

DBMSs are engines to *store*, *combine* and *filter* information.

Filtering is achieved via selection and projection.

The *join* operation (\Join) is the primary means of *combining* information.

Because *join* is

- such an important operation in database applications/systems
- potentially very expensive to execute

many methods have been developed for its implementation.

(We use a running example to compare costs of the various join processing methods)

Types of join:

- simple equijoin (single-equality condition)

```
select * from R,S where R.i = S.j
```

- partial-match join (conjunction of equality conditions)

```
select * from R,S where R.a = S.b and R.c = S.d ...
```

- theta join (arbitrary expression as condition)

```
select * from R,S where R.a < S.b and R.c <> S.d ...
```

Focus on simple equijoin, since common in practice ($R.pk=S.fk$)

Consider a university database with the schema:

```
create table Student(  
    id      integer primary key,  
    name    text,    ...  
);  
create table Enrolled(  
    stude   integer references Student(id),  
    subj    text references Subject(code),    ...  
);  
create table Subject(  
    code    text primary key,  
    title   text,    ...  
);
```

And the following request on this database:

List names of students in all subjects, arranged by subject.

The result of this request would look like:

Subj	Name
-----	-----
COMP1011	Chen Hwee Ling
COMP1011	John Smith
COMP1011	Ravi Shastri
...	
COMP1021	David Jones
COMP1021	Stephen Mao
...	
COMP3311	Dean Jones
COMP3311	Mark Taylor
COMP3311	Sashin Tendulkar

An SQL query to provide this information:

```
select E.subj, S.name
from   Student S, Enrolled E
where  S.id = E.stude
order  by E.subj, S.name;
```

And its relational algebra equivalent:

$$Sort[subj] (Project[subj,name] (Join[id=stude](Student,Enrolled)))$$

The core of the query is the join $Join[id=stude](Student,Enrolled)$

To simplify writing of formulae, $S = Student$, $E = Enrolled$.

Some database statistics:

Sym	Meaning	Value
r_S	# student records	20,000
r_E	# enrollment records	80,000
C_S	Student records/page	20
C_E	Enrolled records/page	40
b_S	# data pages in Student	1,000
b_E	# data pages in Enrolled	2,000

Also, in cost analyses below, N = number of memory buffers.

$Out = Student \bowtie Enrolled$ relation statistics:

Sym	Meaning	Value
r_{Out}	# tuples in result	80,000
C_{Out}	result records/page	80

b_{Out}	# data pages in result	1,000
-----------	------------------------	-------

Notes:

- r_{Out} ... one result tuple for each `Enrolled` tuple
- C_{Out} ... result tuples have only subj and name

Join via Cross-product

8/87

Join can be defined as a cross-product followed by selection:

$$Join[Cond](R,S) = Select[Cond](R \times S)$$

For the example query, could implement

$$Join[id=stude](Student, Enrolled)$$

as

$$Select[id=stude](Student \times Enrolled)$$

Cross-product contains $20,000 \times 80,000 = 1,600,000,000$ tuples.

... Join via Cross-product

9/87

For `Temp = (Student × Enrolled)`

I/O costs:

- size of `Temp` relation $r = 16 \times 10^8$ records
- assuming $C_{Temp}=16$, then $b_{Temp} = 10^8$
- `Temp` is written once, then scanned once
- total I/O = $10^8 \cdot (T_w + T_r)$

Assuming $T_w=T_r=0.01s$, this will take around 500 hours!

... Join via Cross-product

10/87

Because

- cross-products are infrequent in practice (except to describe join)
- cross-products are large (typically **much** larger than the final join result)

DBMSs do **not** implement join via cross-product.

DBMSs implement only join and provide cross-product as:

$$R \times S = Join[true](R,S)$$

or, in SQL

```
select * from R,S
```

Nested-Loop Join

Nested Loop Join

12/87

The simplest join algorithm:

- iteratively generates the cross-product
- checks join condition on each tuple

Algorithm to compute $Join[Cond](R,S)$:

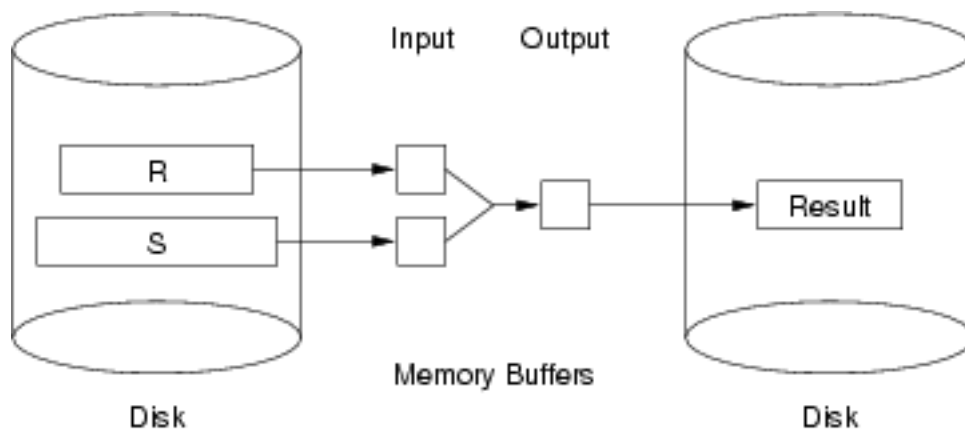
```
for each tuple r in R {
  for each tuple s in S {
    if ((r,s) satisfies join condition) {
      add (r,s) to result
    }
  }
}
```

R is the *outer* relation; S is the *inner* relation.

... Nested Loop Join

13/87

Requires (at least) three memory buffers (2 input, 1 output).



