Discrete Random Variables

A RV is called discrete if its range is finite or countably infinite.

Range: The set of all experimental values the RV can take.

Example: The sample space of tossing a coin is $S=\{H,T\}$. We can introduce a RV to model this:

$$X(\zeta)=\begin{cases}1 & \text{if } \zeta=H\\ 0 & \text{if } \zeta=T\end{cases} \quad \text{or} \quad X=\begin{cases}1 & \text{if } H\\ 0 & \text{if } T\end{cases}$$

Probability Mass Function (PMF)

The PMF is a discrete probability function. It is a function that gives the probability that a discrete RV is exactly equal to some value.

Notation: The PMF is the function $P_X(x)=P(X=x)$

Bernoulli PMF: Toss a coin with $P(\{H\})=p$. We have $P_X(k)=\begin{cases}p & \text{if } k=1\\ 1-p & \text{if } k=0\\ 0 & \text{otherwise}\end{cases}$

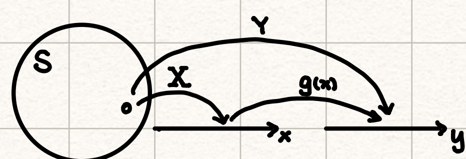
Binomial PMF: $X=$ number of H's in n independent tosses.

$$P_X(k)=\binom{n}{k}p^k(1-p)^{n-k},\ k=0,1,\cdots,n$$

Note: X is the sum of n Bernoulli RVs

$$X_i=\begin{cases}1 & \text{if } i^{th} \text{ toss is H}\\ 0 & \text{if } i^{th} \text{ toss is T}\end{cases} \Rightarrow X=\sum_{i=1}^{n} X_i$$

Functions of RVs



If X is a RV, then $Y=g(X)$ is also a RV.

$$P_Y(y)=\sum_{\{x|g(x)=y\}} P_X(x)$$

Expected Value of a RV

The expected value or "mean" is $m_X=E[X]=\sum_{x\in S_X} xP_X(x)=\sum_k x_k P_X(x_k)$ (k is the countably value, i.e: 0~1: coins 1~6: dice)

Note: we will assume this sum converges, i.e: $\sum_{x\in S_X}|x|P_X(x)<\infty$

Example: Mean of a Bernoulli RV $P_X(k)=\begin{cases}p & \text{if } k=1\\ 1-p & \text{if } k=0\end{cases}$

$$E[X]=1\cdot p+0(1-p)=p$$

Example: Rolling a dice

$$E[X]=1\cdot\frac{1}{6}+2\cdot\frac{1}{6}+3\cdot\frac{1}{6}+\cdots+6\cdot\frac{1}{6}=3.5$$

Notes:
- $E[X]$ is not necessarily the most likely value of X
- It is not guaranteed to be equal to an empirical average.