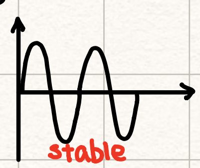
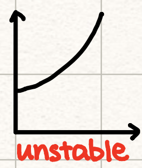
# External Stability

## Bounded Input Bounded Output Stability

**Def:** A LTI system is BIBO stable if any bounded input $u(t)$, the output $y(t)$ is bounded. The system is BIBO unstable if there exists a bounded input $u(t)$ s.t. $y(t)$ is unbounded ("blow-up")

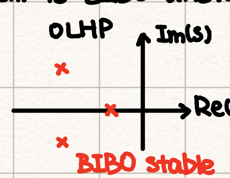
**Notes:** By $u(t)$ bounded we mean there exist ($\exists$) $M > 0$ s.t. $|u(t)| \leq M, \forall t \geq 0$.

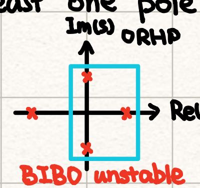


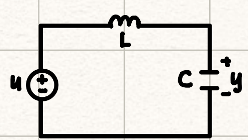

Need a test that involves the model of LTI system. i.e. TF model denoted by $G(s)$

**THM 2:** A LTI system with TF $G(s)$ is BIBO stable if and only if all poles of $G(s)$ are in OLHP. The system is BIBO unstable if $G(s)$ has at least one pole in ORHP or on j-axis.



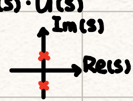




**Example:** LC circuit



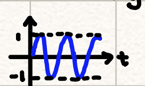

$$G(s) = \frac{\frac{1}{LC}}{s^2+LC} \qquad Y(s) = G(s)\cdot U(s)$$

$\Rightarrow$ poles of $G$: $p_{1,2} = \pm j\sqrt{\frac{1}{LC}}$

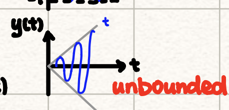


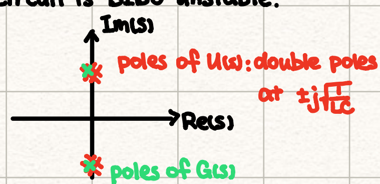

By THM2: LC circuit is BIBO unstable.

Consider $U(s) = \frac{1/\sqrt{LC}}{s^2+1/LC} \xrightarrow{\mathcal{L}^{-1}} u(t) = \sin(\frac{1}{\sqrt{LC}}t)\cdot 1\!\!1(t)$

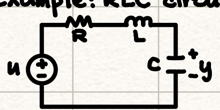



$Y(s) = G(s)\cdot U(s) = \frac{1/(LC)^{\frac{3}{2}}}{(s^2+1/LC)^2} \xrightarrow{\mathcal{L}^{-1}} y(t) = t\cdot\sin(\frac{1}{\sqrt{LC}}t)1\!\!1(t)$




Example: RLC circuit




$G(s) = \frac{1/LC}{s^2+(R/L)s+1/LC}$ poles: $p_{1,2}$ are in OLHP for any $R, L, C > 0$

$\Rightarrow$ By THM 2: the system is BIBO stable

R: dissipates energy, stablize the system