Lecture 14
Segment 3

Factorial ANOVA

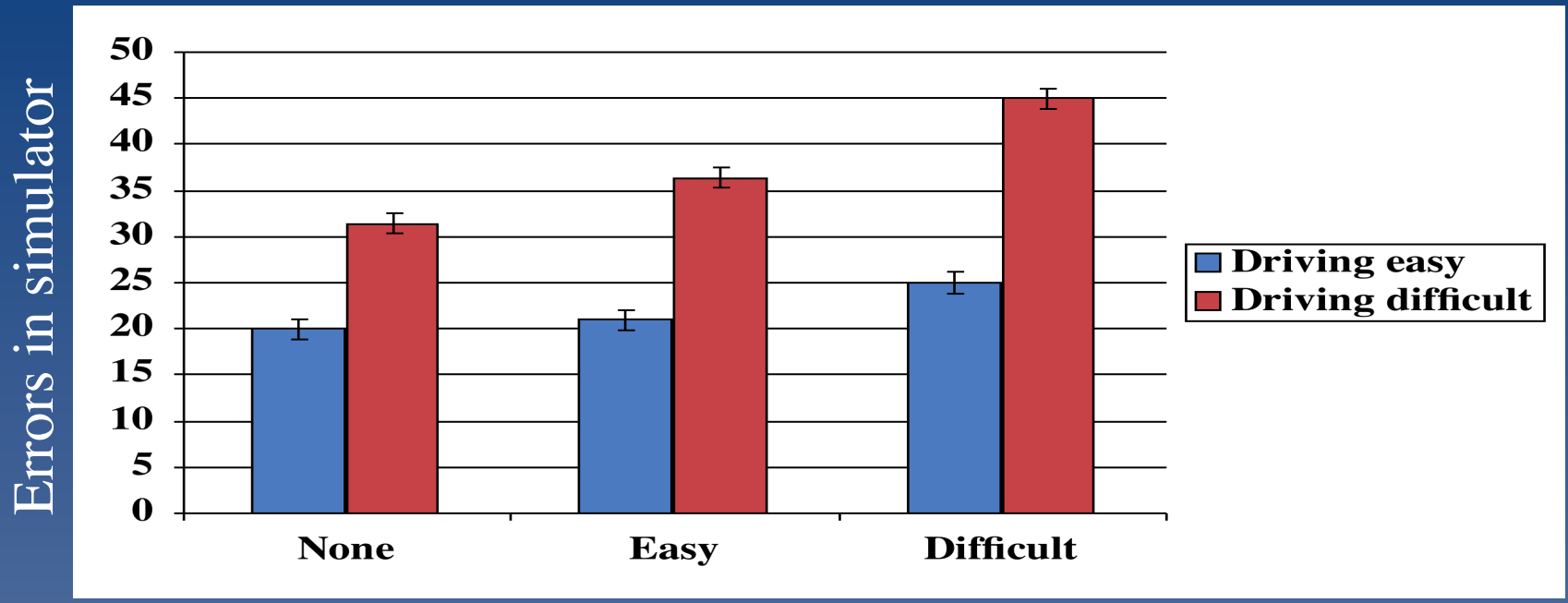
Factorial ANOVA

- Two IVs (treatments)
- One continuous DV (response)

Example

- IV = driving difficulty (easy, difficult)
- IV = conversation difficulty (none, easy, difficult)
- DV = errors made in driving simulator

Driving errors



Factorial ANOVA

- Three hypotheses can be tested in this one experiment:
  - More errors in the difficult simulator?
  - More errors with more difficult conversations?
  - More errors due to the interaction of these factors?

Factorial ANOVA

- Three F ratios
  - F_A
  - F_B
  - F_{AxB}

Factorial ANOVA

- Main effect: the effect of one IV averaged across the levels of the other IV
- Interaction effect: the effect of one IV depends on the other IV (the simple effects of one IV change across the levels of the other IV)
- Simple effect: the effect of one IV at a particular level of the other IV

Factorial ANOVA

- Main effects and interaction effect are independent from one another
  - Note that this is different from studies that don't employ an experimental design
    - For example, in MR, when predicting faculty salary, the effects of publications and years since the Ph.D. were correlated..

Factorial ANOVA

- In fact, factorial ANOVA is just a special case of multiple regression.
  - It is a multiple regression with perfectly independent predictors (IVs).
    - This is why we like it so much!

