Logarithms $a=b^c \Leftrightarrow \log_b a = c$

properties:

$a = b^{\log_b a}$

$\log_b \frac{1}{a} = -\log_b a$

$\log_c (ab) = \log_c a + \log_c b$

$\log_b \frac{a}{c} = \log_b a - \log_b c$

$\log_b a^n = n \log_b a$

$a^{\log_b n} = n^{\log_b a}$

$\log_b a = \frac{1}{\log_a b}$

Note: $\log n = \log_2 n$, when base is missing, base = 2

$\log^{(i)} n = \begin{cases} n & \text{iff } i=0 \\ \log(\log^{(i-1)} n) & \text{if } i>0 \end{cases}$ i.e: $\log^{3} n = \log\log\log n$

such that

$\log^* n = \min\{i : \log^{(i)} n \le 1\}$ i.e: $\log^* 2 = 1$ $\log^* 4 = 1+\log^* 2 = 2$ $\log^* 2^{2^{2^2}} = 1+\log^* 2^{2^2} = 2+\log^* 2^2 = 2+2 = 4$ $\log^* 2^{512} = 5$

log-star grows very slow!

$O(\log^* n) \neq O(1)$ but can be seen as a constant

#atoms in universe $\approx 10^{18} < 2^{512}$

Fibonacci numbers

$F_i = F_{i-1} + F_{i-2}$ where $F_0 = 0$ & $F_1 = 1$,

can be proven that: $F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$ (ϕ: golden conjugate $= \frac{1+\sqrt{5}}{2}$)

($\hat{\phi}$: golden conjugate inverse $= \frac{1-\sqrt{5}}{2}$)

Summations

Arithmatic series: $\sum_{k=1}^{n} k = 1+2+\cdots+n = \frac{n(n+1)}{2} = \Theta(n^2)$

Geometric series: $\sum_{k=0}^{n} x^k = 1+x+\cdots+x^n = \frac{x^{n+1}-1}{x-1}$

Infinite series: $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ if $|x|<1$

example: show that $\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}$ when $|x|<1$

Proof: $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$

Differenciate both sides over x: $\sum_{k=0}^{\infty} kx^{k-1} = \frac{1}{(1-x)^2}$

Multiply both sides by x: $\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}$

Telescoping

Assume a_i = something, it holds:

- $\sum_{i=1}^{n} a_i - a_{i-1} = a_n - a_0$

Proof:

$\sum_{i=1}^{n} a_i - a_{i-1} = \cancel{a_1} - a_0 + \cancel{a_2} - \cancel{a_1} + \cdots a_n - \cancel{a_{n-1}}$

$= a_n - a_0$

- $\sum_{k=0}^{n-1} a_k - a_{k-1} = a_0 - a_n$

Example: Prove $\sum_{k=1}^{n-1} \frac{1}{k(k+1)} = 1 - \frac{1}{n}$

$\sum_{k=1}^{n-1} \frac{1}{k(k+1)} = \sum_{k=1}^{n-1} \frac{1}{k} - \frac{1}{k+1} = \underset{a_0}{1} - \underset{a_n}{\frac{1}{n}}$

Binomial Coefficient

$(x+y)^r = \sum_{i=0}^{r} \binom{r}{i} x^i y^{r-i}$

r choose i / Binomial coefficient $= \frac{r!}{i!(r-i)!}$