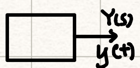
Pole location and Transient Time-Response



$Y(s)$ can be decomposed via PFE into $\sum_i \frac{c_i}{s-p_i}$ (p_i = poles of $Y(s)$)

We could have poles:

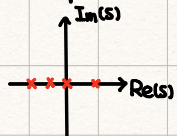
(a). real poles

(b). a pair of complex conjugate poles

(c). ==repeated== real or complex conjugate poles

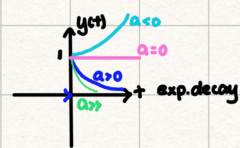
we focus on (a). (b).

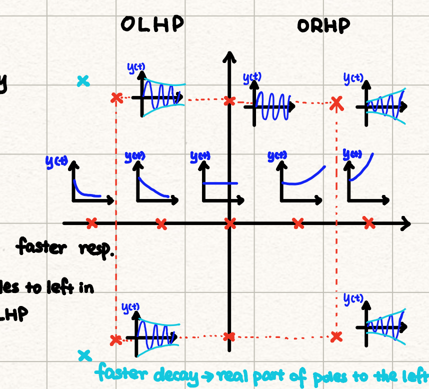
(a). Real pole: $Y(s) = \frac{1}{s+a}$, $a \in \mathbb{R}$



pole: $p = -a$

$y(t) = e^{-at} \cdot 1\!\!1(t)$





$a>0$ $\quad \tau = \frac{1}{a}$ = time-constant, $a \gg \Leftrightarrow \tau \ll$ (faster decay)

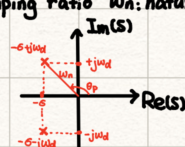
$a<0$ $\quad$ expo. increasing response. (blowing up)

$a=0$ $\quad$ step response

(b). A pair of complex conj. poles: $Y(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ $\quad \zeta$: damping ratio ω_n: natural frequency

$$= \frac{\omega_n^2}{(s+\underbrace{\zeta\omega_n}_{\sigma})^2+\underbrace{\omega_n^2(1-\zeta^2)}_{\omega_d^2}}$$

$$= \frac{\omega_n^2}{(s+\sigma)^2+\omega_d^2}$$

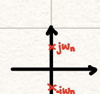


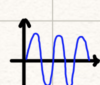
$\Rightarrow$ poles: $p_{1,2} = -\sigma \pm j\omega_d \Rightarrow y(t) = \frac{\omega_n^2}{\omega_d} \underbrace{e^{-\sigma t}}_{\text{like an expo. envelope}} \sin\omega_d t \cdot 1\!\!1(t)$

$\begin{cases} \sigma = \zeta\omega_n \\ \omega_d^2 = \omega_n^2(1-\zeta^2) \end{cases}$ $= re^{j\theta_p}$ $\begin{cases} r = \omega_n \\ \cos\theta_p = \frac{-\sigma}{\omega_n} = -\zeta \end{cases}$

Diff. cases:

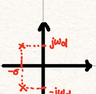
$\zeta = 0$ (undamped): $\begin{cases} \sigma = 0 \\ \omega_d^2 = \omega_n^2 \end{cases}$

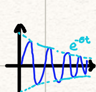




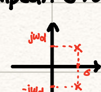
$y(t) = \omega_n \sin\omega_n t \; 1\!\!1(t)$

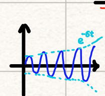
$0<\zeta<1$ (underdamped): $\sigma>0$





$-1<\zeta<0$: $\sigma<0$





$\zeta = 1$ $\quad \frac{\omega_n^2}{(s+\omega_n)^2}$ (double pole)