Rope Physics

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Part of the documentation for project light on github. https://github.com/Catraq/Light. A prerequisite is to read "Contraint based physic solver" by Marijn Tamis, Giuseppe Maggiore.

Description

Fast computation of rope physic. A mass spring method could have been used but this is better in my opinion. However as the method uses the derivative only will it fall apart after some time, have to be addressed before it can be considered finished.

Math

Let J be the matrix with the constraints and M be the matrix with the masses. Let q be the positions. There are in total m constraints and n positions, but $m+1=n$, use your imagination - it is a rope. Keep in mind that q_i is a vector. With $f_{ext}=0$ is (6.2.8) equal to

$$JM^{-1}J^T\vec{\lambda} = -J\dot{\vec{q}}$$

With the constraint $C_i(q_i, q_j) = \frac{q_i - q_j}{||q_j - q_i||}$.
Then J is following

$$J = \begin{bmatrix} \frac{\partial C_1}{\partial q_1} & \frac{\partial C_1}{\partial q_2} & ... & \frac{\partial C_1}{\partial q_n} \\ ... & ... & & ... \\ \frac{\partial C_m}{\partial q_1} & \frac{\partial C_m}{\partial q_2} & ... & \frac{\partial C_m}{\partial q_n} \end{bmatrix} = \begin{bmatrix} \frac{q_1}{||q_1-q_2||} & -\frac{q_2}{||q_1-q_2||} & ... & ... & ... \\ 0 & \frac{q_2}{||q_2-q_3||} & -\frac{q_3}{||q_2-q_3||} & ... & ... \\ ... & ... & ... & ... & ... \\ ... & ... & ... & \frac{q_{n-1}}{||q_{n-1}-q_n||} & -\frac{q_n}{||q_{n-1}-q_n||} \end{bmatrix}$$

Then

$$J\dot{\vec{q}} = \begin{bmatrix} \dot{\vec{q}}_1 \cdot \frac{q_1}{||q_1-q_2||} - \dot{\vec{q}}_2 \cdot \frac{q_2}{||q_1-q_2||} \\ ... \\ \dot{\vec{q}}_{n-1} \cdot \frac{q_{n-1}}{||q_{n-1}-q_n||} - \dot{\vec{q}}_m \cdot \frac{q_n}{||q_{n-1}-q_n||} \end{bmatrix}$$

and

$$JMJ^T = \begin{bmatrix} m_1\frac{\partial C_1}{\partial q_1} & m_2\frac{\partial C_2}{\partial q_2} & ... & ... \\ 0 & m_2\frac{\partial C_2}{\partial q_1} & m_3\frac{\partial C_3}{\partial q_3} & ... \\ ... & ... & ... & ... \end{bmatrix} J^T$$

$$= \begin{bmatrix} m_1\frac{\partial C_1}{\partial q_1}\cdot\frac{\partial C_1}{\partial q_1} + m_2\frac{\partial C_1}{\partial q_2}\cdot\frac{\partial C_2}{\partial q_2} & m_1\frac{\partial C_1}{\partial q_1}\cdot\frac{\partial C_2}{\partial q_1} + m_2\frac{\partial C_2}{\partial q_2}\cdot\frac{\partial C_2}{\partial q_2} & ... & ... \\ m_2\frac{\partial C_2}{\partial q_2}\cdot\frac{\partial C_1}{\partial q_1} & m_2\frac{\partial C_2}{\partial q_2}\cdot\frac{\partial C_2}{\partial q_1} + m_3\frac{\partial C_3}{\partial q_3}\cdot\frac{\partial C_2}{\partial q_1} & m_2\frac{\partial C_2}{\partial q_2}\cdot\frac{\partial C_3}{\partial q_2} + m_3\frac{\partial C_3}{\partial q_3}\cdot\frac{\partial C_3}{\partial q_3} & ... \\ ... & ... & ... & ... \end{bmatrix}$$