... Nested Loop Join

14/87

Abstract algorithm for $Join[Cond](R,S)$ (with 3 memory buffers):

```
for each page of relation R {
  read into buffer rBuf
  for each page of relation S {
    read into buffer sBuf
    for each record r in rBuf {
      for each record s in sBuf {
        if ((r,s) satisfies Cond) {
          add combined(r,s) to OutBuf
          write Outbuf when full
        }
      }
    }
  }
}
```

... Nested Loop Join

15/87

Detailed algorithm for $Join[Cond](R,S)$ (with 3 memory buffers):

```
// rf: file for R, sf: file for S, of: output file
outp = 0; clearBuf(oBuf);
for (rp = 0; rp < nPages(rf); rp++) {
  readPage(rf, rp, rBuf);
  for (sp = 0; sp < nPages(sf); sp++) {
    readPage(sf, sp, sBuf);
    for (i = 0; i < nTuples(rBuf); i++) {
      rTup = getTuple(rBuf, i);
      for (j = 0; j < nTuples(sBuf); j++) {
        sTup = getTuple(sBuf, j);
        if (satisfies(rTup,sTup,Cond)) {
          rsTup = combine(rTup,sTup);
          addTuple(oBuf, rsTup);
        }
      }
    }
  }
}
```

```

        if (isFull(oBuf)) {
            writePage(of, outp++, oBuf);
            clearBuf(oBuf);
        }
    }
}

```

... Nested Loop Join

16/87

The three-memory-buffer nested loop join requires:

- read all b_R pages of R once
- for each of page of R , read b_S pages of S

$$\text{Cost} = b_R + b_R b_S$$

If we use S as the outer relation in the join

$$\text{Cost} = b_S + b_S b_R$$

It is (slightly) better to use smaller relation as outer relation.

Nested Loop Join on Example

17/87

If `Student` is outer relation and `Enrolled` is inner:

$$\begin{aligned} \text{Cost} &= b_S + b_S b_E \\ &= 1,000 + 1,000 \times 2,000 = 2,001,000 \end{aligned}$$

If `Enrolled` is outer relation and `Student` is inner:

$$\begin{aligned} \text{Cost} &= b_E + b_E b_S \\ &= 2,000 + 2,000 \times 1,000 = 2,002,000 \end{aligned}$$

Cost of nested-loop join is too high (5 hours, if $T_f=0.01$ sec)

Implementing Join Better

18/87

Aims of effective join computation:

- generate only relevant tuples from the cross-product
- generate these tuples with minimal disk I/O

Range of costs for $Join(R,S)$

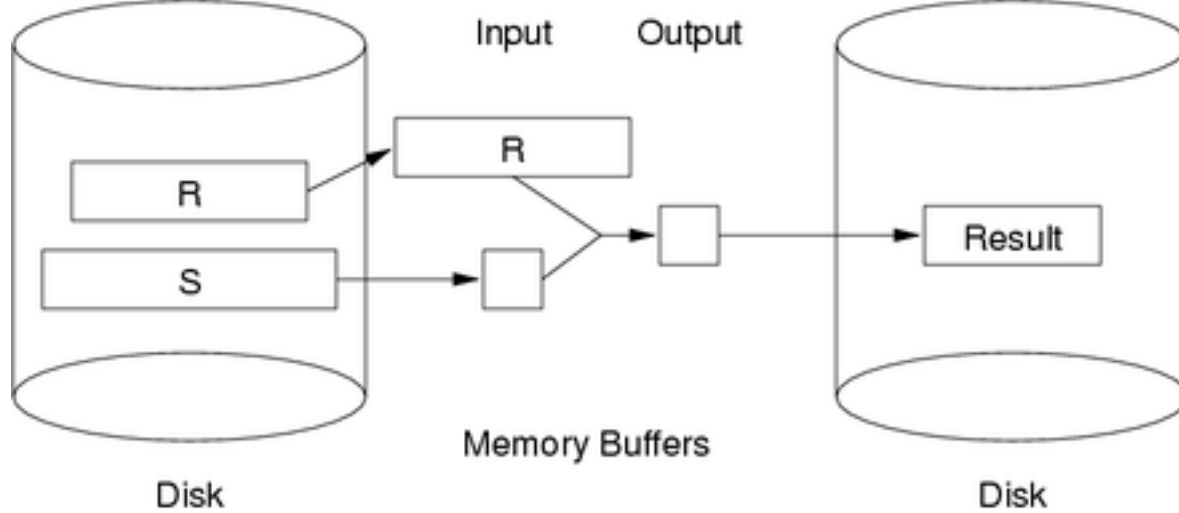
- worst case cost = $b_R + b_R b_S$ (nested loop join)
 - best case cost = $b_R + b_S$ (read each page once)
-

Block Nested Loop Join

19/87

If at least b_R+2 memory buffers available:

- read the entire R relation into memory
- for each S page, check join condition on all (r, s) pairs



... Block Nested Loop Join

20/87

Algorithm for nested loop join with b_R+2 memory buffers:

```

read all of R's pages into memory buffers
for each page of relation S {
  read page into S's input buffer
  for each tuple s in S's buffer {
    for each tuple r in R's memory buffers {
      if ((r,s) satisfies JoinCond)) {
        add (r,s) to output buffer
      }
    }
  }
}

```

Note that R effectively becomes the inner relation in this scheme.

... Block Nested Loop Join

21/87

This method requires:

- read b_R pages of relation R into buffers
- while R is buffered, read b_S pages of S

Cost = $b_R + b_S$

Notes:

- minimal I/O cost, but considers all (r,s) pairs
- thus, requires $r_R \cdot r_S$ checks of the join condition

... Block Nested Loop Join

22/87

Further performance improvements:

- must reduce number of R tuples matched against each S tuple
- use access method to find small set of R tuples matching S tuple

Example:

- each S joins with $k \ll r_R$ tuples of R
- R tuples are stored in sorted array of memory buffers
- for each S tuple, use binary search to find matching buffer
- scan around that buffer to find all matching (R,S) pairs
- requires approx $C_R \cdot r_S$ checks of join condition

23/87

Block Nested Loop Join on Example

If ≥ 1002 memory buffers are available:

- read `Student` relation into memory
- scan `Enrolled` relation, computing join

$$\begin{aligned} \text{Cost} &= b_S + b_E \\ &= 1,000 + 2,000 = 3,000 \end{aligned}$$

This is considerably better than 10^6 (30 secs vs 5 hours).

But what if we have only N memory buffers, where $N < b_R$, $N < b_S$?

