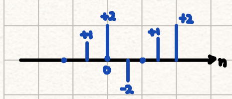
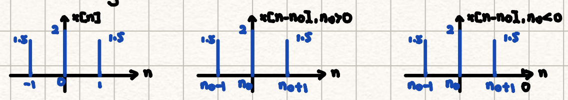
Discrete Time Signal $x[n]\ n \in \mathbb{Z}$

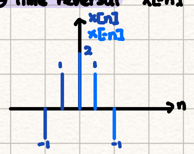


Basic Transformation of the independent variable

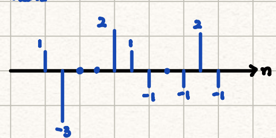
① Time shifting $x[n-n_0]\ n_0 \in \mathbb{Z}$



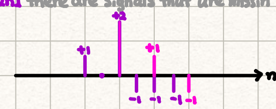
② Time reversal $x[-n]$



③ Time scaling $x[\alpha n]$



$x[2n]$ there are signals that are missin



$x[0.5n]$ there are signals that doesn't exist.

Periodicity

$x[n]$ is periodic if for every n there is an N that satisfies $x[n]=x[n+N]$. N is the period of $x[n]$. The smallest positive N is called the fundamental period. N is always an integer.

example:

Check $x[n]=\sin(10n+\frac{\pi}{3})$ for periodicity. If it is periodic, find N.

$y[n]=\sin(w_0 n+\alpha)\quad y[n]=y[n+N]$

$\Rightarrow \sin(\underbrace{w_0 n+\alpha}_{\theta})=\sin(\underbrace{w_0(n+N)+\alpha}_{\beta}) \quad \Rightarrow \beta=\theta+2\pi k$

$\Rightarrow w_0 n+w_0 N+\alpha = w_0 n+\alpha+2\pi k$

$\Rightarrow w_0 N=2\pi k \Rightarrow \frac{2\pi}{w_0}=\frac{N}{k}\ ,\ N,k\in\mathbb{Z}$

$\Rightarrow y[n]$ is periodic if $\frac{2\pi}{w_0}$ is a rational number

$N=\frac{2\pi k}{w_0}\ ,\ k\in\mathbb{Z} \Rightarrow$ fundamental period $N_0=\frac{2\pi}{w_0}$

$\Rightarrow x[n]$ is not periodic

Repeat per

$x[n]=2\cos(\frac{\pi}{4}n+2)+\sin(\frac{\pi}{8}n)-2\cos(4\pi n) = x_1[n]+x_2[n]-x_3[n]$

for $x_1[n]$ $\frac{2\pi}{w_0}=8 \rightarrow$ periodic

$N_1=\frac{2\pi k}{w_0}=8k \Rightarrow N_{0_1}=8$

$x_2[n]$ $N_2=\frac{2\pi k}{\pi/8}=16k \Rightarrow N_{0_2}=16$

$x_3[n]$ $N_3=\frac{2\pi k}{4\pi}=\frac{k}{2} \Rightarrow N_{0_3}=1$

$\Rightarrow LCM(N_{0_1},N_{0_2},N_{0_3})=16$

Energy and Power of DT signals

Energy of $x[n]$ for $n_1\le n\le n_2$: $E=\sum_{n=n_1}^{n_2}|x[n]|^2$

Power of $x[n]$ for $n_1\le n\le n_2$: $P=\frac{1}{n_2-n_1+1}\sum_{n=n_1}^{n_2}|x[n]|^2$

Total energy: $E_\infty=E_{total}=\lim_{N\to\infty}\sum_{n=-N}^{N}|x[n]|^2$

Total Power: $P_\infty=P_{total}=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x[n]|^2$

Sidenote for midterm: (to think)

Trigonometric FS

$x(t)$: Real

Real & even

Real & odd

Odd and even signals

$x[n]$ is an odd signal if $x[-n] = -x[n]$.

$x[n]$ is an even signal if $x[-n] = x[n]$.

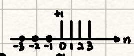
For any signal: $x[n] = x_e[n] + x_o[n]$

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

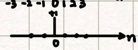
$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

Basic signals

① unit step function: $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

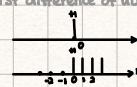


② unit impulse function: $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$



Relation between $u[n]$ and $\delta[n]$: $\delta[n] = u[n] - u[n-1]$ (the first difference of $u[n]$)

$u[n] = \sum_{m=-\infty}^{n} \delta[m]$ proof:



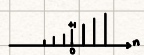
$\sum_{m=-\infty}^{n} \delta[m]$

③ Complex Exponential Signal

$x[n] = C\alpha^n$ Without loss of generality: $C=1$

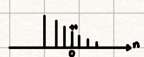
a). $\alpha \in \mathbb{R}$

1. $\alpha > 1$



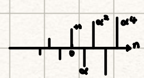
Growing Exponential

2. $0 < \alpha < 1$



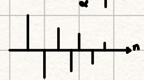
$x[n]$

3. $\alpha < -1$



$x[n]$

4. $-1 < \alpha < 0$



$x[n] = \alpha^n$

b). $\alpha = e^{j\omega_0}$

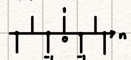
$x[n] = e^{j\omega_0 n} = \cos(\omega_0 n) + j\sin(\omega_0 n)$

$e^{j\omega_0 n}$ is periodic when $\frac{2\pi}{\omega_0}$ is rational, $N = \frac{2\pi k}{\omega_0}$, $k \in \mathbb{Z} \Rightarrow N \in \mathbb{Z}$

* $x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$

$\mathrm{Re}\{e^{j\omega_0 n}\}$, $\omega_0 = 0$ $\mathrm{Re}\{e^{j\omega_0 n}\}$, $\omega_0 = \frac{\pi}{2}$ $\mathrm{Re}\{e^{j\omega_0 n}\}$, $\omega_0 = \pi$ $\omega_0 = 2\pi$, $\cos(2\pi n)$

$\Rightarrow$



For CT complex exponential signal, increasing the frequency means a higher rate of oscillation.

For DT signals, however, increasing the frequency does not necessarily increase the rate of oscillation

The highest rate of oscillations happens at $\omega_0 = (2k+1)\pi$, $k \in \mathbb{Z}$

The lowest rate of oscillations happens for $\omega_0 = 2k\pi$, $k \in \mathbb{Z}$.

$$e^{j(\omega_0 + 2\pi)n} = e^{j\omega_0 n} e^{j2\pi n} = e^{j\omega_0 n}(\cancel{\cos(2\pi n)}^{0} + j\cancel{\sin(2\pi n)}^{0}) = e^{j\omega_0 n}$$

c). $\alpha = e^{\sigma + j\omega_0}$