Diffusion Model

To generate new fake images being the same kind as the input images.

Forward: ① Given a ground-truth image to be learned to output

a mean to ensure feature closeness between generated & ground-truth

each step's output provides a snapshot for the reverse process to learn

② Given a designed form to add steps of noises while each noise is randomly sampled from a decide distribution

$$X_t = \mu_t + \sigma_t \odot \epsilon,$$
$$= \sqrt{1-\beta_t} \cdot x_{t-1} + \sqrt{\beta_t} \cdot \epsilon,$$
$$\epsilon \sim N(0,1)$$

a designed form

a Randomly sampled noise (from any also decided distribution)

a mean to introduce stochasticity to later model of the reverse process

③ Get pure-noises image

(meaning the entire map is of the decided distribution (structural info. is completely lost, has no statistical difference compared to any other pure-noise map of the same distribution)) → a standardized form of the starting state of the reverse process

Reverse: ① Start from the pure-noises image

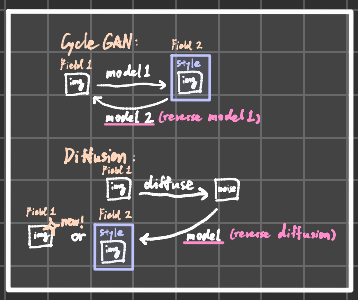
② In each step:

a. use x_t (to represent) & $\overline{\epsilon}^{(t)}$ (the ground-truth) known from `Forward` to learn $\epsilon_\theta(x_t, t)$ and get the distribution of x_{t-1}

b. sample a $\tilde{x}_{t-1}$

a mean to introduce stochasticity to later model of the reverse process

③ Reach the distribution of x_0 in the end then sample any new image



Forward: embed noises (Diffusion)

Assume that: $p(x_t|x_0, x_1, \dots, x_{t-1}) = p(x_t|x_{t-1})$

Designed → ① Markov Chain; of ② $q(x_t|x_{t-1}) = N(x_t; \underbrace{\sqrt{1-\beta_t}\cdot x_{t-1}}_{\mu}, \underbrace{\beta_t I}_{\sigma^2})$

where ③ $\beta_t = S(t)$ (a Variance Schedule <diffusion rate> that $t\uparrow \Rightarrow \beta_t\uparrow$)

Use sinusoidal position embeding

$$\Rightarrow q(x_{1:T}|x_0) = \frac{q(x_{0:T})}{q(x_0)} = \frac{q(x_{0:T-1})\cdot q(x_T|x_{0:T-1})}{q(x_0)} = \frac{q(x_{0:T-2})\cdot q(x_{T-1}|x_{0:T-2})\cdot q(x_T|x_{0:T-1})}{q(x_0)} = \dots$$

$$= \frac{q(x_0,x_1)\cdot q(x_2|x_0,x_1)\cdot \dots \cdot q(x_T|x_{0:T-1})}{q(x_0)} = \frac{q(x_0)\cdot q(x_1|x_0)\cdot \dots \cdot q(x_T|x_{0:T-1})}{q(x_0)} \quad \text{(Markov Chain Assumption)} \quad \frac{q(x_0)\cdot q(x_1|x_0)\cdot q(x_2|x_1)\cdot \dots \cdot q(x_T|x_{T-1})}{q(x_0)}$$

$$= \prod_{t=1}^{T} q(x_t|x_{t-1}) = \prod_{t=1}^{T} N(x_t; \sqrt{1-\beta_t}\cdot x_{t-1}, \beta_t I) \Rightarrow \text{the Forward Proc. (Diffusion)}$$

Use the Multi-parameter Method instead of the original N to be differentiable.

$$x_t = \sqrt{1-\beta_t}\cdot x_{t-1} + \sqrt{\beta_t}\cdot \epsilon^{(1)} \quad (\epsilon^{(1)} \text{ sample once from } N(0,I))$$

$$= \sqrt{1-\beta_t}\left(\sqrt{1-\beta_{t-1}}\cdot x_{t-2} + \sqrt{\beta_{t-1}}\cdot \epsilon^{(2)}\right) + \sqrt{\beta_t}\cdot \epsilon^{(1)} \quad (\epsilon^{(2)} \text{ sample once from } N(0,I))$$

$$= \sqrt{(1-\beta_t)\cdot(1-\beta_{t-1})}\cdot x_{t-2} + \sqrt{(1-\beta_t)\beta_{t-1}}\cdot \epsilon^{(2)} + \sqrt{\beta_t}\cdot \epsilon^{(1)}$$

$$= \sqrt{(1-\beta_t)\cdot(1-\beta_{t-1})}\cdot x_{t-2} + \sqrt{1-(1-\beta_t)(1-\beta_{t-1})}\cdot \bar{\epsilon}^{(2)} \quad (\bar{\epsilon}^{(2)} \Leftrightarrow \text{embed noise from } N(0,I) \text{ since } \alpha_t = 1-\beta_t)$$

$$= \sqrt{\alpha_t\cdot(1-\alpha_{t-1})}\cdot x_{t-2} + \sqrt{1-\alpha_t\alpha_{t-1}}\cdot \bar{\epsilon}^{(2)}$$

$$= \dots = \sqrt{\prod_{i=1}^{t}\alpha_i}\cdot x_0 + \sqrt{1-\prod_{i=1}^{t}\alpha_i}\cdot \bar{\epsilon}^{(t)}$$