... Block Nested Loop Join on Example

24/87

In general case, read *outer* relation in runs of $N-2$ pages

```
for each run of N-2 pages from R {
  read N-2 of R's pages into memory buffers
  for each page of relation S {
    read page into S's input buffer
    for each tuple s in S's buffer do
      for each tuple r in R's memory buffers {
        if ((r,s) satisfies JoinCond)) {
          add (r,s) to output buffer
          write output buffer when full
        }
      }
    }
  }
}
```

... Block Nested Loop Join on Example

25/87

Block nested loop join requires

- read $\lceil b_R/N-2 \rceil$ runs from R
- for each run, scan b_S pages of S

$$\text{Cost} = b_R + b_S \cdot \lceil b_R/N-2 \rceil$$

Notes:

- the final run will typically be "short" (i.e. $< N-2$ pages)
- unless index/hash is used, we still do $r_R.r_S$ tuple comparisons

... Block Nested Loop Join on Example

26/87

Costs for various buffer pool sizes:

N	Inner	Outer	#runs	Cost
22	Student	Enrolled	50	101,000
52	Student	Enrolled	20	41,000
102	Student	Enrolled	10	21,000
1002	Student	Enrolled	1	3,000
22	Enrolled	Student	100	102,000
52	Enrolled	Student	40	42,000

102	Enrolled	Student	20	22,000
1002	Enrolled	Student	2	4,000

Block Nested Loop Join in Practice

Why block nested loop join is very useful in practice ...

Many queries have the form

```
select * from R,S where r.i=s.j and r.x=k
```

This would typically be evaluated as

$$\text{Join } [i=j] ((\text{Sel}[r.x=k](R)), S)$$

If $|\text{Sel}[r.x=k](R)|$ is small \Rightarrow may fit in memory (in small #buffers)

Join Conditions and Methods

Nested loop join makes no assumptions about join conditions.

```
for each pair of tuples (r,s) {
    check join condition on (r,s)
    if satisfied, add to results
}
```

To improve join:

- reduce the number of tuple pairs considered
- but not easy to do for arbitrary join condition

As noted above, simple equijoin is a common join condition.

Thus, a range of other join algorithms has been developed specifically for equality join conditions.

Index Nested Loop Join

Most joins considered so far have a common problem:

- repeated scans of *entire* inner relation S are required

If there is an index on S , we can avoid such repeated scanning.

Consider $\text{Join}[R.i=S.j](R,S)$:

```
for each tuple r in relation R {
    use index to select tuples
      from S where s.j = r.i
    for each selected tuple s from S {
        add (r,s) to result
    }
}
```

(For ordered indexes (e.g. Btree), this also assists join conditions like $R.i < S.j$)

... Index Nested Loop Join

This method requires:

- one scan of R relation (b_R)

- only one buffer needed, since we use R tuple-at-a-time
- for each $tuple$ in $R(r_R)$, one index lookup on S
 - cost depends on type of index and number of results
 - best case is when each $R.i$ matches few S tuples

Cost = $b_R + r_R \cdot Sel_S$ (Sel_S is the cost of performing a select on S).

... Index Nested Loop Join

31/87

For index lookup:

- cost of locating first matching tuple
 - for B+ trees, typically 2-4 page reads
 - for hashing, typically 1-2 page reads
- cost of finding other matching tuples
 - if clustered, typically 1-2 page reads
 - if unclustered, up to b_q page reads

Note: building an index "on the fly" to perform a join can be very cost-effective.

Index Nested Loop Join on Example

32/87

Case 1: $Join[id=stude](Student, Enrolled)$

- `Student` is outer and `Enrolled` is inner
- `Enrolled` has a clustered B+ tree index on `stude` field
- B+ tree has depth 3 (root + internal + leaf)
- most of the time, the four matching records are in a single page

$$\begin{aligned} \text{Cost} &= b_S + r_S \cdot btree_E \\ &= 1,000 + 20,000 \times (3+1.01) = 80,000 \end{aligned}$$

... Index Nested Loop Join on Example

33/87

Case 2: $Join[id=stude](Student, Enrolled)$

- `Student` is outer and `Enrolled` is inner
- `Enrolled` has an unclustered B+ tree index on `stude` field
- B+ tree has depth 3 (root + internal + leaf)
- assume worst case; matching records are all on different pages

$$\begin{aligned} \text{Cost} &= b_S + r_S \cdot btree_E \\ &= 1,000 + 20,000 \times (3+4) = 150,000 \end{aligned}$$

... Index Nested Loop Join on Example

34/87

Case 3: $Join[id=stude](Student, Enrolled)$

- `Enrolled` is outer and `Student` is inner
- `Student` is hashed on `id` field (e.g. linear hashing)

- there may be (short) overflow chains (e.g. 1.1 page reads/bucket)

$$\begin{aligned}\text{Cost} &= b_E + r_E \text{hash}_S \\ &= 2,000 + 80,000 \times 1.1 = 90,000\end{aligned}$$

Optimised Index Nested Loop Join

35/87

Consider the following scenario for $\text{Join}[R.i=S.j](R,S)$:

- $R.i$ is not a primary key (so many tuples have same $R.i$ value)
- R is sorted on $R.i$ (or could be efficiently sorted on $R.i$)
- each $R.i$ value does not match very many tuples

Could save repeated index scans with the same $R.i$ value

- cache results of index scan for $R.i=k$ in buffer
- if next R tuple also has $R.i=k$, re-use scan results

... Optimised Index Nested Loop Join

36/87

Abstract algorithm for optimised index nested loop join:

```
for each tuple r in relation R {
  if (prev == r.i)
    use selected tuples in buffer(s)
  else {
    use index to select tuples
    from S where s.j = r.i
    store selected tuples in buffer(s)
  }
  for each selected tuple s from S
    add (r,s) to result
  prev = r.i
}
```

Cost savings depend on repetition factor, #buffers, size of index scans

Sort-Merge Join

Sort-Merge Join

38/87

Basic approach:

- sort both relations on join attribute (reminder: $\text{Join}[R.i=S.j](R,S)$)
- scan together using merge to form result (r, s) tuples

Advantages:

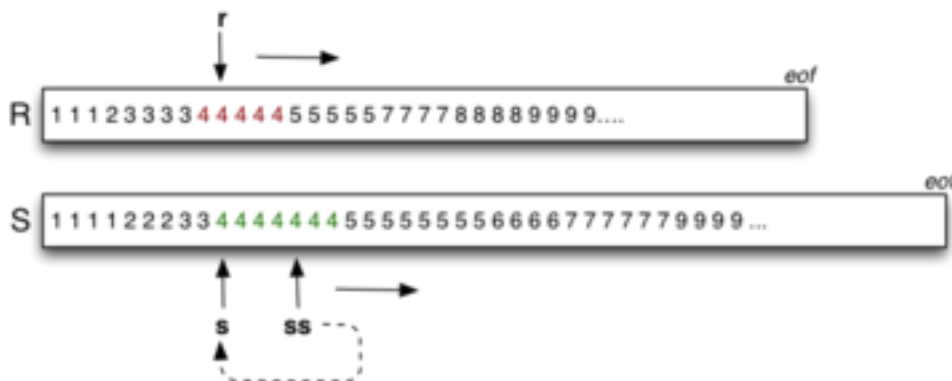
- no need to deal with "entire" S relation for each r tuple
- deal with runs of matching R and S tuples

