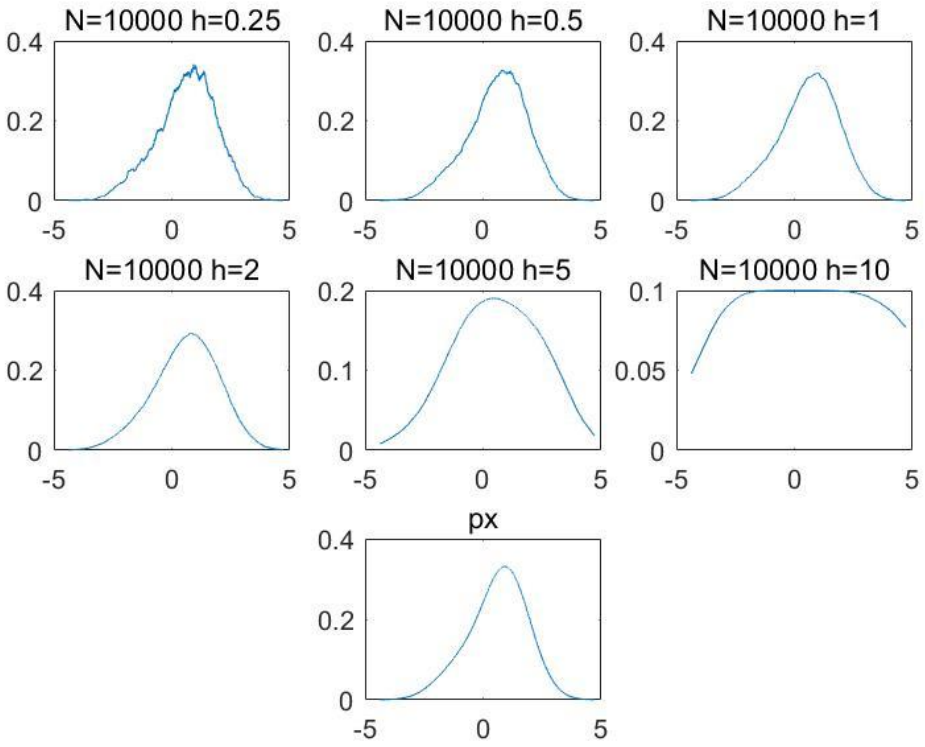
programming report

李逸思 自动化系 2016310707

Case1 retangular window

（a）N=10000 时，取窗宽 a 分别为 0.25,0.5,1,2,5,10 时，得到估计的 p(x)绘图如下



（b）N=1000 时，取窗宽 a 分别为 0.25,0.5,1,2,5,10，50 次随机取样，得到 ε (pn)如下

	a=0.25	a=0.5	a=1	a=2	a=5	a=10
50 次随机取样	1.397115	118.2525	118.2418	119.4488	118.6407	117.6499
	0.756027	118.8615	116.9117	119.2341	121.1588	120.8169
	0.809089	117.6989	117.3114	119.6996	118.7223	118.5354
	1.065019	121.7436	119.0736	117.1799	119.0297	118.0665
	0.716641	119.8869	118.0159	116.8123	118.7288	117.588
	0.604896	119.2847	117.2109	119.5334	122.4142	120.3415
	0.604274	118.5707	119.4668	116.3816	117.9276	119.375
	0.63684	120.1066	117.8945	119.6617	117.7004	120.0977
	0.844577	115.9563	119.0638	122.957	117.7611	119.5661
	0.844082	120.4281	119.7455	121.6042	118.457	117.5924
	0.720393	121.3121	117.5743	117.373	121.1753	117.4872
	0.720217	119.1	118.3941	117.1908	118.5433	119.2056