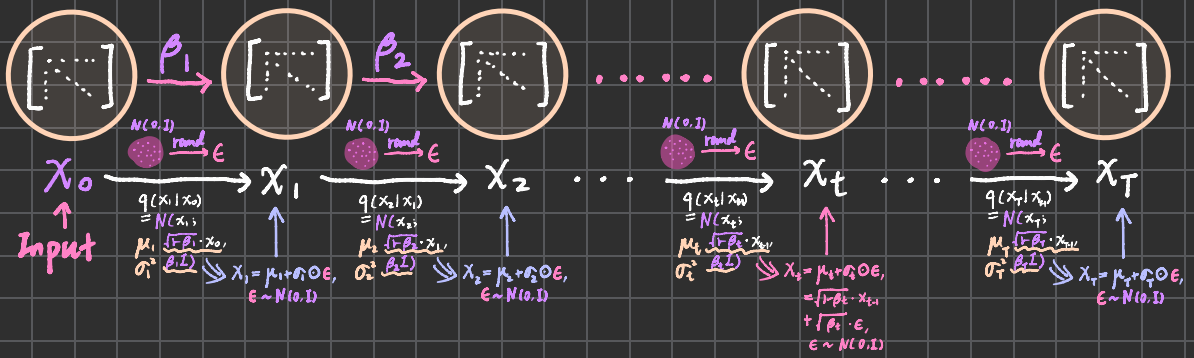
$$\bar{\alpha}_t = \prod_{i=1}^{T}\alpha_i \qquad \sqrt{\bar{\alpha}_t}\cdot x_0 + \sqrt{1-\bar{\alpha}_t}\cdot \bar{\epsilon}^{(t)}$$

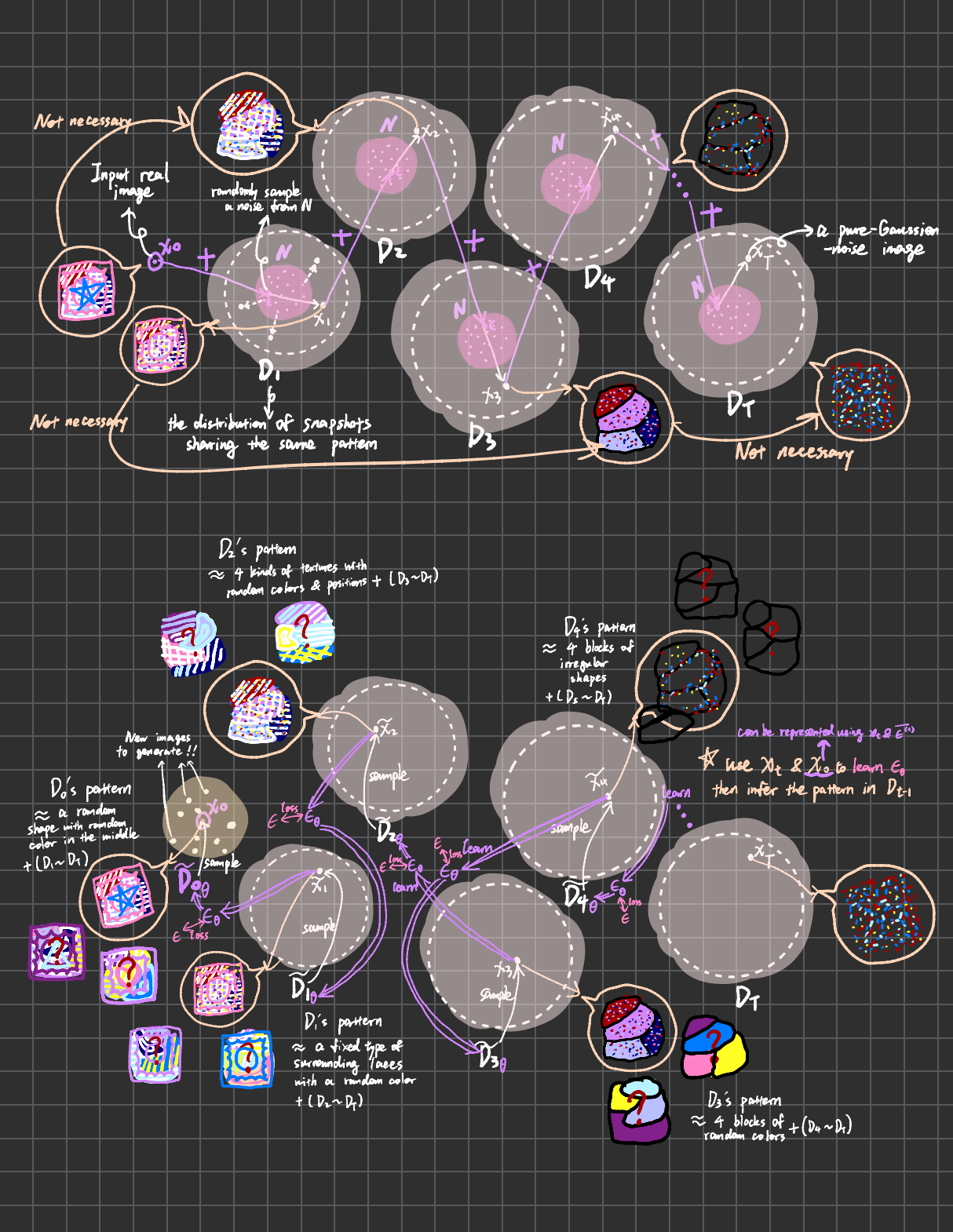
($\bar{\epsilon}^{(t)}$: embed noise from $N(0,I)$ 't' times)

