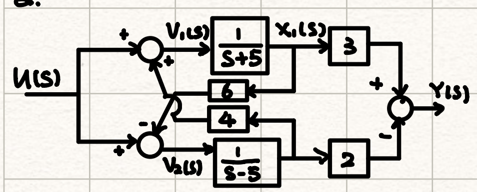
2.

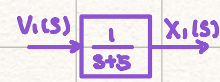


(a). s.s. model for each block w. st. var. x_1, x_2. input V_1, V_2 - for whole sys. st. $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. input u, output y

(b). sys. int. asy. stable?

(c). sys. is BiBo stable either use (a) or find direct TF $\frac{Y(s)}{U(s)} = G(s)$

(a). • Use CCF: $A=-5,\ B=1,\ C=1$



• Direct Method: $X_1(s) = \frac{V_1(s)}{s+5} \Rightarrow sX_1(s) + 5X_1(s) = V_1(s) \xrightarrow{\mathcal{L}^{-1}} \dot{x}_1 = -5x_1 + v_1$ (1) $\Rightarrow A=-5,\ B=1,\ C=1$

For 2^{nd} block: $\dot{x}_2 = +5x_2 + v_2$ (2)

Want: $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

Sub-in v_1, v_2 by a func. of u & x → using the interconnection equ. in block-diagram.

At the top summing junction we have: $v_1 = u + 4x_2$ (3)

At the bottom ... : $v_2 = u - 6x_1$ (4)

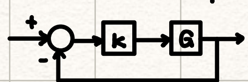
Using (3),(4) into (1),(2): $\begin{cases} \dot{x}_1 = -5x_1 + 4x_2 + u \\ \dot{x}_2 = 5x_2 - 6x_1 + u \end{cases} \Rightarrow A = \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix}\ \ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

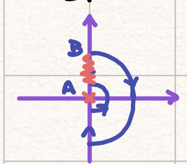
At 3^{rd} summing junc. $y = 3x_1 - 2x_2 \Rightarrow C = [3\ \ -2]\ \ D = 0$

(b). eig(A): $\det(\lambda I - A) = 0 \Leftrightarrow \lambda_{1,2} = +1, -1$ sys. int. unstable

(c). $G(s) = C(sI - A)^{-1}B = [3\ \ -2]\begin{bmatrix} s+5 & -4 \\ +6 & s-5 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{s^2-1}\cdot(s-1) = \frac{1}{s+1} \Rightarrow$ BiBo stable

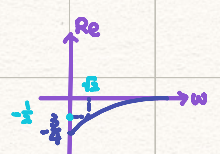
4. Nyquist plot of $G(s) = \frac{s+1}{s(s-2)}$. Find K s.t. feedback loop is BiBo stable. For $K=20$, find $e(\infty)$ for $r = t\cdot\mathbb{1}(t)$ (unit ramp)

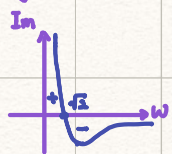


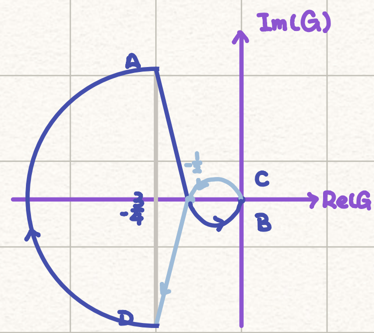


Want $n=1$, $N=1$ ccw @ $-\frac{1}{K}$

$G(j\omega) = \frac{-3}{\omega^2+4} + j\,\frac{2-\omega^2}{\omega(\omega^2+4)}$







$s = \varepsilon \cdot e^{j\theta};\ \theta: -\frac{\pi}{2} \uparrow +\frac{\pi}{2}$ (0)

rad. ∞

$G(\varepsilon e^{j\theta}) \approx \frac{1}{-2\varepsilon e^{j\theta}} = \frac{1}{2\varepsilon} e^{-j\theta-\pi}$

$\to \angle: -\theta-\pi: -\frac{\pi}{2}, -\frac{3\pi}{2}$ $(-\pi)$