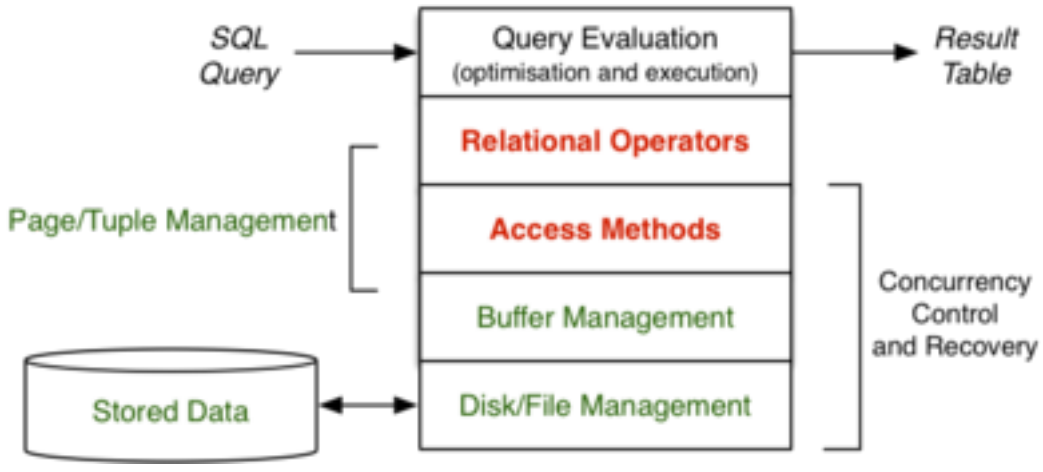
Week 06 Lectures

Implementing Relational Operators

Implementation of relational operations in DBMS:



... Implementing Relational Operators

So far, have considered file structures ...

- *heap file* ... tuples added to any page which has space
- *sorted file* ... tuples arranged in file in key order
- *hash file* ... tuples placed in pages using hash function

with relational algebra operations ...

- scanning (e.g. `select * from R`)
- sorting (e.g. `select * from R order by x`)
- projection (e.g. `select x,y from R`)
- selection (e.g. `select * from R where` *Cond*)

and now ...

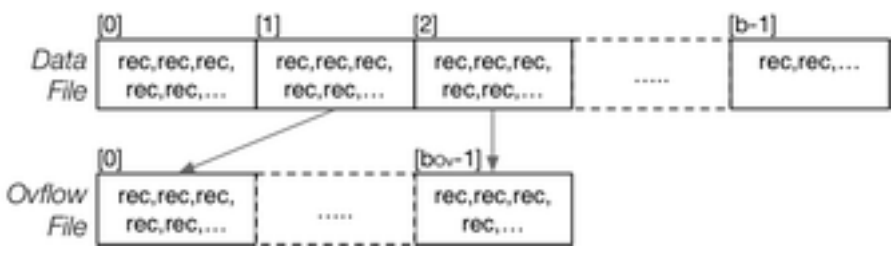
- *indexes* ... search trees based on pages/keys
- *signatures* ... bit-strings which "summarize" tuples

... Implementing Relational Operators

File/query Parameters ...

- r tuples of size R, b pages of size B, c tuples per page
- *Rel.k* attribute in `where` clause, b_q answer pages for query q
- b_{Ov} overflow pages, average overflow chain length Ov

File structures ...



Reminder on Cost Analyses

When showing the cost of operations ...

- for queries, simply count number of pages read
- for updates, use n_r and n_w to distinguish reads/writes

When comparing two methods for same query

- ignore the cost of writing the result (same for both)

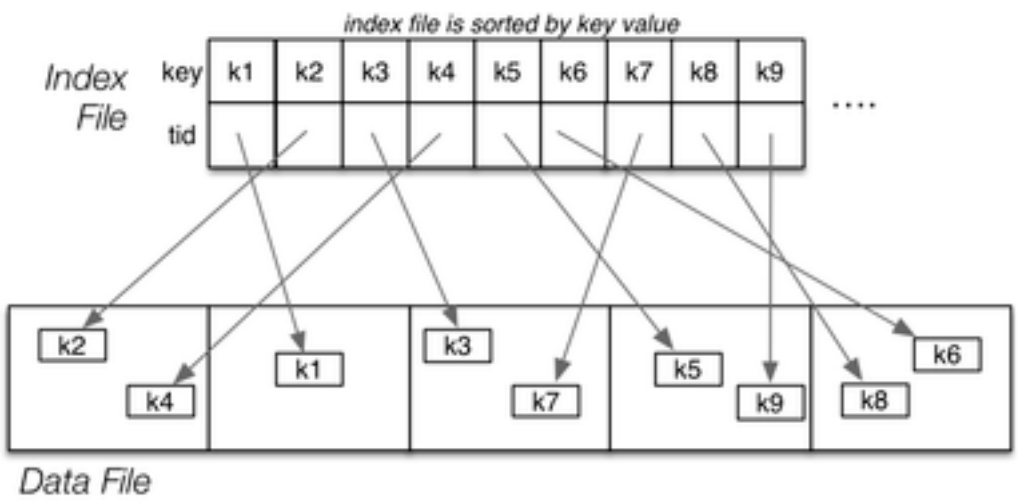
In counting reads and writes, assume minimal buffering

- each `request_page()` causes a read
- each `release_page()` causes a write (if page is dirty)

Indexing

Indexing

An index is a file of (keyVal,tupleID) pairs, e.g.



Indexes

A 1-d *index* is based on the value of a single attribute *A*.

Some possible properties of *A*:

- may be used to sort data file (or may be sorted on some other field)
- values may be unique (or there may be multiple instances)

Taxonomy of index types, based on properties of index attribute:

primary	index on unique field, may be sorted on *A*
clustering	index on non-unique field, file sorted on *A*
secondary	file *not* sorted on *A*

A given table may have indexes on several attributes.

