1. $P = \dfrac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8}{15^8} = $ 0.10124

2. $\underline{\text{odd}}\ \underline{\text{odd}}\ \_\ \_\ \underline{\text{even}}$ $5 \times 4 \times 5 \times {}_7C_1 \times {}_6C_1 = 4200$

$P_{1\ out\ of\ 1} = \dfrac{4200}{100000} = 0.042$ $1 - 0.042 = 0.958$

$P_{5\ out\ of\ 8} = {}_8C_5 \times (0.042)^5 \times (0.958)^3$

$\qquad = 56 \times (0.042)^5 \times (0.958)^3 = $ 0.00000643

$(4,5,6) \to 5\#$

3. $P_A = \dfrac{6 \times 3 \times 3}{6^3} \times 2 = \dfrac{18}{36} = \dfrac{1}{2}$

$P_B = \dfrac{6}{6^3} = \dfrac{1}{36}$

$P(A \cap B) = \dfrac{3}{6^3} = \dfrac{1}{72} \ne$ $P_A \cdot P_B = \dfrac{1}{2} \cdot \dfrac{1}{36} = \dfrac{1}{72}$

$\Rightarrow$ independent

1. $P = \dfrac{{}_4C_1 \cdot {}_{13}C_5}{{}_{52}C_5} = \dfrac{4 \times 1287}{2598960} = \dfrac{5148}{2598960} = 0.00198$

Expected: $1 \div P = 504.8 \approx$ 505

2. $P(\text{played } 5 \mid \text{won } 4)$

$= \dfrac{P(\text{played } 5 \cap \text{won } 4)}{P(\text{won } 4)} = \dfrac{P(\text{played } 5 \cap \text{won } 4)}{P(\text{played } 5 \cap \text{won } 4) + P(\text{not played } 5 \cap \text{won } 4)}$

$= \dfrac{{}_5C_1 \times 0.75 \times (0.7)^4 \times 0.3}{{}_5C_1 \times 0.75 \times (0.7)^4 \times 0.3 + {}_5C_1 \times 0.25 \times (0.5)^5}$

$=$ 0.874