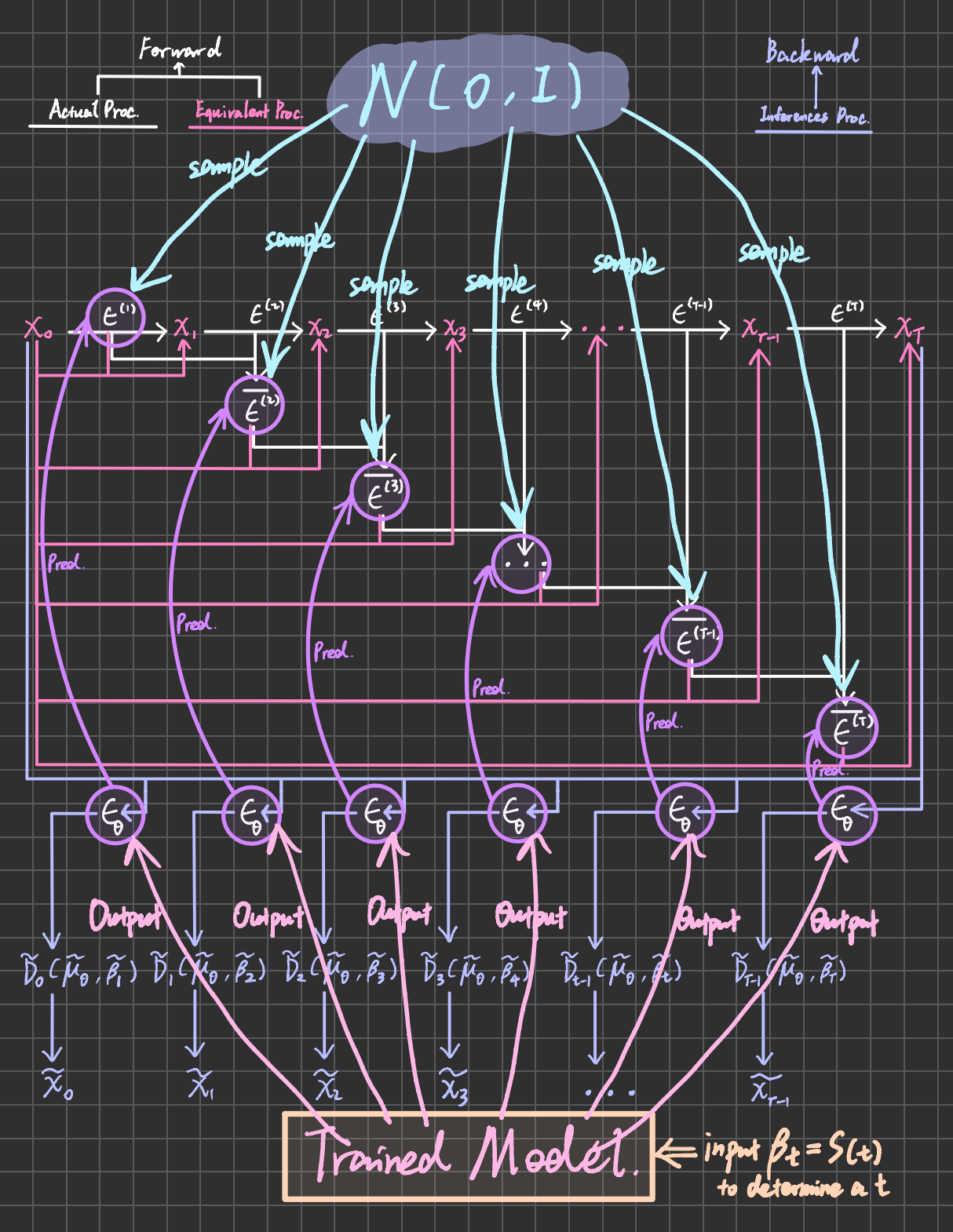
$$\Rightarrow q(x_t|x_0) = N(x_t; \sqrt{\bar{\alpha}_t}\cdot x_0, 1-\bar{\alpha}_t) \rightarrow \text{For Reverse Process ("Snapshots")}$$

$$N(0,I) \sim \sqrt{(1-\beta_t)\beta_{t-1}}\cdot \epsilon^{(2)} + \sqrt{\beta_t}\cdot \epsilon^{(1)} \sim N(0, (1-\beta_t)\beta_{t-1} + \beta_t) = N(0, (1-\beta_t)\beta_{t-1} + 1\cdot\beta_t) = \sqrt{(1-\beta_t)\beta_{t-1}+\beta_t}\cdot N(0,I) \Leftrightarrow \sqrt{(1-\beta_t)\beta_{t-1}+\beta_t}\cdot \epsilon^{(2)} \sim N(0,I)$$

⚠ The snapshots are not for exactly anchoring the targets in Reversing!! They're only the clues for the reverse model to land in their own distributions unknow to the model! → ① Record a $\bar{\epsilon}^{(t)}$; ② Visual features of x_t.




