# Axioms of probability $A = \{K\}$

**Probability law**: Assign a number $P(A)$ to an event $A$ satisfying the following axioms.

1. $0 \le P(A)$

2. $P(S) = 1$

3. If $A$ and $B$ are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$

↳ If $A \cap B = \phi$, the $P(A \cup B) = P(A) + P(B)$

↳ More generally, if $A_1, A_2, \cdots A_k$ are disjoint, then $P(\bigcup_{j=1}^{k} A_j) = \sum_{j=1}^{k} P(A_j)$

**Example**: Three coins tosses

- $S = \{HHH, HHT, HTT, \dots, TTT\}$
- These outcomes are mutually exclusive
- Suppose all outcomes are equally likely, i.e, $P(\{HHH\}) = P(\{HHT\}) = \cdots = P(\{TTT\}) = \frac{1}{8}$
- By axioms 2 and 3, we have:

$$P(S) = P(\{HHH\}) + \cdots + P(\{TTT\}) = 8 \cdot \frac{1}{8} = 1$$

- Consider the event A "at least 2 tails in a row" By axiom 3, we have:

$$P(\{HTT, TTH, TTT\}) = \frac{3}{8}$$

Let us partition the sample space into two mutually exclusive events, $A$ and $A^c$

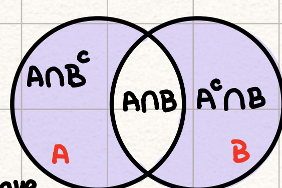
**Corollary 1**: Since $S = A \cup A^c$ and $A \cap A^c = \phi$, by axiom 2 and 3 we must have: $P(S) = P(A^c) + P(A) = 1 \Rightarrow P(A^c) = 1 - P(A)$

**Corollary 2**: By axiom 1, we have that $P(B) \geq 0$. Thus, from corollary 1, we have that:

$$P(A) = 1 - P(A^c) \le 1 \rightarrow 0 \le P(A) \le 1$$

**Corollary 3**: The empty set has probability zero. $P(\phi) = 1 - P(S) = 0$

**Corollary 4**: Union of 2 events $P(A \cup B)$ that are not necessarily mutually exclusive.




Decompose $A \cup B$, $A$ and $B$ as unions of disjoint events. By axiom 3, we have:

$$P(A \cup B) = P(A \cap B^c) + P(B \cap A^c) + P(A \cap B)$$

$$P(A) = P(A \cap B^c) + P(A \cap B)$$

$$P(B) = P(B \cap A^c) + P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary 5: Since $P(A \cap B) \geq 0$, from corollary 4, we must have that $P(A \cup B) \leq P(A) + P(B)$