Def: Close Loop System (feedback loop) in BiBO stable if the transfer function from r,d to e,u are BiBO stable.

Consider: first ($d \equiv 0$)

TF: $r \to e$: $E(s) = \frac{1}{1 + C \cdot G(s)} \cdot R(s)$ ①

$r \to u$: $U(s) = \frac{1}{1 + C \cdot G(s)} \cdot R(s)$ ②

Consider now ($r \equiv 0$)

TF: $d \to e$: $E(s) = \frac{-G(s)}{1 + C \cdot G(s)} \cdot D(s)$ ③

$d \to u$: $U(s) = \frac{1}{1 + C \cdot G(s)} \cdot D(s)$ ④

In general, when both u,d are present. we have superposition: ("add" them up)

①&③: $E(s) = \begin{bmatrix} \frac{1}{1+CG} & -\frac{G}{1+CG} \end{bmatrix} \cdot \begin{bmatrix} R(s) \\ D(s) \end{bmatrix}$

②&④: $U(s) = \begin{bmatrix} \frac{C}{1+CG} & \frac{1}{1+CG} \end{bmatrix} \begin{bmatrix} R(s) \\ D(s) \end{bmatrix}$

Put stacked together:

$$\begin{bmatrix} E \\ U \end{bmatrix} = \begin{bmatrix} \frac{1}{1+CG} & \frac{-G}{1+CG} \\ \frac{C}{1+CG} & \frac{1}{1+CG} \end{bmatrix} \begin{bmatrix} R \\ D \end{bmatrix}$$

It turns out 1 single TF is enough to check if it is BiBO stable

THM: The CL system is BiBO stable if and only if

a). $\frac{1}{1+CG}$ is BiBO stable $\Leftrightarrow$ all poles in OLHP

b). $C \cdot G$ have no "unstable" (on j-axis or in ORHP) pole/zero cancellation (extra condition)

Note: Let's show the necessity of (b) by contradiction

Assume: $G(s) = \frac{G'(s)}{s-1}$; $C(s) = C'(s) \cdot (s-1)$

$C \cdot G$ has "unstable" p/z cancellation in $s=1$ $\Rightarrow$ (b) is not met

$\frac{1}{1+CG} = \frac{1}{1+C'(s)G'(s)}$ $\Rightarrow$ assu. this is BiBO stable $\Rightarrow$ (a) met

Look at (1, 2) TF: $\frac{G}{1+CG} = \frac{G'(s)}{s-1} \cdot \frac{1}{1+C'G'(s)}$ $\Rightarrow$ has pole in $S=1$ $\Rightarrow$ NOT BiBo stable

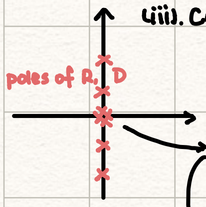
Design guideline for solving TP based on Internal Model Principle (IMP)

Given $G(s)$, $r(t)$, $d(t)$ signal such that $R(s)$, $D(s)$ are strictly proper function with poles on the j-axis

then $C(s)$ makes $e(t) \to 0$ as $t \to \infty$ (solves the TP) for any such $r(t)$, $d(t)$ iff the following are met:

(i). $C(s)$ makes C.L. system BiBO stable (stablizes the CL system) (ii): $C \cdot G(s)$ contains all poles of $R(s)$ and $D(s)$ counting their multiplicity.

(iii). $C(s)$ itself contains all poles of $D(s)$ counting their multiplicity



$\frac{1}{s}, \frac{1}{s^2}, \dots, \frac{1}{s^k}$

$r(t)$, $d(t)$

$r_0 \cdot 1\!\!1(t), t1\!\!1(t) \cdots, t^{k-1} 1\!\!1(t)$ (step) (ramp) $\longrightarrow$ polynomial signals

$r(t)$	$R(s)$
$r_0 \sin t \, 1\!\!1(t)$	$\frac{r_0}{s^2+1}$, $\frac{r_0 s}{s^2+1}$
$r_0 \cos t \, 1\!\!1(t)$	$\frac{r_0}{(s^2+1)^2} \longrightarrow r_0 t \sin t$

Ex: $R(s) = \frac{r_0}{s}$ (step track) $G(s) = \frac{1}{s}$ Candiate $C(s)$?

$C(s) = k \Rightarrow C \cdot G$ already meets (ii)

$\downarrow$

a proper controller might work for $e(\infty) = 0$