Not necessary
Input real image
randomly sample a noise from N
a pure Gaussian noise image
Not necessary
the distribution of snapshots sharing the same pattern
Not necessary
D_2's pattern
≈ 4 kinds of textures with random colors & positions + (D_3~D_T)
D_4's pattern
≈ 4 blocks of irregular shapes
+ (D_5~D_T)
can be represented using x_t & $\hat{x}_0^{(t)}$
☆ use x_t & $\hat{x}_0$ to learn ϵ_θ then infer the pattern in D_{t-1}
New images to generate !!
D_0's pattern
≈ a random shape with random color in the middle
+ (D_1~D_T)
sample
learn
loss
D_1's pattern
≈ a fixed type of surrounding trees with a random color
+ (D_2~D_T)
D_3's pattern
≈ 4 blocks of random colors + (D_4~D_T)


Forward
Actual Proc.
Equivalent Proc.
N(0, I)
Backward
Inferences Proc.
sample
sample
sample
sample
sample
sample
Output
Output
Output
Output
Output
Output
Trained Model.
input $\beta_t = S(t)$
to determine a t

Backward: denoise (inferences)

Designed → Markov. Chain.

Assume that: $p(x_{t+1}|x_t, x_{t+1}, \dots, x_T) = p(x_{t+1}|x_t)$

Assumed → $q(x_{t-1}|x_t) = N(x_{t-1}; \tilde{\mu}, \tilde{\beta}_{t-1} I)$ because β_t is small enough.

> diffusion (limit of small step size β) the reversal of the diffusion process has the identical functional form as the forward process (Feller, 1949). Since $q(x^{(t)}|x^{(t-1)})$ is a Gaussian (binomial) distribution, and if β_t is small, then $q(x^{(t-1)}|x^{(t)})$ will also be a Gaussian (binomial) distribution. The longer the trajectory, the smaller the diffusion rate β can be made.

⇒ Need to be learned!! Known from forward proc.: $x_0, \bar{\epsilon}^{(t)}, x_t, \beta_t \Rightarrow q(x_t|x_{t-1}), q(x_t|x_0), x_T$

$$q(x_{t-1}|x_t, x_0) = \frac{q(x_{t-1}, x_t, x_0)}{q(x_t, x_0)} = \frac{q(x_t|x_{t-1}, x_0)\cdot q(x_{t-1}, x_0)}{q(x_t, x_0)} = \frac{q(x_t|x_{t-1})\cdot q(x_{t-1}|x_0)}{q(x_t|x_0)}$$

(eliminate the unknown $q(x_t)$ using Markov chain's assumption; keep the known x_0 & x_t (in $q(x_t|x_0)$))

$q(x_t|x_{t-1}) \to N(x_t; \sqrt{1-\beta_t}\cdot x_{t-1}, \beta_t I)$; $q(x_{t-1}|x_0) \to N(x_{t-1}; \sqrt{\bar{\alpha}_{t-1}}\cdot x_0, 1-\bar{\alpha}_{t-1})$; $q(x_t|x_0) \to N(x_t; \sqrt{\bar{\alpha}_t}\cdot x_0, 1-\bar{\alpha}_t)$

$$= \frac{1}{\sqrt{2\pi\cdot\beta_t}}\cdot e^{\frac{(x_t-\sqrt{1-\beta_t}\cdot x_{t-1})^2}{-2\beta_t}} \cdot \frac{\frac{1}{\sqrt{2\pi(1-\bar{\alpha}_{t-1})}}\cdot e^{\frac{(x_{t-1}-\sqrt{\bar{\alpha}_{t-1}}\cdot x_0)^2}{-2(1-\bar{\alpha}_{t-1})}}}{\frac{1}{\sqrt{2\pi(1-\bar{\alpha}_t)}}\cdot e^{\frac{(x_t-\sqrt{\bar{\alpha}_t}\cdot x_0)^2}{-2(1-\bar{\alpha}_t)}}} = \frac{\sqrt{1-\bar{\alpha}_{t-1}}}{\sqrt{2\pi\cdot\beta_t\cdot(1-\bar{\alpha}_t)}}\cdot e^{-\frac{1}{2}\cdot\left[\frac{(x_{t-1}-\sqrt{\bar{\alpha}_{t-1}}\cdot x_0)^2}{1-\bar{\alpha}_{t-1}} - \frac{(x_t-\sqrt{\bar{\alpha}_t}\cdot x_0)^2}{1-\bar{\alpha}_t} + \frac{(x_t-\sqrt{\alpha_t}\cdot x_{t-1})^2}{\beta_t}\right]}$$

