

Part a

$i = 2$

while ($i < n$)

$i = i^2$

$k : 0 \quad 1 \quad 2 \quad 3 \quad \dots \quad k$
 $i : 2^1 \quad 2^2 \quad 2^4 \quad 2^8 \quad \dots \quad 2^{2^k}$

$$2^{2^k} = n$$

$$k = \log \log n \Rightarrow \Theta(\log \log n)$$

Part b

for (int $i = 1$, $i \leq n$, $i++$)

if ($i \% \text{sqrt}(n) == 0$)

worst case: if statement always true

for (int $k = 0$, $k < \text{pow}(i, 3)$, $k++$)

t increment by 1

$$\begin{aligned} & \sum_{i=1}^n (\Theta(1) + O(\sum_{k=0}^{i^3} \Theta(1))) \\ &= \sum_{i=1}^n \Theta(1) + \sum_{i=1}^n i^3 \cdot \sum_{k=0}^{i^3-1} \Theta(1) \\ &= \Theta(n) + \sum_{t=1}^{\sqrt{n}} \cdot \sum_{i=t\sqrt{n}}^{(t\sqrt{n})^3-1} \Theta(1) \\ &= \Theta(n) + \sum_{t=1}^{\sqrt{n}} \cdot \Theta((t\sqrt{n})^3) \\ &= \Theta(n) + \Theta(\sqrt{n} \cdot (\sqrt{n} \cdot \sqrt{n})^3) \\ &= \Theta(n) + \Theta(n^{\frac{7}{2}}) = \Theta(n^{\frac{7}{2}}) \end{aligned}$$

$t = 1 \quad 2 \quad 3 \quad \dots \quad t$
 $i = \sqrt{n} \quad 2\sqrt{n} \quad 3\sqrt{n} \quad \dots \quad t\sqrt{n}$

stop when $i = n$

$$i = t\sqrt{n} = n \quad t = \sqrt{n}$$

$$n = 4, i = 2, 4$$

$$n = 9, i = 3, 6, 9$$

$$n = 16, i = 4, 8, 12, 16$$

Part c

for ($i = 1$, $i \leq n$, $i++$)

for ($k = 1$, $k \leq n$, $k++$)

if ($A[k] == i$)

worst case: always true, not possible
 only true once in each iteration of i 's for loop

for (int $m = 1$, $m \leq n$, $m = m + m$)

$$T = \sum_{i=1}^n \sum_{k=1}^n \Theta(1) + \sum_{i=1}^n \sum_{m=1}^{\log n} \Theta(1)$$

$t = 0 \quad 1 \quad 2 \quad 3 \quad \dots \quad t$
 $m = 1 \quad 2 \quad 4 \quad 8 \quad \dots \quad 2^t$

stop when $m = n = 2^t$

$$= \Theta(n^2) + \Theta(n \cdot \log n) = \Theta(n^2)$$

$$t = \log(n)$$

Part d

$\Theta(1)$ $\text{int}^* a = \text{new int } [10]$

$\Theta(1)$ $\text{int size} = 10$

for (int i = 0; i < n; i++) n-1 iterations

if i == size \leftarrow worst case. always true, not possible

$\Theta(1)$ $\text{int newSize} = 3 * \text{size} / 2$ \rightarrow increase by 1
 $\text{int}^* b = \text{new int } [\text{newSize}]$ \rightarrow size increase by $\frac{3}{2}$
 \Rightarrow true only once

(size-1) iterations for (j = 0; j < size; j++)

$\Theta(1)$ $b[j] = a[j]$

$\Theta(1)$ delete [] a

a = b

size = newSize

$\Theta(1)$ $a[i] = i * i$

size
 $i=0, 10$
 $i=1, 10 \cdot \frac{3}{2}$
 $i=2, 10 \cdot (\frac{3}{2})^2$
 \dots

stop before $i = n, 10 (\frac{3}{2})^i$

size = $10 (\frac{3}{2})^i = i = n$

$i = \log \frac{n}{10}$ (when)

$$T(n) = \cancel{2\Theta(1)} + \sum_{i=0}^{n-1} 2\Theta(1) + \cancel{2\Theta(1)} + \sum_{j=0}^{\text{size}-1} \Theta(1)$$

$$= \Theta(n) + \sum_{j=0}^{\log \frac{n}{10} - 1} \Theta(1)$$

$$= \Theta(n) + \Theta(\log \frac{n}{10}) = \Theta(n) + \Theta(\log n) - \Theta(\log 10)$$

$$= \Theta(n)$$