... Indexes

Indexes themselves may be structured in several ways:

dense	every tuple is referenced by an entry in the index file

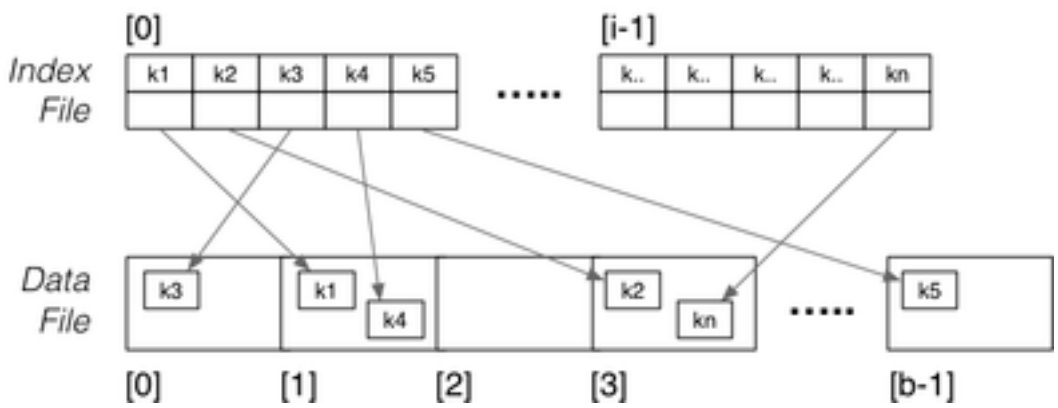
sparse	only some tuples are referenced by index file entries
single-level	tuples are accessed directly from the index file
multi-level	may need to access several index pages to reach tuple

Index file has total i pages (where typically $i \ll b$)

Index file has page capacity c_i (where typically $c_i \gg c$)

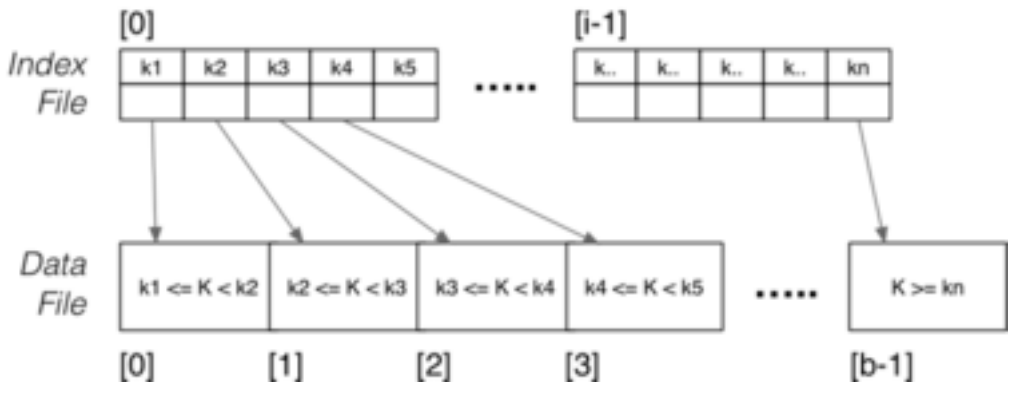
Dense index: $i = ceil(r/c_i)$ Sparse index: $i = ceil(b/c_i)$

Dense Primary Index



Data file unsorted; one index entry for each tuple

Sparse Primary Index



Data file sorted; one index entry for each page

Exercise 1: Index Storage Overheads

Consider a relation with the following storage parameters:

- *B = 8192, R = 128, r = 100000*
- header in data pages: 256 bytes
- key is integer, data file is sorted on key
- index entries (keyVal,tupleID): 8 bytes
- header in index pages: 32 bytes

How many pages are needed to hold a dense index?

How many pages are needed to hold a sparse index?

Selection with Primary Index

For *one* queries:

```
ix = binary search index for entry with key K
if nothing found { return NotFound }
b = getPage(pageOf(ix.tid))
t = getTuple(b,offsetOf(ix.tid))
   -- may require reading overflow pages
return t
```

Worst case: read $log_2 i$ index pages + read $1+Ov$ data pages.

Thus, $Cost_{one,prim} = log_2\ i + 1 + Ov$

Assume: index pages are same size as data pages ⇒ same reading cost

---

... Selection with Primary Index

For *range* queries on primary key:

- use index search to find lower bound
- read index sequentially until reach upper bound
- accumulate set of buckets to be examined
- examine each bucket in turn to check for matches

For *pmr* queries involving primary key:

- search as if performing *one* query.

For queries not involving primary key, index gives no help.

... Selection with Primary Index

Method for range queries (when data file is not sorted)

```
// e.g. select * from R where a between lo and hi
pages = {}   results = {}
ixPage = findIndexPage(R.ixf,lo)
while (ixTup = getNextIndexTuple(R.ixf)) {
   if (ixTup.key > hi) break;
   pages = pages U pageOf(ixTup.tid)
}
foreach pid in pages {
   // scan data page plus ovflow chain
   while (buf = getPage(R.datf,pid)) {
      foreach tuple T in buf {
         if (lo<=T.a && T.a<=hi)
            results = results U T
}  }  }
```

Insertion with Primary Index

Overview:

```
tid = insert tuple into page P at position p
find location for new entry in index file
```

```
insert new index entry (k,tid) into index file
```

Problem: order of index entries must be maintained

- need to avoid overflow pages in index
- either reorganise index file or mark entries

Reogranisation requires, on average, read/write half of index file:

$Cost_{insert,prim} = (log_2 i)_r + i/2.(1_r+1_w) + (1+Ov)_r + (1+\delta)_w$

Deletion with Primary Index

Overview:

```
find tuple using index
mark tuple as deleted
delete index entry for tuple
```

If we delete index entries by marking ...

