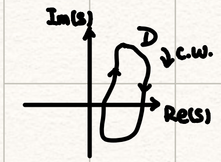
Principle of Arguement

Relates # of poles and zeros of a TF inside of a contour to the change in its argument ("angle") or equiv to the # of encirclements of the origin by some graphical image of the contour under the TF.

Let's consider a closed contour with no self-intersections traversed in clock-wise (cw) direction



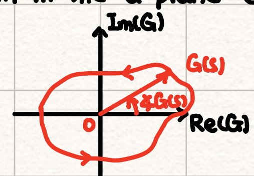
Assume $G(s)$ is a rational func. with no poles or zeros on D. Then the (net) change in $\arg G(s)$

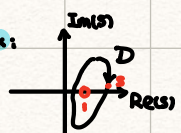
when s traverses D is equal to: $\text{chg} \measuredangle G|_{s\in D} = 2\pi(n-m)$

where: $\begin{cases} n = \#\text{ of poles of } G(s) \text{ inside } D \\ m = \#\text{ of zeros of } G(s) \text{ inside } D \end{cases}$

Equivalently, the image of D under $G(s)$, called $\gamma = \{G(s) \mid s \in D\}$ encircles the origin in the G-plane exactly

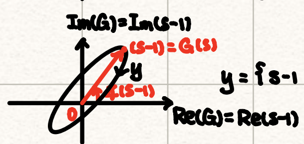
$N = (n-m)$ in ccw (counterclock wise)



Ex: 

$G(s) = s-1$; zero of G in $s=1 \Rightarrow 1$ zero inside D (no pole)

Pr. of Arg says that: $\measuredangle G|_{s\in D} = -2\pi$ or # of encirclements of origin by $G = -1$ c.c.w $= 1$ c.w



$\gamma = \{s-1 \mid s\in D\}$: just a translation of D by 1 to the left.

$G(s)=0 \Rightarrow s=1$ G-plane change $\measuredangle(s-1)|_{s\in D} = -360° = -2\pi$ # of encircle of 0 by G is $= 1$ cw

Ex: $G(s) = \frac{1}{s-1} \Rightarrow G$ has a pole inside D (@ $s=1$)

$\measuredangle G|_{s\in D} = \measuredangle \frac{1}{s-1}|_{s\in D} = \measuredangle 1 - \measuredangle(s-1)|_{s\in D} = -\text{chg}$ in prev. Ex $= +2\pi$

# of encirclements of 0 by $G = -1$cw (prev Ex) $= +1$ ccw

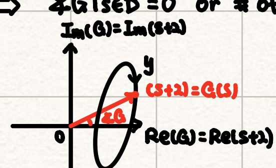
Note: a zero inside D contributes $\Rightarrow -2\pi$

a pole inside D contributes $\Rightarrow 2\pi$

$$G(s) = \frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)} \qquad \measuredangle G(s) = \overbrace{\sum_{i=1}^{m} \measuredangle(s-z_i)}^{-2\pi} - \overbrace{\sum_{i=1}^{n} \measuredangle(s-p_i)}^{2\pi} = 2\pi(n-m)$$

Ex: $G(s) = s+2$, same D. has 0 zeros, 0 poles inside $D \xRightarrow{\text{prf. of Arg}} \measuredangle G|_{s\in D} = 0$ or # of encirclements of origin by γ is 0

$\gamma = \{s+2 \mid s\in D\}$ = D translated by 2 to right



(γ does NOT encircle the origin)

chg. $\measuredangle G = 0$ # of encircle. is 0

counting # of encirclements = # of intersect of a ray by γ (+, − from L/R or R/L)