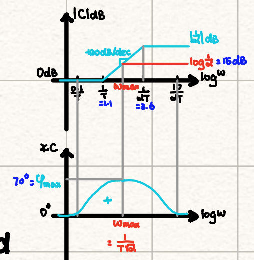
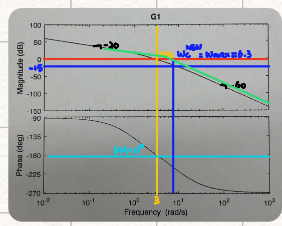
LEAD CONTROL DESIGN



Example: $G(s)=\frac{100}{s(s+1)(s+10)}$ Design $C(s)$ s.t. feedback loop is BiBo stable.

$e_{ss}=e(\infty)=0$ for step ref. input and a PMspec of $\simeq 40^\circ$ and ω_c spec of at least 3 rad/s is achieved

For e_{ss}, a K controller would work (by IMP) as long as feedback sys. is BiBo stable. If $K=1 \Rightarrow$ feedback sys. is NOT stable. I could use a $K<1$ to ↑ the PM by ↓ω_c but ω_c spec. will not be met!



We can use a LEAD controller for PM.

$\Rightarrow$ Start with $K=1$ and find $C(s)=\frac{Ts+1}{\alpha Ts+1}$, $0<\alpha<1$, $T>0$

- Want to have φ_{max} at freq. where we read the new PM, which means we want ω_{max} to be the new ω_c^{NEW}

- We will use extra margin (because $\angle G$ in decreasing as ω_c increases due to $|C|$)

(1) $\sin\varphi_{max}=\frac{1-\alpha}{1+\alpha}$

(2) $\omega_{max}=\frac{1}{T\sqrt{\alpha}}$

(3) $|C(j\omega_{max})|_{dB}=10\lg\frac{1}{\alpha}$

$\Rightarrow$ set $\varphi_{max} = \underbrace{40^\circ}_{\text{PMspec}} + \underbrace{\text{extra margin}}_{\text{typ: }\simeq 30^\circ} < 90^\circ$

$\overset{(2)}{\Rightarrow} \omega_{max}=\frac{1}{T\sqrt{0.03}}$

Use $\varphi_{max}=70^\circ \overset{(1)}{\Rightarrow} \frac{1-\alpha}{1+\alpha}=\sin 70^\circ \Rightarrow \alpha \cong 0.03 \overset{(3)}{\Rightarrow} |C(j\omega_{max})|_{dB}=10\lg\frac{1}{0.03}\cong 15\text{ dB}$

- To set ω_{max} the new ω_c^{NEW}, it means that at that freq. we need to have:

$$0\text{dB} = |G(j\omega_{max})|_{dB} + |C(j\omega_{max})|_{dB} \Rightarrow |G(j\omega_{max})|_{dB} = -15\text{dB}$$

$\Rightarrow$ we read ω_{max} from $|G|_{dB} \Rightarrow \omega_{max} \simeq 6.3$ rad/s $\Rightarrow T \cong 0.92$

Final steps would be:

- Draw Bode plot of $C\cdot G$ with the designed $C(s)$.
- Draw Nyquist plot to check feedback loop is BiBo stable.