Design guidelines are based on approximations of these indicators

* T_r = rise-time $\simeq \frac{1.8}{\omega_n} = \frac{1.8}{|p_{1,2}|}$ → magnitude of poles

** T_s = settling time $\simeq \frac{4}{\zeta\omega_n} = \frac{4}{\sigma}$ → $|Re(p_{1,2})|$

*** T_p = peak-time $= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$ → $|Im(p_{1,2})|$

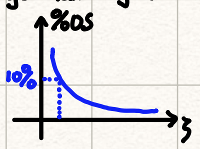
**** %OS = percent-overshoot $= \frac{y(T_p)-1}{1}\,\% = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \underset{\text{proportional}}{\propto} \frac{1}{\zeta}$

* From last lec: $y(t) = 1\!\!1(t) - \frac{1}{\sqrt{1-\zeta^2}} \cdot e^{-\sigma t} \cdot \sin(\omega_d t + \phi) \cdot 1\!\!1(t)$, where $\cos\phi = \zeta$

T_1 @ $y(T_1) = 0.1$ → no cleared-form solution

$T_r = T_2 - T_1$ ← T_2 @ $y(T_2) = 0.9$ (transcendental eqn) ⇒ rough approx. (numerically)

*** $y(t)_{max} \Leftrightarrow \dot{y}(t) = 0 \Rightarrow T_p$

**** 

Control Specifications based on these indicators

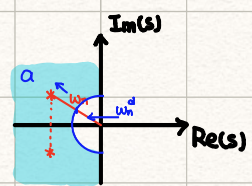
Ass. that goal is to achieve $T_r \le T_r^d$ ← desire ; $T_s \le T_s^d$; $\%OS \le \%OS^d$ ← specs.

Want to determine the admissible region where the poles of TF G should be

that will guarantee these ctrl specs. are met

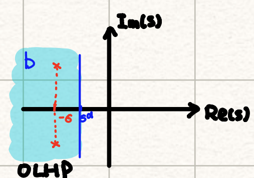
$T_r \le T_r^d \Leftrightarrow \frac{1.8}{\omega_n} \le T_r^d \Leftrightarrow \underset{|mag.(p_{1,2})|}{\omega_n} \ge \frac{1.8}{T_r^d} = \omega_n^d$

outside the semicircle of radius $\frac{1.8}{T_r^d}$, in OLHP



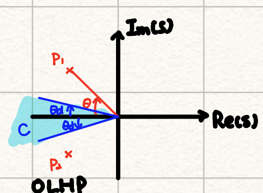
$T_s \le T_s^d \Leftrightarrow \frac{4}{\sigma} \le T_s^d \Leftrightarrow \underset{|Re(p_{1,2})|}{\sigma} \ge \frac{4}{T_s^d} = \sigma^d$

to the left of the line $-\sigma^d$



$\%OS \le \%OS^d \Leftrightarrow e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \le \%OS^d \Rightarrow \zeta \ge \frac{-\ln(\%OS^d)}{\sqrt{\pi^2 + \ln^2(\%OS^d)}} = \zeta^d$

$\cos\theta = \zeta \ge \zeta^d = \cos\theta^d \Leftrightarrow \theta \le \theta^d$: cone with max angle θ^d)



All specs are met by pole placed in the intersection of regions a. b. c

Example: $G(s) = \frac{1}{s^2+2s+2} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \cdot \boxed{\frac{1}{\omega_n^2}}$ to cancel out the top that doesn't match

$\omega_n = \sqrt{2}$

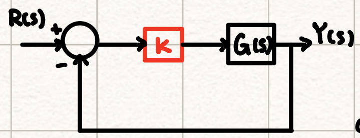
$\Rightarrow \zeta\omega_n = 1 \Rightarrow \zeta = \frac{1}{\sqrt{2}} = 0.707$

$T_s = \frac{4}{\zeta\omega_n} = 4$ sec.

$\%OS = e^{-\frac{1}{\sqrt{2}}\pi/\frac{\sqrt{2}}{2}} = e^{-\pi} = 0.04 = 4\%$

$T_r \cong \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{2}}$

Q: Can we improve these specs by using feedback?



controller: K (proportional type)

$\frac{Y(s)}{R(s)} = \frac{K\cdot G(s)}{1+K\cdot G(s)} = G_f(s)$ $\quad R(s) \to \boxed{G_f} \to Y(s)$

$G_f(s) = \frac{K}{s^2+\underbrace{2s}_{2\zeta\omega_n}+\underbrace{2+K}_{\omega_n^2}} \Rightarrow \begin{cases} \omega_n = \sqrt{2+K} \\ \zeta = \frac{1}{\sqrt{2+K}} \end{cases}$, for any $K>0,\ \zeta < \frac{1}{\sqrt{2}}$

$\Rightarrow T_s = \frac{4}{\zeta\omega_n} = 4\text{sec.}$

$\left.\begin{array}{l} \%OS > 4\% \\ T_r = \frac{1.8}{\sqrt{2+K}} < \frac{1.8}{\sqrt{2}} \end{array}\right]$ trade-off