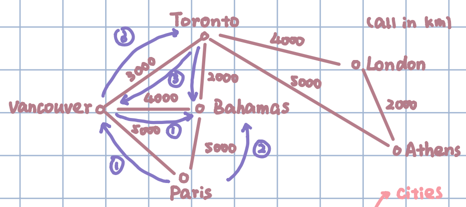
Introductions to Graphs and Trees

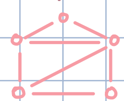
Graphs



cities, direct flights

Graph G(V, E) defined over set of vertices V and set of edges E

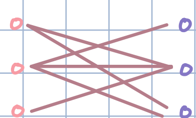
- directed or undirected (edges)
  - directed → only go one way
  - undirected → edges don't have direction
- weighted or unweighted
  - i.e. cost, distance, profits...
- path: sequence of edges, between adjacent vertices i.e in the graph
- simple path: no vertex is repeated i.e: path ③
- cycle: simple path with same start/end vertex
- connected G: ∃ path between every two vertices (otherwise G is disconnected)



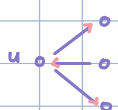
G is connected

- bipartite graphs: V can be divided into 2 sets V_1 & V_2 s.t. $V_1 \cap V_2 = 0$ and $V_1 \cup V_2 = V$ and adjacencies exist only between elements of V_1 and V_2

  example: V_1 V_2



- Vertex degree = # of edges adjacent to the vertex
  - Undirected G: only degree
  - Directed G: in-degree and out-degree

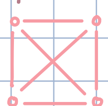


u has degree 3

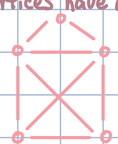
u has in-degree 1

out-degree 2

- Clique: every two vertices have an edge

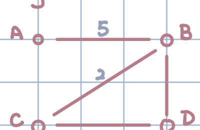


K4



K5 # edge = $\frac{V(V-1)}{2}$

- Representations
  1. Adjacency List
  2. Adjacency matrix



Adjacency List

A → B

B → A → C → D

C → B → D

D → C → B

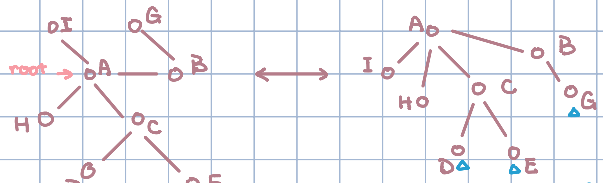
Adjacency matrix (assume every edge is 1)

	A	B	C	D
A		1 5		
B	1 5		1 2	1
C		1 2		1
D		1	1	

$\lvert V \rvert = n$	List	Matrix
time	O(n)	O(1) ⇒ there's a trade-off
memory	O(V+E) → worst case: O(n^2)	O(n^2)

Trees

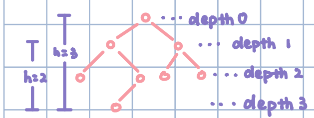
A tree is a graph that is
- connected
- acyclic
- undirected



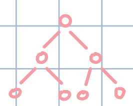
D, E & G are leaves (no child)

C is the parent of D & E ⇔ D & E children of C

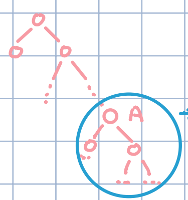
- Binary tree: every node has up to 2 children (generalize to k children → k-ary tree)
- Depth of a node: length of path from root to that node
- Height: # of edges of longest path from the node to a leaf



- Complete k-ary tree: every internal node has exactly k children and all leaves have same depth



- Subtree rooted at a node:



→ sub-tree rooted at A

- Theorem: every two of the following statements are equivalent
  1. G is a tree
  2. every two vertices in G are connected by a unique simple path
  3. G is connected but if any edge is removed, resulting graph is disconnected
  4. G is connected and $|E| = |V| - 1$
  5. G is acyclic and $|E| = |V| - 1$
  6. G is acyclic but if edge is added then new graph has a cycle