Calculating Biological Quantities

CSCI 2897

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Lecture 2

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Lecture 2 Plan

1. **One minute review of the basics:**
    1. Website
    2. Syllabus
    3. Canvas
    4. Slack
2. **Office Hours?**
3. **Asking “modeling” questions**
4. **Some vocabulary**
5. **Steps to modeling a biological problem (1-4)**

Last Time on CBQ…

- Website: https://github.com/dblarremore/CSCI2897
  - Homework & reading posted, Code examples, Class notes
- Syllabus: https://github.com/dblarremore/CSCI2897#syllabus
- Canvas: Turn in homework, Check grades
- Slack: **Didn't get the invite? Stick around after class—we'll get you set up!**
- Textbook: See Slack.

- First assignment already posted on Canvas — due Tuesday. [Easy!]

The Quiz

Universe	Votes
Star Wars ❌	6 (4)
Star Trek ✅	2 (4)
Marvel ❌	2
IDK ❌	1

Thornton
Colorado Springs (2)
Durango
Fort Collins
Highlands Ranch
Wheat Ridge

Normal, IL
Dearborn, MI
Austin, TX
San Francisco, CA
Fremont, CA

"Boring AF"
"Suburbia Hell but in the middle of nowhere"
"Really fun because I love the outdoors"
"Great!"
"Hot, but alright"

"A very nice place with world class mountain biking."
"Surfing & traveling"
"Got a bit boring by the time I was 16"
"I am going to be brutally honest — it's kinda boring.

Dynamical Models 101: Ask a question

- Think about a problem that puzzles you.
- Draw a diagram that illustrates the various processes at work.
- *Dynamical* models describe how a system changes over time.

Models, Vocab, and 7 Steps

Deterministic vs Stochastic dynamical models

- **Deterministic** models assume that the future is entirely predicted (i.e. determined) by the model.

- **Stochastic** models assume that random (stochastic) events affect the system.

Otto & Day: 7 steps to modeling a biological problem

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5. Analyze the equations

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6. Checks & balances

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7. Relate the results back to the question

1. Formulate the question

2. Determine the basic ingredients

- **Variables**: what entities might change over time?
- Assign a letter to each variable. (Hint: use “intuitive” letters!)
- Write down *fundamental* constraints on your variables.
- Write down *reasonable* constraints on your variables.

Discrete time vs Continuous time

- **Discrete time models**:

- **Continuous time models**:

- **Note:**

Be clear about your time scale

- **Time scale**: the unit of time between $t = 0$ and $t = 1$.
  - How much time is in the *tick of the clock?*
- **Discrete time models**:
- **Continuous time models**:

btw…

- You’ll have to decide whether your variables are discrete or continuous too!
    - Often, discrete values get **SO BIG** that you can model a discretized population using a continuous variable.
    - Sometimes, you can reinterpret a discrete variable in continuous units.
    - Why might we do this?

Equations!

Recursion Equations

- A **recursion equation** describes the value of a variable in the next time step.

  $n(t+1) =$ "some function of $n(t)$"

- Examples.

Difference Equations

- A **difference equation** describes the difference between a variable's values in two successive time steps

  $\Delta n = n(t+1) - n(t) =$ "some function of $n(t)$"

- Examples.

Differential Equations

- A **differential equation** describes the rate of change of the variable over time

$$\frac{dn(t)}{dt} = \text{"some function of } n(t)\text{"}$$

- Examples.

Intuition?

Example 1

Suppose that $\dfrac{dn(t)}{dt} = 0$

(A) Sketch the derivative $\dfrac{dn(t)}{dt}$ vs. $n(t)$.

(B) Sketch the variable $n(t)$ vs time.

Example 2

Suppose that $\dfrac{dn(t)}{dt} = 1$

(A) Sketch the derivative $\dfrac{dn(t)}{dt}$ vs. $n(t)$.

(B) Sketch the variable $n(t)$ vs time.

Example 3

Suppose that $\dfrac{dn(t)}{dt} = -k$

(A) Sketch the derivative $\dfrac{dn(t)}{dt}$ vs. $n(t)$.

(B) Sketch the variable $n(t)$ vs time.

Example 4

Suppose that $\dfrac{dn(t)}{dt} = \sqrt{n(t)}$

(A) Sketch the derivative $\dfrac{dn(t)}{dt}$ vs. $n(t)$.

(B) Sketch the variable $n(t)$ vs time.

Parameters

- The **parameters** of the model are quantities that influence the dynamics but remain fixed over time.
- Examples:

Parameters

- The **parameters** of the model are quantities that influence the dynamics but remain fixed over time.
- When we fix parameters and look at a trajectory of the equation, that's called **forward simulation** or **forward integration**. Model + Parameters → Data
- When we have data and a model, and we determine the values of the parameters that best fit the data, that's **parameter inference.** Model + Data → Parameters

- Note: parameters' units need to match the kind of model we're using.
- Note: parameters may have *reasonable* ranges in addition to *fundamental* ranges.

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Diagrams: Life Cycle

- Keep track of the events occurring during a single time step *and their order*.

Diagrams: Flow

- Keep track of the events occurring during a single time step *and their order*.

Diagrams: Table of Events

- Discrete-time models with **multiple events** per time step and **multiple variables**.

Pros and Cons?

- See Otto & Day, Chapter 2.4

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Example: tree branching

- Use the life cycle diagram to derive a recursion, and use that to create a difference equation.

Example: mouse model

- Use the life cycle diagram to derive the stages of the recursion.

Recipes: recursion & difference equations from life cycle diagrams

1. Use $n'(t), n''(t), n'''(t)$ etc to denote the variable's value after each life cycle event.

2. Set $n(t+1)$ to the value of n after the final event in the cycle.

3. Substitute, and get $n(t+1)$ in terms of $n(t)$ by eliminating $n'(t)$ etc.

4. [Bonus] Subtract $n(t)$ from both sides and simplify to get the difference equation $\Delta n = n(t+1) - n(t) = \ldots$

Example: COVID-19

- Use the flow diagram to create the recursion equations for COVID-19 spread.

Recipes: differential equations from flow diagrams

$$\frac{d(n(t))}{dt} = \ldots$$

the flow rates along arrows *entering* the circle

+ the flow rates along arrows leaving & returning to the circle

– the flow rates along arrows exiting the circle

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