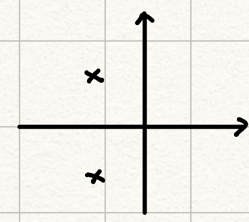
Observations on Transient Control & Pole Location

Additional Poles/Zeros

The analysis we had (Ts, Tr, %OS) is for a TF with $\begin{cases} 2 \text{ cplx. conj. poles} \\ \text{no zeros} \end{cases}$

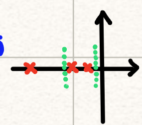


This remains approximately valid under the following assumptions:

(1). Any additional poles are far to the left of the 2 "dominant" cplx. conj. poles (|real part| is at least 5 times larger)

(2). The real part of all zeros is < 0, and is much different than the real part of the 2 dominant poles.

Ex: $G(s) = \dfrac{(s+3.1)}{(s+5)(s+3)(s+20)} \approx \dfrac{1}{(s+3)(s+20)} \approx \dfrac{1}{(s+3)}$



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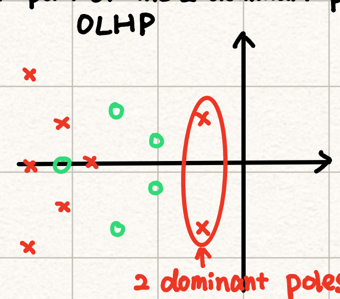
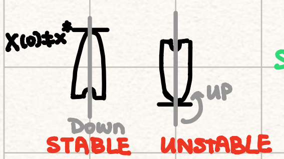


Stability — a Prelude

Ex 1: DOF link (internal stability)

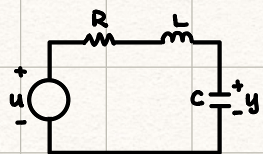


state-space model

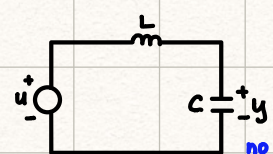
perturbing I.C. & being able to recover

Ex 2: RLC & LC circuit (External BIBO stability 0I.C.)

TF model



ability of system to sustain bonded input without blowing up



no R dissipative

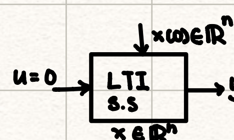
resonance → for a certain u i or y blow up

Defining stab. concepts, char. mathematically and giving tests to check them.

Internal Stability

motion that applies to state-space models

Consider L.T.I system: $\begin{cases} \dot{x} = Ax + Bu & x \in \mathbb{R}^n \quad x(0) \in \mathbb{R}^n \\ y = Cx + Du \end{cases}$ and we assume there is no input acting $u = 0$



x(t) solu. for ∀x(0)

Then $\dot{x} = Ax$, $x(0) \in \mathbb{R}^n$ and $y = Cx$ $\qquad x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_i(t) \\ \vdots \\ x_n(t) \end{bmatrix} \in \mathbb{R}^n$

Definition:

(a): Sys. is internally stable if:

$\forall x(0) \in \mathbb{R}^n$, state $x(t)$ is bonded for any $t \geq 0$ i.e. $\exists M > 0$, s.t. $|x_i(t)| \leq M$, $\forall t \geq 0$ and $\forall i = \{1, \dots, n\}$

(b): Sys. is internally asymptotically stable if:

it is int. stable AND $\forall x(0) \in \mathbb{R}^n$, $x(t) \to \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = 0$ as $t \to \infty$

there exists

(c): Sys. is int. unstable if it is NOT int. stable, i.e. $\exists x(0) \in \mathbb{R}^n$, s.t. $x_i(t)$ becomes unbonded for some $i \in \{1, \dots, n\}$

$x_i(t) \to \pm\infty$

Want to get a test ⟶ A.