rm* curve points; each point is represented by a tuple `(curveId, point, imageId)`; the pair `(curveId, point)` forms a primary key with an ordering, where $point = C_{curveId}(v_{imageId})$
v_q	feature vector for query *q*

... Data Structures for CurveIx

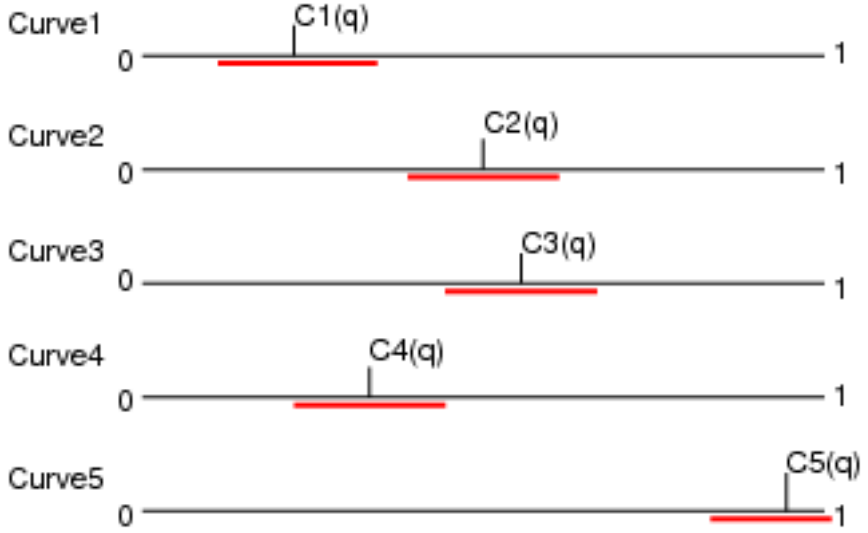


Database Construction

For each image *obj*, insertion requires:

```
id = identifier for obj
for (i in 1..m) {
     p = C_i(v_obj)
     insert (i, p, id) into Index
}
insert (id, v_obj) into Db
```

Example: Search with 5 Curves



Finding *k*-NN in CurveIx

Given: v_q , $C_1 .. C_m$, *Index* , *Db*

```
for (i in 1..m) {
    p = Ci(vq)
    lookup (i,p) in Index
    fetch (i,p1,j)
    while (p1 "close to" p on curve i) {
        collect j as candidate
        fetch next (i,p1,j)
}   }
for each candidate j {
     lookup j in Db
     fetch (j,vj)
     d = D(vj , vq)
     include j in k-NN if d small enough
}
```

CurveIx vs. Linear Scan

Linear scan: Cost = read all *N* vectors + compute *D* for each

CurveIx: Cost = *m* B-tree lookups + compute *D* *fN* times

Some observations ...

- we can't afford to have too many curves *(m < 10?)*
- CurveIx has to do effective filtering *(f ≪ 1)*

Difficult to formally analyse further, so we implemented the system to see how it performed ...

C_i Values as Keys

One problem raised early in the implementation:

- the Hilbert numbers produced by C_i could be expanded to arbitrary precision in order to distinguish between $C_i(v_1)$ and $C_i(v_2)$

To use such values as database keys, we need them as fixed precision, so we limit expansion (to 4 levels).

Problems:

- gives keys that are 96 bytes long (producing very large Index files)

- different v_j can be mapped to same point (candidate set size artificially inflated)

Solution:

- use a "prefix" index structure (something like a trie)

Performance Analysis

Since CurveIx does not guarantee to deliver the *k*-NN among its candidates, we set an "acceptable" accuracy level of 90%.

In other words, CurveIx must deliver *0.9 k* nearest-neighbours to be considered useful.

The initial experiments aimed to answer the questions:

- How many curves are needed to achieve 90% accuracy?
- How many curve-neighbours do we need to examine?
- Can all of this be done reasonably efficiently?