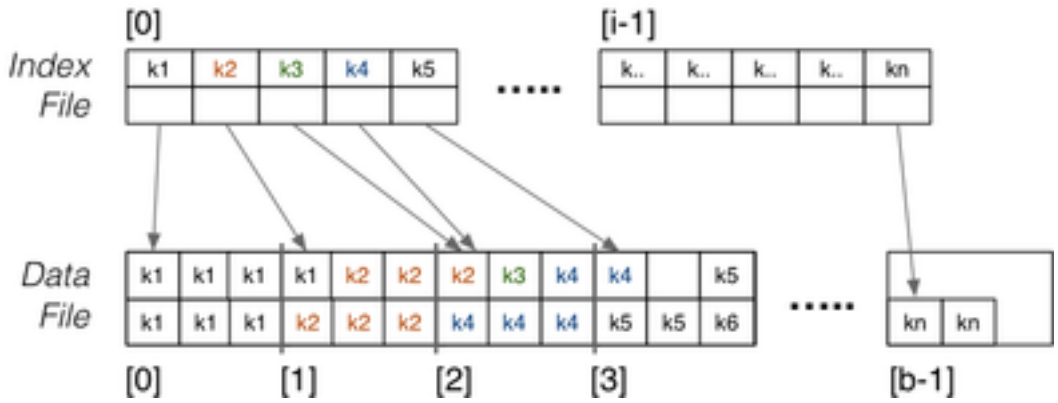
- $Cost_{delete,prim} = (log_2\ i)_r + (1 + Ov)_r + 1_w + 1_w$

If we delete index entry by index file reorganisation ...

- $Cost_{delete,prim} = (log_2\ i)_r + (1 + Ov)_r + i/2.(1_r+1_w) + 1_w$

Clustering Index

Data file sorted; one index entry for each key value

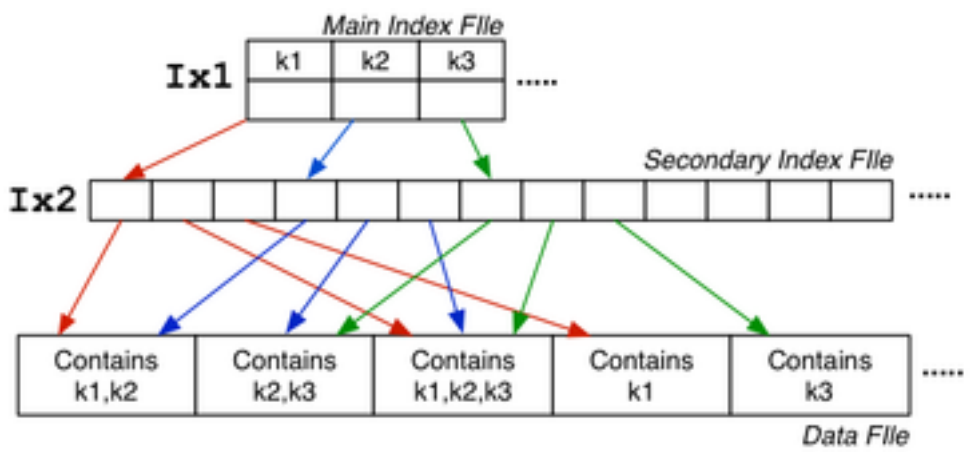


Cost penalty: maintaining both index and data file as sorted

(Note: can't mark index entry for value X until all X tuples are deleted)

Secondary Index

Data file not sorted; want one index entry for each key value



$Cost_{pmr} = (log_2 i_{ix1} + a_{ix2} + b_q.(1 + Ov))$

Multi-level Indexes

Above Secondary Index used two index files to speed up search

- *by keeping the initial index search relatively quick*
- `Ix1` *small (depends on number of unique key values)*
- `Ix2` *larger (depends on amount of repetition of keys)*
- *typically,* $b_{Ix1} \ll b_{Ix2}$

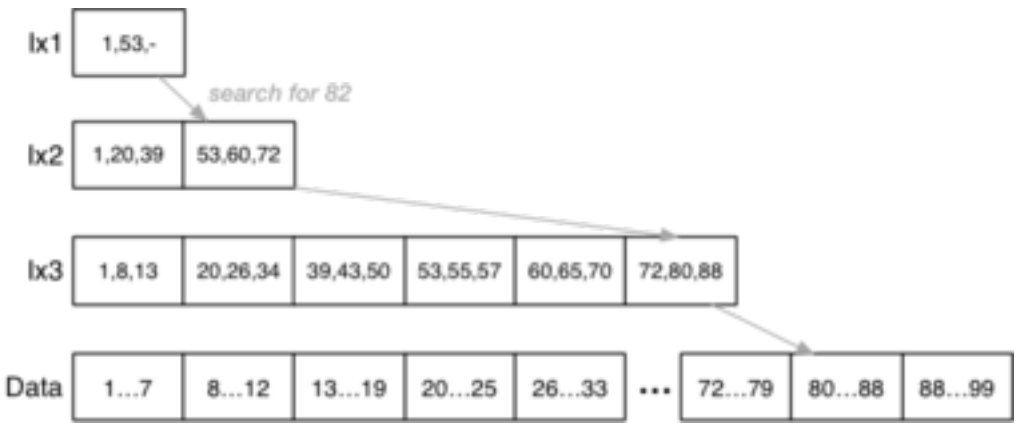
Could improve further by

- *making* `Ix1` *sparse, since* `Ix2` *is guaranteed to be ordered*
- *in this case,* $b_{Ix1} = ceil(b_{Ix2} / c_i)$
- *if* `Ix1` *becomes too large, add* `Ix3` *and make* `Ix2` *sparse*
- *if data file ordered on key, could make* `Ix3` *sparse*

Ultimately, reduce top-level of index hierarchy to one page.

... Multi-level Indexes

Example data file with three-levels of index:



Assume: not primary key, $c = 100$, $c_i = 3$

Select with Multi-level Index

For one query on indexed key field:

```
xpid = top level index page
for level = 1 to d {
    read index entry xpid
    search index page for J'th entry
        where index[J].key <= K < index[J+1].key
    if (J == -1) { return NotFound }
    xpid = index[J].page
}
pid = xpid  // pid is data page index
search page pid and its overflow pages
```

$Cost_{one,mli} = (d + 1 + Ov)_r$

(Note that $d = ceil(log_{c_i} r)$ *and* c_i *is large because index entries are small)*

B-Trees

B-trees are MSTs with the properties:

- *they are updated so as to remain balanced*
- *each node has at least (n-1)/2 entries in it*
- *each tree node occupies an entire disk page*

