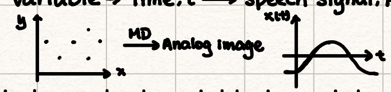
Signal : a function of an independent variable that typically carries some form of information.

continuous-time $\begin{cases} 1\text{ D} \\ \text{MD} \end{cases}$

discrete-time $\begin{cases} 1\text{D} \\ \text{MD} \end{cases}$

Continuous-time signal: a signal whose independent variable is a real number.

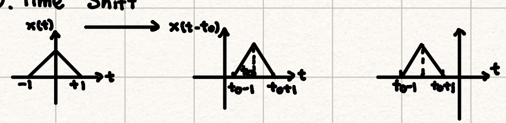
The independent variable → Time, t $\xrightarrow{1D}$ speech signal. Air pressure at different altitudes for an airplane.



discrete-time signal: a signal whose independent variable is an integer.

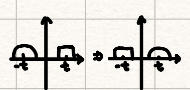
These transformations can be applied to the independent variable.

①. Time Shift

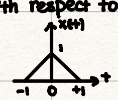


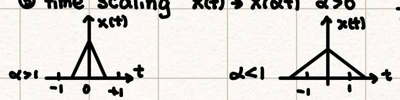
② Time reversal

x(-t) is the mirrored image of x(t) with respect to the vertical axis

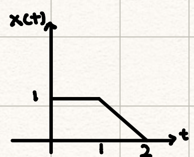


③ time scaling $x(t) \Rightarrow x(\alpha t)$ $\alpha > 0$



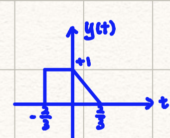


example:



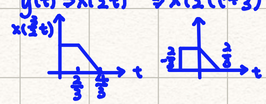
, $x(\frac{3}{2}t+1) = ?$

①. $x(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t<1 \\ -t & 1<t<2 \\ 0 & t>2 \end{cases}$ $x(\frac{3}{2}t+1) = \begin{cases} 0 & \frac{3}{2}t+1<0 \\ 1 & 0<\frac{3}{2}t+1<1 \\ -(\frac{3}{2}t+1)+2 & 1<\frac{3}{2}t+1<2 \\ 0 & \frac{3}{2}t+1>2 \end{cases}$ $\Rightarrow$ $x(\frac{3}{2}t+1) = \begin{cases} 0 & t<-\frac{2}{3} \\ 1 & -\frac{2}{3}<t<0 \\ -\frac{3}{2}t+1 & 0<t<\frac{2}{3} \\ 0 & t>\frac{2}{3} \end{cases}$

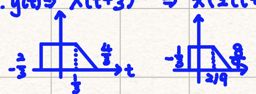


⇒ verify by choosing random numbers

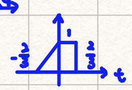
②. $y(t) \Rightarrow x(\frac{3}{2}t)$ $\Rightarrow x(\frac{3}{2}(t+\frac{2}{3}))$



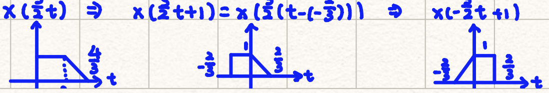
③. $y(t) \Rightarrow x(t+\frac{2}{3})$ $\Rightarrow x(\frac{3}{2}(t+\frac{2}{3}))$ ✗



Example 2: with the same figure, $y(t) = x(-\frac{3}{2}t+1) = ?$ Ans:



① $x(\frac{3}{2}t)$ $\Rightarrow$ $x(\frac{3}{2}t+1) = x(\frac{3}{2}(t-(-\frac{2}{3})))$ $\Rightarrow$ $x(-\frac{3}{2}t+1)$



$\frac{2}{3}$

time-scaling → time-shifting* → time reversal

*: considering the effect of the scaling factor.

Periodicity: $x(t)$ is a periodic signal if for every t, there is a T that satisfies $x(t) = x(t+T)$. T is called the **period**.

