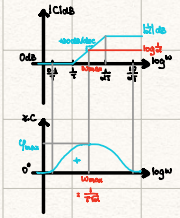
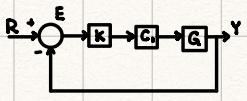
LEAD COMPENSATOR DESIGN



(1) $\sin\varphi_{max} = \frac{1-\alpha}{1+\alpha}$

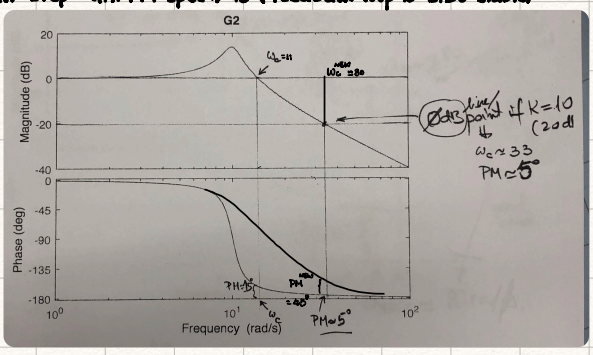
(2) $\omega_{max} = \frac{1}{T\sqrt{\alpha}}$

(3) $|C(j\omega_{max})|_{dB} = 10\lg\frac{1}{\alpha}$



$C_1(s) = \frac{Ts+1}{\alpha Ts+1}$, $0<\alpha<1$, $T>0$

Example 2: $G(s) = \frac{100}{s^2+2s+100}$; Control spec: $e_{ss} = e(\infty) \le 10\%$ (i. when $r(t)$ = step (ii. PM spec $\ge 45°$ (feedback loop is BiBo stable)



For e_{ss} spec., want to understand what do we need.

→ use FVT: $e_{ss} = e(\infty) = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0} s\cdot\frac{1}{1+kC_1\cdot G}\cdot\frac{1}{s}$ ($R(s)$) $= \frac{1}{1+\underbrace{kC_1(0)\cdot G(0)}_{\text{DC gain}}}$

⇒ From plot: $G(0)=1$, select: $C_1(0)=1$ ⇒ $e_{ss} = \frac{1}{1+k}$

⇒ For steady-state spec: $e(\infty) \le 0.1$ ⇒ $\frac{1}{1+k} \le 0.1$ $k \ge 9$

⇒ Let $k=10$ (ii. spec used) ⇒ $|k|_{dB} = 20dB$

⇒ new: ω_c↑ & PM↓ ($\le 3°-6°$)

We will design C_1 lead s.t. $PM^{new} \ge 45°$ (spec iii.)

Step 1: Find φ_{max} needed s.t. $PM^{new} \simeq 45°$

$\varphi_{max} = 45° + \underbrace{30°}_{\text{extra margin}} = 75° \overset{②}{\Rightarrow} \alpha = \frac{1-\sin\varphi_{max}}{1+\sin\varphi_{max}} \simeq 0.071 \overset{②}{\Rightarrow} |C_1(j\omega_{max})|_{dB} = 10\lg\frac{1}{\alpha} = 17.7dB$

Step 2: Set ω_{max} as the ω_c^{new} (gain crossover freq.) s.t. we get max benefit φ_{max}.

⇒ $|kC_1(j\omega_{max})G(j\omega_{max})| = 1$ ⇒ $0 = |k|_{dB} + |C_1(j\omega_{max})|_{dB} + |G(j\omega_{max})|_{dB}$

⇒ $|G(j\omega_{max})| = -17.7$ ⇒ we will read ω_{max} from $|G|_{dB}$ plot ⇒ $\omega_{c\,max} \simeq 80$

Step 3: From $\omega_{max} = \frac{1}{T\sqrt{\alpha}}$ ⇒ $\frac{1}{T} \simeq 10.3$ $\frac{1}{\alpha T} \simeq 600$

⇒ $C_1(s) = \frac{0.1s+1}{(10.1)*(0.01)s+1}$ lead compensator

Overall: $C(s) = k\cdot C_1(s)$; $k=10$

Last step: do Nyquist and check feedback loop is BiBo stable