B-tree insertion and deletion methods

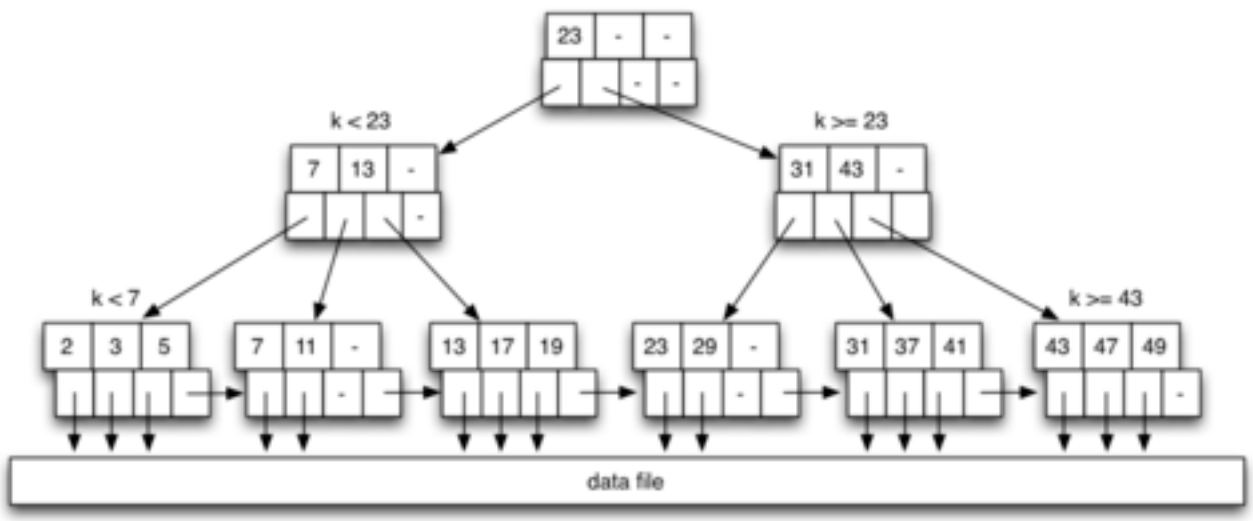
- *are moderately complicated to describe*
- *can be implemented very efficiently*

Advantages of B-trees over general MSTs

- *better storage utilisation (around 2/3 full)*
- *better worst case performance (shallower)*

... B-Trees

Example B-tree (depth=3, n=3) (actually B+ tree)



(Note: in DBs, nodes are pages ⇒ large branching factor, e.g. n=500)

B-Tree Depth

Depth depends on effective branching factor (i.e. how full nodes are).

Simulation studies show typical B-tree nodes are 69% full.

Gives load $L_i = 0.69 \times c_i$ and depth of tree ~ ceil($\log_{L_i} r$).

Example: c_i=128, L_i=88

Level	#nodes	#keys
root	1	87
1	88	7656
2	7744	673728
3	681472	59288064

Note: c_i is generally larger than 128 for a real B-tree.

Insertion into B-Trees

Overview of the method:

1. *find leaf node and position in node where entry would be stored*

2. *if node is not full, insert entry into appropriate spot*
3. *if node is full, split node into two half-full nodes*
   and promote middle element to parent
4. *if parent full, split and promote upwards*
5. *if reach root, and root is full, make new root upwards*

Note: if duplicates not allowed and key exists, may stop after step 1.

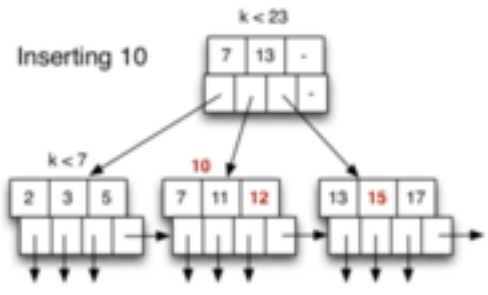
Example: B-tree Insertion

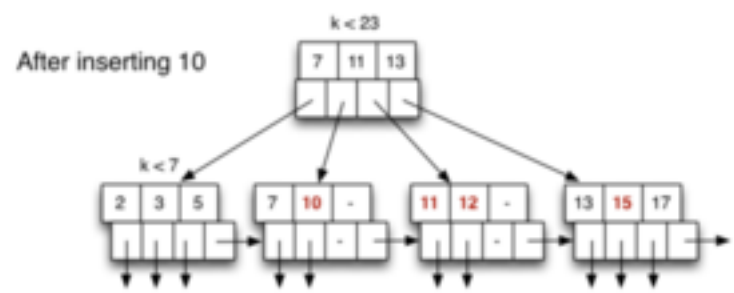
Starting from this tree:

insert the following keys in the given order `12 15 30 10`

... Example: B-tree Insertion

... Example: B-tree Insertion





B-Tree Insertion Cost

Insertion cost = $Cost_{treeSearch}$ + $Cost_{treeInsert}$ + $Cost_{dataInsert}$

Best case: write one page (most of time)

- *traverse from root to leaf*
- *read/write data page, write updated leaf*

$Cost_{insert} = D_r + 1_w + 1_r + 1_w$

Common case: 3 node writes (rearrange 2 leaves + parent)

- *traverse from root to leaf, holding nodes in buffer*
- *read/write data page*
- *update/write leaf, parent and sibling*

$Cost_{insert} = D_r + 3_w + 1_r + 1_w$

... B-Tree Insertion Cost

Worst case: 2D-1 node writes (propagate to root)

- *traverse from root to leaf, holding nodes in buffers*
- *read/write data page*
- *update/write leaf, parent and sibling*
- *repeat previous step D-1 times*

