

1) Describe each of the following sets as a list

- a) $\{x \in \mathbb{N} \mid |x| < 6\}$
 $= \{1, 2, 3, 4, 5\}$
- b) $\{x \in \mathbb{N} \mid x < 30 \wedge (\forall y \in \mathbb{N})(x \neq 2y)\}$
 $= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$
- c) $\{x \in \mathbb{R} \mid 6x^4 + 43x^3 + 78x^2 + 5x - 12 = 0\}$
 $= \{-4, -3, -\frac{1}{2}, \frac{1}{3}\}$
- d) $\{x \in \mathbb{N} \mid x < 30 \wedge (\forall y \in \mathbb{N})(x \neq 2y)\}$
 $= \{5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

2) Given $A, B, C \subseteq S$ where

$$\begin{aligned} S &= \{x \in \mathbb{N} \mid x \leq 20\} \\ &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \\ A &= \{x \in S \mid (\exists y \in \mathbb{Z})(\exists z \in \mathbb{Z})(20y + xz = 1)\} \\ &= \{1, 3, 7, 9, 11, 13, 17, 19\} \\ B &= \{x \in S \mid (\exists y \in \mathbb{Z})(x = 2y + 1)\} \\ &= \{1, 3, 5, 7, 9, 11, 15, 17, 19\} \\ C &= \{x \in S \mid (\exists y \in \mathbb{N})(x = 5y)\} \\ &= \{5, 10, 15, 20\} \end{aligned}$$

Find

$$\begin{aligned} \text{a) } (A \cap B' \cap C)' &= \\ (A \cap B' \cap C) &= \{\text{empty}\} \\ \{\text{empty}\}' &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \end{aligned}$$

$$\begin{aligned} \text{b) } C \cup (A' - (S - B)) &= \\ (S - B) &= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \\ (A' - B') &= \{5, 15\} \\ (C \cup (A' - B')) &= \{5, 10, 15, 20\} \end{aligned}$$

3) Given $A, B, C \subseteq S$ where

$$\begin{aligned} S &= \{x \in \mathbb{N} \mid x \leq 50\} \\ &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, \dots, 30, \dots, 40, \dots, 50\} \\ A &= \{x \in S \mid (\exists y \in \mathbb{Z})(x = 4y)\} \\ &= \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\} \\ B &= \{x \in S \mid (\exists y \in \mathbb{Z})(\exists z \in \mathbb{Z})(5y + xz = 3)\} \\ &= \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24, 26, 27, 28, 29, 31, 32, \\ &\quad 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49\} \end{aligned}$$

$$C = \{x \in S \mid (\exists y \in N)(x = 3y + 2)\}$$

$$= \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50\}$$

Find

a) $|A \cup B \cup C|$

$$(A \cup B) = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49\}$$

$$((A \cup B) \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50\}$$

$$|A \cup B \cup C| = 45$$

b) $|P(A \cup B) \times C'|$

$$|(A \cup B)| = 42$$

$$|C'| = 34$$

$$|P(A \cup B) \times C'| = 2^{42} * 34$$