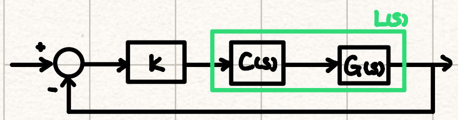
Nyquist Stability Criteria

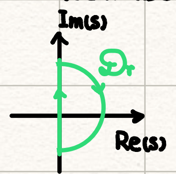


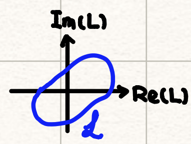
Feedback loop BIBO stable $\Leftrightarrow \begin{cases} \text{poles of } \frac{1}{1+k\cdot C\cdot G(s)} \in \text{OLHP} \\ \text{no "unstable" poles/zeros cancel in } C\cdot G \text{ (in ORHP or j-axis)} \end{cases}$

Fig. 1 k-variable gain (const.) For $k=1$, unit feedback loop

want a graphical test for this

We'll use the Priciple of Arg. for a special TF and for a special contour $\mathcal{L}$, called Nyquist contour. (all in ORHP & j-axis)





Let: $L(s) = C\cdot G(s)$ and $\mathcal{L} = \{L(s) \mid s \in \mathcal{D}_r\}$ be the Nyquist plot of $L(s)$ (image of $\mathcal{D}_r$ under $L(s)$)

THM: Suppose $L(s) = C\cdot G(s)$ a strictly proper TF (rational) which has NO poles on j-axis and has n poles in ORHP

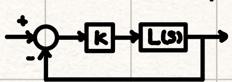
(in $\{Re(s) > 0\}$) and suppose $C\cdot G$ has no "unstable" pole-zero cancellation (in $\{Re(s) \geq 0\}$) (ORHP, j-axis)

Then the feedback loop (or the closed loop (C.L.) system) is BIBO stable if and only if the Nyquist plot $\mathcal{L}$ of $L(s)$:

(i) does NOT pass through the point $-\frac{1}{k}$ ("critical point")

(ii) it encircles $-\frac{1}{k}$ a # of times $N = n$ c.w.

Note: feedback loop $\Leftrightarrow \begin{cases} \text{poles of } \frac{1}{1+k\cdot C\cdot G(s)} \in \text{OLHP} \\ \text{no "unstable" poles/zeros cancel in } C\cdot G \text{ (in ORHP or j-axis)} \end{cases}$



C.L poles $\equiv$ zeros of $(1+k\cdot L(s) = \tilde{L}_k(s))$ $\in$ OLHP

"O.L. poles" $\equiv$ poles of $\tilde{L}_k(s)$ $\equiv$ poles of $L(s)$

Note: if $\mathcal{L}$ passes through $-\frac{1}{k} \Leftrightarrow \exists s \in \mathcal{D}_r$ s.t. $L(s) = -\frac{1}{k} \Rightarrow 1+kL(s) = 0 = \tilde{L}_k(s)$

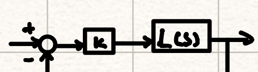
$s \in \mathcal{D}_r \Rightarrow \begin{cases} \text{j-axis} \\ \text{big semi-circles in ORHP} \end{cases} \Rightarrow$ C.L. poles $\in s \Rightarrow$ ILLEGAL

Apply Pr. of Arg to $\tilde{L}_k(s)$:

$N = $ # of encircle of the origin in $\tilde{L}$-plane $=$ #poles inside $\mathcal{D}_r$ (ORHP) $\overset{n \text{ (by cond. in THM)}}{=}$ $-$ # zeros inside $\mathcal{D}_r$ (ORHP) $\Rightarrow$ should be ϕ

$N = n$ ccw

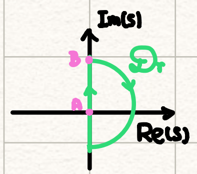
But: $\tilde{L}_k(s) = 1+kL(s) \Rightarrow \frac{1}{k}\tilde{L}(s) = \frac{1}{k} + L(s) \Rightarrow -\frac{1}{k} + \frac{1}{k}\tilde{L}(s) = L(s) \Rightarrow$ the origin in $\tilde{L}$ plane $\equiv -\frac{1}{k}$ in the L-plane (they are translated / scaled)

Example: 

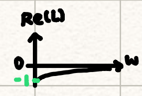
$L(s) = \frac{1}{s-1}$ $\quad \mathcal{D}_r = \{2 \text{ portions}\}$: ① j-axis ② big semi-circle

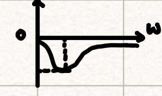
① $s = j\omega$, $\omega \in [0, +\infty)$ (A,B)

Im(L)



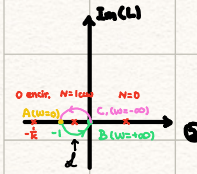
$L(s)|_{s=jw} = \frac{1}{jw-1} = \frac{jw+1}{(jw-1)(jw+1)} = \underbrace{\frac{-1}{1+w^2}}_{Re[L(jw)]} + j\underbrace{\frac{-w}{1+w^2}}_{Im[L(jw)]}$





For $s=jw$, $w: (-\infty, 0]$

$L(-jw) = \frac{1}{-jw-1}$, $w = (0, +\infty) = \frac{1}{L(jw)}$, $w \in (0, +\infty)$ ⇒ by reflection



② $s = r \cdot e^{j\theta}$, $r >> 1$; $L(s) = \frac{1}{r \cdot e^{j\theta} - 1} \overset{r \to \infty}{\simeq} 0$

n=1 ⇒ By Nyquist, $-1 < -\frac{1}{K} < 0$ ⇒ $K > 1$