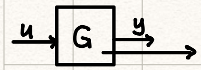
Course Review: Intro to Control System

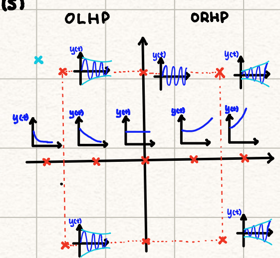


plant: model: $x \in \mathbb{R}^n$ state-space ODE: $\begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases}$

I/O: n^{th} ODE: $y^{(n)} + a_1 y^{(n-1)} + \cdots = b_0 u^{(m)} + \cdots$

linearization @(x^*, u^*): $\delta x = x - x^*$

LTI S.S model $\begin{cases} \delta\dot{x} = A\delta x + B\delta u \\ \delta y = C\cdot\delta x + D\cdot\delta u \end{cases} \overset{u,y \in \mathbb{R}}{\longleftrightarrow}$ G: L.T. SISO $\begin{cases} \dot{x} = Ax + Bu & x(0) \in \mathbb{R}^n \\ y = C\cdot x + Du & x \in \mathbb{R}^n \end{cases} \overset{\mathcal{L}}{\longleftrightarrow}$ TF model $\begin{cases} C(s) = C(s-IA)^{-1}B + D = \frac{N(s)}{D(s)} \text{ poly} \\ \text{If } x(0)=0: \\ Y(s) = G(s)\cdot U(s) \end{cases}$



Time-response y(t): pole location of $Y(s) = \mathcal{L}[y(t)]$

$$y(t) = e^{-at}\mathbb{1}(t) \overset{a>0}{\longleftrightarrow} Y(s) = \frac{1}{s+a}$$

Transient resp. spec. for 2^{nd} order step-response for $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$U(s) = \frac{1}{s} \Rightarrow Y(s) \Rightarrow y(t) \Rightarrow \%OS = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}, \quad T_s = \frac{4}{\omega_n \zeta}$$

Stability: ① If $u \equiv 0$, sys. int. asy. stable: if $\forall x(0) \in \mathbb{R}^n$: $x(t) \to 0$ as $t \to \infty$

unstable: if $\exists x(0)$ s.t. $x(t)$ is unbounded

Test: iff all eig$(A) \subset$ OLHP

② If $x(0)=0$: sys. is BIBO stable if for $\forall u$ bounded. The output y is bounded

Test: iff all poles of $G(s) \subset$ OLHP

Note: ① → ② only if there are no unstable p/z cancellation

Introduction → Tracking Problem

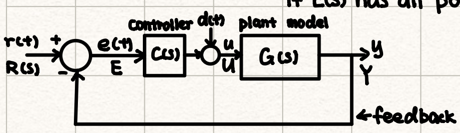
Want $y(t)$ to asy. track a desired ref. signal $r(t)$

Error: $e(t) = r(t) - y(t)$; $\lim_{t\to\infty} e(t) = e(\infty) = 0$ $\forall r$ in a certain class

Open-loop Control: based on final value theorem (FVT)

$e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) \xrightarrow{\text{finite}}$ find $\bar{u}$ s.t. $e(\infty)=0$, not robusting G or disturb.

if E(s) has all poles in OLHP or at most one pole in $s=0$



Intro to Tracking Problems

To design C ⇒ simplest: K (proportional) $u = k\cdot e$ ⇒ solves the TP example if $k \to \infty$, introduced robustness

Internal Model Principle (IMP)

TP solved iff: (i) feedback loop is BiBo stable ← use to select parameters of $C(s)$

give a structure for a candidate $C(s)$ $\begin{cases} \text{(ii) } C \cdot G \text{ has poles of } R.D \\ \text{(iii) } C \text{ has poles of } D \end{cases}$ TF: $\frac{1}{1+CG}$, no p/z unstable in CG : BiBo stable $R \to E$

To test (i) we can use:

Routh array → stability criteria: $p(s)$ = numerator of $1 + CG$

(Algebraic) roots of $p(s) \in$ OLHP → find param. of C

Nyquist stability criteria: zeros of $(1+CG) \in$ OLHP, $1 + CG \neq 0$; $L(s) = C \cdot G(s)$

(Graphical) $\Rightarrow 1 + L(s) \neq 0 \Rightarrow L(s) \neq -1$ (poles of C.L. sys. on j-axis)

Feedback loop is BiBo stable iff L does NOT pass through -1 ($-\frac{1}{K}$) and encircles -1 ($-\frac{1}{K}$) exactly $N = n$ ccw where n = # of poles of $L(s)$ in OLHP

What was missing?

Had $e(\infty) = 0$ (tracking solved). Feedback loop BiBo stable.

Steady-state performance but NO transient spec./perf.

%OS ↓ ⇔ PM ↑ = phase margin

→ freq. domain spec.

T_s ↓ ⇔ ω_c ↑ = gain crossover freq.

Nyquist not amenable for design ⇒ Bode plots are simple to use → $L(j\omega) = |L(j\omega)| \cdot e^{j \angle L(j\omega)}$ ⇒ $|L(j\omega)|_{dB} = 20\lg|L(j\omega)|$, $\angle L(j\omega)$ vs $\log \omega$

Types of C: $\begin{cases} \text{LEAD} \to \text{PD} \\ \text{LAG} \to \text{PI} \end{cases}$ & combination: loopshaping design to achieve both steady-state & transient spec.