$Cost_{insert} = D_r + 3D_w + 1_r + 1_w$ *(naive method)*

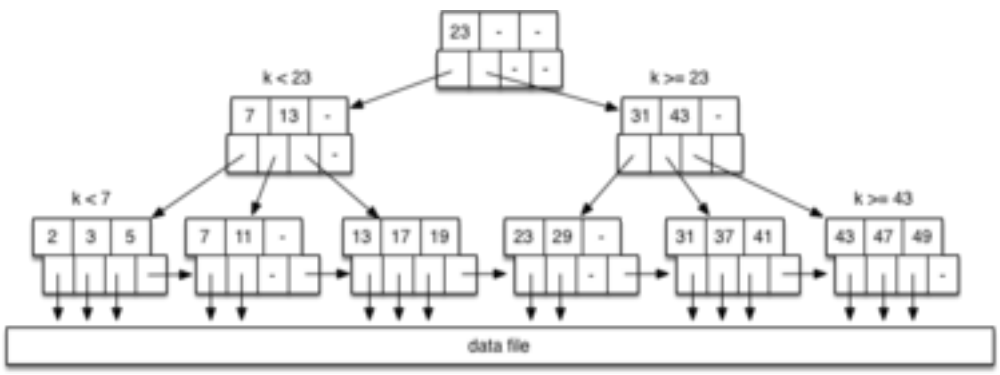
$Cost_{insert} = D_r + (2D-1)_w + 1_r + 1_w$ *(optimised method)*

Selection with B-Trees

For one queries:

```
N = B-tree root node
while (N is not a leaf node)
   N = scanToFindChild(N,K)
tid = scanToFindEntry(N,K)
access tuple T using tid
```

$Cost_{one} = (D + 1)_r$



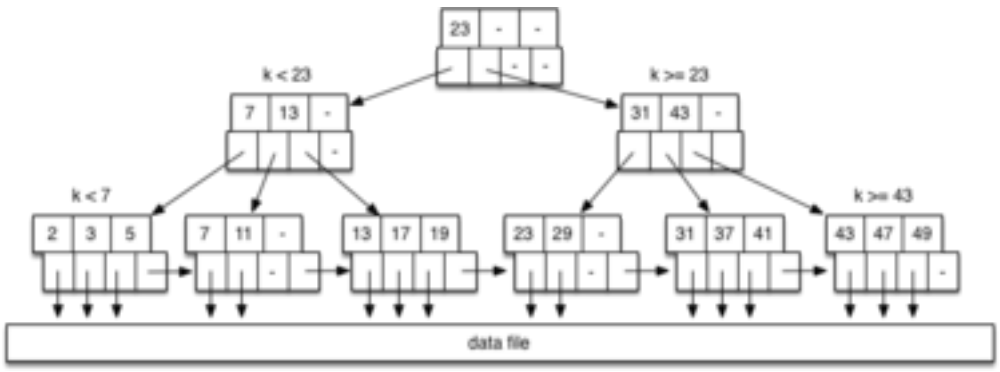
... Selection with B-Trees

For range queries (assume sorted on index attribute):

```
search index to find leaf node for Lo
for each leaf node entry until Hi found {
```

```
        access tuple T using tid from entry
}
```

$Cost_{range} = (D + b_i + b_q)_r$



B-trees in PostgreSQL

PostgreSQL implements ≅ Lehman/Yao-style B-trees

- *variant that works effectively in high-concurrency environments.*

B-tree implementation: **`backend/access/nbtree`**

- **`README`** *... comprehensive description of methods*
- **`nbtree.c`** *... interface functions (for iterators)*
- **`nbtsearch.c`** *... traverse index to find key value*
- **`nbtinsert.c`** *... add new entry to B-tree index*

Notes:

- *stores all instances of equal keys (dense index)*
- *avoids splitting by scanning right if key = max(key) in page*
- *common insert case: new key is max(key) overall; handled efficiently*

... B-trees in PostgreSQL

Changes for PostgreSQL v12

- *indexes smaller*
  - *for composite keys, only store first attribute*
  - *index entries are smaller, so c_i larger, so tree shallower*
- *include TID in index key*
  - *duplicate index entries are stored in "table order"*
  - *makes scanning table files to collect results more efficient*

To explore indexes in more detail:

- **`\di+ IndexName`**
- **`select * from bt_page_items(IndexName,BlockNo)`**

... B-trees in PostgreSQL

Interface functions for B-trees

```
// build Btree index on relation
Datum btbuild(rel,index,...)
// insert index entry into Btree
Datum btinsert(rel,key,tupleid,index,...)
// start scan on Btree index
Datum btbeginscan(rel,key,scandesc,...)
// get next tuple in a scan
Datum btgettuple(scandesc,scandir,...)
```

```
// close down a scan
Datum btendscan(scandesc)
```

N-dimensional Selection

N-dimensional Queries

Have looked at one-dimensional queries, e.g.

```
select * from R where a = K
select * from R where a between Lo and Hi
```

and heaps, hashing, indexing as ways of efficient implementation.

Now consider techniques for efficient multi-dimensional queries.

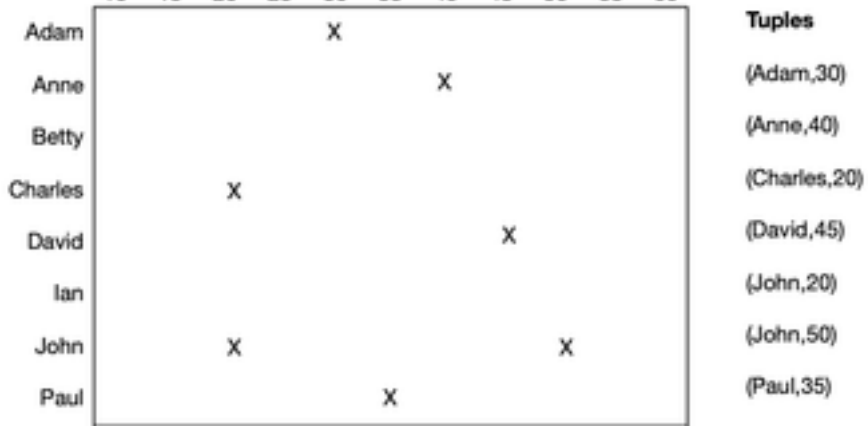
Compared to 1-d queries, multi-dimensional queries

- *typically produce fewer results*
- *require us to consider more information*
- *require more effort to produce results*

Tuple Space

It is possible to view tuples as points in an Nd space.

Consider the relation `Person(name,age)` *and its tuple-space.*



Operations for Nd Select

N-dimensional select queries = condition on ≥1 attributes.

