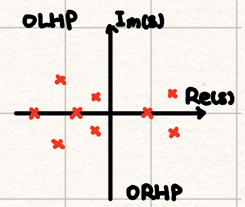
Test in-terms of "A"



THM 1: (i) Sys is int. asy. stable if and only if all eigen value of A are in OLHP $[\mathrm{Re}(\lambda_i(A)) < 0]$

(ii) Sys is int. unstable if at least 1 eigenvalue of A is in ORHP

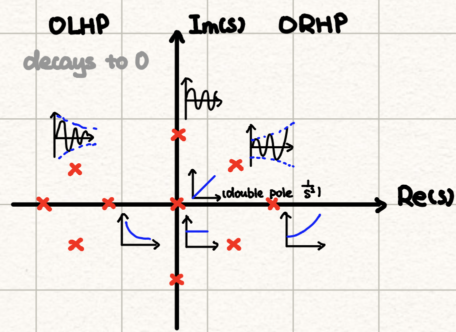
(iii) If A has eigenvalue on the j-axis the system can be either int. stable or int. unstable

Sketch of Proof:

$x(t)$ is solu. of $\dot{x} = Ax$, $x(0)$ I.C.

Use LT to analyse it: $X(s) = \mathcal{L}[x(t)]$; $X(s) = \begin{bmatrix} \vdots \\ X_i(s) \\ \vdots \end{bmatrix}$

SO: $sX(s) - x(0) = AX(s)$



$(sI_n - A)X(s) = x(0)$

$X(s) = (sI_n - A)^{-1} x(0) = \dfrac{\mathrm{adj}(sI_n - A)\cdot x(0)}{\det(sI_n - A)}$ (nxn) $[\,]_n$

$X_i(s) = \dfrac{N_i(s)}{\det(sI_n - A)}$; $i \in \{1, \dots, n\}$ — $N_i(s)$: poly of deg(n-1); $\det(sI_n-A)$: poly of deg n

poles of $X_i(s)$ are the roots of determinator: $\det(sI_n - A) = 0$ $\Leftrightarrow$ $\lambda_i(A)$: eigenvalue of A

$x_i(t) = \mathcal{L}^{-1}[X_i(s)]$

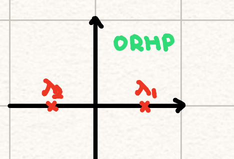
EX: 1 DOF robot link

UP  linearized model @ $\left(\begin{bmatrix}\pi\\0\end{bmatrix}, 0\right)$; $\delta x = x - \begin{bmatrix}\pi\\0\end{bmatrix}$; $n=2$

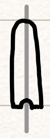
Intuition: unstable

$\delta\dot{x} = \overbrace{\begin{bmatrix} 0 & 1 \\ \frac{mgl}{I} & 0 \end{bmatrix}}^{A} \delta x \Rightarrow \mathrm{eig}(A) : \det(\lambda I_2 - A) = 0$

$\Rightarrow \begin{vmatrix} \lambda & -1 \\ -\frac{mgl}{I} & \lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \frac{mgl}{I} = 0 \Rightarrow \begin{array}{l} \lambda_1 = +\sqrt{mgl/I} \\ \lambda_2 = -\sqrt{mgl/I} \end{array}$

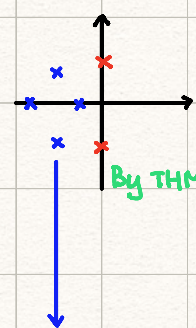


By THM(ii): unstable

DOWN: NO friction 

linearized @ $\left(\begin{bmatrix}0\\0\end{bmatrix}, 0\right)$; $\delta x = x$

$\delta\dot{x} = \overbrace{\begin{bmatrix} 0 & 1 \\ -mgl/I & 0 \end{bmatrix}}^{A} \delta x \Rightarrow \mathrm{eig}(A)$ in this case: $\lambda^2 + \frac{mgl}{I} = 0 \Rightarrow \lambda_{1,2} = \pm j\sqrt{mgl/I}$



By THM: stable

If we account for friction: $\delta\dot{x} = \begin{bmatrix} 0 & 1 \\ -mgl/I & -b/I \end{bmatrix} \delta x$ — b: friction coeff >0

$\Rightarrow \mathrm{eig}(A): \lambda^2 + \frac{b}{I}\lambda + \frac{mgl}{I} = 0 \Rightarrow \lambda_{1,2} = -\frac{b}{2I} \pm \frac{1}{2}\sqrt{\left(\frac{b}{I}\right)^2 - 4(mgl/I)} \Rightarrow \forall\, m, I, l, b > 0,\ \lambda_{1,2} \in \mathrm{OLHP}$

$\Rightarrow$ by THM: int. asy. stable