Probability Density Function (PDF)

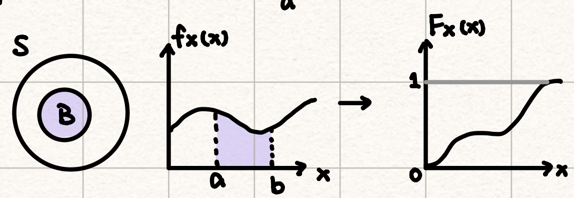
Consider this: the mass of some volume is given by the integral of the density over that volume. However, any single point in space has zero mass.

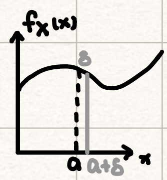
$F_X(x) = P(X \leq x) = \int_{-\infty}^{\infty} f_X(t)\,dt \qquad f_X(x) = \frac{d}{dx} F_X(x)$

If we integrate the PDF over some event, we will find the probability mass for that event:

$$P(X \in B) = \int_B f_X(x)\,dx \qquad B: \{a \leq X \leq b\}$$

Example: $P(a \leq X \leq b) = \int_a^b f_X(x)\,dx$





for a small δ. $P(a \leq X \leq a+\delta) = \int_a^{a+\delta} f_X(t)\,dt \approx f_X(a)\,\delta \approx 0$

$P(a \leq X \leq b) = P(a < X < b)$

The integral of the PDF over the entire line is $P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f_X(x)\,dx = 1$

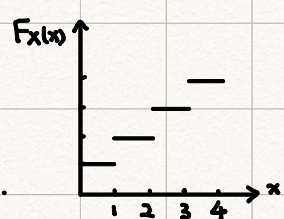
Example: Recall our previous example: $P(X > x) = e^{-\lambda x}$ for $x > 0$

CDF: $F_X(x) = P(X \leq x) = 1 - e^{-\lambda x}$ for $x \geq 0$

PDF: $f_X(x) = F_X'(x) = \lambda e^{-\lambda x}$ for $x \geq 0$

Types of RVs:

- Discrete RVs
  - ↳ Probability defined by the PMF
  - ↳ CDF is a "staircase" function of discrete x.



$F_X(x) = P(X \leq x)$

- Continuous RVs
  - ↳ CDF is continuous for all values of x, $P(X = x) = 0$
  - ↳ The CDF is described as the integral of the PDF, $P(X \in B) = \int_B f_X(x)\,dx$
- Mixed type RVs
  - ↳ RV with a CDF that has "jumps" on a countable set of points

↳ It also increases continuously over at least one interval in x.

↳ Consider the weight sum of both continuous and discrete RVs :

$F_X(x) = \lambda F_1(x) + (1-\lambda) F_2(x)$, where $\lambda \in (0,1)$, $F_1(x)$ and $F_2(x)$ are the CDFs of discrete and continuous RVs, respectively

Conditional CDF and PDF

If some event A is given, then the conditional CDF is $F_X(x|A) = \dfrac{P(\{X \le x\} \cap A)}{P(A)}$, $P(A) > 0$

the conditional PDF is $f_X(x|A) = \dfrac{d}{dx} F_X(x|A)$