$L(j\omega) \Rightarrow$ frequency response $= Re(L) + j\,Im(L) = |L(j\omega)|\,e^{j\measuredangle L(j\omega)}$

Bode Plots mag & phase plots vs. freq. $\omega : [0, +\infty)$

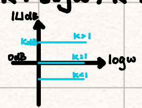
$$\begin{cases} |L(j\omega)|_{dB} = 20\log|L(j\omega)| \text{ vs. } \omega \text{ on log-scale.} \\ \measuredangle L(j\omega) \text{ vs. } \omega \text{ on log-scale} \end{cases}$$

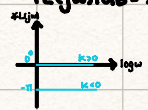
Note: $L(s) = \underbrace{C(s)}_{\text{controller to be designed}} \cdot \underbrace{G(s)}_{\text{plant}} \Leftrightarrow L(j\omega) = C(j\omega)\cdot G(j\omega)$ (multiplication) $\Rightarrow \begin{cases} |L|_{dB} = 20\log|C(j\omega)|\cdot|G(j\omega)| = |C|_{dB} + |G|_{dB} & \text{addition} \\ \measuredangle L(j\omega) = \measuredangle C(j\omega) + \measuredangle G(j\omega) & \text{addition} \end{cases}$

Review of Bode Plots for elementary terms : ex: $L(s) = \frac{2(s+1)}{s(s+10)} \Rightarrow \begin{cases} |L|_{dB} = |2|_{dB} + |s+1|_{dB} - |s|_{dB} - |s+10|_{dB} \\ \measuredangle L = \measuredangle 2 + \measuredangle(s+1) - \measuredangle s - \measuredangle(s+10) \end{cases}$

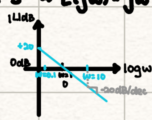
↳ for O.L. stable plants with poles @ $s=0$ only.

$L(s) = K$: $s = j\omega$; $K > 0$ $\quad |L(j\omega)|_{dB} = 20\log K \Rightarrow |L|_{dB} = 0\text{ dB} \Rightarrow |L| = 1 \quad \measuredangle L(j\omega) = \measuredangle K$

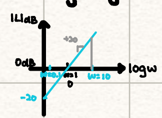


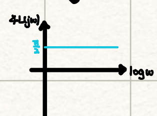


$L(s) = \frac{1}{s} \Leftrightarrow L(j\omega) = \frac{1}{j\omega} \Rightarrow |L(j\omega)| = \frac{1}{\omega} \quad \measuredangle L(j\omega) = -\frac{\pi}{2} \Rightarrow |L|_{dB} = -20\log\omega \rightarrow$ line w slop 20 dB/dec

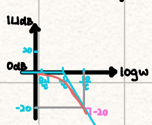


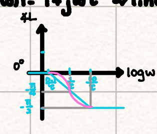
$L(s) = s \Leftrightarrow L(j\omega) = j\omega \Rightarrow |L(j\omega)| = \omega \quad \measuredangle L(j\omega) = \frac{\pi}{2} \Rightarrow |L|_{dB} = 20\log\omega$



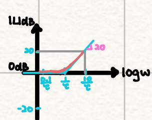


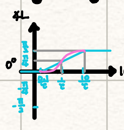
$L(s) = \frac{1}{1+s\tau}$; $\tau > 0$ pole at $-\frac{1}{\tau} \Rightarrow |L(j\omega)| = \frac{1}{1+j\omega\tau} \Rightarrow$ linear approx for 2 regions : $\begin{cases} \omega \gg \frac{1}{\tau} : L(j\omega) = \frac{1}{j\omega\tau} \Rightarrow |L|_{dB} = -20\log\omega - 20\log\tau & \measuredangle L = -\frac{\pi}{2} \\ \omega \ll \frac{1}{\tau} : L(j\omega) \simeq 1 \Rightarrow |L|_{dB} = 0\text{ dB} & \measuredangle L = 0° \end{cases}$





$L(s) = 1 + s\tau$; $\tau > 0$ zero at $-\frac{1}{\tau} \Rightarrow L(j\omega) = 1 + j\omega\tau \Rightarrow \begin{cases} \omega \ll \frac{1}{\tau} : |L|_{dB} \simeq 1 & \measuredangle L \simeq 0 \\ \omega \gg \frac{1}{\tau} : |L|_{dB} = 20\log\omega + 20\log\tau & \measuredangle L = \frac{\pi}{2} \end{cases}$





$|L(j0)| = |L(0)| = $ DC gain : pole @ $-\frac{1}{\tau} \rightarrow \frac{1}{1+s\tau} \Rightarrow$ DC gain $= 1$

pole @ $0 \rightarrow \frac{1}{s} \Rightarrow$ DC gain $= 0$