The smallest positive T is **the fundamental period T_0.**

e.g: $x(t) = A\sin(wt+\alpha)$ V $\qquad y(t) = \begin{cases} 0 & t<0 \\ A\sin(wt+\alpha) & t>0 \end{cases}$ X

e.g: Find the fundamental period of $x(t) = A\sin(wt+\alpha)$

$x(t) = x(t+T)$

$A\sin(wt+\alpha) = A\sin(w(t+T)+\alpha)$

$= A\sin(wt+wT+\alpha)$

$\Rightarrow \sin(\beta) = \sin(\beta + wT) = \sin(\beta + 2\pi k)$, $k \in \mathbb{Z}$.

$wT = 2\pi k \Rightarrow T = \frac{2\pi k}{w}$, $k \in \mathbb{Z}$

If $k=1$, $T_0 = \frac{2\pi}{w}$

Remark: If $x(t)$ and $y(t)$ are periodic signals with fundamental periods T_x and T_y, respectively, then $x(t)+y(t)$ is a periodic signal if $\frac{T_x}{T_y}$ is a rational number. And the fundamental period of $x(t)+y(t)$ is usually the least common multiplier (LCM) of T_x and T_y. $LCM(T_x, T_y)$

e.g: Are these signal periodic? If so, what's the period?

1. $x(t) = \cos(t) + \sin(\pi t)$

2. $y(t) = \cos(\frac{3\pi}{4}t + \frac{\pi}{6}) + \sin(\frac{2\pi}{3}t - \frac{\pi}{4})$

1. $x(t) = x_1(t) + x_2(t)$ $\quad T_{x_1} = 2\pi \quad T_{x_2} = 2 \Rightarrow$ no LMC $\Rightarrow x(t)$ is not periodic

2. $y(t) = y_1(t) + y_2(t)$ $\quad T_{y_1} = \frac{8}{3} \quad T_{y_2} = 3 \Rightarrow \frac{T_{y_1}}{T_{y_2}} = \frac{8}{9} \Rightarrow$ rational $\Rightarrow y(t)$ is periodic, $T_y = LCM(T_{y_1}, T_{y_2}) = 24$

Fundamental Frequency = 1/fundamental period = $\frac{1}{T_0}$ [Hz]

Harmonic: If $x(t) = \sin(w_0 t + \alpha)$ and $x_m(t) = \sin(m w_0 t + \alpha)$, $m \in \mathbb{Z}^+$, then the frequency of $x_m(t)$ is m times larger than that of $x(t)$, and $x_m(t)$ is referred to as the m-th harmonic of $x(t)$.

Energy and Power

Energy of $x(t)$ between t_1 and t_2 is defined as $E_x = \int_{t_1}^{t_2} |x(t)|^2 dt$.

Power of $x(t)$ between t_1 and t_2 is defined as $P_x = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} |x(t)|^2 dt$

Total energy and power of a signal are obtained when $t_1 \to -\infty$ & $t_2 \to +\infty$:

$E_{tot} = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt \qquad P_{tot} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$

Energy signal: $x(t)$ is an energy signal if E_{tot} for $x(t)$ is finite and non-zero. The total power of an energy signal is zero.

Power signal : $x(t)$ is a power signal if P_{tot} is finit and non-zero. E_{tot} of a power signal is not finite.

Remark: Periodic signals are power signals. The total power is equal to the power over period.

example: Find P_{tot} of $x(t) = A\sin(w_0 t + \alpha)$ $\Rightarrow t_1 = 0,\ t_2 = T = \frac{2\pi}{w_0}$

$$P_{tot} = \frac{1}{T}\int_0^T A^2 \sin^2(w_0 t + \alpha)\,dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1}{2} - \frac{1}{2}\cos(2w_0 t + 2\alpha)\,dt$$

$$= \frac{A^2}{2T}\left(t - \frac{1}{2w_0}\sin(2w_0 t + 2\alpha)\right)\Big|_0^T$$

$$= \frac{A^2}{2T}\left[T - \frac{1}{2w_0}\sin(4\pi + 2\alpha) - 0 + \frac{1}{2w_0}\sin(0 + 2\alpha)\right]$$

$$= \frac{A^2}{2}$$