	0.787693	118.6286	120.1487	119.8019	119.553	120.6595
	1.088132	118.8804	119.3811	117.7977	121.3014	119.4763
	0.633501	119.8239	119.5796	118.1114	117.5396	117.8224
	0.875222	119.086	120.7558	118.5864	120.1545	116.1723
	0.494414	118.477	117.5483	116.8166	117.749	119.9714
	0.831312	119.9875	118.245	119.0029	116.5468	118.8978
	1.48849	118.1939	118.2571	118.2762	118.5597	117.6535
	0.832224	118.6038	119.38	120.3112	118.6904	118.6754
	0.491516	121.309	120.1238	120.1205	116.9954	119.8087
	0.646839	116.7377	117.861	120.0171	120.2892	119.9145
	1.462457	120.0591	119.8177	117.9526	117.5221	120.5191
	1.396426	118.6316	117.952	117.0447	118.7198	119.0415
	0.913498	117.6146	120.2164	118.2391	120.4424	118.4186
	0.718012	119.9362	118.9792	117.3171	116.6736	118.2558
	1.214755	122.9904	117.5798	117.4682	119.5586	118.6453
	1.099684	117.6054	119.9931	119.7396	118.1169	119.5777
	0.990538	118.2934	119.4499	117.2894	119.3086	117.1196
	0.493743	118.6981	119.9385	117.8317	119.2773	119.2402
	0.556991	118.6013	121.8053	117.9118	117.8043	119.3996
	0.637545	119.577	119.0879	117.9311	116.4915	122.0062
	0.652789	121.3719	119.2982	118.2535	118.9611	118.5033
	0.786526	117.4164	118.362	120.352	117.1206	119.3145
	0.638259	116.153	118.7358	119.8187	116.4068	119.6465
	0.631523	118.7347	118.0058	121.1018	119.6337	118.7605
	0.768942	117.0226	119.0087	120.7722	118.5399	120.5756
	0.859681	117.7659	121.1267	118.7416	120.4185	119.2771
	0.495038	116.5449	118.9938	117.8919	117.7142	116.9441
	1.162715	119.0622	121.0075	119.8847	119.7514	119.5728
	1.022101	121.9292	119.6552	119.1696	118.4625	117.8678
	0.502892	117.6543	119.037	119.3492	117.5277	117.8821
	0.575808	117.4286	118.3256	120.645	118.2683	116.1422
	1.003953	118.7039	121.5239	120.2475	118.1881	120.1977
	1.056918	115.3824	119.7851	120.2058	120.355	119.3856
	0.758351	120.499	117.1989	115.3344	117.4888	116.9959
	0.791599	118.3779	117.8517	120.9361	120.3669	120.1158
	0.948575	119.6098	118.2454	119.5355	118.7653	119.0089
	0.35176	118.2825	115.3628	120.1133	119.1328	120.9705
	1.02154	117.4463	118.1358	117.6414	117.4832	118.8138

（c）取 N 分别为 10,50,100,1000,2000，取窗宽 a 分别为 0.25,0.5,1,2,5,10 时，50 次随机取样，得到 ε (pn)的均值与方差如下

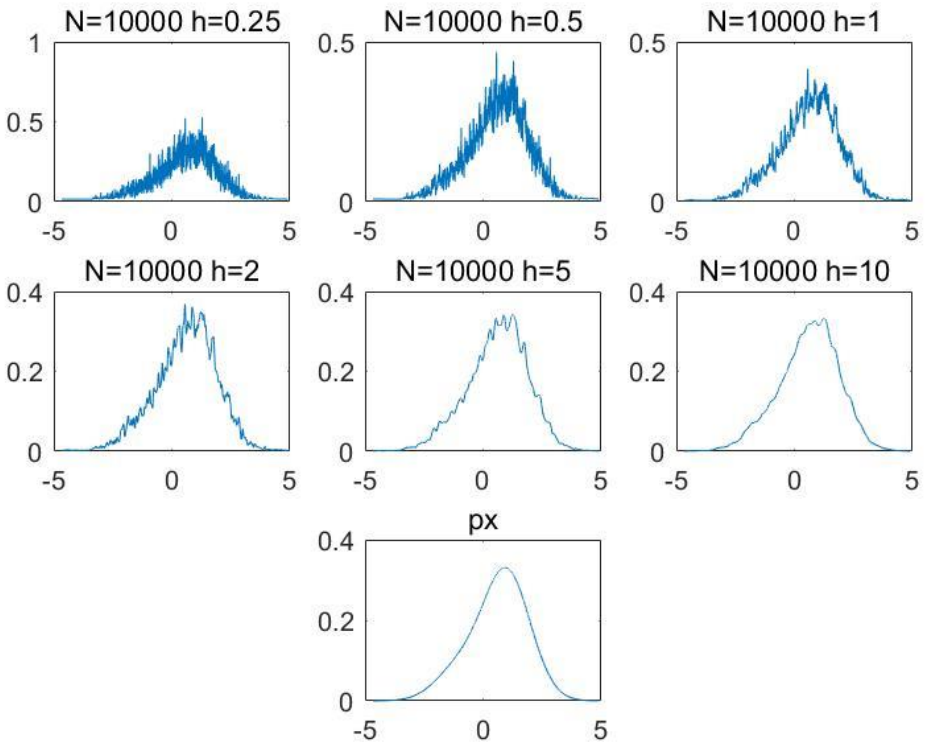
期望（方窗）						
	a=0.25	a=0.5	a=1	a=2	a=5	a=10

N=10	2.015938	0.565173	5.92E-01	0.617063	0.584517	0.601398
N=50	1.040665	2.924534	2.993305	2.952223	3.008282	2.992557
N=100	0.965687	6.00896	5.976998	5.887836	5.965782	5.993199
N=1000	0.820075	59.43296	59.70203	59.43262	59.01572	59.54193
N=2000	0.828023	118.8464	118.8933	118.8928	118.7228	118.9515

