Ex: Pole Location & Qualitative Transient Response.

$$G(s) = \frac{s+12}{(s+10)(s^2+2s+9)[(s+20)^2+5]}$$

step: $\mathbb{1}(t) = u(t) \xrightarrow{U(s)} \boxed{G(s)} \xrightarrow{Y(s)} y(t)$ = ? qualitative elementary components

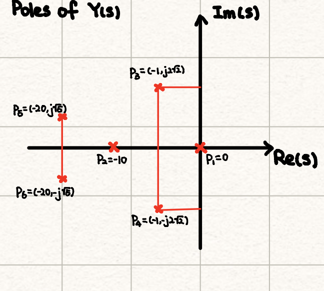
$Y(s)$ poles $\Rightarrow G(s) \cdot \frac{1}{s} \Rightarrow p_1 = 0,\ p_2 = -10,\ p_{3,4} = \underbrace{-1 \pm j2\sqrt{2}}^{-\sigma \pm \omega_d}$ $\quad p_{5,6} = -20 \pm j\sqrt{5}$

elementary component

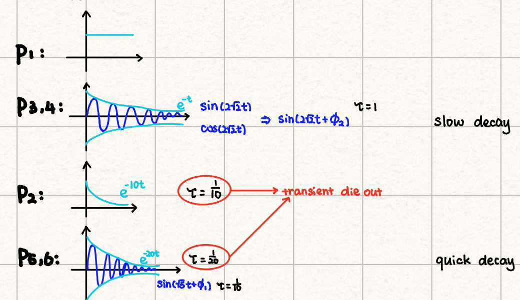
$y(t) = \sum_i c_i y_{u_i}(t)$

y_{ss}: a constant

Poles of $Y(s)$



(dominant behavior)



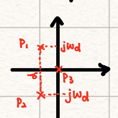
Transient response can be approximated by the slowest component ⇒ due to the pole with the slowest absolute value of real part (closest to j-axis)

Quantitative Time Response for 2nd order Systems

$$G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+\sigma)^2+\omega_d^2}\ ; \begin{cases} \sigma = \zeta\omega_n \\ \omega_d^2 = \omega_n(1-\zeta^2) \end{cases}$$

$u(t) \xrightarrow{U(s)} \boxed{G(s)} \xrightarrow{Y(s)} y(t)$ = ? exact(quantitative) response, dep on ζ, ω_n $(0<\zeta<1)$

$u(t) = \mathbb{1}(t)$ (step)

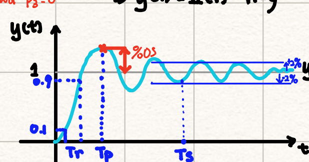


$$y(t) = \mathcal{L}^{-1}\left[\frac{\omega_n^2}{(s+\sigma)^2+\omega_d^2} \cdot \frac{1}{s}\right]$$

$$\sum_i \mathrm{Res}[Y(s)e^{st}, s = p_i]$$

$p_{1,2} = -\sigma \pm j\omega_d \quad p_3 = 0$

$$\Rightarrow y(t) = \mathbb{1}(t) - \frac{1}{\sqrt{1-\zeta^2}} \cdot e^{-\sigma t} \cdot \sin(\omega_d t + \phi) \cdot \mathbb{1}(t),\ \text{where } \cos\phi = \zeta$$



y_{ss}: steady-state value

Quantitative indecators:

T_r = rise-time: time it takes to get from 10% to 90% of y_{ss}

T_p = peak-time: time where $y(t)$ is max (@ peak)

T_s = setting-time: time for $y(t)$ to reach and stay within a ±2% band around y_{ss}

%OS = % overshoot : $\frac{y(T_p)-y_{ss}}{y_{ss}}$ %

We will relate this to ζ, ω_n or alternatively to (σ, ω_d)