Disadvantages:

- cost of sorting both relations (relations may be sorted on join key?)
- some rescanning required when long runs of S tuples

Method requires several cursors to scan sorted relations:

- r = current record in R relation
- s = start of current run in S relation
- ss = current record in current run in S relation



... Sort-Merge Join

40/87

Abstract algorithm for merge phase of $Join[R.i=S.j](R,S)$:

```

r = first tuple in R
s = first tuple in S
while (r != eof and s != eof) {
    // align cursors to start of next common run
    while (r != eof and r.i < s.j) { r = next tuple in R }
    while (s != eof and r.i > s.j) { s = next tuple in S }
    // scan common run, generating result tuples
    while (r != eof and r.i == s.j) {
        ss = s    // set to start of run
        while (ss != eof and ss.j == r.i) {
            add (r,s) to result
            ss = next tuple in S
        }
        r = next tuple in R
    }
    s = ss    // start search for next run
}

```

Sidetrack: Iterators

41/87

Sort-merge join implementation is simplified by use of iterators.

- iterators give the appearance of tuple-at-a-time
- even when the underlying data is page-by-page
- and even in the presence of auxiliary index structures

Typical usage of iterator:

```

Iterator iter; Tuple tup;
iter = startScan("Rel", "i=5");
while ((tup = nextTuple(iter)) != NULL) {
    process(tup);
}
endScan(iter);

```

... Sidetrack: Iterators

42/87

```

typedef struct {

```

```

File    inf;    // input file
Buffer buf;    // buffer holding current page
int     curp;   // current page during scan
int     curr;   // index of current record in page
} Iterator;

// simple linear scan; no condition
Iterator *startScan(char *relName) {
    Iterator *iter = malloc(sizeof(Iterator));
    iter->inf = openFile(fileName(relName), READ);
    iter->curp = 0;
    iter->curr = -1;
    readPage(iter->inf, iter->curp, iter->buf);
}

```

... Sidetrack: Iterators

43/87

```

Tuple nextTuple(Iterator *iter) {
    // check if reached end of current page
    if (iter->curr == nTuples(iter->buf)-1) {
        // check if reached end of data file
        if (iter->curp == nPages(iter->inf)-1)
            return NULL;
        iter->curp++;
        iter->buf = readPage(iter->inf, iter->curp);
        iter->curr = -1;
    }
    iter->curr++;
    return getTuple(iter->buf, iter->curr);
}
// curp and curr hold indexes of most recently read page/record

```

... Sidetrack: Iterators

44/87

```

TupleID scanCurrent(Iterator *iter) {
    // form TupleID for current record
    return iter->curp + iter->curr;
}

void setScan(Iterator *iter, int page, int rec) {
    assert(page >= 0 && page < nPages(iter->inf));
    if (iter->curp != page) {
        iter->curp = page;
        readPage(iter->inf, iter->curp, iter->buf);
    }
    assert(rec >= 0 && rec < nTuples(iter->buf));
    iter->curr = rec;
}

void endScan(Iterator *iter) {
    closeFile(iter->buf);
    free(iter);
}

```

Sort-Merge Join

45/87

Concrete algorithm using iterators:

```

Iterator *ri, *si;  Tuple rup, stup;

ri = startScan("SortedR");

```

```

si = startScan("SortedS");
while ((rtup = nextTuple(ri)) != NULL
      && (stup = nextTuple(si)) != NULL) {
    // align cursors to start of next common run
    while (rtup != NULL && rtup.i < stup.j)
        rtup = nextTuple(ri);
    if (rtup == NULL) break;
    while (stup != NULL && rtup.i > stup.j)
        stup = nextTuple(si);
    if (stup == NULL) break;
    // must have (r.i == s.j) here
    ...

```

... Sort-Merge Join

46/87

```

...
// remember start of current run in S
TupleID startRun = scanCurrent(si);
// scan common run, generating result tuples
while (rtup != NULL && rtup.i == stup.j) {
    while (stup != NULL and stup.j == rtup.i) {
        addTuple(outbuf, combine(rtup, stup));
        if (isFull(outbuf)) {
            writePage(outf, outp++, outbuf);
            clearBuf(outbuf);
        }
        stup = nextTuple(si);
    }
    rtup = nextTuple(ri);
    setScan(si, startRun);
}
}

```

... Sort-Merge Join

47/87

Buffer requirements:

- for sort phase:
 - as many as possible (remembering that cost is $O(\log_{\#Bufs})$)
 - if insufficient buffers, sorting cost can dominate
 - for merge phase:
 - one output buffer for result
 - one input buffer for relation R
 - (preferably) enough buffers for longest run in S
-

... Sort-Merge Join

48/87

Cost of sort-merge join.

Step 1: sort each relation (if not already sorted):

- $\text{Cost} = 2 \cdot b_R (1 + \log_{N-1}(b_R / N)) + 2 \cdot b_S (1 + \log_{N-1}(b_S / N))$
(where N = number of memory buffers)

Step 2: merge sorted relations:

- if every run of values in S fits completely in buffers, merge requires single scan, $\text{Cost} = b_R + b_S$
 - if some runs in of values in S are larger than buffers, need to re-scan run for each corresponding value from R
-

Case 1: $Join[id=stude](Student, Enrolled)$

- $Student$ and $Enrolled$ already sorted on $id\#$
- memory buffers $N=4$; all runs are of length < 2

Cost = $b_S + b_E = 3,000$ (i.e. minimal cost)

... Sort-Merge Join on Example

50/87

Case 2: $Join[id=stude](Student, Enrolled)$

- relations are not sorted on $id\#$
- memory buffers $N=32$; all runs are of length < 30

Cost = $sort(S) + sort(E) + b_S + b_E$

= $b_S \lceil \log_{30} b_S \rceil + b_E \lceil \log_{30} b_E \rceil + b_S + b_E$

= $1,000 \times 3 + 2,000 \times 3 + 1,000 + 2,000$

= $12,000$

... Sort-Merge Join on Example

51/87

Case 3: $Join[id=stude](Student, Enrolled)$

- $Student$ and $Enrolled$ already sorted on $id\#$
- memory buffers $N=3$ (S input, E input, output)
- one-quarter of the "runs" in E span two pages
- there are no "runs" in S , since $id\#$ is a primary key

Cost depends on which relation is outer and which is inner.

