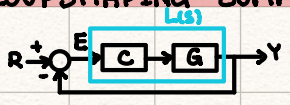
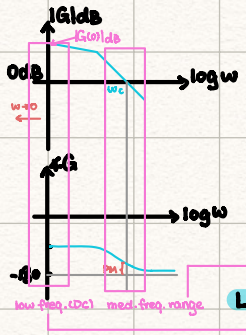
LOOPSHAPING COMPENSATORS



$L = C \cdot G \Rightarrow \begin{cases} |L|_{dB} = |C|_{dB} + |G|_{dB} \\ \angle L = \angle C + \angle G \end{cases}$

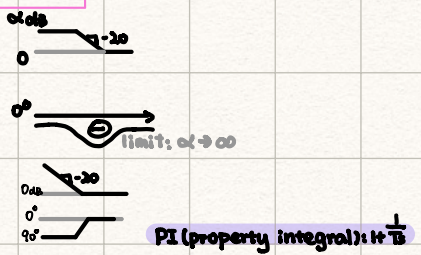


Design C s.t $e_{ss} = e(\infty)$ specs ($\propto \frac{1}{|L(0)|}$ (DC gain)), PM spec, ω_c spec

⇒ Look for C(s) that "shape" loop TF freq. resp in diff freq. region

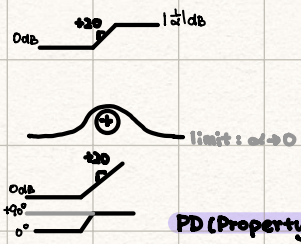
⇒ simple, 1st order compensators that later on can be combined

LAG: $\alpha \cdot \frac{Ts+1}{\alpha Ts+1}$; $\alpha > 1$



PI (property integral): $1 + \frac{1}{Ts}$

LEAD: $\frac{Ts+1}{\alpha Ts+1}$; $0 < \alpha < 1$



PD (Property derivative): $1 + Ts$ not implementable

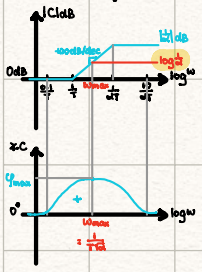
Advantage: for low freq. (DC gain ↑ or slope ↑)

Disadvantage: is lag in phase ⇒ would want to slide them to left (don't affect medium range freq. for PM)

- **No advantage** for low freq.
- **Useful** for medium freq. range to increase PM
- Phase lead ⊕ @ med. freq. and also gain ↑ ⇒ ω_c ↑
- More comlexity in the design i.e: selection of α, T

Use of superposition: i.e: $C = C_1 + C_2 \Rightarrow |C|_{dB} = |C_1|_{dB} + |C_2|_{dB}$ ⇒ combined for eg - PI / PD (operate in diff freq. range) (LEAD / LAG)

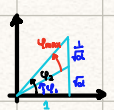
LEAD compensator: $C_s = \frac{Ts+1}{\alpha Ts+1}$ $0 < \alpha < 1$ $T > 0$



⇒ $C(j\omega) = \frac{1 + j\omega T}{1 + j\omega \alpha T}$ ⇒ break points: $\frac{1}{T}$, $\frac{1}{\alpha T}$ (zero, pole)

ω_{max} = mid-point on log scale b/w $\frac{1}{T}$ & $\frac{1}{\alpha T}$ ⇒ $\frac{1}{\alpha T}\left[\frac{1}{2}\left(\log\frac{1}{T} + \log\frac{1}{\alpha T}\right)\right] = \frac{1}{T\sqrt{\alpha}} = \omega_{max}$

$|C(j\omega_{max})|$ = mid-point between $\begin{cases} 0\ dB \\ \frac{1}{\alpha}\ dB \end{cases} = 20\log\frac{1}{\sqrt{\alpha}} = 10\log\frac{1}{\alpha}$



$\varphi_{max} = \angle C(j\omega_{max}) = \angle \frac{1 + j\frac{1}{\sqrt{\alpha}}}{1 + j\sqrt{\alpha}} = \angle\left(1 + j\frac{1}{\sqrt{\alpha}}\right) - \angle(1 + j\sqrt{\alpha}) = \varphi_2 - \varphi_1$

⇒ $\sin\varphi_1 = \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}}$ ⇒ $\sin\varphi_{max} = \sin(\varphi_2 - \varphi_1) = \frac{1-\alpha}{1+\alpha}$

3 basic equations: Goals to "slide" plots s.t. φ_{max} is where PM is read max "boost" in phase ⇒ $\omega_{max} = \omega_c^{NEW}$ → new gain crossover freq.

⇒ we need to work a bit because $|C|_{dB}$