方差（方窗）						
	a=0.25	a=0.5	a=1	a=2	a=5	a=10
N=10	2.163227	0.014645	1.15E-02	0.013744	0.013411	0.00586
N=50	0.258197	0.045013	0.060281	0.057225	0.07614	0.048344
N=100	0.140872	0.133904	0.13708	0.118494	0.142147	0.095233
N=1000	0.085187	0.976744	1.01526	1.264955	1.411819	1.392943
N=2000	0.071165	2.523395	1.610364	2.316613	1.853908	1.600784

Case2 Guassian window

（a）N=10000 时，取窗宽 a 分别为 0.25,0.5,1,2,5,10 时，得到估计的 p(x)绘图如下



（b）N=1000 时，取窗宽 a 分别为 0.25,0.5,1,2,5,10，50 次随机取样，得到 ε (pn)如下

	a=0.25	a=0.5	a=1	a=2	a=5	a=10
50 次随机取样	16.04998	118.0335	118.4675	116.9522	120.2299	120.0373
	15.92308	118.6662	118.3689	119.7031	117.6526	122.1974
	15.18348	119.5446	119.5443	119.3016	119.3739	118.2124

	14.58351	118.7341	118.1987	121.5538	120.4857	119.3263
	14.41306	117.3646	116.9779	119.8696	117.6366	117.4771
	14.64598	121.3483	118.6749	118.9562	119.0373	117.2734
	13.67345	117.5253	118.781	122.0331	120.0588	118.0402
	12.63819	119.519	116.8371	120.2325	116.5873	118.5365
	11.86589	117.1492	119.4153	118.4213	118.8804	119.8464
	12.94853	121.3997	119.1097	118.9258	116.5858	119.3342
	14.81696	120.794	117.9841	116.6904	119.9974	118.1712
	12.09445	117.388	116.1123	118.6917	117.9426	119.7234
	13.22646	117.2503	121.1359	116.9771	119.4026	119.6649
	13.37126	118.0862	118.2803	120.963	119.5825	116.9156
	15.9188	120.2866	115.1901	118.4808	116.257	118.1172
	12.3369	118.1552	118.4259	120.9138	118.7512	122.0948
	12.45567	117.2468	120.59	117.2161	117.3887	120.7109
	12.42923	120.3715	118.7762	117.2322	118.7387	116.7209
	13.1077	119.9823	118.3071	117.4075	116.6065	121.8528
	12.72671	118.4365	121.2039	119.2586	120.097	118.7051
	11.32183	120.4646	117.0716	118.6446	121.1259	121.6224
	13.4019	120.5802	120.1488	118.5549	117.3584	117.7405
	12.4364	117.1807	118.4822	118.9839	119.7127	118.326
	15.01194	120.9238	117.1903	116.7277	118.1319	122.1063
	13.50124	119.2137	121.6191	119.0902	119.1987	116.9149
	13.60859	116.4613	118.9758	118.2581	119.5333	119.3513
	14.01682	117.849	116.9436	120.0809	117.7482	117.7268
	11.23046	116.7571	119.295	117.9579	116.1414	120.2764
	14.74891	117.7757	119.1445	120.4925	119.7622	118.477
	11.78149	120.046	120.0389	118.0112	118.1618	117.681
	10.49938	117.1012	118.9155	118.3278	117.3066	116.8094
	11.87944	117.4449	116.9045	118.4347	116.1818	118.2625
	14.39362	118.3682	120.3962	119.1072	120.392	119.4861
	14.17299	116.6732	117.8345	120.7143	117.7779	120.3682
	12.99459	118.9092	116.5975	120.2108	119.5614	116.013
	12.40631	117.8496	118.0609	118.1672	118.9859	120.0193
	14.91237	119.2049	119.2027	118.37	115.5092	118.958
	15.40048	119.3238	117.9435	119.4808	120.1679	119.3213
	14.52159	116.6925	117.5974	116.0416	117.2969	119.8352
	14.96776	116.7013	119.1539	118.073	118.9427	121.1063
	11.97151	119.8786	117.4131	118.4119	121.8393	118.7209
	14.28784	118.8864	118.552	117.0423	120.0457	117.2865
	14.93154	121.7157	116.9916	119.1825	117.0877	118.25
	13.74847	117.6025	118.3605	117.9368	118.4379	117.3697
	12.47045	119.0696	119.4836	119.6919	118.8797	117.3226
	14.70806	118.9859	116.9123	121.8402	118.5071	119.1614

