Continuous Random Variables

We have seen discrete RVs take values from a countable set $S_X = \{0, 1, 2, \ldots\}$

Continuoust RVs have an uncountably infinite set

Example:

- Let X be any real number between 0 and 1.
- Let x be the time it takes to receive your next phone call.

We have seen the PMF for a discrete variable. $P_X(k) = P(X=k)$

However, for a continuous RV, the probability of $X=k$ is zero, i.e, each outcome has zero prob. mass.

How do we characterize continuous RVs?

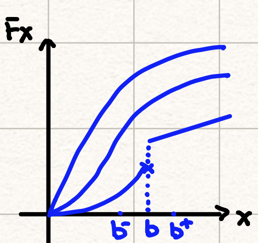
Cumulative Distribution Function (CDF)

Define the probability of the event $\{X \le x\}$: $F_X(x) = P(X \le x)$ for $-\infty < x < \infty$

$F_X(x)$ defines the probability that X takes on a value between $(-\infty, x]$

Recall the axioms of prob. and their corollaries.

1. $0 \le F_X(x) \le 1$
2. $\lim_{x \to \infty} F_X(x) = 1$
3. $\lim_{x \to -\infty} F_X(x) = 0$
4. $F_X(x)$ is a nondecreasing function. if $a < b$, then $F_X(a) \le F_X(b)$
5. $F_X(x)$ is right-continuous. For $h > 0$. $F_X(x) = \lim_{h \to 0} F_X(b+h) = F_X(b^+)$
6. $P(a < X \le b) = F_X(b) - F_X(a)$
7. $P(X=b) = F_X(b) - F_X(b^-)$
8. $P(X > x) = 1 - F_X(x)$



Example:

Let $P(X > x) = e^{-\lambda x}$ for $x > 0$. Find the CDF for X, and find $P(T < X < 2T)$ where $T = \frac{1}{\lambda}$

CDF: $F_X(x) = P(X \le x) = 1 - P(X > x) = 1 - e^{-\lambda x}$

$P(T < X \le 2T) = P(2T) - P(T) = (X - e^{-2}) - (X - e^{-1}) = 0.233$

Can we express the probability of a continuous RV, X, differently?

$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)\,dt$

$f_X(x) = \frac{d}{dx} F_X(x)$ where $f_X(x)$ is the probability density function (PDF)

Note: the CDF is defined for both continuous and discrete RVs.

$$F_X(x) = P(X \leq x) = \begin{cases} \sum_{k \leq x} p_X(x) & \text{if } X \text{ discrete} \\ \int_{-\infty}^{x} f_X(t)\,dt & \text{if } X \text{ continuous} \end{cases}$$