Partition SS in the DV

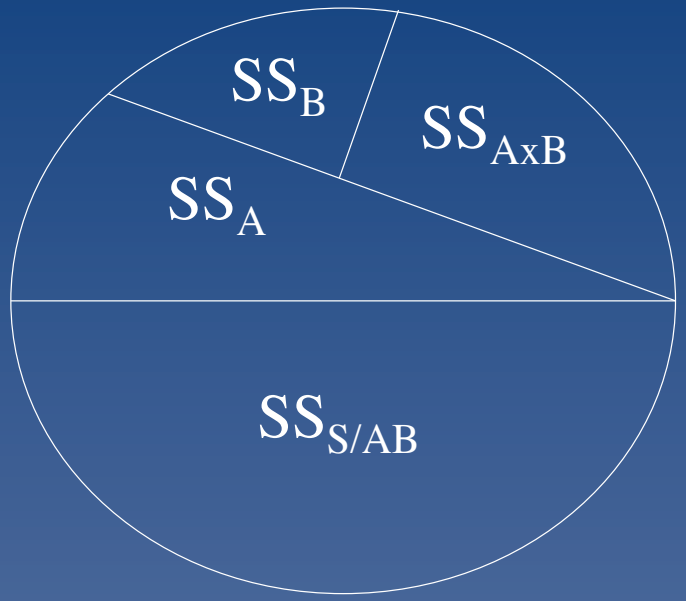


Illustration from multiple regression

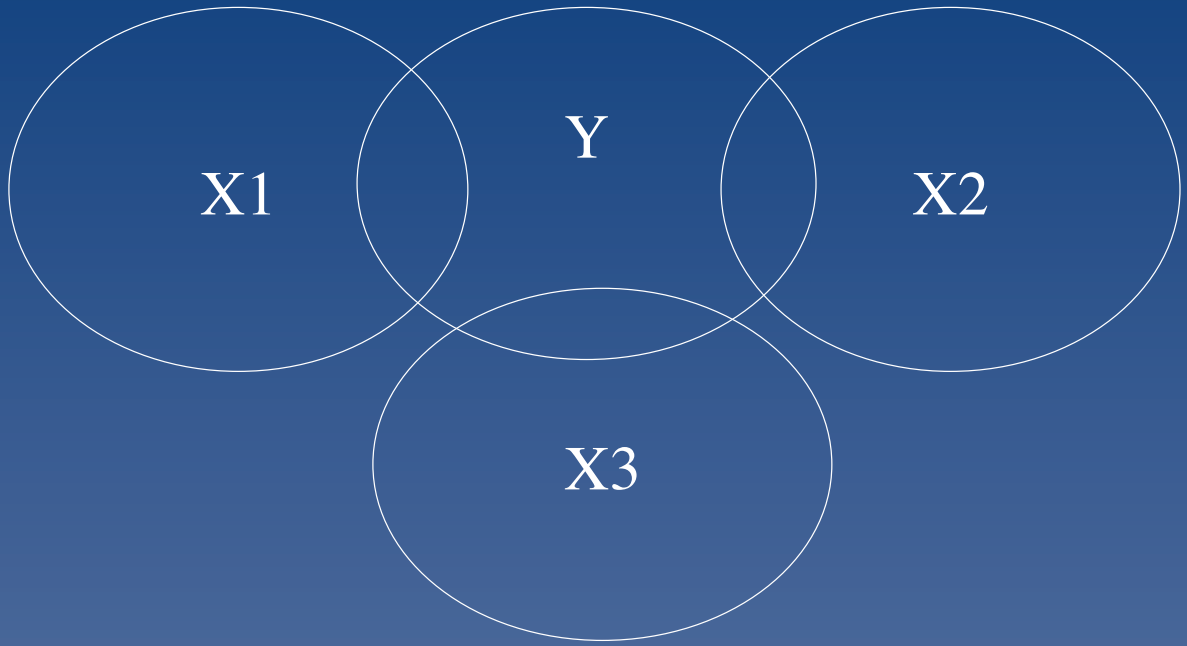
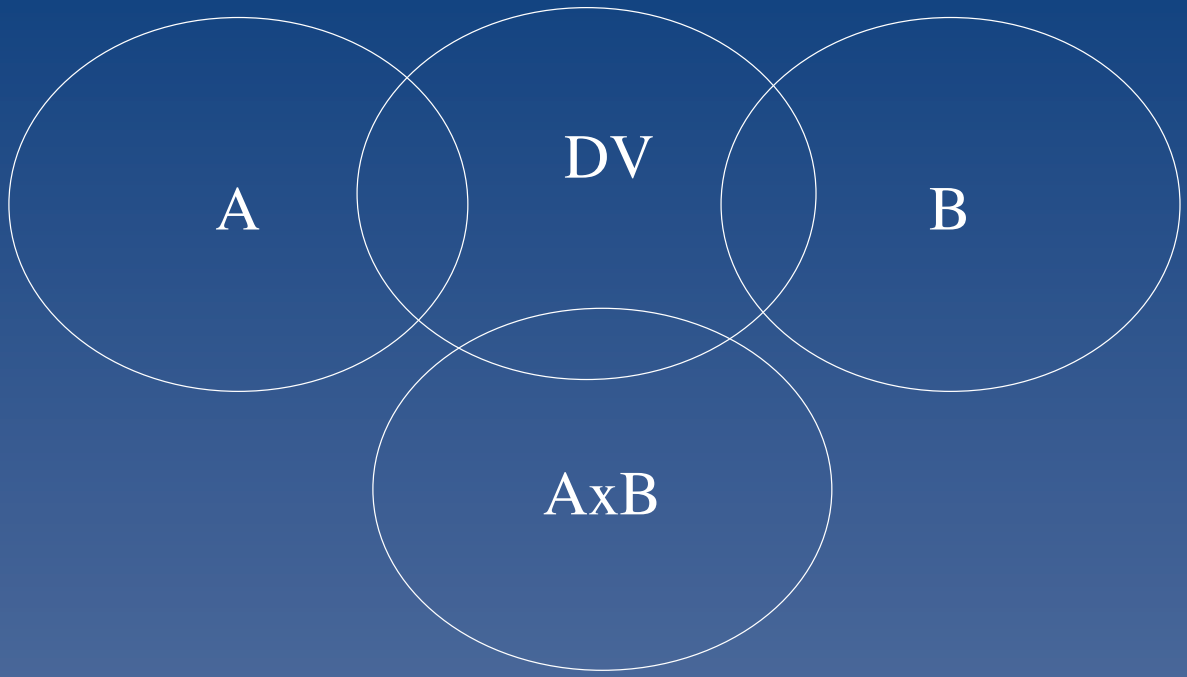