$N(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$$\frac{\sqrt{1-\bar{\alpha}_{t-1}}}{\sqrt{1-\bar{\alpha}_t\cdot\beta_t}} = C_0 \quad C_0\cdot e^{-\frac{1}{2}\cdot\left[\left(\frac{1}{1-\bar{\alpha}_{t-1}}+\frac{\alpha_t}{\beta_t}\right)\cdot x_{t-1}^2 - 2\left(\frac{x_0\sqrt{\bar{\alpha}_{t-1}}}{1-\bar{\alpha}_{t-1}}+\frac{x_t\sqrt{\alpha_t}}{\beta_t}\right)\cdot x_{t-1} + C_1(x_t, x_0)\right]} \Leftrightarrow$$

can be represented with $\beta_t, \bar{\alpha}_{t-1}, x_t$ but not enough!!! NEED TO KNOW THE DISTRIBUTION OF x_t is random!!! ⇒ Model q with p_θ for learning

$$\Leftrightarrow \begin{cases} \frac{1}{1-\bar{\alpha}_{t-1}} + \frac{\alpha_t}{\beta_t} = \frac{1}{\tilde{\sigma}^2} \\ \frac{x_0\sqrt{\bar{\alpha}_{t-1}}}{1-\bar{\alpha}_{t-1}} + \frac{x_t\sqrt{\alpha_t}}{\beta_t} = \frac{\tilde{\mu}}{\tilde{\sigma}^2} \end{cases} \Leftrightarrow \begin{cases} \tilde{\sigma}^2 = \tilde{\beta}_t = \frac{\beta_t(1-\bar{\alpha}_{t-1})}{1-\bar{\alpha}_t} \\ \tilde{\mu}_t = \frac{\beta_t x_0\sqrt{\bar{\alpha}_{t-1}} + (1-\bar{\alpha}_{t-1})x_t\sqrt{\alpha_t}}{1-\bar{\alpha}_t} = \frac{\beta_t(x_0)\sqrt{\bar{\alpha}_{t-1}} + (1-\bar{\alpha}_{t-1})x_t\sqrt{\alpha_t}}{1-\bar{\alpha}_t} \end{cases}$$

(Use the actual input)

☆ The Multi-param method from forward proc.: $x_t = \sqrt{\bar{\alpha}_t}\cdot x_0 + \sqrt{1-\bar{\alpha}_t}\cdot\bar{\epsilon}^{(t)}$ ($x_0 = \frac{x_t - \sqrt{1-\bar{\alpha}_t}\cdot\bar{\epsilon}^{(t)}}{\sqrt{\bar{\alpha}_t}}$)

$\tilde{\mu}_t = \frac{\beta_t(x_0)\sqrt{\bar{\alpha}_{t-1}}}{1-\bar{\alpha}_t} + \frac{(1-\bar{\alpha}_{t-1})x_t\sqrt{\alpha_t}}{1-\bar{\alpha}_t}$; $\tilde{\mu}_t = \frac{x_t}{\sqrt{\alpha_t}} - \frac{\beta_t(\bar{\epsilon}^{(t)})}{\sqrt{1-\bar{\alpha}_t}}$ ← Random!

$\Rightarrow q(x_{t-1}|x_t, x_0) = N(x_{t-1}; \tilde{\mu}_t(x_t, x_0), \tilde{\beta}_t I)$

& $\because \bar{\epsilon}^{(t)} = \frac{x_t - \sqrt{\bar{\alpha}_t}\cdot x_0}{\sqrt{1-\bar{\alpha}_t}}$ is sampled & x_0 is given

$\therefore$ To be learnable, use $\epsilon_\theta(x_t, t)$ as an equivalence to the single random $\bar{\epsilon}^{(t)}$ (introduced by α_t ($\bar{\alpha}_t$))

☆ transform randomness into a structured rule (a function: $\epsilon_\theta = \epsilon_\theta(x_t, t; \theta)$)

$$\begin{cases} \Sigma_\theta\ (:= \tilde{\beta}_t \approx \beta_t \text{ in DDPM}) \\ \mu_\theta := \frac{x_t}{\sqrt{\alpha_t}} - \frac{\beta_t\cdot\epsilon_\theta(x_t, t)}{\sqrt{1-\bar{\alpha}_t}\cdot\sqrt{\alpha_t}} \Rightarrow \text{Loss} \end{cases}$$

(actually equivalent to $\bar{\epsilon}^{(t)}$) ☆ learned

$\Rightarrow p_\theta(x_{t-1}|x_t) := N(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta I)$ learn → $q(x_{t-1}|x_t)$ (Markov Chain Assumption) $q(x_{t-1}|x_t, x_0)$

And $\tilde{x}_{t-1}$ (the next $\tilde{x}_t$) $= \mu_\theta(x_t, t) + \sqrt{\Sigma_\theta}\cdot\epsilon^*$