Experiments

Measures for accuracy:

- *Acc1* average top-10 entries in CurveIx top-10
- *Acc2* how frequently CurveIx gives 10 out of 10

Measures for efficiency:

- *Size* size of *Db* file + *Index* file
- *Dist* number of distance calculations required
- *IO* total amount of I/O performed

... Experiments

To determine how these measures vary:

- built databases of size 5K, 10K, 15K, 20K (supersets)
- for each database, ran 25 query "benchmark" set
- for each query, ran for 3,5,10,20,30,40 curve-neighbours
  (but because of curve-mapping problem, only got 20,30,40)
- for each query, ran for 20,40,60,80,100 curves

Also implemented a linear scan version for comparison and to collect the exact answer sets.

Sample Comparison

CurveIx Indexing

```
QUERY: img00102
0.000000  img00102
0.005571  img00713
0.008826  img00268
0.011413  img05054
0.011811  img00631
0.014259  img04042
0.027598  img00203
0.037471  img00287
0.063639  img00244
0.067583  img00306
```

Linear Scan

```
QUERY: img00102
0.000000  img00102
0.005571  img00713
0.008826  img00268
0.011413  img05054
0.011811  img00631
0.014259  img04042
0.027598  img00203
0.037471  img00287
0.063639  img00244
```

```
                                0.067583  img00306
# dist calcs: 524
1.67user 0.23system ...         # dist calcs: 20000
                                1.93user 1.12system ...
```

Experimental Results

55/65

For fixed database (20K), effect of varying *Range*, *Ncurves*

#Curves	Range	Acc1	Acc2	#Dist
20	20	6.72	0.20	426
20	30	7.28	0.28	695
20	40	7.68	0.36	874
40	30	8.16	0.40	1301
40	40	8.60	0.44	1703
60	30	8.40	0.44	1905
60	40	8.60	0.48	2413
80	30	8.87	0.58	2485
80	40	9.20	0.72	3381
100	30	9.10	0.70	3061
100	40	9.28	0.72	4156

Results: Size vs. Accuracy

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For fixed CurveIx parameters (80 curves, 30 range), effect of varying database size:

#Images	Acc1	Acc2
5K	9.72	0.76
10K	9.44	0.80
15K	9.16	0.70
20K	9.04	0.64

---

iDistance

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iDistance

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iDistance (Jagadish, Ooi, Tan, Yu, Zhang)

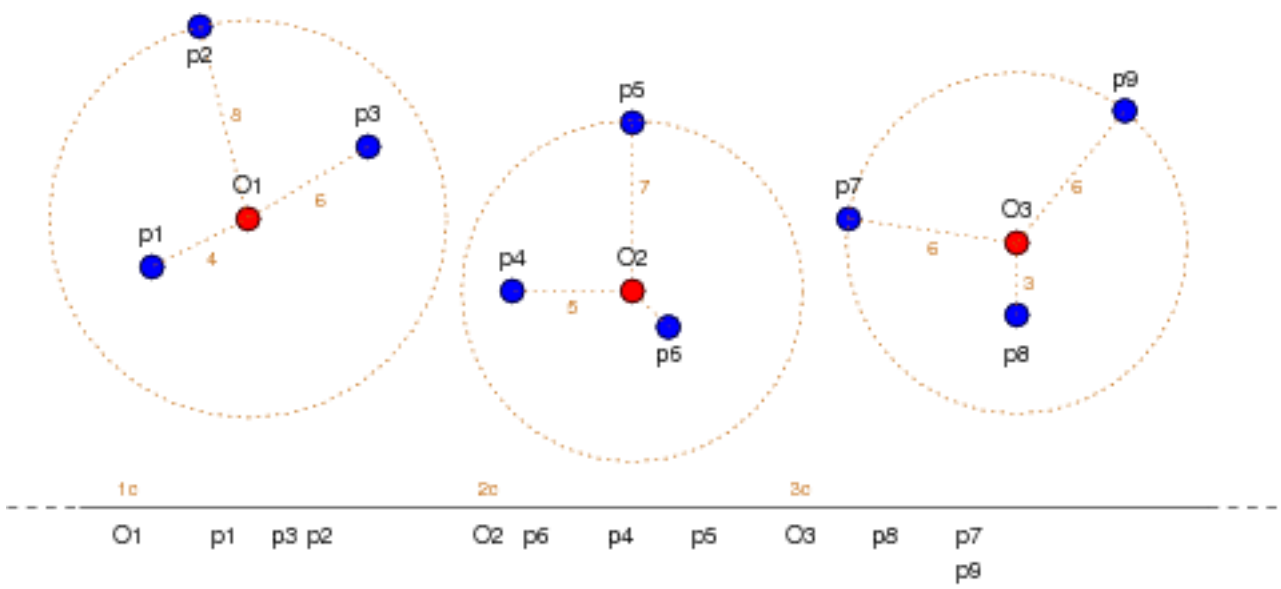
- adaptive B-tree based indexing method
- aimed at handling kNN queries in high-d spaces

