Ex. If $X \sim \text{Binomial}(m, p)$ and $Y \sim \text{Binomial}(n, p)$ are independent, find the distribution of $Z = X + Y$.

The CFs of X and Y are $\Phi_X(\omega) = (pe^{j\omega} + 1 - p)^m$ $\Phi_Y(\omega) = (pe^{j\omega} + 1 - p)^n$

Since X and Y are independent, $\Phi_Z(\omega) = \Phi_X(\omega)\Phi_Y(\omega) = (pe^{j\omega} + 1 - p)^{m+n}$

$\Rightarrow$ It means $Z \sim \text{Binomial}(m+n, p)$

Independent and Identically Distributed RVs

If we have IID RVs, then they are independent with the exact same distribution.

$E[S_n] = E[X_1] + E[X_2] + \cdots E[X_n] = n\mu_x$ $\quad VAR[S_n] = n\sigma_x^2$ $\quad \Phi_{S_n}(\omega) = (\Phi_X(\omega))^n$

Sample Mean

The sample mean (or "average") can be modelled as the sum of multiple IID variables

$$M_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Since Mn is a function of RVs, then the sample mean itself is a random variable. In this case, the expected value of the sample mean is

$$E[M_n] = E\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] = \frac{1}{n}\sum_{i=1}^{n} E[x_i] = \frac{1}{n} \cdot n\mu_x = \mu_x$$

Since Mn is a RV, then we can also calculate its variance

$$VAR[M_n] = \frac{1}{n^2} VAR(S_n) = \frac{1}{n^2} \cdot n\sigma_x^2 = \frac{\sigma_x^2}{n}$$

Therefore, the sample mean has its own variance. The sandard deviation of the sample mean is also called the "standard error".

The variance approaches zero as the number of samples is increased, $\lim_{n\to\infty} \frac{\sigma^2}{n} = 0$.

In turn, this means that the possibility that the sample mean is close to the true mean increases as n increases.

We can use the chebyshev inequability to set an upper bound on this probability.

$$P[|M_n - \mu_x| \geq \varepsilon] \leq \frac{\sigma^2}{n\varepsilon^2} \quad \text{or} \quad P[|M_n - \mu_x| < \varepsilon] \geq 1 - \frac{\sigma^2}{n\varepsilon^2}$$

This conclusion leads to the weak law of large numbers

$\lim_{n\to\infty} P[|M_n - \mu_x| < \varepsilon] = 1$ for any positive ε

[illegible]