1). $X \sim N(m, \sigma^2)$

pdf: $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-m)^2}{2\sigma^2}\right\}$ $\quad E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx = m$ $\quad VAR[X] = \sigma^2$

CDF: $F_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{x} e^{\frac{-(t-m)^2}{2\sigma^2}}\,dt$

2) $Z \sim N(0,1)$ $\quad Z = \frac{x-m}{\sigma}$

$\Phi_Z = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{t^2}{2}}\,dt$ $\quad \Phi_Z(-z) = 1 - \Phi_Z(z)$ $\quad Q(z) = \Phi_Z(-z)$

3) Rayleigh $\quad y = ax+b$

X $\quad$ pdf: $f_X(x) = \begin{cases} \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} & x>0 \\ \phi & \text{otherwise} \end{cases}$

CDF: $F_X(x) = \int_0^x f(x)\,dx = 1 - e^{\frac{-x^2}{2\sigma^2}}$

for $\sigma=1$, $f_X(x) = \begin{cases} x e^{-x^2/2} & x>0 \\ 0 & \text{otherwise} \end{cases}$

$F_X(x) = \begin{cases} 1 - e^{-x^2/2} & x>0 \\ 0 & \text{otherwise} \end{cases}$

Example: Production Line Manufacture

$1\,k\Omega$ resistors that have 10% tolerance. Let X denote the resistance. Assuming that $X \sim N(100, 2500)$

Find the prob. that a resistor picked at random will be rejected.

A: resistor is rejected $\Rightarrow A = \underbrace{\{X<900\}}_{B} \cup \underbrace{\{X>1100\}}_{C}$

B & C mutually exclusive because $B \cap C = \phi$

$P(A) = P(B) + P(C) = \underbrace{P(X<900) + P(X>1100)}_{F_X(900) + [1-F_X(1100)]}$

$X \sim N(1000, 2500)$

$Z \sim N(0,1)$: $Z = \frac{X-1000}{50}$, $\quad x=900 \quad z=-2$, $\quad x=1100 \quad z=2$

$P(A) = F_X(900) + [1-F_X(1100)] = \Phi(-2) + [1-\Phi(2)] = 2\Phi(-2) = 2Q(2) = 0.045$

Example: The radial miss distance (in meters) of the landing point of a drone from the centre of the target one is known to be Rayleigh distributed with $\sigma^2 = 100$

a). Find the prob. that the drone will land within a radius of 10 m from the centre of the target area.

b). Find the radius r such that the probability that $X>r$ is e^{-1}. $[e^{-1}] \approx 0.368$

a). $P(A) = P(X<10) = F_X(10) = 1 - e^{\frac{-x^2}{2\sigma^2}} = 1 - e^{\frac{-100}{2\times100}} = 1 - e^{-\frac{1}{2}} = 0.393$

b). $P(X>r) = 1-P(x<r) = 1-(1-e^{\frac{-x^2}{2\sigma^2}}) = e^{\frac{-r^2}{200}} = e^{-1} \Rightarrow \frac{-r^2}{200} = -1 \Rightarrow r=\sqrt{200}$