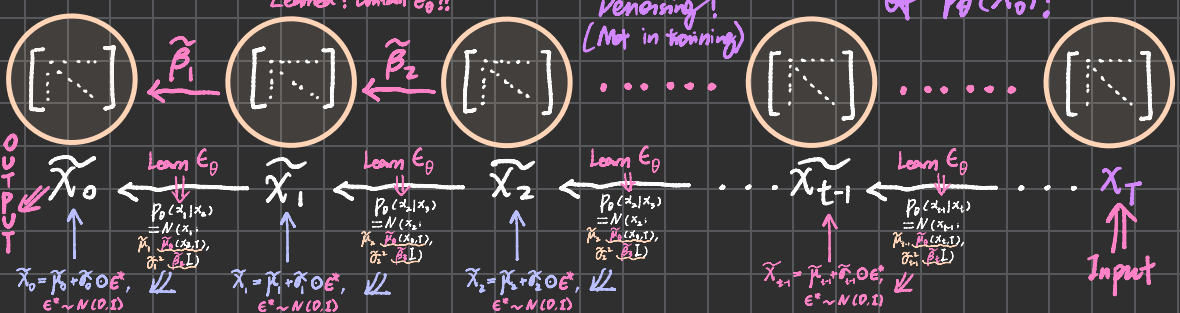
☆ Use the Multi-param Method instead of the original N to be differentiable.

Still sampled from $N(0, I)$! (Added noise)

Learned! Contain ϵ_θ!!

Only used when Denoising! (Not in training)

☆ Ready for the Variational lower bound of $p_\theta(x_0)$!



$$\Rightarrow p_\theta(x_{0:T}) = p_\theta(x_{1:T})\cdot p_\theta(x_0|x_{1:T}) = p_\theta(x_{2:T})\cdot p_\theta(x_1|x_{2:T})\cdot p_\theta(x_0|x_{1:T}) = \dots = p_\theta(x_{T-1:T})\cdot p_\theta(x_{T-2}|x_{T-1}, x_T)\cdot p_\theta(x_{T-3}|x_{T-2}, x_{T-1}, x_T)\dots p_\theta(x_0|x_{1:T})$$

$$= \underbrace{p(x_T)}_{q(x_T)}\cdot p(x_{T-1}|x_T)\cdot p_\theta(x_{T-2}|x_{T-1}, x_T)\cdots p_\theta(x_0|x_{1:T}) \quad \text{(Markov Chain Assumption)} \quad p(x_T)\cdot p_\theta(x_{T-1}|x_T)\cdot p_\theta(x_{T-2}|x_{T-1})\cdot\dots\cdot p_\theta(x_0|x_1)$$

$$= p(x_T)\cdot\prod_{t=1}^{T} p_\theta(x_{t-1}|x_t) = \underbrace{p(x_T)}_{N(x_T; 0, I)\ \text{(Pure noise)}}\prod_{t=1}^{T} N(x_{t-1}; \tilde{\mu}_\theta(x_t, t), \tilde{\beta}_\theta I) \Rightarrow \text{the Reverse Proc. (Denoise)}$$

Training.

Assumption : $q(x_t)$ is Gaussian

Learning : $p_\theta(x_t)$ learns the noise in q_t as a function

Ground-truth : D_t has $p(x_t) \approx 1$, which is the target of p_θ

Initial target : $\epsilon_\theta(x_t, t)$ equivalent to $\epsilon^{(t)} \sim N(t; 0, I)$

(⚠ No ways to calculate loss directly because equivalent is not exact "equal")

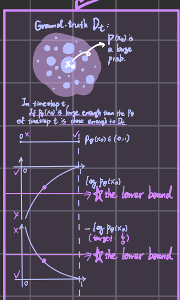
Practical target : $p_\theta(x_0) \rightarrow p(x_0)$ → In actuall Distribution of x_0

(⚠ Still difficult to optimize p_θ directly! Impossible to calculate prior prob.)

More Practical target :

min L where --

$$E[-\log p_\theta(x_0)] \leqslant L := L_{VLB} = E_q\left[-\log \frac{p_\theta(x_{0:T})}{q(x_{1:T}|x_0)}\right]$$



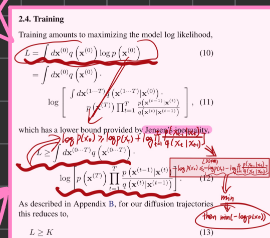
Actually a Cross-entropy loss between:

p: $p(x=x_t)=1$, $p(x=\text{other})=0$ & $q^{(t)}$: $q(x=x_t)=p_\theta(x_t)$, $q(x=\text{other})=1-p_\theta(\text{other})$

$$\Rightarrow H(p,q) = -\sum_x p(x)\cdot\log q(x) = -(1\cdot\log p_\theta(x_t) + 0\cdot\log p_\theta(\text{other})) = -\log p_\theta(x_t)$$

$$= E_q\left[-\log p_\theta(x_T) - \sum_{t\geq 1}\log\frac{p_\theta(x_{t-1}|x_t)}{q(x_t|x_{t-1})}\right]$$

(means that using the simple Gaussian-q to calculate this lower bound by means of which to optimize the black-boxed ground-truth prior p.)