Illustration from multiple regression



Remember, GLM

- Multiple regression
  - $Y = B_0 + B_1X_1 + B_2X_2 + B_3X_3 + e$

  - Y = faculty salary
  - X1 = years since Ph.D.
  - X2 = # of publications
  - X3 = (years x pubs)

Remember, GLM

- Factorial ANOVA
  - $Y = B_0 + B_1X_1 + B_2X_2 + B_3X_3 + e$

  - Y = faculty salary
  - X1 = gender
  - X2 = race
  - X3 = interaction (gender x race)

F ratios

- $F_A = MS_A / MS_{S/AB}$
- $F_B = MS_B / MS_{S/AB}$
- $F_{AxB} = MS_{AxB} / MS_{S/AB}$

MS

- $MS_A = SS_A / df_A$
- $MS_B = SS_B / df_B$
- $MS_{AxB} = SS_{AxB} / df_{AxB}$
- $MS_{S/AB} = SS_{S/AB} / df_{S/AB}$

df

- $df_A = a - 1$
- $df_B = b - 1$
- $df_{AxB} = (a - 1)(b - 1)$
- $df_{S/AB} = ab(n - 1)$
- $df_{Total} = abn - 1 = N - 1$

Follow-up tests

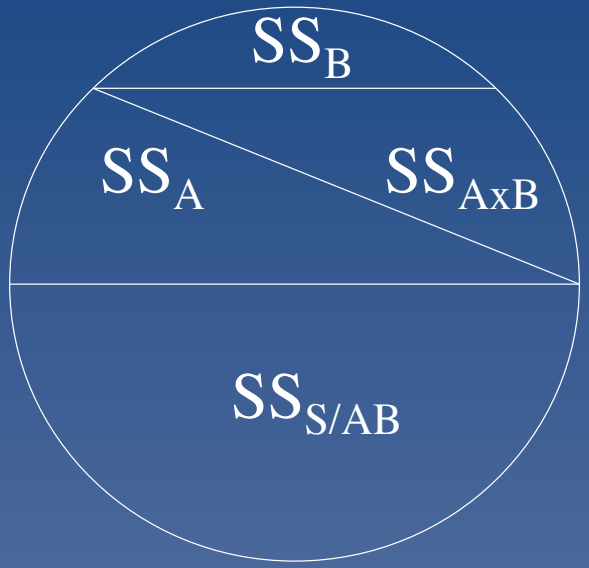
- Main effects
  - Post-hoc tests
- Interaction
  - Analysis of simple effects
    - Conduct a series of one-way ANOVAs
      - For example, we could conduct 3 one-way ANOVAs comparing high and low spans at each level of the other IV

