

Conditional PMF

Let x be a discrete RV with PMF $p_x(x)$. Let C be an event with $P(C) > 0$. The conditional PMF of X given C is:

$$p_x(x|C) = P(X=x|C) = \frac{P(\{X=x\} \cap C)}{P(C)}$$

Example:

Roll a fair dice and let x be the resulting number. Find the conditional PMF of X given that the observed number was less than 5

Condition on the event $C = \{x < 5\}$ and $P(C) = \frac{4}{6}$

$$\text{Therefore, } p_x(x|C) = P(X=x|x < 5) = \frac{P(\{X=x\} \cap \{x < 5\})}{4/6} = \begin{cases} \frac{1}{4} & \text{if } x=1, \dots, 4 \\ 0 & \text{otherwise} \end{cases}$$

Conditional Expected Value and Variance

The conditional expected value, denoted $E[X|B]$ for some event B is

$$E[X|B] = m_{x|B} = \sum_x p_x(x|B) = \sum_{k=1}^{\infty} x_k p_k(x_k|B)$$

The conditional variance is:

$$\text{VAR}(X|B) = E[(X - E[X|B])^2 | B] = \sum_{k=1}^{\infty} (x_k - E[X|B])^2 p_k(x_k|B) = E[X^2|B] - (E[X|B])^2$$