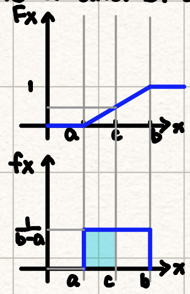
Uniform RV

All values in an interval of the real line are equally likely to occur.

Example: A plane departs between time a and b. Departure time is a uniform RV, X.

$$F_X(x) = P(X \le x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & x > b \end{cases}$$



$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

What is the exp. value?

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_a^b x\left(\frac{1}{b-a}\right)dx = \left(\frac{1}{b-a}\right)\frac{x^2}{2}\Big|_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$$

What is the variance?

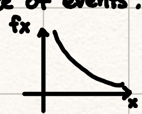
$$VAR[X] = \int_{-\infty}^{\infty} (x-E[X])^2 f_X(x)\,dx = \int_a^b \left(x-\frac{b+a}{2}\right)^2 \frac{1}{b-a}\,dx$$

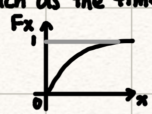
Let $y = x - \frac{b+a}{2}$; $VAR[X] = \frac{1}{b-a}\int_{-\frac{b-a}{2}}^{\frac{b-a}{2}} y^2\,dy = \frac{(b-a)^2}{12}$

Exponential RV

Models the time between occurrence of events, such as the time between your last phone call and your next phone call.

PDF: $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$





CDF: $F_X(x) = \begin{cases} 1-e^{-\lambda x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$

$\lambda > 0$ is the rate parameter

Exp. value $E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^{\infty} x\lambda e^{-\lambda x}\,dx$

Integration by parts: $u = x$ $v = -e^{-\lambda x}$, $\int_a^b uv'\,dx = uv\Big|_a^b - \int_a^b u'v\,dx$

$$E[X] = -xe^{-\lambda x}\Big|_0^{\infty} + \int_0^{\infty} e^{-\lambda x}\,dx = \frac{e^{-\lambda x}}{-\lambda}\Big|_0^{\infty} = \frac{1}{\lambda}$$

Variance: start by computing the second moment

integration by parts: $u = x^2$

$$E[X^2] = \int_0^{\infty} x^2 \lambda e^{-\lambda x}\,dx = -x^2 e^{-\lambda x}\Big|_0^{\infty} + \int_0^{\infty} 2x e^{-\lambda x}\,dx = \frac{2}{\lambda}\int_0^{\infty} x\lambda e^{-\lambda x}\,dx = \frac{2}{\lambda^2}$$

$$VAR[X] = E[X^2] - (E[X])^2 = \frac{1}{\lambda^2}$$