	11.90655	118.9347	119.1138	120.6342	117.6362	121.2267
	15.99144	121.0586	117.4187	119.6246	117.7268	117.396
	14.24897	118.2163	119.7115	120.258	120.097	118.2999
	15.03348	115.1478	120.4615	117.4417	118.2084	120.1005

（c）取 N 分别为 10,50,100,1000,2000，取窗宽 a 分别为 0.25,0.5,1,2,5,10 时，50 次随机取样，得到 ε (pn)的均值与方差如下

期望（高斯窗）						
	a=0.25	a=0.5	a=1	a=2	a=5	a=10
N=10	2.783947	0.586878	5.98E-01	0.615888	0.608323	0.597713
N=50	4.345464	2.964096	2.951577	2.964691	2.964541	2.987247
N=100	5.137519	5.966001	5.935043	5.988158	5.913247	5.918787
N=1000	10.44253	59.38146	59.39566	59.66093	59.54455	59.31658
N=2000	13.61831	118.646	118.5263	118.9115	118.5751	118.9699

方差（高斯窗）						
	a=0.25	a=0.5	a=1	a=2	a=5	a=10
N=10	1.990375	0.010069	1.05E-02	0.00848	0.013151	0.011081
N=50	4.94873	0.043801	0.054525	0.049038	0.042787	0.06264
N=100	3.214371	0.086857	0.117435	0.086308	0.079279	0.077242
N=1000	3.986081	1.645414	0.88972	1.311399	1.246977	1.414335
N=2000	1.960566	2.328832	1.877423	1.990392	2.053472	2.473582

summary

（d）由以上经验分析可知，当给定 n 时，随着窗宽减小误差减小，但由于 parzen window method 估计密度时，需满足以下三个条件:

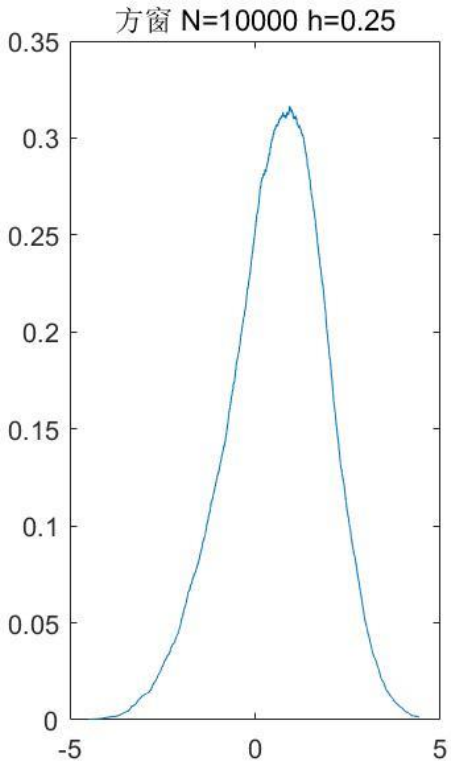
$$\lim_{n\to\infty} V_n = 0,$$

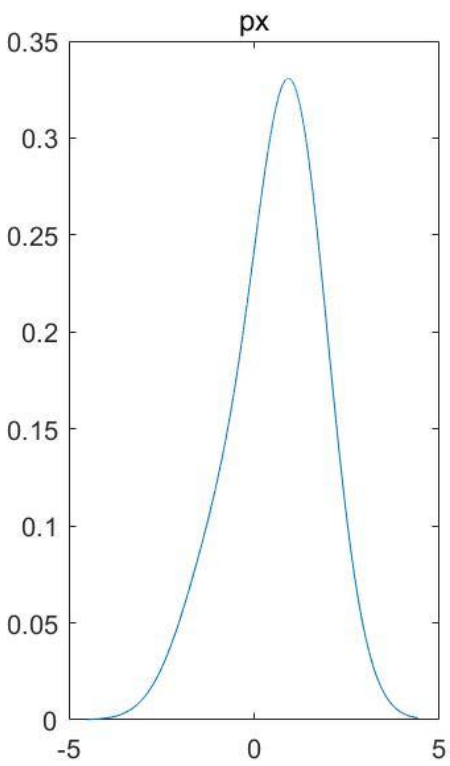
$$\lim_{n\to\infty} k_n = \infty,$$

$$\lim_{n\to\infty} \frac{k_n}{n} = 0$$

那么当样本趋于无穷多时，密度的估计值收敛于其真实值；故不能将窗宽取无限小，实际选取时可以选取误差的期望与方差小于所需阈值的窗宽。

（g）对比以上采用方窗函数，参数选取 N=10000，a=0.25 时，拟合效果最好，分布与实际分布相近， ε (pn)=0.8919


方窗 N=10000 h=0.25
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
0
5


px
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
0
5