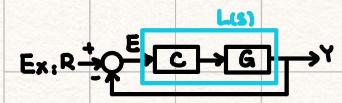
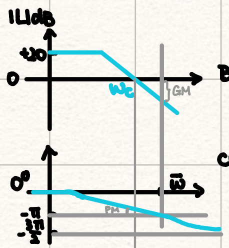
Bode Plots and Performance Specs. $\begin{cases}\text{Steady-state}\\ \text{Transient}\end{cases}$

Ex:



Last ex: $L(s) = \frac{10}{(s+1)(s+10)(s+100)}$



By Nyquist plot: $\frac{1}{1+L(s)}$ is BiBo stable ⇔ all poles in OLHP

consider $C(s)=1$ $G(s) = \frac{10}{(s+1)(s+10)(s+100)}$ Find $e_{ss} = \lim_{t\to\infty} e(t)$ when $r(t) = \mathbb{1}(t)$ and relate it the Bode plot.

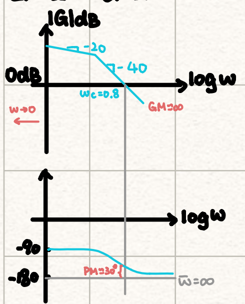
$E(s) = \frac{1}{1+C\cdot G(s)} \cdot R(s) = \frac{1}{1+L(s)} \cdot \frac{1}{s}$ ⇒ poles ∈ OLHP and 1 in s=0

⇒ can apply FVT: $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0} s \cdot \frac{1}{1+L(s)} \cdot \frac{1}{s} = \frac{1}{1+\underbrace{L(0)}_{\text{D.C gain}}} = \frac{1}{1+10} \approx \frac{1}{10} = 0.1 = 10\%$

Bode plot: $|L(0)|dB = 20dB \Leftrightarrow |L(0)| = 10$

It's relation b/w steady-state err. of C.L. system to the D.C gain of O.L system (Bode plot)

Ex 2: $G(s) = \frac{0.8}{s(s+1/2)}$; $C(s) = 1$; $|G(0)| = \infty$

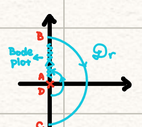


$|G(0)|dB = \infty$ $e_{ss} = \frac{1}{|G(0)|} = 0$

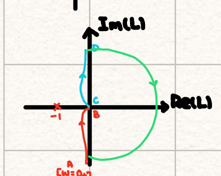
Design C(s) s.t. when $r(t) = \mathbb{1}(t)$ $\overset{R(s)=\frac{1}{s}}{}$ $e_{ss} = 0$ (asy. tracking of step) and a certain $\overset{\text{transient specs.}}{\%OS \ \& \ T_s}$ are met in step response $y(t)$

If only asy. track spec was imposed, we could use IMP to design C(s)

$C(s) = 1$ satisfies the IMP as long as feedback loop is BiBo stable. We check that based on Nyquist plot.



⇒ n=0 (#ORHP poles in CG) N=0 (encircle of -1 critical pt.)



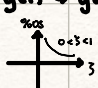
at ×: $s = \varepsilon e^{j\theta}$; $\theta = -\frac{\pi}{2} \nearrow_{(0)} +\frac{\pi}{2}$ ⇒ $G(s) \approx \frac{0.8}{\varepsilon e^{j\theta}\cdot\frac{1}{2}} \approx \frac{1}{\varepsilon} e^{-j\theta}$ → semicircle w ∞ radius $(-\theta: +\frac{\pi}{2} \searrow_{(0)} -\frac{\pi}{2})$

(θ goes from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ through 0)

⇒ N=0 ⇒ Feedback loop is BiBo stable

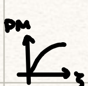
$e_{ss} = 0$ $e(t) = r(t) - y(t)$ ⇒ $y(t) \to 1$, $t \to \infty$

2nd order syst. spec $\begin{cases} \%OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \\ T_s \approx \frac{4}{\omega_n \zeta} \end{cases}$

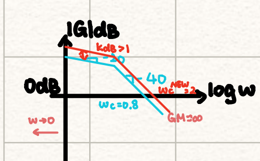


$\frac{CG}{1+CG} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

As guideline: $PM \propto \zeta$



$T_s \propto \frac{1}{\omega_c}$ → gain cross-over freq (where $|L|dB = 0dB$)



⇒ instead of %OS, T_s specs we use $\begin{cases} PM \\ \omega_c \end{cases}$ as specs on top of s.s. specs. (DC gain L(0))

where PM ≃ 40~45% desired $\omega_c^{NEW} \approx 2$ ⇒ $T_s \downarrow$ half (in ex 2) ⇒ $PM^{NEW} \approx 10°$