- *pmr = partial-match retrieval (equality tests), e.g.*

  ```
  select * from Person
  where  name = 'John' and age = 50;
  ```

- *space = tuple-space queries (range tests), e.g.*

  ```
  select * from Person
  where 20 ≤ age ≤ 50 and 'Charles' ≤ name ≤ 'John'
  ```

N-d Selection via Heaps

Heap files can handle pmr or space using standard method:

```
// select * from R where C
r = openRelation("R",READ);
for (p = 0; p < nPages(r); p++) {
    buf = getPage(file(r), p);
    for (i = 0; i < nTuples(buf); i++) {
        t = getTuple(buf,i);
        if (matches(t,C))
            add t to result set
    }
}
```

$Cost_{pmr} = Cost_{space} = b$

N-d Selection via Multiple Indexes

DBMSs already support building multiple indexes on a table.

Which indexes to build depends on which queries are asked.

```
create table R (a int, b int, c int);
create index Rax on R (a);
create index Rbx on R (b);
create index Rcx on R (c);
create index Rabx on R (a,b);
create index Racx on R (a,c);
create index Rbcx on R (b,c);
create index Rallx on R (a,b,c);
```

But more indexes ⇒ space + update overheads.

N-d Queries and Indexes

Generalised view of pmr and space queries:

```
select * from R
where  a1 op1 C1 and ... and an opn Cn
```

pmr : all op_i are equality tests. space : some op_i are range tests.

Possible approaches to handling such queries ...

1. *use index on one a_i to reduce tuple tests*
2. *use indexes on all a_i, and intersect answer sets*

... N-d Queries and Indexes

If using just one of several indexes, which one to use?

```
select * from R
where  a1 op1 C1 and ... and an opn Cn
```

The one with best selectivity for a_i op_i C_i (i.e. fewest matches)

Factors determining selectivity of a_i op_i C_i

- *assume uniform distribution of values in dom(a_i)*
- *equality test on primary key gives at most one match*

- *equality test on larger dom(a_i) ⇒ less matches*
- *range test over large part of dom(a_i) ⇒ many matches*

... N-d Queries and Indexes

Implementing selection using one of several indices:

```
// Query: select * from R where a1op1C1 and ... and anopnCn
// choose ai with best selectivity
TupleIDs = IndexLookup(R,ai,opi,Ci)
// gives { tid1, tid2, ...} for tuples satisfying aiopiCi
PageIDs = { }
foreach tid in TupleIDs
   { PageIDs = PageIDs U {pageOf(tid)} }

// PageIDs = a set of bqix page numbers
...
```

Cost = $Cost_{index}$ + $b_{q_{ix}}$ *(some pages do not contain answers, $b_{q_{ix}} > b_q$)*

DBMSs typically maintain statistics to assist with determining selectivity

---

... N-d Queries and Indexes

Implementing selection using multiple indices:

```
// Query: select * from R where a1op1C1 and ... and anopnCn
// assumes an index on at least ai
TupleIDs = IndexLookup(R,a1,op1,C1)
foreach attribute ai with an index {
   tids = IndexLookup(R,ai,opi,Ci)
   TupleIDs = TupleIDs ∩ tids
}
PageIDs = { }
foreach tid in TupleIDs
   { PageIDs = PageIDs U {pageOf(tid)} }
// PageIDs = a set of bq page numbers
...
```

Cost = k.$Cost_{index}$ + b_q *(assuming indexes on k of n attrs)*

Exercise 2: One vs Multiple Indices

Consider a relation with r = 100,000, B = 4096, defined as:

```
create table Students (
    id       integer primary key,
    name     char(10), -- simplified
    gender   char(1),  -- 'm','f','?'
    birthday char(5)   -- 'MM-DD'
);
```

Assumptions:

- *data file is not ordered on any attribute*
- *has a dense B-tree index on each attribute*
- *96 bytes of header in each data/index page*

For `Students(id,name,gender,birthday)` ...

- *calculate the size of the data file and each index*
- *describe the selectivity of each attribute*

Now consider a query on this relation:

```
select * from Students
where  name='John' and birthday='04-01'
```

- *estimate the cost of answering using `name` index*
- *estimate the cost of answering using `birthday` index*
- *estimate the cost of answering using both indices*

Bitmap Indexes

Alternative index structure, focussing on sets of tuples:

Data File

	Part#	Colour	Price
[0]	P7	red	$2.50
[1]	P1	green	$3.50
[2]	P9	blue	$4.10
[3]	P4	blue	$7.00
[4]	P5	red	$5.20
[5]	P5	red	$2.50
	.....		

Colour Index

red	100011...
blue	001100...
green	010000...

Price Index

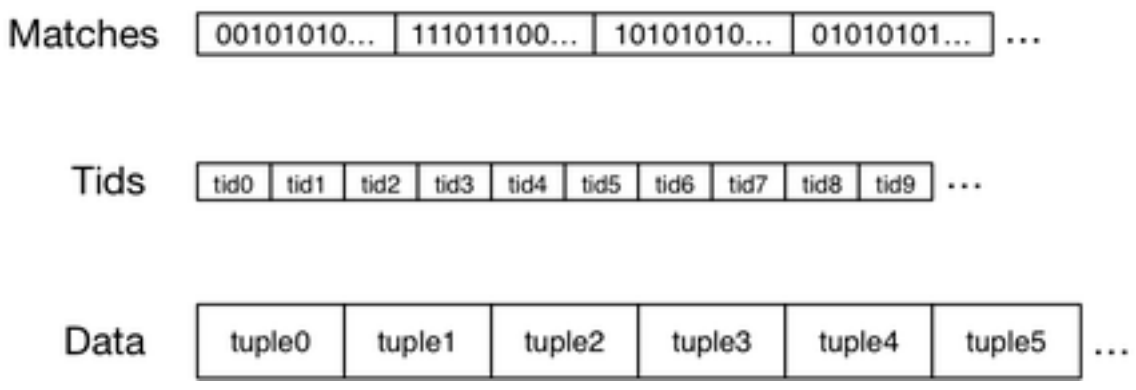
< $4.00	110001...
>= $4.00	001110...