The basic idea:

- determine a set of reference points O_j
  (reference points partition the data space)
- build index on $(p_i, D(p_i))$
  (distance of each point p_i to its nearest reference point O_j)
- use index to quickly find p_i likely to be close to q

... iDistance

Computing the iDistance: $D(p) = j.c + dist(p,O_j)$



where *c* is a constant to "spread" the partitions over the iDistance line

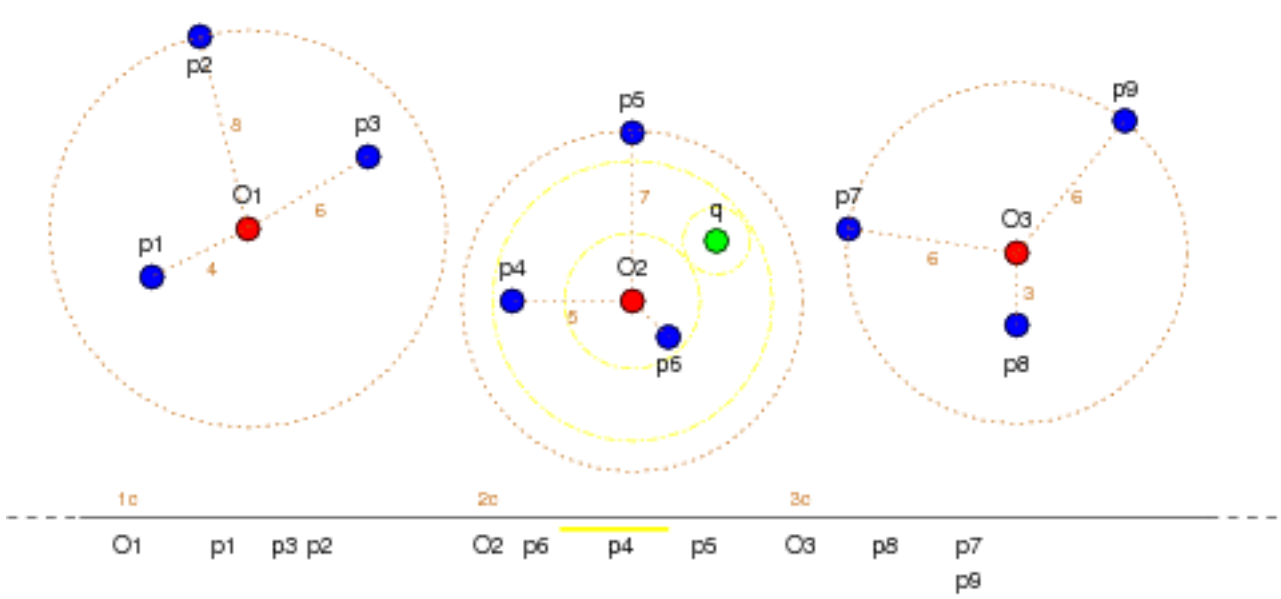
Searching with iDistance

The approach:

- start with query point *q*
- choose a radius *r* around query, giving hypersphere *S*
- for all partitions intersected by *S*
  - determine $a = min(dist(S,O_j))$ and $b = max(dist(S,O_j))$
  - find all data points *p* with *D(p)* in range *j.c+a .. j.c+b*
  - maintain a sorted list of *(p,dist(p,q))* pairs
- increase radius by delta-*r* and repeat above steps
- continue until kNN are found