... Sort-Merge Join on Example

52/87

Case 3 (continued) ...

If E is outer relation:

- Cost = $b_E + b_S = 3,000$

If S is outer relation:

- one-quarter of E runs require two page reads
- each E run is processed once for matching $S.id$ value
- Cost = $b_S + b_E + r_S/4 = 8,000$

Sidetrack 2: More on Iterators

53/87

Above description of iterators:

- involved simple scan of a single table

- with no condition to select tuples

In the general case, an iterator involves:

- one (selection) or two (join) tables
- with a condition to determine relevant tuples

A typical SQL query involves many iterators

- one for each relational operator in query plan
- connected in a demand-driven network of query nodes

... Sidetrack 2: More on Iterators

54/87

Requires a more general definition of execution state:

```
typedef struct {
    Oper    op;      // operation (sel,sort,join,...)
    Reln    r1;      // first relation
    Reln    r2;      // second relation (if any)
    Buffer   *bufs;   // buffers used by operation
    int     curp1;   // index of current page for r1
    int     curr1;   // index of current record in page
    int     curp2;   // index of current page for r2
    int     curr2;   // index of current record in page
    Cond    cond;    // condition for choosing tuple(s)
} Iterator;
```

For PostgreSQL details, see [include/nodes/execnodes.h](#)

Hash Join

Hash Join

56/87

Basic idea:

- use hashing as a technique to partition relations
- to avoid having to consider all pairs of tuples

Requires sufficient memory buffers

- to hold substantial portions of partitions
- (preferably) to hold largest partition of outer relation

Other issues:

- works only for equijoin $R.i=S.j$ (but this is a common case)
- susceptible to data skew (or poor hash function)

Variations: *simple*, *grace*, *hybrid*.

Simple Hash Join

57/87

Basic approach:

- hash the inner relation R into memory buffers (build)
- scan the outer relation S , using hash to search (probe)
 - if $R.i=S.j$, then $h(R.i)=h(S.j)$ (hash to same buffer)
 - only need to check one memory buffer for each S tuple

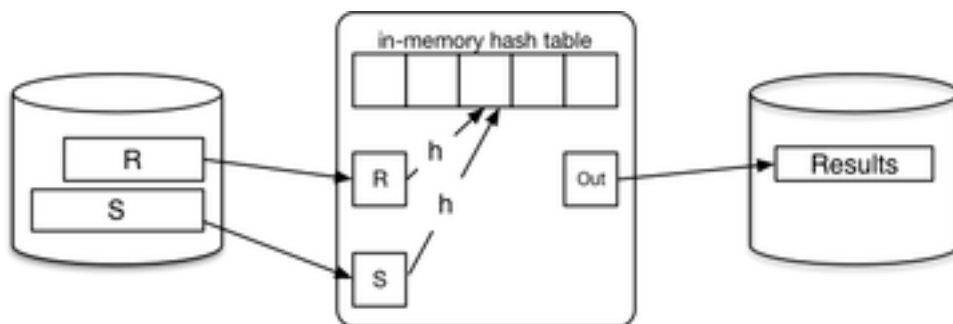
Makes the assumption: whole of S hashes into memory

- requires R to be smaller than memory buffers
- requires a uniform hash function (no overflows)

... Simple Hash Join

58/87

Data flow:



... Simple Hash Join

59/87

Algorithm for ideal simple hash join $Join[R.i=S.j](R,S)$:

```
for each tuple r in relation R
  { insert r into buffer[h(R.i)] }
for each tuple s in relation S {
  for each tuple r in buffer[h(S.j)] {
    if ((r,s) satisfies join condition) {
      add (r,s) to result
    }
  }
}
```

Cost = $b_R + b_S$ (minimum possible cost)

... Simple Hash Join

60/87

Consider that we have N buffers available.

If $b_R \leq N-2$ buffers, no need to hash (use nested loop).

In practice, size of hash table $b_{hR} > b_R$ (e.g. data skew)

\Rightarrow hash table for R is even less likely to fit in memory

Can be handled by a variation on above algorithm:

- scan R , making hash table with $N-2$ buffers
- once hash table built, scan S (standard probe phase)
- if more R tuples, build new table and repeat

... Simple Hash Join

61/87

Algorithm for realistic simple hash join $Join[R.i=S.j](R,S)$:

```
for each tuple r in relation R {
  if (buffer[h(R.i)] is full) {
    for each tuple s in relation S {
      for each tuple rr in buffer[h(S.j)] {
        if ((rr,s) satisfies join condition) {
          add (rr,s) to result
        }
      }
    }
  }
}
```



```

    }
    }
    clear all hash table buffers
  }
  insert r into buffer[h(R.i)]
}

```

Note: requires multiple passes over the S relation.

... Simple Hash Join

62/87

Cost depends on N and on properties of data/hash.

Worst case:

- $h(i)=k$ so read only C_R tuples before hash table "full"
- each hash table for R occupies one buffer with C_R tuples
- degenerates to nested-loop-with-3-buffers case $\Rightarrow b_R + b_R b_S$

Best case:

- perfect uniform distribution of hash values
- each hash table of R holds $(N-2)C_R$ tuples from $N-2$ pages
- number of hash tables built = $n_{hR} = \lceil b_R / (N-2) \rceil$
- read all of S for each hash table $\Rightarrow b_R + n_{hR} \cdot b_S$

Grace Hash Join

63/87

Basic approach:

- partition both relations on join attribute using hashing
- scan through corresponding pairs of partitions to form results

Similar approach to sort-merge join, except:

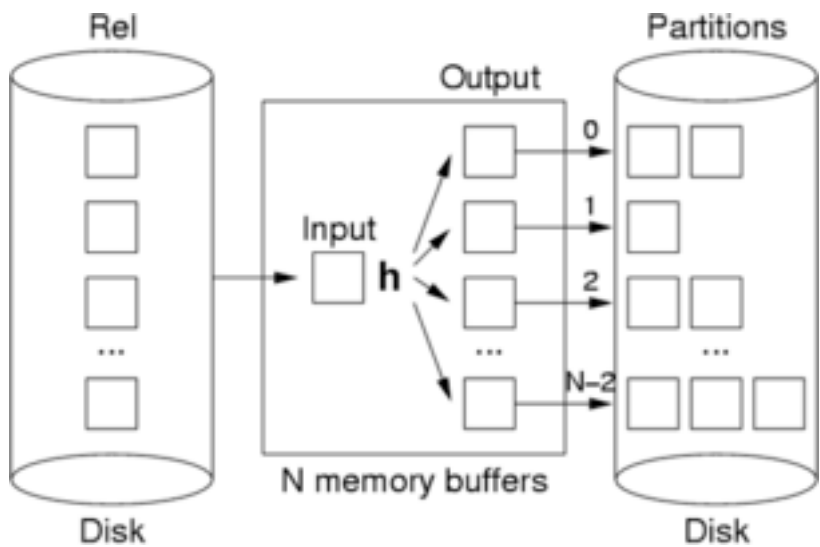
- sort-merge: partitioning achieved by sorting (runs)
- hash: partitioning achieved by hashing

Requires enough buffer space to hold largest partition of inner relation.

