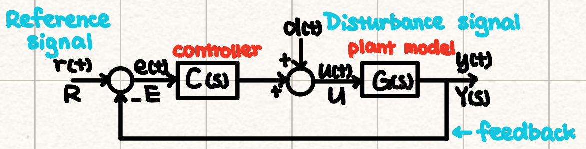
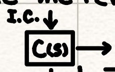


Note: Name – Internal Model Priciple (IMP)

If G does not have poles of R(s) or D(s), the controller itself C should have them by ② ③. i.e. as if C should be able to genearate the reference signal r(t) and disturbance signal d(t) for some I.C.

C has internal model of r(t), d(t)



$\frac{1}{s}, \frac{1}{s^2+1} \Leftrightarrow u(t), \sin(t)$

Corollary: If G has zeros at the poles of R(s) and D(s) then the tracking problem is unsolvable.

Proof: Assume $R(s) = \frac{r_0}{s} \Rightarrow$ pole at $s=0$ and $G(s)$ has a zero at $s=0$, $G(s) = s \cdot G'(s)$ for some $G'(s)$

Assume $D(s) = 0$, by ② need $C \cdot G$ to have at least 1 pole at $s=0 \Rightarrow C$ has to have a pole in $s=0$.

$\Rightarrow C(s) = \frac{1}{s} \cdot C'(s) \Rightarrow C \cdot G = \frac{1}{\cancel{s}} \cdot C'(s) \cdot \cancel{s} \cdot G'(s)$ no pole at $s=0$

Try $C(s) = \frac{1}{s^2} \cdot C'(s) \Rightarrow CG = \frac{1}{s} C'(s) \cdot G'(s)$ satisfies ②

Need to meet ① and that is impossible because of "unstable" pole/zero cancellation between C & G at $s=0$

Design Guideline

Given $R(s), D(s), G(s)$ select $C(s)$ (a candidate TF) s.t. it is proper TF of minimal order, and it satisfies ②, ③ in IMP and the coeff. of C_1 will be selected s.t. ① is met (BIBO stable for C.L system) using Routh Array.

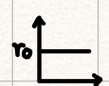
E.g. $C(s) = \frac{C_1(s)}{P(s)}$ → proper, degree of C_1 → poles of $C(s)$

As a mean of checking / verifying

compute: $E(s)$ as a function of $R(s), D(s)$ for the selected / designed $C(s)$ and given $G(s) \Rightarrow E(s) = \frac{1}{1+CG} \cdot R(s) - \frac{G}{1+CG} D(s)$

and check that we can apply FVT (poles of $E(s)$ in OLHP and at most 1 in $s=0$) and $e(\infty) = \lim_{s \to 0} s \cdot E(s) = 0$, for any r, d.

Ex: $G(s) = \frac{1}{s+1}$, $R(s) = \frac{r_0}{s} \Leftrightarrow r(t) = r_0 \cdot 1(t)$



$D(s) = \frac{d_0}{s^2+1} \Leftrightarrow d(t) = d_0 \cdot \sin t$

where r_0, d_0 unknown

Want $e(t) \to 0$ as $t \to \infty$ no matter what r_0, d_0

By ② in IMP: C needs poles $s=0$ & $s^2+1=0 \Rightarrow$ ③ met $\Rightarrow C(s) = \frac{k_1 s^3 + k_2 s^2 + k_3 s + k_4}{s(s^2+1)}$

where k_i coeff will be selected s.t. ① is met i.e. BIBO stable C.L.

Ex: $G(s) = \frac{1}{s(s+1)}$ $R(s) = \frac{r_0}{s}$ $D(s) = \frac{d_0}{s}$ both $r(t), d(t)$ are step signals

By ② CG needs a pole in $s=0$, note that the plant G has already a pole in $s=0$, i.e. C can be proportional

$C(s) = K$ (minimal order)

But we also need ③, i.e. C itself needs to have a pole in $s=0$ then $C(s) = \frac{k_1 s + k_2}{s}$ ← proper TF is a candidate

that meets both ②, ③. The coeff k_1, k_2 will be selected s.t. ① is met, i.e.: $\begin{cases} \frac{1}{1+CG} \text{ is BIBO stable} \\ \text{no pole/zero cancellation in } C \cdot G \text{ (unstable)} \end{cases}$

$$\frac{1}{1+CG} = \frac{1}{1 + \frac{k_1 s + k_2}{s} \cdot \frac{1}{s(s+1)}}$$ needs poles in OLHP

$$= \frac{s^2(s+1)}{s^3 + s^2 + k_1 s + k_2}$$ use Routh array $\Rightarrow k_2 > 0 \quad k_2 - k_1 < 0 \Rightarrow 0 < k_2 < k_1$

Let $k_2 = 1$, $k_1 = 2$: $C(s) = \frac{2s+1}{s} = \underbrace{2}_{\text{"P"}} + \underbrace{\frac{1}{s}}_{\text{"I"}}$ controller

Check that TP obj. are met