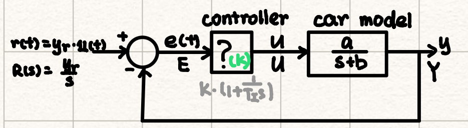
Cruise Control Example



y_r = des. velocity (steady state)
y = velocity

$e(t) = y_r 1(t) - y(t)$ tracking err: $e(\infty) = 0$

proportional control: $u = k\cdot e$, $k>0$ $e(\infty) = y_r - y(\infty) = \frac{b}{b+ka}\cdot y_r$ as $k \gg$, $e(\infty) \simeq 0$

"high-gain" control (by using FVT) — does not need precise knowledge of param. / robust to variant in param.

Note: for k finite, $e(\infty) \neq 0$ ← offset in tracking)

Try: proportional - integral control ("PI" control)

① $u(t) = \underbrace{Ke(t)}_{\text{"P" term}} + \underbrace{K\cdot\frac{1}{T_I}\int_0^t e(\tau)\,d\tau}_{\text{"I" term}}$ accumulating small err. ⇒ u(t) will compensate

and make $e(\infty)=0$

Find TF of controller: from e to u: using ① (take LT with ∅ I.C

$U(s) = K\cdot E(s) + K\cdot\frac{1}{T_I}\cdot\frac{1}{s}\cdot E(s)$

$U(s) = \underbrace{K(1+\frac{1}{T_I s})}_{\text{TF}}\cdot E(s)$

$Y(s) = \frac{K(1+\frac{1}{T_I s})(\frac{a}{s+b})}{1+K(1+\frac{1}{T_I s})(\frac{a}{s+b})}\cdot \boxed{\frac{y_r}{s}}\, R(s)$

② $E(s) = R(s) - Y(s) = \frac{1}{1+K(1+\frac{1}{T_I s})\cdot\frac{a}{s+b}}\cdot\frac{y_r}{s}$

$E(s) = \frac{\cancel{s}(s+b)}{s^2+s(b+Ka)+\frac{Ka}{T_I}}\cdot\frac{y_r}{\cancel{s}}$, want to use FVT

Poles of E(s) ≡ roots of this quaratic poly. where all coeff. are >0, are both in OLHP

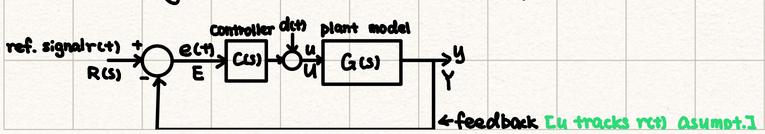
Then by FVT:

$e(\infty) = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0} s\cdot\frac{s+b}{s^2+s(b+Ka)+Ka/T_I}\cdot y_r = 0$

⇒ "PI" controller actually achieves $e(\infty)=0$. perfect asympt tracking

Take this as intro. to General Tracking Problem & Basic Feedback Loop.

General Tracking Problem & Basic Feedback Loop



$e(t) = r(t) - y(t)$ tracking err: $e(\infty) = 0$ $d(t)$: disturbance signal

Given general TF $G(s)$, design controller with TF $C(s)$, find $C(s)$ s.t. the following are satisfied:

Spec. a: For any bounded $r(t)$, $d(t)$ both $y(t)$ or $e(t)$ and $u(t)$ are bounded

Note: $e(t) = r(t) - y(t) \leftrightarrow y(t) = r(t) - e(t)$

(feedback loop or closed-loop sys is BIBO stable)

Spec. b: When $d(t) \equiv 0$ (no disturb.), for all $r(t)$ in a class $\mathcal{R}$ of signals the tracking err. $e(t)$ achieves $e(\infty) = 0$

(perfect asympt. tracking)

Spec. c: When there is disturbance, for all $d(t)$ in a class $\mathcal{D}$ of signals, and for all $r(t)$ in the class $\mathcal{R}$,

the tracking err. achieves $e(\infty) = 0$

(disturbance rejection) (optional requirement)