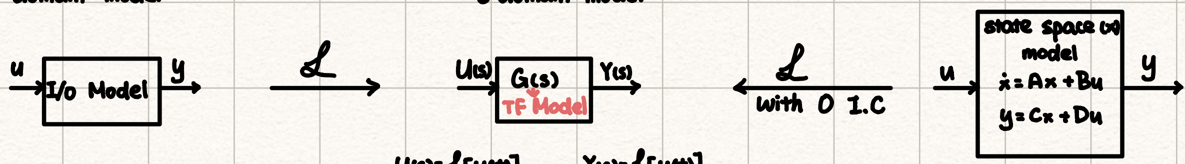
Transfer function

LTI, time domain model   s-domain model



$U(s) = \mathcal{L}[u(t)]$ $Y(s) = \mathcal{L}[y(t)]$

$Y(s) = G(s) \cdot U(s)$ $G(s) = \frac{N(s)}{D(s)}$ (real rational, proper function in ⓢ)

$G(s) = C(sI_n - A)^{-1} \cdot B + D$

Ex: RLC circuit:

roots of $D(s)$ = poles of G

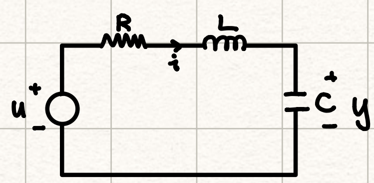
roots of $N(s)$ = zeros of G

I/O model: $LC\ddot{y} + RC\dot{y} + y = u$

normalization, coeff = 1 : $\ddot{y} + \frac{R}{L}\dot{y} + \frac{1}{LC}y = \frac{1}{LC}u$

$\mathcal{L}\downarrow$ $\Big\}\ \mathcal{L}$ w. 0 I.C.

$(s^2 + \frac{R}{L}s + \frac{1}{LC})Y(s) = \frac{1}{LC}U(s)$



Then : $G(s) = \frac{Y(s)}{U(s)} = \frac{\frac{1}{LC}}{(s^2 + \frac{R}{L}s + \frac{1}{LC})} \triangleq \frac{N(s)}{D(s)}$ ⇒ 2 poles no zeros

• state-space model (we obtained this: with. $\begin{cases} x_1 = y \\ x_2 = i \end{cases}$; $x \in \mathbb{R}^2$)

$$(**)\quad \begin{cases} \dot{x} = \overset{A}{\begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}} \cdot x + \overset{B}{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}} \cdot u \\ y = \overset{C}{[1 \quad 0]} \cdot x \ (+ \overset{0}{\underset{\shortparallel}{}}D) \end{cases}$$

$M^{-1} = \frac{1}{\det M} \operatorname{adj} M$

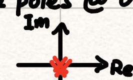
$$\Rightarrow G(s) = \overset{C}{[1 \quad 0]} \underbrace{\begin{bmatrix} s & -\frac{1}{C} \\ \frac{1}{L} & s+\frac{R}{L} \end{bmatrix}^{-1}}_{(s-I_2A)^{-1}} \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}_{B} = [1 \quad 0] \frac{1}{s(s+\frac{R}{L}) + \frac{1}{LC}} \begin{bmatrix} s+\frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & s \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} = \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

(the same TF func. from I/O model)

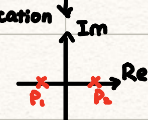
Ex: 1 DOF robot arm linearized @ $(x^*, u^*) = ([\begin{smallmatrix} \frac{\pi}{2} \\ 0 \end{smallmatrix}], mgl)$

① st.sp.model $\begin{cases} \delta\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\delta x + \begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix}\delta u \\ \delta y = [1 \quad 0]\delta x \end{cases}$

② $\Rightarrow G(s) = [1 \quad 0]\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix} = \frac{1}{s^2}[1 \quad 0]\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}\begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix} = \frac{1}{I}/s^2 = \frac{N(s)}{D(s)}$ ⇒ 2 poles @ 0



totally different location



Next: 1 DOF robot arm linearized @ $(x^*, u^*) = ([\begin{smallmatrix} \pi \\ 0 \end{smallmatrix}], 0)$

③ st.sp.model $\begin{cases} \delta\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{mgl}{I} & 0 \end{bmatrix}\delta x + \begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix}\delta u \\ \delta y = [1 \quad 0]\delta x \end{cases}$

④ $\Rightarrow G(s) = [1 \quad 0]\begin{bmatrix} s & -1 \\ -\frac{mgl}{I} & s \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix} = \frac{1/I}{\underset{\det(sI_2 - A)^{-1}}{s^2 - mgl/I}} = \frac{N(s)}{D(s)}$ ⇒ 2 poles @ $\pm\sqrt{\frac{mgl}{I}}$

Find ⋯ @ $(x^*, u^*) = ([\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}], 0)$

From TF model ⟹ I/O model state space model

Ex: RLC circuit

Given: $G(s) = \dfrac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$ ⇒ what is I/O model? state model?

TF model: $Y(s) = G(s) \cdot U(s)$ ⇒ $s^2 Y(s) + \frac{R}{L} s Y(s) + \frac{1}{LC} Y(s) = \frac{1}{LC} U(s)$

$\mathcal{L}^{-1}$, w 0 I.C.: ⇒ $\ddot{y} + \frac{R}{L}\dot{y} + \frac{1}{LC} y = \frac{1}{LC} u$ (*), 2^{nd} order ODE as I/O model → need 2 state components x_1, x_2

What's state model?

Let: $\begin{cases} x_1 = y \\ x_2 = \dot{y} \end{cases}$ ⇒ then $\begin{cases} \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} \overset{(*)}{=} \begin{bmatrix} x_2 \\ -\frac{R}{L}x_2 - \frac{1}{LC}x_1 + \frac{1}{LC}u \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \cdot x + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} \cdot u \\ y = [1 \quad 0]\, x \end{cases}$

compare to (* *): A & B are different, b/c state-vari. is different

(state model NOT unique)