Effect size

- Complete η^2
  - $\eta^2 = SS_{effect} / SS_{total}$
- Partial η^2
  - $\eta^2 = SS_{effect} / (SS_{effect} + SS_{S/AB})$

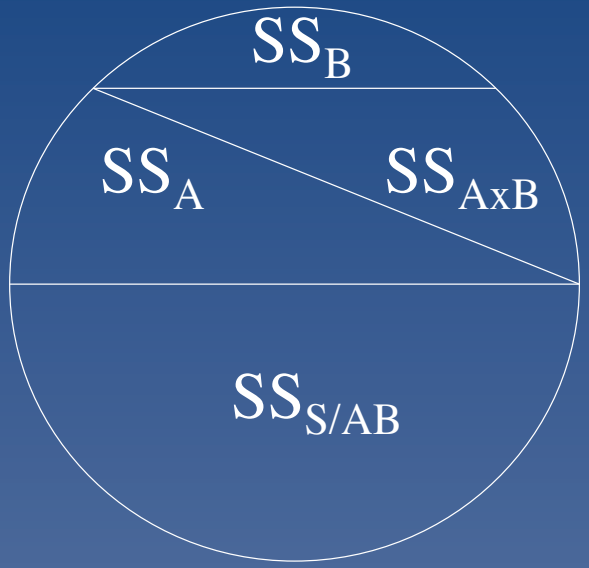
Effect size (complete)

η^2 for the interaction = SS_{AxB} / SS_{total}



Effect size (partial)

η^2 for the interaction = $SS_{AxB} / (SS_{AxB} + SS_{S/AB})$



Assumptions

- Assumptions underlying the factorial ANOVA are the same as for the one-way ANOVA
  - DV is continuous
  - DV is normally distributed
  - Homogeneity of variance

Back to the driving example

- Strayer and Johnson (2001) conducted an experiment to examine the effect of talking on a cell-phone on driving
- They tested subjects in a driving simulator
- Here's the interesting part…
  - They manipulated the difficulty of the driving
  - AND the difficulty of the conversation

Back to the driving example

- To manipulate driving difficulty, they simply made the driving course in the simulator more or less difficult

