Sample space: The set of all possible outcomes

↳ denoted by $S$

↳ List all the elements, e.g: toss a coin $S = \{H, T\}$
$= \{x \in \mathbb{Z} : \text{"} \quad \text{"}\}$

↳ Specify the elements, e.g: $S = \{x : 0 \leq x \leq 3\}$

↳ Discrete sample space: $S$ is countable

- Finite: $S_1 = \{1, 2, \cdots 10\}$

- Countably infinite: $S_2 = \{1, 2, 3, \cdots\}$

↳ Continuous sample space: $S$ is not countable

- Uncountably infinite: $S_3 = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$

Event: An event is a subset of the sample space $S$ where the outcome satisfies a specific condition

↳ An event $A$ corresponds to an experiment $E$, e.g:

- $E$: Select a ball from an urn (缸) containing balls numbered from 1 to 10

- $A$: An even-numbered ball is selected

  $S = \{1, 2, 3, \cdots 10\}$ $\quad A = \{2, 4, 6, 8, 10\}$

↳ Certain events: when $A = S$, i.e: an event that always occurs

↳ Impossible or null event: when $A = \emptyset$

Event classes: A collection of events, i.e: a set of sets

↳ If the class $C$ consists of sets $A_1, \cdots A_k$, then $C = \{A_1, \cdots, A_k\}$

Probability Models:

Probability as a frequency: Consider probability as a measure of the frequency of occurrence

↳ If we have a probability $P(k)$ that experiment will result in the outcome $k$, then after repeating this experiment a large number of times, the fraction of times that $k$ occurs will approximate $P(k)$.

↳ Example: We can define the probability of observing "heads" in a coin flip by counting the number of heads observed after $n$ trials. As $n$ approaches infinity, we have $P(H) = \lim_{n \to \infty} \frac{\#\text{ of heads}}{n}$

↳ Let $N_k(n)$ be the number of times in which outcome is equal to $k$ after $n$ trials. We must have that

$0 \leq N_k(n) \leq n, \ \forall k \in S$

↳ Dividing by n, and as $n \to \infty$, we have $0 \leq P(k) \leq 1$

↳ The sum of the number of occurrences must sum up to $\sum_{k \in S} N_k(n) = n$

↳ As before, this leads to the conclusion $\sum_{k \in S} P(k) = 1$