Lecture 13
Segment 2

Dependent & Independent t-tests

Dependent means t

The formulae are actually the same as the single sample t but the raw scores are difference scores, so the mean is the mean of the difference scores and SE_M is based on the standard deviation of the difference scores

Dependent means t

- Suppose a researcher is testing a new technique to help people quit smoking. The number of cigarettes smoked per day is measured before and after treatment. Is the difference significant?

Dependent means t

Subject #	X1	X2	D
1	19	12	-7
2	35	36	1
3	20	13	-7
4	31	24	-7

Dependent means t

Subject #	D	$(D - M_D)$	$(D - M_D)^2$
1	-7	-2	4
2	1	6	36
3	-7	-2	4
4	-7	-2	4

Dependent means t

$SD^2 = \Sigma(D - M_D)^2 / (N - 1) = SS/df = 48 / 3 = 16$

$SE^2_{MD} = SD^2/N = 16 / 4 = 4$

$SE_{MD} = 2$

$t = (M_D - \mu) / SE_{MD} = (-5 - 0) / 2 = -2.5$

$t = M_D / SE_{MD} = -5 / 2 = -2.5$

For a directional test with alpha = .05, df = 3, p = .044

$\therefore$ Reject H_0

Dependent means t

- Effect size
  - $d = (M_D - \mu) / SD = -5/4 = -1.25$
  - Note: $\mu = 0$

Independent means t

- Compares two independent groups
  - For example, males and females, control and experimental, patients and normals, etc.

Independent means t

$t = (M_1 - M_2) / SE_{Difference}$

$SE^2_{Difference} = SE^2_{M1} + SE^2_{M2}$

$SE^2_{M1} = SD^2_{Pooled} / N1$

$SE^2_{M2} = SD^2_{Pooled} / N2$

$SD^2_{Pooled} = df_1/df_{Total}(SD^2_1) + df_2/df_{Total}(SD^2_2)$

Notice that this is just a weighted average of the sample variances

Independent means t

- Group 1 (young adults)
    - $M_1 = 350$
    - $SD_1 = 20$
    - $N_1 = 100$
- Group 2 (elderly adults)
    - $M_2 = 360$
    - $SD_2 = 30$
    - $N_2 = 100$

    - Null hypothesis: $\mu_1 = \mu_2$
    - Alternative hypothesis: $\mu_1 < \mu_2$

Independent means t

$SD^2_{Pooled} = df_1/df_{Total}(SD^2{}_1) + df_2/df_{Total}(SD^2{}_2)$

$SD^2_{Pooled} = 99/198(400) + 99/198(900)$

$SD^2_{Pooled} = 650$

Independent means t

$SE^2_{M1} = SD^2_{Pooled} / N1 = 650 / 100 = 6.5$

$SE^2_{M2} = SD^2_{Pooled} / N2 = 650 / 100 = 6.5$

$SE^2_{Difference} = SE^2_{M1} + SE^2_{M2} = 13$

$SE_{Difference} = \sqrt{SE^2_{Difference}} = 3.61$

Independent means t

$t = (M_1 - M_2) / SE_{Difference}$

$t = (350 - 360) / 3.61 = -2.77$

$p = .003$ (based on $df = 198$, $\alpha = .05$, directional test)

$\therefore$ Reject H_0

Independent means t

- Now let's suppose that $N_1 = 10$ and $N_2 = 10$

Independent means t

$SD^2_{Pooled} = df_1/df_{Total}(SD^2_1) + df_2/df_{Total}(SD^2_2)$

$SD^2_{Pooled} = 9/18(400) + 9/18(900)$

$SD^2_{Pooled} = 650$

Independent means t

$SE^2_{M1} = SD^2_{Pooled} / N1 = 650 / 10 = 65$

$SE^2_{M2} = SD^2_{Pooled} / N2 = 650 / 10 = 65$

$SE^2_{Difference} = SE^2_{M1} + SE^2_{M2} = 130$

$SE_{Difference} = \sqrt{SE^2_{Difference}} = 11.4$

Independent means t

$t = (M_1 - M_2) / SE_{Difference}$

$t = (350 - 360) / 11.4 = -.88$

$p = .20$ (based on $df = 18$, $\alpha = .05$, directional test)

$\therefore$ Retain H_0

Independent means t

Effect size

$d = (M_1 - M_2) / SD_{pooled}$

$d = (350 - 360) / 25.5 = -.39$

Student’s t-test summary

- Single sample t
  - compares a sample mean to a population mean
- Dependent samples t
  - compares two dependent means
- Independent samples t
  - compares two independent means