

Transformations of Two RVs

Let's say we define two new RVs, $Z = g_1(X, Y)$ and $W = g_2(X, Y)$.

This is considered a transformation of variables.

Even though we do not know the joint distribution of Z and W , we can infer it through their relationship to X and Y . The joint CDF of Z and W is

$$F_{ZW}(z, w) = P(Z \leq z, W \leq w) = P(g_1(X, Y) \leq z, g_2(X, Y) \leq w) = \iint_{R_{ZW}} f_{XY}(x, y) dx dy$$

where R_{ZW} is some 2D region in the (x, y) plane. $R_{ZW} = \{x, y : g_1(x, y) \leq z, g_2(x, y) \leq w\}$

PDF of Linear Transformations

Let's say $Z = g_1(X, Y)$ and $W = g_2(X, Y)$ are linear transformations

$$\begin{aligned} Z &= aX + bY \\ W &= cX + dY \end{aligned} \Rightarrow \begin{bmatrix} Z \\ W \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \Rightarrow \bar{X} = \bar{A}^{-1} \bar{Z}$$