... Grace Hash Join

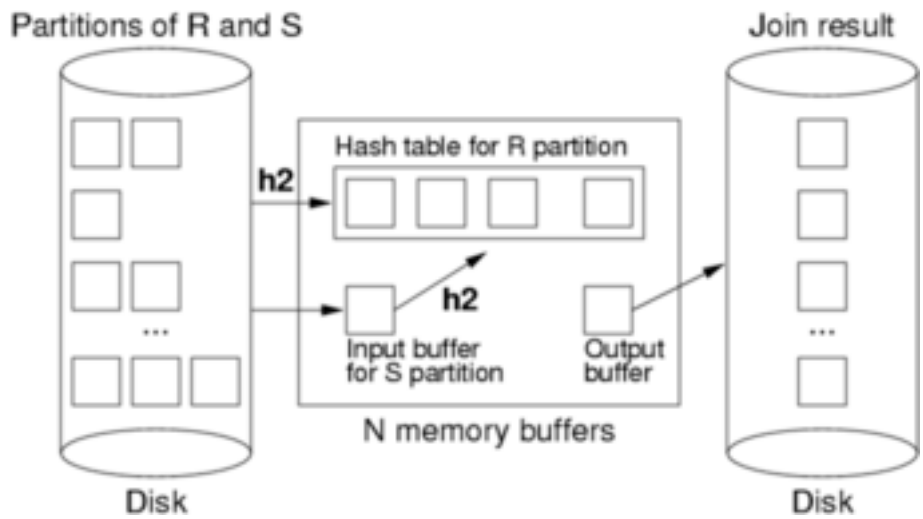
64/87

Partition phase:



This is applied to each relation R and S .

Probe/join phase:



The second hash function (h_2) simply speeds up the matching process. Without it, would need to scan entire R partition for each record in S partition.

... Grace Hash Join

Abstract algorithm for $Join[R.i=S.j](R,S)$:

```
// assume  $h(val)$  generates  $[0..N-2]$ 
// assume  $h_2(val)$  generates  $[0..N-3]$ 

// Partition phase (each relation  $\rightarrow$   $N-1$  partitions)
// 1 input buffer,  $N-1$  output buffers

for each tuple  $r$  in relation  $R$ 
    add  $r$  to partition  $h(r.i)$  in output file  $R'$ 
for each tuple  $s$  in relation  $S$ 
    add  $s$  to partition  $h(s.j)$  in output file  $S'$ 
...
```

... Grace Hash Join

Abstract algorithm for $Join[R.i=S.j](R,S)$ (cont.)

```
// Probe/join phase
// 1 input buffer for  $S$ , 1 output buffer
//  $N-2$  buffers to build hash table for  $R$  partition

for each partition  $p = 0 \dots N-2$  {
    // Build in-memory hash table for partition  $p$  of  $R'$ 
    for each tuple  $r$  in partition  $p$  of  $R'$ 
        insert  $r$  into buffer  $h_2(r.i)$ 

    // Scan partition  $p$  of  $S'$ , probing for matching tuples
    for each tuple  $s$  in partition  $p$  of  $S'$  {
         $b = h_2(s.j)$ 
        for all matching tuples  $r$  in buffer  $b$ 
            add  $(r,s)$  to result
    }
}
```

... Grace Hash Join

Concrete algorithm for partitioning:

```

Buffer iBuf, oBuf[N-1];
File inf, outf[N-1]; char rel[100];
int i, r, h, ip, op[N-1]; Tuple tup;
for (i = 0; i < N-1; i++) {
    clearBuf(oBuf[i]); op[i] = 0;
    rel = sprintf("%s%d", "Rel", i);
    outf[i] = openFile(fileName(rel), WRITE));
}
inf = openFile(fileName("Rel"), READ);
for (ip = 0; ip < nPages(inf); ip++) {
    iBuf = readPage(inf, ip);
    for (r = 0; r < nTuples(iBuf); r++) {
        tup = getTuple(iBuf, r);
        h = hash(tup.i, N-1);
        addTuple(oBuf[h], tup);
        if (isFull(oBuf[h])) {
            writePage(outf[h], op[h]++, oBuf[h]);
            clearBuf(oBuf[h]);
        }
    }
}

```

... Grace Hash Join

69/87

Cost of grace hash join:

- #pages in all partition files of $Rel \approx b_{Rel}$ (maybe slightly more)
- partition relation R ... Cost = $b_R.T_r + b_R.T_w = 2b_R$
- partition relation S ... Cost = $b_S.T_r + b_S.T_w = 2b_S$
- probe/join requires one scan of each (partitioned) relation
Cost = $b_R + b_S$
- all hashing and comparison occurs in memory $\Rightarrow \approx 0$ cost

Total Cost = $3(b_R + b_S)$

... Grace Hash Join

70/87

The above cost analysis assumes:

- every partition of R fits in memory buffers at once

We achieve this situation if:

- data has uniform distribution
- hash function gives uniform distribution
(all partitions are similar size)
- we have $N-1 \geq \lceil \sqrt{b} \rceil$ memory buffers
(giving $N-1$ partitions, each with $\approx b_R/(N-1)$ pages)

... Grace Hash Join

71/87

Possibilities for dealing with "over-long" partitions of R

- handle each over-long partition via scanning
 - requires over-long partitions to be scanned multiple times
 - essentially, such partitions are treated via nested loop join
- apply hash join recursively to over-long partitions
 - increases i/o by needing to partition parts of file multiple times
- use a different hash function with better distribution properties
 - but difficult to find such hash functions "on the fly"
- use the relation with the best partitioning as the "outer" relation

Grace Hash Join on Example

72/87

For the example $Join[id=stude](Student, Enrolled)$:

- assume that we have a good hash function and $N = \sqrt{1000} = 32$

$$\begin{aligned}\text{Cost} &= 3(b_S + b_E) \\ &= 3(1,000 + 2,000) = 9,000\end{aligned}$$

Hybrid Hash Join

73/87

An optimisation if we have $\sqrt{b_R} < N < b_R + 2$

- create k partitions using N buffers where $k \ll N$
- with grace join, would use k output buffers (one per partition)
- what to do with $N-k$ remaining buffers? (ignore input buffer)
- use them to hold m partitions of R in memory (no disk writes)
- other partitions are handled as before (using $k-m$ output buffers)

When we come to scan and partition S relation

- any tuple with hash in range $0..m-1$ can be resolved
- other tuples are written to one of $k-m$ partition files for S

Final phase is same as grace join, but with only $k-m$ partitions.

