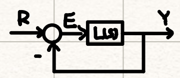
Nyquist Criteria & Stability Margins

Margins: how far we are from the onset of instability?

Consider: L = strictly proper TF no poles in ORHP or on j-axis ⇒ "n=0"



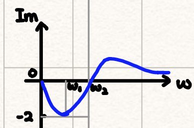
$L(s) = C(s)\cdot G(s)$ = O.L. TF.

(K=1, unit feedback loop) Apply Nyquist criteria to determine if C.L (feedback loop) is BiBo stable and introduce Stability Margins.

Assume: we have the poles of $Re(L(j\omega))$, $Im(L(j\omega))$

$s = re^{j\theta}$, $r \to \infty$, $\theta: \frac{\pi}{2} \to -\frac{\pi}{2}$ $\mathcal{D}_r$ = Nyquist contour

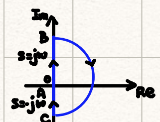
Want to obtain Nyquist plot $\mathcal{L} = \{L(s) \mid s \in \mathcal{D}_r\}$ and apply Nyq. criteria:



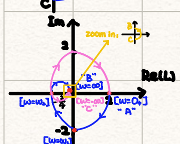
Feedback loop (C.L) system is BiBo stable if and only if:

i). $\mathcal{L}$ DOES NOT pass thru critical point -1

ii) $\mathcal{L}$ encircles -1 point N = 0 ccw. ⇒ $\mathcal{L}$ DOES NOT encircle -1



The two plots corespond to L(s) for $s \in AB$



Nyquist plot of L(s)

For $s \in CA$, use $L(s) = L(\bar{s})$, the image of CA can be obtained as the reflection across Re-axis of the image of AB.

For $s \in BC$ semicircle $L(s) = \frac{N(s)}{D(s)}$, e.g: $\frac{s+1}{(s+3)^2}$; $s = r\cdot e^{j\theta}$, $r \to \infty$, $s \simeq 0$ in the L-plane

Revisit: Why is it -1 a critical point? What if (i) is NOT met?

i.e. $\mathcal{L}$ passes thru -1 ⇒ $L(s) = -1$ for some $s \in \mathcal{D}_r$ ($s \in$ j-axis, $s \in$ Big semicircle) → here $L(s) \simeq 0$

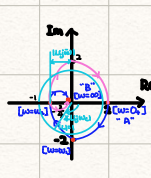
⇒ $1 + L(s) = 0$ for some $s \in$ j-axis ⇒ $\frac{1}{1+L(s)} = \pm\infty$ for some $s \in$ j-axis

⇒ C.L. TF: $\frac{1}{1+L(s)}$ has a pole on j-axis → ILLEGAL for BiBo stability

⇒ How far we are from -1 tells us how far we are from the onset of instability

We want to measure how far we are: pass thru it: $L(j\omega) = 1$ for some ω ⇒ $\begin{cases} |L(j\omega)| = 1 \\ \measuredangle L(j\omega) = -\pi \end{cases}$ for some ω

⇒ First: assume $|L(j\omega)| = 1$ for some ω_c ⇒ Secondly, assume $\measuredangle L(j\omega) = -\pi$ for some $\bar{\omega}$



Let: $\measuredangle L(j\omega_c) - (-\pi) =$ PM (phase margin) $= \measuredangle L(j\omega_c) + \pi$

Let: $|L(j\bar{\omega})|$ mgn. for $\bar{\omega}$

and gain margin $GM = \frac{1}{|L(j\bar{\omega})|} = \frac{4}{3}$