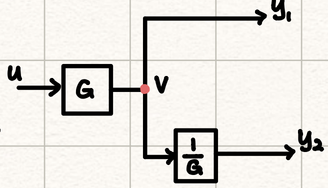
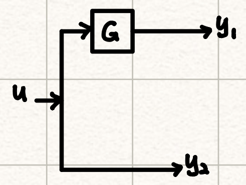
Block Diagram Manipulation & TF

Junction:



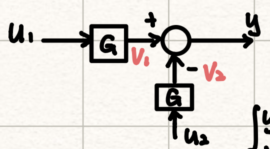
$$\begin{cases} y_2 = \frac{1}{G}\cdot V \\ V = G\cdot u \end{cases}$$

$\Longleftrightarrow$



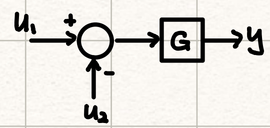
short-cut → push through G

Summing Junction:



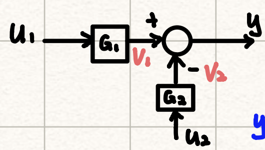
$$\begin{cases} y = V_1 - V_2 \\ V_1 = Gu_1 \\ V_2 = Gu_2 \end{cases}$$

$\Longleftrightarrow$

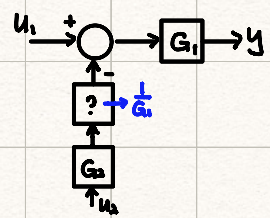


$y = G(u_1 - u_2)$

special case:



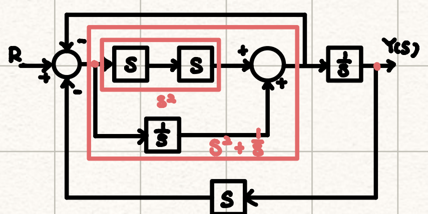
$y = G_1u_1 - G_2u_2$



$y = G_1u_1 - G_1\cdot ?\cdot G_2\cdot u_2$

$\Rightarrow ? = \frac{1}{G_1}$

Example:

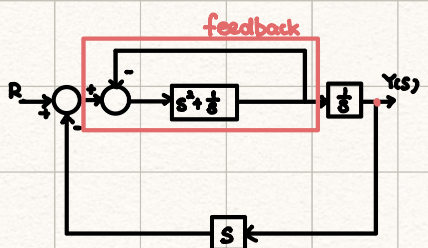


Find TF: $\frac{Y(s)}{R(s)} = ?$

Check: $TF = \frac{s^3+1}{s(2s^3+s+2)}$

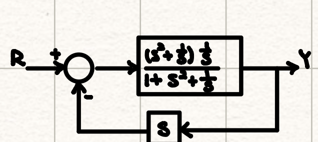
Think:

- From the inside out



fdback TF $= \frac{s^2+\frac{1}{s}}{1+(s^2+\frac{1}{s})(1)}$ ← feed forward path ← fdbk path

fw path



fdbk TF $= \frac{G_{fw}}{1+G_{fw}G_{fd}}$

How do we use TF to predict time-response of system?

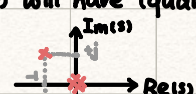
Example:

$G(s) = \frac{1}{s(s^2+2s+2)}$

Input is step $1\!\!1(t)$

Want to predict what type of components (elementary) the response y(t) will have (qualitatively) without full $\mathcal{L}^{-1}$?

$Y(s) = \frac{1}{s^2\cdot[(s+1)^2+1]}$, Poles of Y(s): s = 0 (double pole) , s = −1 ± i



$\downarrow$ PFE for some c_1, c_2, c_3, c_4

$$Y(s) = \frac{c_1}{s} + \frac{c_2}{s^2} + \frac{c_3 s + c_4}{(s+1)^2+1}$$

$$\downarrow \mathcal{L}^{-1}$$

$\mathbb{1}(t)$ + t + $e^{-t}\sin t$, $e^{-t}\cos t$

step, ramp, decaying sinusoids