Let's say we partition S into $A_1, A_2, \cdots, A_n$. Suppose event B occurs.

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} = \frac{P(B \mid A)P(A)}{\sum_{i=1}^{n} P(B \mid A_i)P(A_i)}$$

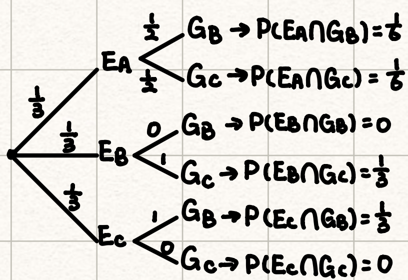
We have used the total prob. theorem to rewrite the denominator

Example: Prisoner's dilemma

There are 3 prisoner (A, B and C). One will be executed, 2 will be released. "A" asks the guard, who will be released between B and C. " B" will be released. "A" now thinks: before asking, my chance to be executed were $\frac{1}{3}$, but now they are $\frac{1}{2}$. Why is he wrong?

Let $E_i = \{\text{prisoner } i \text{ will be executed}\}$, for $i = A, B, C$

Let $G_j = \{\text{guard names prisoner } j\}$, for $j = A, B, C$



We know B will be released. What is the prob. that A will be executed?

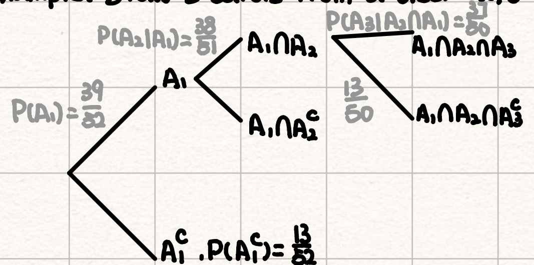
$$P(E_A \mid G_B) = \frac{P(G_B \mid E_A)P(E_A)}{P(G_B)} = \frac{\frac{1}{2} \times \frac{1}{3}}{(\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1)} = \frac{1}{3}$$

Multiplication Rule

We have seen that $P(A_1 \cap A_2) = P(A_1 \mid A_2)P(A_2) = P(A_2 \mid A_1)P(A_1)$. What if we wish to define multiple events?

$$P(\bigcap_{i=1}^{n} A_i) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_2 \cap A_1) \cdots P(A_n \mid \bigcap_{i=1}^{n} A_i)$$

Example: Draw 3 cards from a deck w/o replacement. What is the probability that none of them is a heart?



$$\Rightarrow P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_2 \cap A_1) = \frac{39}{52} \times \frac{38}{51} \times \frac{37}{50} \approx 0.41$$

Independence of events

Two events are independent if $P(A \cap B) = P(A)P(B)$.

Recall conditional probability: $P(A \cap B) = P(A \mid B)P(B)$. If two events are independent, $P(A \mid B) = P(A)$

[illegible]