Bernoulli Trials

A Bernoulli trial is any random experiment with only 2 possible outcomes.

- Flipping a coin: H or T

- Will Tesla become insolvent during the next 12 months

"success" with probability P

"failure" with probability $1-P = q$

Binomial Probability Law

Consider a sequence of independent Bernoulli trialls.

$$P(A_1 \cap A_2 \cap A_3 \cdots A_n) = P(A_1) P(A_2) \cdots P(A_n)$$

Let X be the number of successes in n trials. The probability of X is given by the binomial prob. law.

$$P(X=k) = \binom{n}{k} P^k (1-P)^{n-k}$$

Example: We toss an unfair coin 3 times, where $P(H) = p$. What is the probability of getting 2 heads?

$$P(\{HHT\}) = p \times p \times (1-p)$$

$$P(X=2) = P(\{HHT, HHH, THH\}) = 3p^2(1-p) = \binom{3}{2} p^2 (1-p)$$

Geometric probability laws

Consider a sequence of independent Bernoulli trials until the first success occurs.

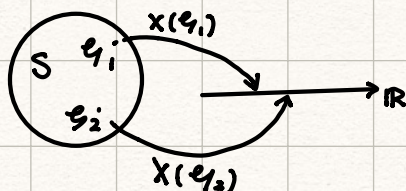Let X be the number of trials until our first success.

$$P(X=k) = (1-p)^{k-1} p \quad \text{for } k = 1, 2, \cdots$$

The probabilities of all outcomes must sum up to 1.

$$\sum_{k=1}^{\infty} P(X=k) = p \sum_{k=1}^{\infty} q^{k-1} = p \frac{1}{1-q} = 1$$
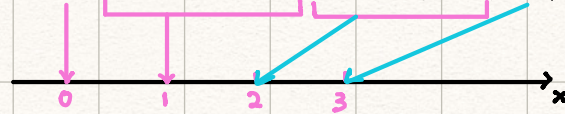
Random variables

A random variable (RV) is a function that assigns a real number to every outcome in the sample space

Example: 3 tosses of a coin

RV X = # of T's $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

- We will denote RVs with uppercase letters (e.g X)

- out-comes are denoted by lowercase letters (e.g x)