2.4. Training

Training amounts to maximizing the model log likelihood,

$$L = \int d\mathbf{x}^{(0)} q\left(\mathbf{x}^{(0)}\right) \log p\left(\mathbf{x}^{(0)}\right) \quad (10)$$

$$= \int d\mathbf{x}^{(0)} q\left(\mathbf{x}^{(0)}\right) \cdot \log\left[\begin{array}{c}\int d\mathbf{x}^{(1\cdots T)} q\left(\mathbf{x}^{(1\cdots T)}|\mathbf{x}^{(0)}\right)\cdot \\ p\left(\mathbf{x}^{(T)}\right)\prod_{t=1}^{T}\frac{p\left(\mathbf{x}^{(t-1)}|\mathbf{x}^{(t)}\right)}{q\left(\mathbf{x}^{(t)}|\mathbf{x}^{(t-1)}\right)}\end{array}\right], \quad (11)$$

which has a lower bound provided by Jensen's inequality,

$$\log p(x_0) \geqslant \log p(x_T) + \log\prod_{t=1}^{T}\frac{p(x_{t-1}|x_t)}{q(x_t|x_{t-1})}$$

$$L \geq \int d\mathbf{x}^{(0\cdots T)} q\left(\mathbf{x}^{(0\cdots T)}\right)\cdot \log\left[p\left(\mathbf{x}^{(T)}\right)\prod_{t=1}^{T}\frac{p\left(\mathbf{x}^{(t-1)}|\mathbf{x}^{(t)}\right)}{q\left(\mathbf{x}^{(t)}|\mathbf{x}^{(t-1)}\right)}\right]. \quad (12)$$

(DDPM) $-\log p(x_0) \leqslant -\log p(x_T) - \log\prod_{t=1}^{T}\frac{p(x_{t-1}|x_t)}{q(x_t|x_{t-1})}$ → min → then min$(-\log p(x_0))$

As described in Appendix B, for our diffusion trajectories this reduces to,

$$L \geq K \quad (13)$$

$$= E_q\left[-\log p_\theta(x_T) - \sum_{t\geq 2}\log\frac{p_\theta(x_{t-1}|x_t)}{q(x_t|x_{t-1})} - \log\frac{p_\theta(x_0|x_1)}{q(x_1|x_0)}\right]$$

$$q(x_t|x_{t-1}) = \frac{q(x_t, x_{t-1}, x_0)}{q(x_{t-1}, x_0)} = \frac{q(x_{t-1}|x_t, x_0)\cdot q(x_t|x_0)\cdot q(x_0)}{q(x_{t-1}|x_0)\cdot q(x_0)}$$

$$\left(\sum_{t\geq 2}\log\frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t, x_0)} + \sum_{t\geq 2}\log\frac{q(x_{t-1}|x_0)}{q(x_t|x_0)}\right)$$

$$= \left(\sum_{t\geq 2}\log\frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t, x_0)} + \log\prod_{t\geq 2}\frac{q(x_{t-1}|x_0)}{q(x_t|x_0)}\right)$$

$$= \left(\sum_{t\geq 2}\log\frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t, x_0)} + \log\frac{q(x_1|x_0)}{q(x_T|x_0)}\right)$$

$$= E_q\left[-\log p_\theta(x_T) - \left(\sum_{t\geq 2}\log\frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t, x_0)} + \log\frac{q(x_1|x_0)}{q(x_T|x_0)}\right) - \log\frac{p_\theta(x_0|x_1)}{q(x_1|x_0)}\right]$$

$$= E_q\left[-\log p_\theta(x_T) - \sum_{t\geq 2}\log\frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t, x_0)} - \log\frac{q(x_1|x_0)}{q(x_T|x_0)}\cdot\frac{p_\theta(x_0|x_1)}{q(x_1|x_0)}\right]$$

$$= E_q\left[-\log p_\theta(x_T) + \sum_{t\geq 2}\log\frac{q(x_{t-1}|x_t, x_0)}{p_\theta(x_{t-1}|x_t)} + \log\frac{q(x_T|x_0)}{p_\theta(x_0|x_1)}\right]$$

$$= E_q\left[\sum_{t\geq 2}\log\frac{q(x_{t-1}|x_t, x_0)}{p_\theta(x_{t-1}|x_t)} + \log\frac{q(x_T|x_0)}{p_\theta(x_T)} - \log p_\theta(x_0|x_1)\right]$$

variance reduction

using KL div.

$$E_q\Big[\underbrace{D_{KL}(q(x_T|x_0)\,\|\,p_\theta(x_T))}_{L_T} + \sum_{t\geq 2}\underbrace{D_{KL}(q(x_{t-1}|x_t, x_0)\,\|\,p_\theta(x_{t-1}|x_t))}_{L_{t-1}} \underbrace{-\log p_\theta(x_0|x_1)}_{L_0}\Big]$$

L_T (constant)

L_{t-1} (to be learnt)

L_0 (modeled saperately with a discrete decoder derived from $N(x_0; \mu_\theta(x_1, 1), \Sigma_\theta(x_1, 1))$)

L_t : $D_{KL}(P\|Q) = \frac{1}{2}\left(\log\frac{|\Sigma_Q|}{|\Sigma_P|} - \dim + tr(\Sigma_Q^{-1}\Sigma_P) + (\mu_Q - \mu_P)^T\Sigma_Q^{-1}(\mu_Q - \mu_P)\right)$

$$E_{x_0,\epsilon}\left[\frac{1}{2\|\Sigma_\theta(x_t, t)\|_2^2}\left\|\tilde{\mu}_t(x_t, x_0) - \mu_\theta(x_t, t)\right\|^2\right]$$