Index contains bit-strings of r bits, one for each value/range

---

... Bitmap Indexes

Also useful to have a file of `tids`, one for each tuple:



... Bitmap Indexes

Answering queries using bitmap index:

```
Matches = AllOnes(r)
foreach attribute A with index {
   // select i^th bit-string for attribute A
   // based on value associated with A in WHERE
   Matches = Matches & Bitmaps[A][i]
}
// Matches contains 1-bit for each matching tuple
foreach i in 0..r-1 {
   if (Matches[i] == 0) continue;
   Pages = Pages U {pageOf(Tids[i])}
```

```
}
foreach pid in Pages {
   P = getPage(pid)
   extract matching tuples from P
}
```

Exercise 3: Bitmap Index

Using the following file structure:

Data File

	Part#	Colour	Price
[0]	P7	red	$2.50
[1]	P1	green	$3.50
[2]	P9	blue	$4.10
[3]	P4	blue	$7.00
[4]	P5	red	$5.20
[5]	P5	red	$2.50
	.....		

Colour Index

red	100011...
blue	001100...
green	010000...

Price Index

< $4.00	110001...
>= $4.00	001110...

Show how the following queries would be answered:

```
select * from Parts
where colour='red' and price < 4.00

select * from Parts
where colour='green' or colour ='blue'
```

... Bitmap Indexes

Storage costs for bitmap indexes:

- *one bitmap for each value/range for each indexed attribute*
- *each bitmap has length ceil(r/8) bytes*
- *e.g. with 50K records and 8KB pages, bitmap fits in one page*

Query execution costs for bitmap indexes:

- *read one bitmap for each indexed attribute in query*
- *perform bitwise AND on bitmaps (in memory)*
- *read pages containing matching tuples*

Note: bitmaps could index pages rather than tuples (shorter bitmaps)

Hashing for N-d Selection

Hashing and *pmr*

For a pmr query like

```
select * from R where a1 = C1 and ... and an = Cn
```

- *if one a_i is the hash key, query is very efficient*
- *if no a_i is the hash key, need to use linear scan*

Can be alleviated using multi-attribute hashing (mah)

- *form a composite hash value involving all attributes*

- *at query time, some components of composite hash are known*
  (allows us to limit the number of data pages which need to be checked)

MA.hashing works in conjunction with any dynamic hash scheme.

... Hashing and *pmr*

Multi-attribute hashing parameters:

- *file size = b = 2^d pages ⇒ use d-bit hash values*
- *relation has n attributes: a_1, a_2, ...a_n*
- *attribute a_i has hash function h_i*
- *attribute a_i contributes d_i bits (to the combined hash value)*
- *total bits $d = \sum_{i=1}^{n} d_i$*
- *a choice vector (cv) specifies for all k ...*
  bit j from $h_i(a_i)$ contributes bit k in combined hash value

---

MA.Hashing Example

Consider relation `Deposit(branch,acctNo,name,amount)`

Assume a small data file with 8 main data pages (plus overflows).

Hash parameters: d=3 d_1=1 d_2=1 d_3=1 d_4=0

Note that we ignore the `amount` *attribute (d_4=0)*

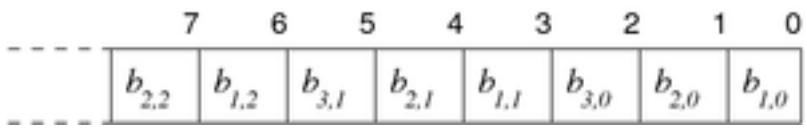
Assumes that nobody will want to ask queries like

```
select * from Deposit where amount=533
```

Choice vector is designed taking expected queries into account.

... MA.Hashing Example

Choice vector:



This choice vector tells us:

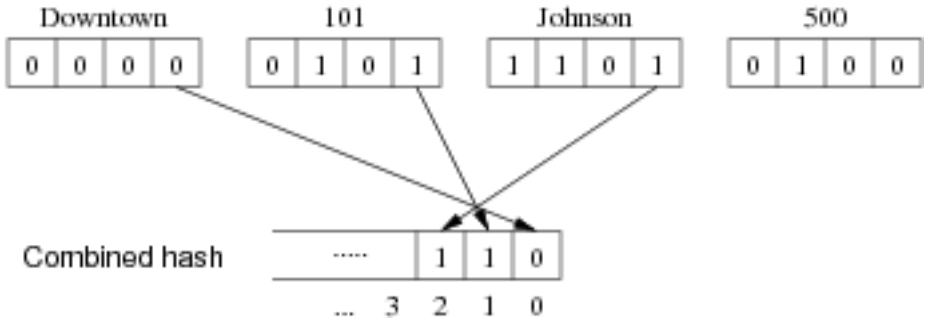
- *bit 0 in hash comes from bit 0 of $hash_1(a_1)$ ($b_{1,0}$)*
- *bit 1 in hash comes from bit 0 of $hash_2(a_2)$ ($b_{2,0}$)*
- *bit 2 in hash comes from bit 0 of $hash_3(a_3)$ ($b_{3,0}$)*
- *bit 3 in hash comes from bit 1 of $hash_1(a_1)$ ($b_{1,1}$)*
- *etc. etc. etc. (up to as many bits of hashing as required, e.g. 32)*

... MA.Hashing Example

Consider the tuple:

branch	acctNo	name	amount
Downtown	101	Johnston	512

Hash value (page address) is computed by:



MA.Hashing Hash Functions

Auxiliary definitions:

```
#define MaxHashSize 32
typedef unsigned int HashVal;

// extracts i'th bit from hash value
#define bit(i,h) (((h) & (1 << (i))) >> (i))

// choice vector elems
typedef struct { int attr, int bit } CVelem;
typedef CVelem ChoiceVec[MaxHashSize];

// hash function for individual attributes
HashVal hash_any(char *val) { ... }
```

... MA.Hashing Hash Functions

Produce combined d-bit hash value for tuple t:

```
HashVal hash(Tuple t, ChoiceVec cv, int d)
{
    HashVal h[nAttr(t)+1];  // hash for each attr
    HashVal res = 0, oneBit;
    int     i, a, b;
    for (i = 1; i <= nAttr(t); i++)
        h[i] = hash_any(attrVal(t,i));
    for (i = 0; i < d; i++) {
        a = cv[i].attr;
        b = cv[i].bit;
        oneBit = bit(b, h[a]);
        res = res | (oneBit << i);
    }
    return res;
}
```