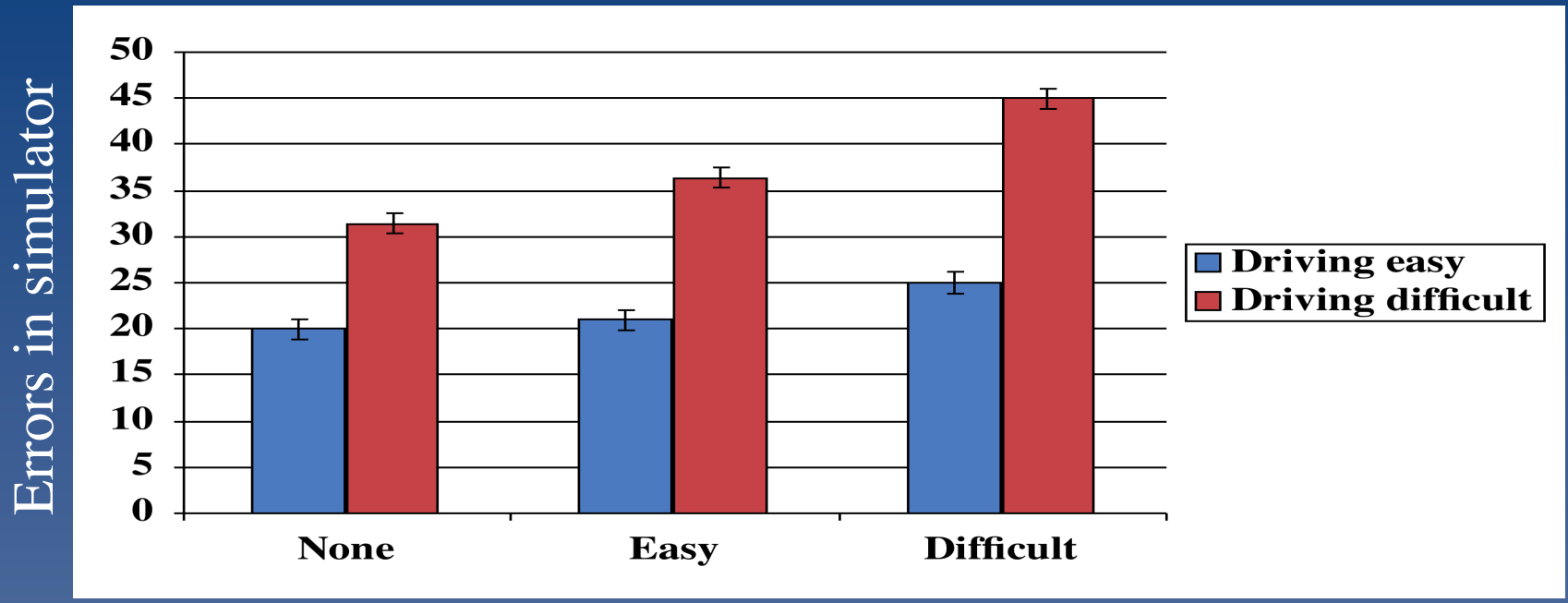
Back to the driving example

- To manipulate conversation difficulty, they included two "talking" conditions:
  - In one, the subject simply had to repeat what they heard on the other line of the phone
  - In the other, the subject had to think of and then say a word beginning with the last letter of the last word spoken on the phone
    - e.g., if you hear "ship" then respond with a word that begins with p - "pear"
  - And there was a no-talking condition (control)

Summary of the design

- IV 1 = driving difficulty (easy, difficult)
- IV 2 = conversation difficulty (none, easy, difficult)
- DV = errors made in driving simulator

Driving errors



Summary table

Source	SS	df	MS	F	p	η^2
Intercept	26641.200	1	26641.200	4251.255	.000	.994
Converse	450.600	2	225.300	35.952	.000	.750
Driving	1825.200	1	1825.200	291.255	.000	.924
Converse* Driving	92.600	2	46.300	7.388	.003	.381
Error	150.400	24	6.267			
Total	2518.800	29				

Follow-up tests

- Post-hoc tests
  - Need to conduct post-hoc tests on the conversation IV
  - No need for driving difficulty because there's only 2 levels
- Simple effects
  - Simple effect of conversation at each level of driving difficulty OR
  - Simple effect of driving difficulty at each level of conversation

Simple effects

- $F_{a \text{ at } bk} = MS_{a \text{ at } bk} / MS_{S/AB}$
- $MS_{a \text{ at } bk} = SS_{a \text{ at } bk} / df_{a \text{ at } bk}$
- $df_{a \text{ at } bk} = a - 1$
- $SS_{a \text{ at } bk} = n \Sigma(Y_{jk} - Y_{Bk})^2$

Simple effects

- Simple effect of driving difficulty at each level of conversation

Simple effects

- No conversation
  - $F = MS_{A \text{ at } b1} / MS_{S/AB}$
  - $F = 324.9 / 6.267 = 51.84$
- Easy
  - $F = MS_{A \text{ at } b2} / MS_{S/AB}$
  - $F = 592.9 / 6.267 = 94.61$
- Difficult
  - $F = MS_{A \text{ at } b3} / MS_{S/AB}$
  - $F = 1000 / 6.267 = 159.57$

Simple effects

- No conversation
  - $\eta^2 = SS_{A \text{ at } b1} / (SS_{A \text{ at } b1} + SS_{S/AB})$
  - $\eta^2 = 324.9 / (324.9+150.4) = .68$
- Easy
  - $\eta^2 = SS_{A \text{ at } b2} / (SS_{A \text{ at } b2} + SS_{S/AB})$
  - $\eta^2 = 592.9 / (592.9+150.4) = .80$
- Difficult
  - $\eta^2 = SS_{A \text{ at } b3} / (SS_{A \text{ at } b3} + SS_{S/AB})$
  - $\eta^2 = 1000 / (1000+150.4) = .87$