Like any other PMF, its sum over all possible outcomes is equal to 1, $\sum_{k=0}^{\infty} \frac{e^{-\alpha}\alpha^k}{k!} = e^{-\alpha} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} = e^{-\alpha} e^{\alpha} = 0$

Poisson Mean and Variance

$$E[X] = \sum_{k=0}^{\infty} k \frac{e^{-\alpha}\alpha^k}{k!} = \sum_{k=1}^{\infty} k \frac{e^{-\alpha}\alpha^k}{k!} = \sum_{k=1}^{\infty} \frac{e^{-\alpha}\alpha^k}{(k-1)!} = e^{-\alpha}\alpha \sum_{k=1}^{\infty} \frac{\alpha^{k-1}}{(k-1)!} = e^{-\alpha}\alpha \sum_{k=0}^{\infty} \frac{\alpha^k}{k!}$$

$$\Rightarrow E[X] = \alpha e^{-\alpha} e^{\alpha} = \alpha$$

The variance is $VAR(x) = \alpha$

Approximating A Binomial RV

We can approximate a binomial PMF with Poisson when n is large and p small.

Let $\alpha = n \cdot p$. We have $P_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \approx \frac{e^{-\alpha}\alpha^k}{k!}$

Binomial RV

Total # of successes in n trials of independent Bernoulli RVs.

$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k},\ k = 0,1,2 \cdots n.\ E[X] = n \cdot p,\ VAR(X) = np(1-p)$$

Bernoulli RV

We only have 2 outcomes, "success" and "failure"

$$P_X(k) = \begin{cases} p & k=1 \\ 1-p & k=0 \end{cases};\ E[X] = 1 \cdot p + 0(1-p) = p;\ VAR(X) = p(1-p)^2 + (1-p)(0-p)^2 = p(1-p)$$

Geometric RV

of trials until the 1^{st} success, $S_X = 1, 2, \ldots$

$$P_X(k) = p(1-p)^{k-1};\ E[X] = \frac{1}{p};\ VAR(X) = \frac{1-p}{p^2}$$

# of failures before our first success, $S_X = 0, 1, 2, \cdots$

$$P_X(k) = p(1-p)^k;\ E[X] = \frac{1-p}{p};\ VAR(X) = \frac{1-p}{p^2}$$

Uniform RV

Let x be a variable with n possible outcomes. If x is uniform,

$$P_X(x) = \begin{cases} \frac{1}{n} & x = x_1, x_2 \ldots \\ 0 & \text{otherwise} \end{cases};\ E[X] = \frac{x_1}{n} + \frac{x_2}{n} + \cdots + \frac{x_n}{n}$$

If the sample space is $S_X = 1, 2, \ldots, n$

$$P_X(k) = \frac{1}{n},\ k = 1, 2, \ldots n;\ E[X] = \frac{n+1}{2};\ VAR(X) = \frac{n^2-1}{12}$$

Q.3. $\boxed{\text{10 marks}}$ A high school student is anxiously waiting to receive an email telling her whether she has been accepted to UofT engineering program. Notification emails are due to be sent out from Monday to Wednesday of next week. She estimates the conditional probabilities of receiving an email, given that she is accepted or rejected, as follows:

Day	$P(\text{email}\mid\text{accepted})$	$P(\text{email}\mid\text{rejected})$
Monday	0.55	0.35
Tuesday	0.20	0.30
Wednesday	0.25	0.35

She also estimates that, given her GPA, the probability of being accepted is 0.6. Given this information, answer the following questions

(a). P(email on Mon) = P(email on Mon | A)P(A) + P(email on Mon | R)P(R) = 0.47

(b). P(email on Tue & not on Mon) = ?

$$P(T \mid M^c) = \frac{P(T \cap M^c)}{P(M^c)} = \frac{P(T)}{P(M^c)} = 0.453$$

(c). P(accepted on Wed) = ?

$$P(A \mid W) = \frac{P(W \mid A)P(A)}{P(W \mid A)P(A) + P(W \mid R)P(R)} = 0.517$$