HOMEWORK 3

Question 1:

+ bowling balls are sporting equipment:

  $\forall x\ ball(x) \wedge bowling(x) \longrightarrow equipment(x, sporting)$

+ All dosmeticated horses have an owner:

  $\forall x\ (horse(x) \wedge dosmeticated(x) \rightarrow \exists y\ Owner(x, y))$

+ the rider of a horse can be different than the owner.

  $\exists x\ rider(horse, x) \wedge \neg owner(horse, x)$.

+ horses move faster than frogs

  $\forall x, y\ horse(x) \wedge frog(y) \rightarrow isfaster(x, y)$

+ a finger is any digit on a hand other than the thumb.

  $\forall x\ finger(x) \leftrightarrow digit(x) \wedge \neg isthumb(x)$

+ an isosceles triangle is defined as a polygon w 3 edges, which are connected at 3 vertices, where 2 (but not 3) edges have the same length.

  $\forall a\ triangle(a, isosceles) \longrightarrow \exists x, y, z\ edge(x) \wedge edge(y) \wedge edge(z) \wedge connected(x, y) \wedge connected(y, z) \wedge connected(x, z) \wedge equal(x, y) \wedge \neg equal(x, z) \wedge \neg equal(y, z)$

Question 2:

$\forall x\ person(x) \wedge [\exists z\ petOf(x, z) \wedge \forall y\ petOf(x, y) \rightarrow dog(y)] \rightarrow doglover(x)$.

$\neg [\forall x\ person(x) \wedge [\exists z\ petOf(x, z) \wedge \forall y\ petOf(x, y) \rightarrow dog(y)]]\ \vee\ doglover(x)$

$\approx\ \exists x\ \neg person(x) \vee [\forall z\ \neg petOf(x, z) \vee \exists y\ \neg(petOf(x, y) \rightarrow dog(y))]\ \vee\ doglover(x)]$

$\approx\ \exists x\ \exists z\ \forall y\ [(\neg person(x) \vee \neg petOf(x, z) \vee \neg(petOf(x, y) \rightarrow dog(y))) \vee doglover(x)]$

$\approx\ \exists x\ \exists z\ \forall y\ [(\neg person(x) \vee \neg petOf(x, z) \vee (\neg petOf(x, y) \vee dog(y))) \vee doglover(x)]$

$\approx\ \exists x\ \exists z\ \forall y\ [(\neg person(x) \vee \neg petOf(x, z) \vee \neg petOf(x, y) \vee dog(y) \vee doglover(x)))]$

$\approx\ \exists x\ \forall y\ [(\neg person(f(z, y)) \vee \neg petOf(f(z, y), z) \vee \neg petOf(f(z, y), y) \vee dog(y) \vee doglover(f(z, y)))]$

. Question 3:

- owes(owner(X), citibank, cost(X))   owes(owner(ferrari), Z, cost(Y))

  unifiable ✓ u={X/Y, X/ferrari, Z/citibank}.

- gives(bill, jerry, book21)   . gives(X, brother(X), Z)

  not unifiable ✓ because you can't replace constant with function.

- opened(X, result(open(X), s0))) and opened(toolbox, Z)

  unifiable ✓ u={X/toolbox, result(open(X), s0)/Z}

Question 4:

(a) Marcus is a Pompeian: Pompeian (Marcus) (1)

All pompeians are Romans: $\forall x$ Pompeian$(x) \rightarrow$ Romans(x) (2)

Ceasar is a ruler: Ruler (Ceasar). (3)

All Romans are either loyal to Caesar or hate Caesar (not both)

$\forall x$ Romans$(x) \rightarrow$ Loyal$(x, Caesar) \wedge \neg H(x, Caesar) \wedge \forall y (L(x,y) \vee H(x,y)) \wedge y \neq Caesar))$. (4)

Everyone is loyal to someone:

$\forall x$ person$(x) \rightarrow \exists y$ loyal(x, y). (5)

People only try to assassinate rulers they are not loyal to:

$\forall x,y$ person$(x) \wedge$ ruler$(y) \wedge$ assasinate$(x,y) \wedge \neg$loyal(x,y) (6)

Marcus tries to assassinate Caesar:

assasinate (Marcus, Caesar). (7)

(b) Prove that Marcus hates Caesar by Natural Deduction.

8. Romans (Marcus) (MP, 1, 2) $\theta = \{x / Marcus\}$
9. Loyal (Marcus, Caesar) v Hates (Marcus, Caesar) (MP, 4, 8) $\theta = \{x / Marcus\}$
10. $\exists y$ Loyal (Marcus, y) (5, existential) $\theta = \{x / Marcus\}$
11. $\neg$Loyal (Marcus, Caesar) (MP, 6, 7) $\theta = \{x / Marcus, y / Caesar\}$
12. Hates (Marcus, Caesar) (MP, 9, 11) $\theta = \{x / Marcus\}$

(c) Convert all the sentences into CNF. (need to do Skolem)

* Convert to conjunctive form:

- P(marcus) : $\neg$ Marcus v P(x) (1)
- $\forall x$ $P(x) \rightarrow R(x)$: $\neg P(x) \vee R(x)$ (2)
- Ruler(Caesar): $\neg$Causar v Ruler(x) (3)
- $\neg R(x) \vee L(x, Caesar) \vee H(x, caesar)$ (4)
- $\forall x \exists y\ L(x,y)$ / $L(x, f(x))$ (5) $\rightarrow$ Skolem
- $\forall x \forall y (\neg A(x,y) \vee \neg L(x,y)) \wedge (\neg A(x,y) \vee R(y)) \wedge (\neg A(x,y) \vee R(x))$ (6)
- A (marcus, causar) (7)

(d) Prove that Marcus hates Caesar using Resolution Refutation.