... Hybrid Hash Join

74/87

Some observations:

- for k partitions, each partition has expected size $\text{ceil}(b_R/k)$
- holding m partitions in memory needs $m \times \text{ceil}(b_R/k)$ buffers
- since we have $k-m$ output buffers, we must have $mb_R/k + (k-m) \leq N$
- for every partition/block held in memory, we save on disk i/o
- saving is $m/k \times 2(b_R + b_S)$

Other notes:

- if $N = b_R + 2$, using block nested loop join is simpler
- cost depends on N (but less than grace hash join)

... Hybrid Hash Join

75/87

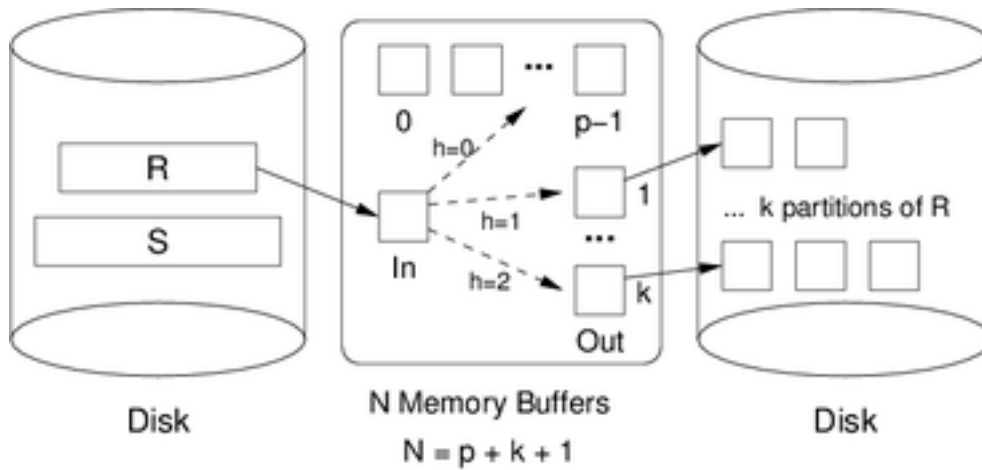
Need to choose appropriate m and k to minimise cost

- base cost: $3 \times (b_R + b_S)$ (grace join)
- i/o saving: $m/k \times 2(b_R + b_S)$
- constraint: $mb_R/k + (k-m) \leq N$

Approach to maximise saving:

- have one large in-memory partition ($m = 1$)
- use as many as possible of N buffers for partition
- use as few output buffers as possible (minimise k)

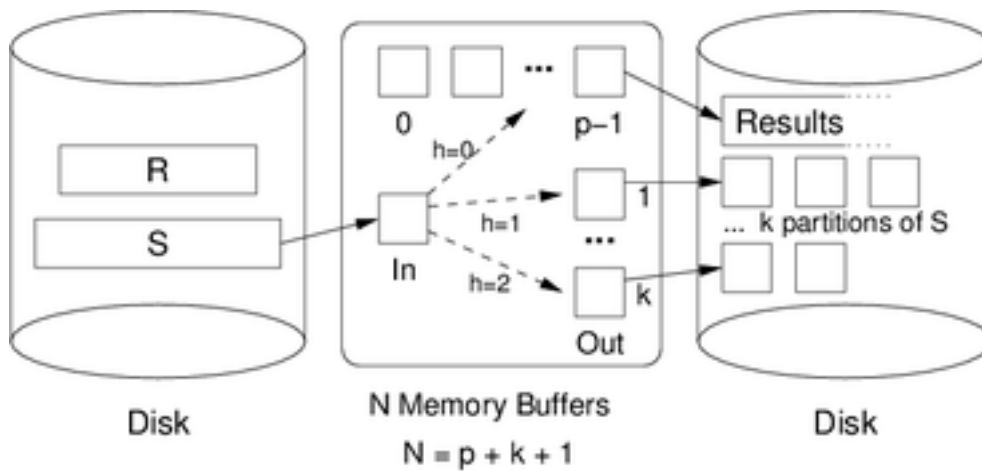
Data flow for hybrid hash join (partitioning R):



... Hybrid Hash Join

77/87

Data flow for hybrid hash join (partitioning S):



After this, proceed as for grace hash join.

... Hybrid Hash Join

78/87

Cost of hybrid hash join:

- assume: large N total buffers, m partitions in memory, k partitions on disk
- read both tables: $b_R + b_S$
- total partitions for each table: $m+k$
- assuming uniform hashing, #pages in each R partition $P_R = \text{ceil}(b_R/(m+k))$
- assuming uniform hashing, #pages in each S partition $P_S = \text{ceil}(b_S/(m+k))$
- in Pass 1, $k \cdot P_R + k \cdot P_S$ pages written to disk partitions
- all joining of m in-memory partitions is handled in memory
- in Pass2, $k \cdot P_R + k \cdot P_S$ pages read back from disk partitions

$$\begin{aligned}
 \text{Cost} &= b_R + b_S + k \cdot P_R + k \cdot P_S + k \cdot P_R + k \cdot P_S \\
 &= b_R + b_S + 2 \cdot k \cdot (P_R + P_S) \\
 &= b_R + b_S + 2 \cdot k \cdot (\text{ceil}(b_R/(m+k)) + \text{ceil}(b_S/(m+k)))
 \end{aligned}$$

How to determine k :

- set $m=1$ and so size of partition $\approx N \Rightarrow k \approx b_R/N$
- need to ensure that $\lceil b_R/k \rceil + k \leq N$ (allowing for input buffer)
- choose k close to b_R/N but satisfying constraint