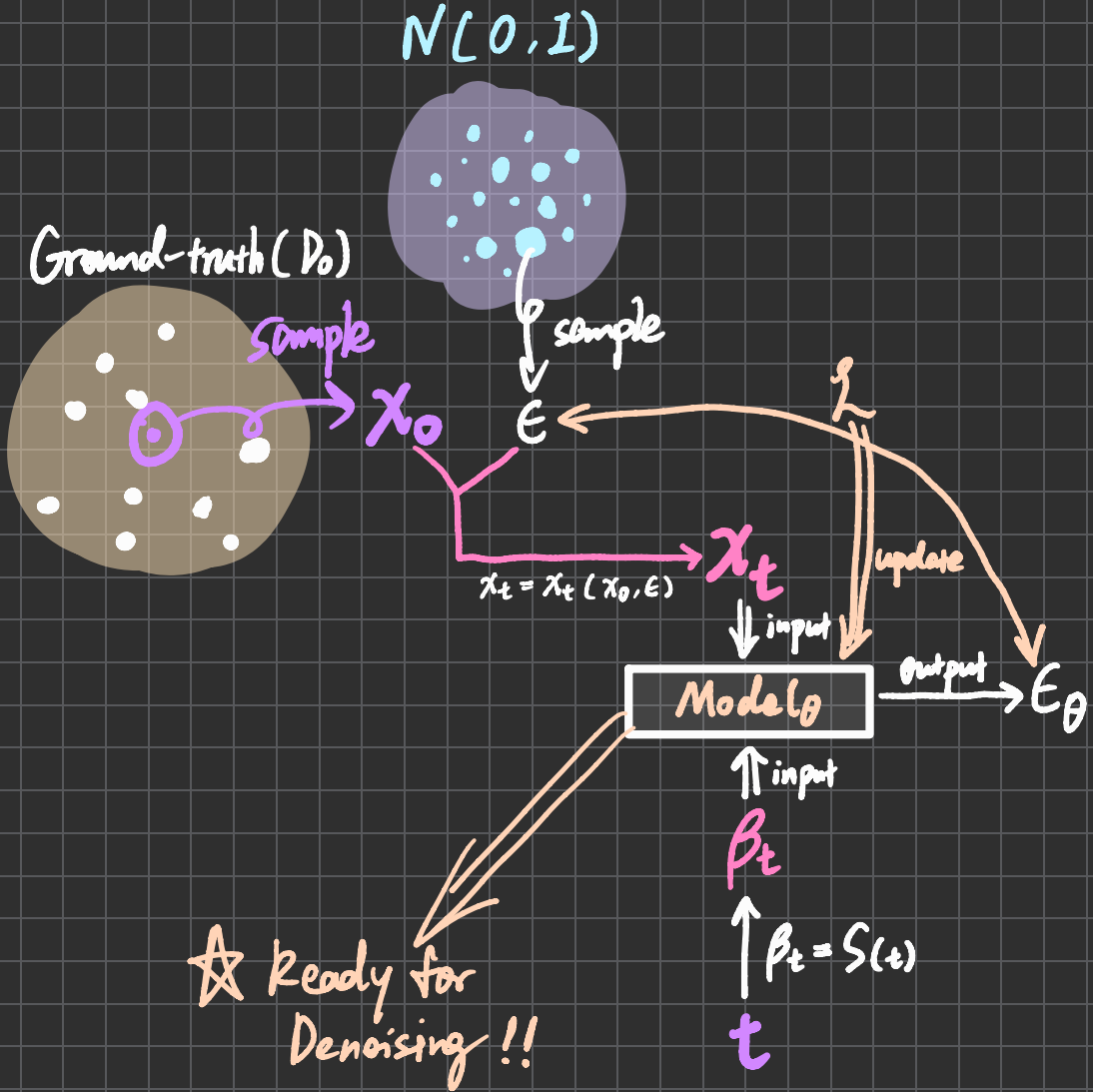
$\tilde{\mu}_t = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t\cdot\epsilon^{(t)}}{\sqrt{1-\bar{\alpha}_t}}\right)$

$\mu_\theta = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t\cdot\epsilon_\theta(x_t, t)}{\sqrt{1-\bar{\alpha}_t}}\right)$

$$E_{x_0,\epsilon}\left[\frac{1}{2\|\Sigma_\theta(x_t, t)\|_2^2}\left\|\frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t\cdot\epsilon}{\sqrt{1-\bar{\alpha}_t}}\right) - \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t\cdot\epsilon_\theta(x_t, t)}{\sqrt{1-\bar{\alpha}_t}}\right)\right\|^2\right]$$

Multi-param method for x_t ($x_t = x_t(x_0, \epsilon)$)

$$E_{x_0,\epsilon}\left[\frac{(1-\alpha_t)^2}{2\alpha_t(1-\bar{\alpha}_t)\|\Sigma_\theta(x_t, t)\|_2^2}\left\|\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t}x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon, t)\right\|^2\right]$$


N(0,1)
Ground-truth (p_0)
sample
sample
x_0
ϵ
x_t
$x_t = x_t(x_0, \epsilon)$
update
input
Model$_\theta$
output
ϵ_θ
input
β_t
$\beta_t = S(t)$
t
Ready for Denoising !!