Why is $G(j\omega)$ called frequency response?

If $u(t)=\delta(t)$ impulse $\xrightarrow{U(s)=1u}$ $\boxed{G}$ $\xrightarrow{Y(s)=G(s)\cdot 1}$ $y(t)=\mathcal{L}^{-1}\{G(s)\}=g(t)$ where $g(t)$ = impulse response

G = BIBO stable system $=\mathcal{L}[g(t)]$

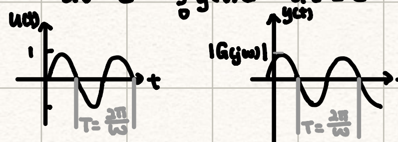
$U(s)=\frac{1}{s}$

If $u=\mathbb{1}(t)$ (step) $\Rightarrow y(t)$ is called step-response, $Y(s)=G(s)\cdot\frac{1}{s} \Rightarrow y(t)=\mathcal{L}^{-1}\{Y(s)\}$

Consider: $u(t)=e^{j\omega t}$ ← cplx. sine $\quad G(s)=\int_0^\infty g(\tau)e^{-s\tau}d\tau$

$e^{j\omega t}\rightarrow\boxed{g}\rightarrow y(t)=G(j\omega)\cdot e^{j\omega t}=g * e^{j\omega t}=\int_0^\infty g(\tau)e^{j\omega(t-\tau)}d\tau=e^{j\omega t}\cdot\int_0^\infty g(\tau)e^{-j\omega\tau}d\tau=e^{j\omega t}G(j\omega)=|G(j\omega)|e^{j\measuredangle G(j\omega)}e^{j\omega t}$

$=|G(j\omega)|e^{j(\omega t+\measuredangle G(j\omega))}$

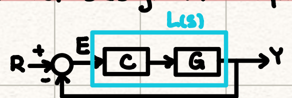


⇒ if $u(t)=\sin(\omega t)=(e^{j\omega t}-e^{-j\omega t})/2j$, $y(t)=|G(j\omega)|\cdot\sin(\omega t+\measuredangle G(j\omega))$ for sin input of freq ω, the output in steady-state is sine of ω (same freq) with amplitude $=|G(j\omega)|$ and phase shift $=\measuredangle G(j\omega)$

When varying $\omega \Rightarrow |G(j\omega)|$ and $\measuredangle G(j\omega)$ will characterize the response of system to sinusoidal input of freq. ω

$G(j\omega)$: called freq. response of system ⇒ this is how spectum analyzers obtain the Bode plots

Control Design in Frequency Domain by LOOPSHAPING of freq. response (Bode plots) of O.L TF

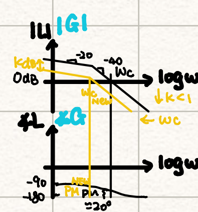


loop TF: $L(s)=C(s)\cdot G(s)$

Given G plant TF stable. i.e: all poles in OLHP or at most poles in $s=0$ and either its TF or Bode plots (freq. response). Design $C(s)$ controller ("compensator") s.t. feedback loop (C.L. system) as well as some transient perf. specs, given in terms of freq. domain specs. PM (phase design) (typ: 40°-45°) and analyze ω_c (given cross freq.) on the O.L. TF $L(s)$. Note: As PM↑, %OS↓. As ω_c↑, T_s↓

We will do design using Bode plots and their properties.

Recall: $|e_{ss}|=|e(\infty)|=\frac{1}{|L(0)|}$ ⇒ as DC gain↑, e_{ss}↓



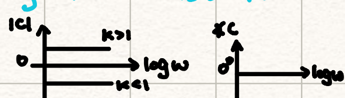
superposition

Note: $L(j\omega)=C(j\omega)\cdot G(j\omega) \Leftrightarrow \begin{cases}|L(j\omega)|_{dB}=|C(j\omega)|_{dB}+|G(j\omega)|_{dB}\\ \measuredangle L(j\omega)=\measuredangle C(j\omega)+\measuredangle G(j\omega)\end{cases}$ given plant

Typ: $PM^{NEW}\simeq 45^\circ$ (spec.)

→ to be designed s.t. $|L|_{dB}$ & $\measuredangle L$ have a desired "shape" ⇒ "LOOPSHAPING"

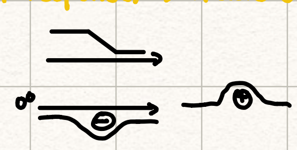
Note: consider $C(s)=K \Rightarrow \begin{cases}|C(j\omega)|_{dB}=K_{dB}\\ \measuredangle C(j\omega)=0^\circ\end{cases}$



⇒ $|L|$, $\measuredangle L$, obtained superposition

If use $K<1$ ⇒ PM↑ (to an acceptable) but ω_c↓ to ω_c^{NEW} i.e. T_s↑ (slower response) ⇒ trade off

to achieve both, we will look at $C(s)$ having a non-uniform freq. resp.



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