Hybrid Hash Join on Example

Case 1: $N = 100$ buffers, $b_R = 1000$

- $k = 10 \Rightarrow 1000/10 + 10 = 110$ buffers; not less than 100
- $k = 12 \Rightarrow 1000/12 + 12 = 96$ buffers
- $\text{Cost} = (3 - 2/12) \cdot (1000 + 2000) = 8500$

Case 2: $N = 200$ buffers, $b_R = 1000$

- $k = 5 \Rightarrow 1000/5 + 5 = 205$ buffers; not less than 200
- $k = 6 \Rightarrow 1000/6 + 6 = 173$ buffers
- $\text{Cost} = (3 - 2/6) \cdot (1000 + 2000) = 8000$

Case 3: $N = 502$ buffers, $b_R = 1000$

- $k = 2 \Rightarrow 1000/2 + 2 = 502$ buffers
- $\text{Cost} = (3 - 2/2) \cdot (1000 + 2000) = 6000$

Pointer-based Join

80/87

Conventional join algorithms set up $R \leftrightarrow S$ connections via attribute values.

Join could be performed faster if direct connections already existed.

- in OODBMSs, they generally already exist in the form of object references (oids)
- in RDBMSs, they could be introduced via extra rid attributes

Such a modification to conventional RDBMS structure would be worthwhile:

- if we know in advance what kind of joins will be required
- adding the extra rid attributes into tuples is feasible

... Pointer-based Join

81/87

The basic idea for pointer-based join is:

```
for each tuple r in relation R {
  for each rid associated with r {
    fetch tuple s from S via rid
    add (r,s) to result relation
  }
}
```

Often, each R tuple is associated with only one rid, so the inner loop is not needed.

... Pointer-based Join

82/87

The advantage over value-based joins:

- rather than find S tuples via value-based lookup (e.g. hashing, index)
- we find S tuples by direct fetch with rid (much faster per tuple)
- requires no assumption about sorted-ness of relations
- does not require large numbers of buffers

The (potential) disadvantages:

- every fetch goes to a different page of S
(this essentially returns us to the worst-case scenario for nested-loop join)
- the join only works in "one direction" (from R to S)

- requires additional data for each different join type
- requires tuples to be larger $\Rightarrow b_R$ is larger

General Join Conditions

83/87

Above examples all used simple equijoin e.g. $\text{Join}[i=j](R,S)$.

For theta-join e.g. $\text{Join}[i<j](R,S)$:

- index nested loop join: need B+ tree index on inner relation
- sort-merge join can be adapted, but is not very effective
- hash join is inapplicable
- other methods are essentially unchanged

... General Join Conditions

84/87

For multi-equality (pmr) join e.g. $\text{Join}[i=j \wedge k=l](R,S)$

- index nested loop join:
 - build index on all join fields of inner relation
 - e.g. if S is inner, build index on $(S.j, S.l)$
- sort-merge join:
 - sort both relations on combined join fields
 - e.g. sort R on $(R.i, R.k)$, sort S on $(S.j, S.l)$
- hash-join:
 - use multi-attribute hashing on combined join fields
- other methods are essentially unchanged

Join Summary

85/87

No single join algorithm is superior in some overall sense.

Which algorithm is best for a given query depends on:

- sizes of relations being joined, size of buffer pool
- any indexing on relations, whether relations are sorted
- which attributes and operations are used in the query
- number of tuples in S matching each tuple in R

Choosing the "best" join algorithm is critical because the cost difference between best and worst case can be very large.

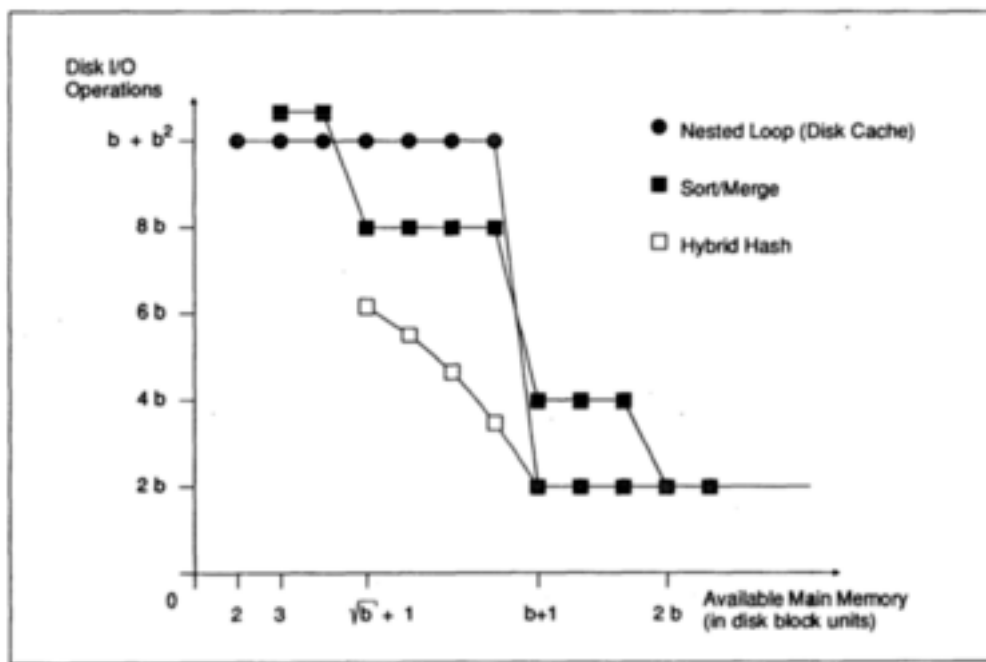
E.g. $\text{Join}[id=stude](\text{Student}, \text{Enrolled})$: 3,000 ... 2,000,000

In some cases, it may be worth modifying access methods "on the fly" (e.g. add index) to enable an efficient join algorithm.

... Join Summary

86/87

Comparison of join costs (from Zeller/Gray VLDB90, assumes $b_R = b_S = b$)



Join in PostgreSQL

87/87

Join implementations are under: [src/backend/executor](#)

PostgreSQL supports three kinds of join:

- nested loop join ([nodeNestloop.c](#))
- sort-merge join ([nodeMergejoin.c](#))
- hash join ([nodeHashjoin.c](#)) (hybrid hash join)

Query optimiser chooses appropriate join, by considering

- physical characteristics of tables being joined
- estimated selectivity (likely number of result tuples)