Exercise 4: Multi-attribute Hashing

Compute the hash value for the tuple

`('John Smith','BSc(CompSci)',1990,99.5)`

where d=6, d_1=3, d_2=2, d_3=1, and

- *cv = <(1,0), (1,1), (2,0), (3,0), (1,2), (2,1), (3,1), (1,3), ...>*
- *$hash_1$('John Smith') =* `...0101010110110100`
- *$hash_2$('BSc(CompSci)') =* `...1011111101101111`
- *$hash_3$(1990) =* `...0001001011000000`

Queries with MA.Hashing

In a partial match query:

- *values of some attributes are known*
- *values of other attributes are unknown*

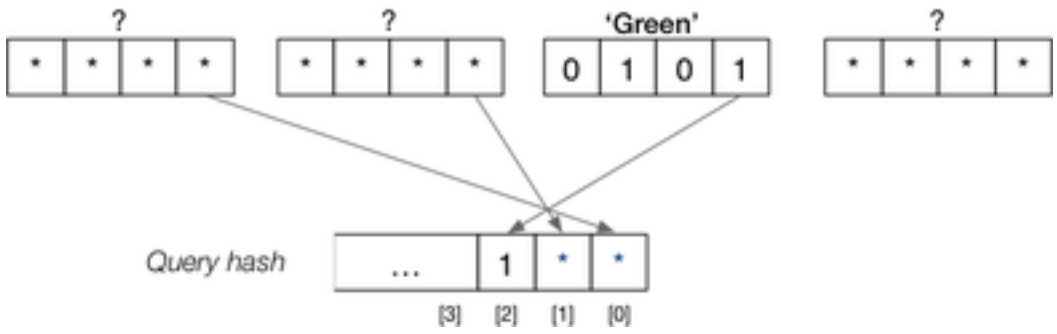
E.g.

```
select amount
from   Deposit
where  branch = 'Brighton' and name = 'Green'
```

for which we use the shorthand `(Brighton, ?, Green, ?)`

... Queries with MA.Hashing

Consider query: select amount from Deposit where name='Green'



Matching tuples must be in pages: `100`, `101`, `110`, `111`.

Exercise 5: Partial hash values in MAH

Given the following:

- *d=6, b=2^6, CV = <(0,0),(0,1),(1,0),(2,0),(1,1),(0,2), ...>*
- *hash (a) = ...00101101001101*
- *hash (b) = ...00101101001101*
- *hash (c) = ...00101101001101*

What are the query hashes for each of the following:

- *(a,b,c), (?,b,c), (a,?,?), (?,?,?)*

MA.Hashing Query Algorithm


```
// Builds the partial hash value (e.g. 10*0*1)
// Treats query like tuple with some attr values missing
nstars = 0;
for each attribute i in query Q {
    if (hasValue(Q,i)) {
        set d[i] bits in composite hash
            using choice vector and hash(Q,i)
    } else {
        set d[i] *'s in composite hash
            using choice vector
        nstars += d[i]
    }
}
...
```

```
...
// Use the partial hash to find candidate pages

r = openRelation("R",READ);
for (i = 0; i < 2^nstars; i++) {
    P = composite hash
    replace *'s in P
        using i and choice vector
    Buf = readPage(file(r), P);
    for each tuple T in Buf {
        if (T satisfies pmr query)
            add T to results
    }
}
```

Exercise 6: Representing Stars

Our hash values are bit-strings (e.g. `100101110101`)

MA.Hashing introduces a third value (`*` = unknown)

How could we represent "bit"-strings like `1011*1*0**010`?

Exercise 7: MA.Hashing Query Cost

Consider `R(x,y,z)` using multi-attribute hashing where

$d = 9 \quad d_x = 5 \quad d_y = 3 \quad d_z = 1$

How many buckets are accessed in answering each query?

1. `select * from R where x = 4 and y = 2 and z = 1`
2. `select * from R where x = 5 and y = 3`
3. `select * from R where y = 99`
4. `select * from R where z = 23`
5. `select * from R where x > 5`

Query Cost for MA.Hashing

Multi-attribute hashing handles a range of query types, e.g.

```
select * from R where a=1
select * from R where d=2
select * from R where b=3 and c=4
select * from R where a=5 and b=6 and c=7
```

A relation with n attributes has 2^n different query types.

Different query types have different costs (different no. of `*`'s)

Cost(Q) = 2^s where $s = \sum_{i \notin Q} d_i$ (alternatively Cost(Q) = $\prod_{i \notin Q} 2^{d_i}$)

Query distribution gives probability p_Q of asking each query type Q.

... Query Cost for MA.Hashing

Min query cost occurs when all attributes are used in query

Min $Cost_{pmr} = 1$

Max query cost occurs when no attributes are specified

Max $Cost_{pmr} = 2^d = b$

Average cost is given by weighted sum over all query types:

Avg $Cost_{pmr} = \Sigma_Q\, p_Q\, \Pi_{i \notin Q}\, 2^{d_i}$

Aim to minimise the weighted average query cost over possible query types

---

Optimising MA.Hashing Cost

For a given application, useful to minimise $Cost_{pmr}$.

Can be achieved by choosing appropriate values for d_i *(cv)*

Heuristics:

- *distribution of query types (more bits to frequently used attributes)*
- *size of attribute domain (≤ #bits to represent all values in domain)*
- *discriminatory power (more bits to highly discriminating attributes)*

Trade-off: making Q_j *more efficient makes* Q_k *less efficient.*

This is a combinatorial optimisation problem
(solve via standard optimisation techniques e.g. simulated annealing)

Exercise 8: MA.Hashing Design

Consider relation `Person(name,gender,age)` *...*

p_Q	Query Type Q
0.5	`select name from Person where gender=X and age=Y`
0.25	`select age from Person where name=X`
0.25	`select name from Person where gender=X`

Assume that all other query types have $p_Q=0$.

Design a choice vector to minimise average selection cost.