Negation: $\neg$ Hate(Marcus, Caesar)

(8) $\neg$Roman(Marcus) v Loyal(Marcus, Caesar)

(9) Loyal (Marcus, Caesar) V $\neg$Pompeian (Marcus)

(10) Loyal (Marcus, Caesar)

(11) $\neg$Ruler(Caesar) V $\neg$ Assasinate (Marcus, Caesar)

(12) $\neg$ Ruler (Caesar) $\rightarrow$ Contradiction $\rightarrow$ Marcus hates Caesar.

. Question 5:

a. Map-Coloring:

. Assume that there are five states: WA, NT, SA, Q, NSW.

. Assume these neigboring states: WA & NT, WA & SA, NT & SA, NT & Q, SA & Q, SA & NSW, Q & NSW

KB= { $\forall c$ color(c), $\forall s$ state(s), $\forall s\ \exists! c$ color(s,c), adjacent(WA,NT), adjacent(WA,SA), adjacent(NT,SA),
adjacent(NT,Q), adjacent(SA,Q), adjacent(SA,NSW), adjacent(Q,NSW),
$\forall s_1 \forall s_2 (adjacent(s_1,s_2) \rightarrow (\forall c_1 \forall c_2 ((color(s_1,c_1) \wedge color(s_2,c_2)) \rightarrow \neg(c_1=c_2))))$}

b. Sammy's Sport Shop

Facts: { obs(1,Y), obs(2,W), obs(3,Y), label(1,W), label(2,Y), label(3,B) }

. $\forall x$ Contains(x,W) $\vee$ Contains(x,B)

1. box1-3 represents 3 boxes
2. W,Y,B: white, yellow, both

Predicates: Contains(x,c) → x contains balls color c.

obs(x,c): observe box have balls color c.

label(x,c): label box have balls color c.

→ each box contains either y, w, both.

Axioms: { $\forall b_1, b_2, b_3$ (contains(b_1,W) $\vee$ contains(b_2,Y) $\vee$ contains(b_3,B))

$\forall b$(contains(b,B) → ($\neg$(contains(b,W) $\wedge$ contains(b,Y)) $\wedge$ $\neg$(contains(b,W) $\vee$ contains(b,Y) $\vee$ contains(b,B))))

→ unique color for each box

→ label wrong

$\forall x,c$ label(x,c) → $\neg$contains(x,c),

facts from hw2 → obs(1,Y), obs(2,W), obs(3,Y), label(1,W), label(2,Y), label(3,B) }.

c. Wumpus World:

. Room(x,y): room @ location (x,y)

. Pit(x,y): a pit @ (x,y)

. Wumpus(x,y): wumpus @ (x,y)

. Gold(x,y): Gold @ (x,y)

. Adjacent(x_1,y_1,x_2,y_2): rooms are adjacent.

. Safe(x,y): safe to explore.

KB: { $\forall x,y$ (Pit(x,y) → $\exists x_2,y_2$(Adjacent(x,y,x_2,y_2) $\wedge$ Breeze(x_2,y_2))),
$\forall x,y$ (Wumpus(x,y) → $\exists x_2,y_2$(Adjacent(x,y,x_2,y_2) $\wedge$ Stench(x_2,y_2))),
$\forall x,y$ (Gold(x,y) → Glitter(x,y)), $\forall x,y$ (Safe(x,y) → $\neg$Pit(x,y) $\wedge$ $\neg$Wumpus(x,y)),
$\forall x,y$ $\neg$(Pit(x,y) $\wedge$ Wumpus(x,y)), $\forall x,y,z$ (Adjacent(x,y,z,y) → $\neg$Pit(x,y) → Breeze(z,y)),
$\forall x,y,z$ (Adjacent(x,y,z,y) → $\neg$Wumpus(x,y) → Stench(z,y)),
$\forall x,y$ Safe(x,y) → ($\exists z$ Adjacent(x,y,z,y) → Safe(z,y)) $\wedge$ ($\exists z$ Adjacent(x,y,x,z) → Safe(x,z)). }.

(d) A queen can only be in 1 row + column :

$\forall r\ \forall c\ \forall c' ((queen(r,c) \wedge queen(r,c') \wedge c \neq c') \rightarrow False)$

$\forall r\ \forall r'\ \forall c\ ((queen(r,c) \wedge queen(r',c) \wedge r \neq r') \rightarrow false)$

2. No two queens can be on the same diagonal:

$\forall r\ \forall c\ \forall r'\ \forall c'\ (((queen(r,c) \wedge queen(r',c')) \wedge abs(r-r') = abs(c-c')) \rightarrow false)$

3. There must be exactly 4 queens in total:

$\exists r_1\ \exists r_2\ \exists r_3\ \exists r_4\ (queen(r_1, col(1)) \wedge queen(r_2, col(2)) \wedge queen(r_3, col(3)) \wedge (queen(r_4, col(4))).$