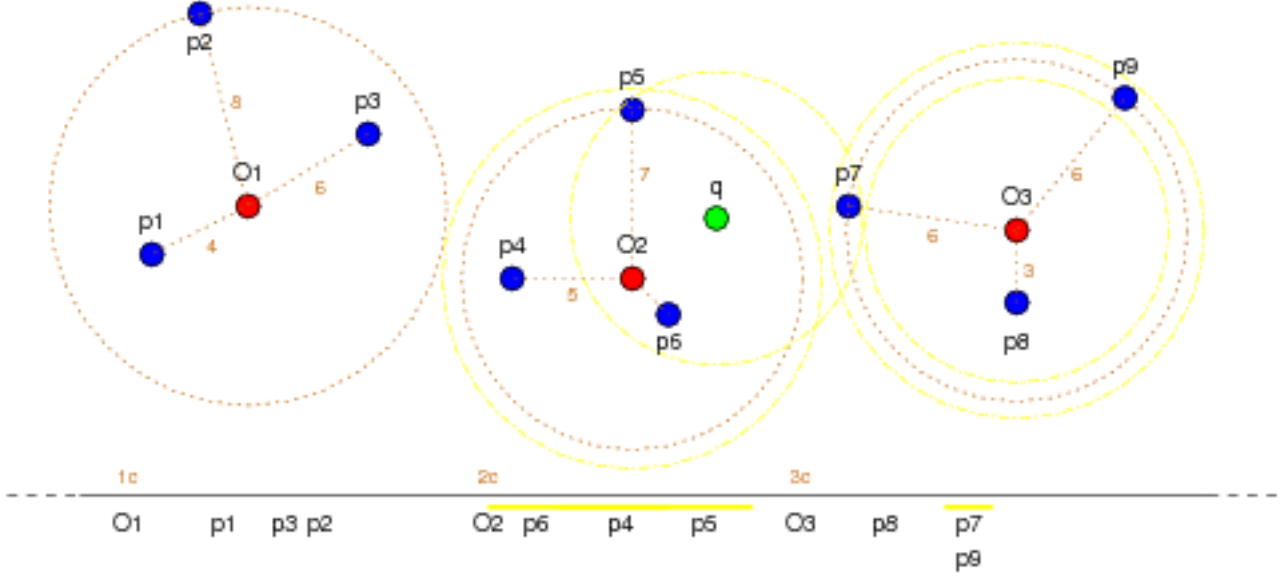
... Searching with iDistance

First iteration of search (small *r*):



... Searching with iDistance

Later iteration of search (larger *r*):



... Searching with iDistance

Details of search method:

```
//Inputs:
// q = query point, k = # nearest-neighbours
// O[m] = set of reference points
// rad[m] = radius of partition around O[i]
// deltaR = search radius increase on each step
// maxR   = maximum search radius
int r = 0;  // search radius around q
int stop = 0;  // flag for when to halt
int seen[];  // partition i already searched?
Obj S[];  // result set, k nearest neighbours
Obj lp[], rp[];  // lists of limits with partitions

while (!stop) {
   r = r + deltaR
   stop = SearchRefs(q,r)
}
...
```

... Searching with iDistance


```
int SearchRefs(q,r) {
   f = furthest(S,q)
   stop = (dist(f,q) < r && |S| == k)
   for (i = 0; i < m; i++) {
      d = dist(O[i],q)
      if (!seen[i]) {
         hs[i] = sphere(O[i],rad[i])
         if (hs[i] contains q) {
            seen[i] = 1
            leaf = BtreeSearch(i*c+d)
            lp[i] = SearchInward(leaf, i*c+d-r)
            rp[i] = SearchOutward(leaf, i*c+d+r)
         }
         elseif (hs[i] intersects sphere(q,r)) {
            seen[i] = 1
            leaf = BtreeSearch(rad[i])
            lp[i] = SearchInward(leaf, i*c+d-r)
         }
      }
      else {
         if (lp[i] != null)
            lp[i] = SearchInward(left(lp[i]), i*c+d-r)
```

```
         if (rp[i] != null)
            rp[i] = SearchOutward(right(rp[i]), i*c+d+r)
      }
   }
   return stop
}
```

... Searching with iDistance

Cost depends on data distribution and position of O_i

See analysis in ACM Trans on DB Systems, v.30, Jun 2005, p.364

Determining the set of reference points:

- space-partitioning ... divide space up using geometric partitions
- data-partitioning ... try to have equal # points in each partition

Determining the size of `deltaR`:

- too small ... more interations, more access to B-tree index
- too large ... "overshoot" and fetch unnecessary pages (in last iteration)
