Substitution (induction)

Mergesort: $T(n) = 2T(\lceil \frac{n}{2} \rceil) + n$

We "guess" the answer, and then use induction to prove it.

We guess $T(n) = O(n\log n)$ and we will prove it.

Base case: later...

Hypothesis: Assume it holds $\forall$ strictly smaller than n (and we will prove n)

$$T(\tfrac{n}{2}) \leq c\lceil \tfrac{n}{2} \rceil \log \lceil \tfrac{n}{2} \rceil$$

Steps: We need prove that: $T(n) \leq cn\log n$

Apply hypothesis:

$$\begin{aligned} T(n) &\leq 2c(\tfrac{n}{2})\log(\tfrac{n}{2}) + n \\ &= cn\log n - cn\log 2 + n \\ &= cn\log n - cn + n \\ &= cn\log n + (1-c)n \leq cn\log n \text{ for } c \geq 1 \end{aligned}$$

$\Rightarrow T(n) \leq cn\log n$

Back to base:

We try to prove that $T(n) \leq cn\log n$

For $n = 1$: $T(1) \leq c1\log 1 = \emptyset$

"Arbitrarily" set:

$T(2) = 4$

$T(3) = 5$ ... and see if it works!

$T(2) \leq c \cdot 2 \cdot \log 2 = 2 \cdot c$

$T(3) \leq c \cdot 3 \cdot \log 3 = 3 \cdot c \cdot 1.584 \simeq 4.752c$

$\Rightarrow$ they work with $c \geq 3$

$\Rightarrow$ start base $n = 2$ (double base)

Erroneous Method

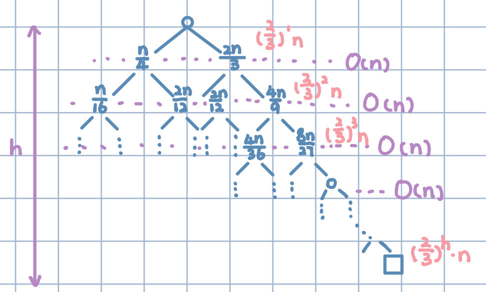
We guess $T(n) = O(n)$ ... we build proof (step)

$T(n) \leq 2c(\frac{n}{2}) + n \Leftrightarrow T(n) \leq cn + n = (c+1)n$ QED!

Error: different constant! ($c+1$ not c)

Recursion Tree

$T(n) = T(\frac{n}{4}) + T(\frac{2}{3}n) + n$



Total amount of work $= T(n) = h \cdot O(n) \Rightarrow h = ?$

As there's only 1 element left: $(\frac{2}{3})^h \cdot n = 1 \Leftrightarrow n = (\frac{3}{2})^h \Leftrightarrow h = \log^{\frac{3}{2}} \cdot n = O(\log^n)$

$\Rightarrow h = \log n \Rightarrow T(n) = O(n\log^n)$