

# Data Abstraction

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# Announcements

# Dictionaries

{ 'Dem': 0}

## Dictionary Comprehensions

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{<key exp>: <value exp> for <name> in <iter exp> if <filter exp>}

Short version: {<key exp>: <value exp> for <name> in <iter exp>}

## Example: Multiples

Implement **multiples**, which takes two lists of positive numbers **s** and **factors**. It returns a dictionary in which each element of factors is a key, and the value for each key is a list of the elements of **s** that are multiples of the key.

```
def multiples(s, factors):  
    """Create a dictionary where each factor is a key and each value  
    is the elements of s that are multiples of the key.
```

```
>>> multiples([3, 4, 5, 6, 7, 8], [2, 3])  
{2: [4, 6, 8], 3: [3, 6]}  
>>> multiples([1, 2, 3, 4, 5], [2, 5, 8])  
{2: [2, 4], 5: [5], 8: []}  
.....
```

```
return {d: [x for x in s if x % d == 0] for d in factors}
```

# Recursion

# Recursion so far

**double\_eights**(s: list[int]) -> bool:

**Strategy:** Check if the first two elements are both 8s

Call `double_eights` on everything except the first element

**streak**(n: int) -> bool:

Return whether n is a dice integer in which all digits the same

**Strategy:** Check if last digit is a dice integer, and matches the previous

Call `streak` on everything except the last digit

Deal with one item or digit;  
recurse for the rest

**reverse**(s: list) -> list:

**Strategy:** Get the first element into place

Call `reverse` on the rest

**count\_partitions**(n: int, m: int) -> int:

Return how many ways we can count to n, using pieces of up to size m

**Strategy:** Use a piece of size m; recurse for the rest

Don't use any pieces of size m; recurse for the rest

**Tree recursion:**  
Make a **SMALL** choice;  
for each choice, recurse

# Recursion and Strings

## Spring 2023 Midterm 2 Question 5(a) [modified a bit]

**Definition.** When parking vehicles in a row, a motorcycle takes up 1 parking spot and a car takes up 2 adjacent parking spots. A string of length  $n$  can represent  $n$  adjacent parking spots using `%` for a motorcycle, `<>` for a car, and `.` for an empty spot.

For example: `'.%%.<><>'` (Thanks to the Berkeley Math Circle for introducing this question.)

Implement `count_park`, which returns the number of ways that vehicles can be parked in  $n$  adjacent parking spots for positive integer  $n$ . Some or all spots can be empty.

```
def count_park(n):
    """Count the ways to park cars and motorcycles in n adjacent spots.
    >>> count_park(1) # '.' or '%'
    2
    >>> count_park(2) # '..', '.%', '%.', '%%', or '<>'
    5
    >>> count_park(4) # some examples: '<><>', '.%%.', '%<>%', '%.<>'
    29
    ....
```

We haven't parked anything yet. What's a first decision we can make?

## Spring 2023 Midterm 2 Question 5(a) [modified a bit]

3 spaces left

2 spaces left

1 space left

Choice: how do  
we fill the  
first space?

(Haven't parked anything)

count\_park(n-1) %

count\_park(n-1) ■

count\_park(n-2) <>

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    >>> count_park(2) # '...', '.%', '%.', '%%', or '<>'  
    5  
    >>> count_park(4) # some examples: '<><>', '.%%.', '%<>%', '%.<>'  
    29  
    """  
  
    if n < 0:  
        return _____  
    elif n == 0:  
        return _____  
    else:  
        return _____
```

One way to think about these base cases:  
which recursive calls lead to these cases,  
and what should their values be?

```
count_park(3):  
    %%%  
    %%.  
    %.%  
    %.  
    %<>  
    .%%  
    .%.  
    ...%  
    ...  
    .<>  
    <>%  
    <>.
```

## Spring 2023 Midterm 2 Question 5(a) [modified a bit]

3 spaces left

Choice: how do we fill the first space?

(Haven't parked anything)

2 spaces left

count\_park(n-1) %

1 space left

%%

% .

0 spaces left

%%% %%

% . % % . .

% <>

-1 spaces left

What choices did we make?  
motorcycle  
motorcycle  
motorcycle

% % <>

% . <>

## Spring 2023 Midterm 2 Question 5(a) [modified a bit]

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    if n < 0:  
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    else:  
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```

One way to think about these base cases:  
which recursive calls lead to these cases,  
and what should their values be?

```
count_park(3):  
    %%%  
    %%.  
    %.%  
    %.  
    %<>  
    .%%  
    .%.  
    ...%  
    ...  
    .<>  
    <>%  
    <>.
```

## Recursion so far

**double\_eights**(s: list[int]) -> bool:

**Strategy:** Check if the first two elements are both 8s  
Call **double\_eights** on everything except the first element

**streak**(n: int) -> bool:

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**Strategy:** Check if last digit is a dice integer, and matches the previous  
Call **streak** on everything except the last digit

Deal with one item or digit;  
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**reverse**(s: list) -> list:

**Strategy:** Get the first element into place  
Call **reverse** on the rest

**count\_partitions**(n: int, m: int) -> int:

Return how many ways we can count to n, using pieces of up to size m

**Strategy:** Use a piece of size m; recurse for the rest  
Don't use any pieces of size m; recurse for the rest

**Tree recursion:**  
Make a **SMALL** choice; recurse

**park**(n: int) -> int: Return the ways to park in n adjacent spots

**Strategy:** Use a motorcycle; recurse for the rest  
Use nothing; recurse for the rest  
Use a car; recurse for the rest

## Quick Review: Adding Lists & Strings

```
>>> x = 'cal'  
>>> y = 'bears'  
>>> u = [x]  
>>> v = [y]
```

```
>>> x + y  
'calbears'
```

```
>>> u + v  
['cal', 'bears']
```

```
>>> ['go ' + x for x in [x, y]]  
['go cal', 'go bears']
```

```
>>> ['cal' + x for x in s]
```

What s will result in ['cal']?

[pollev.com/cs61a](http://pollev.com/cs61a)

What s will result in []?

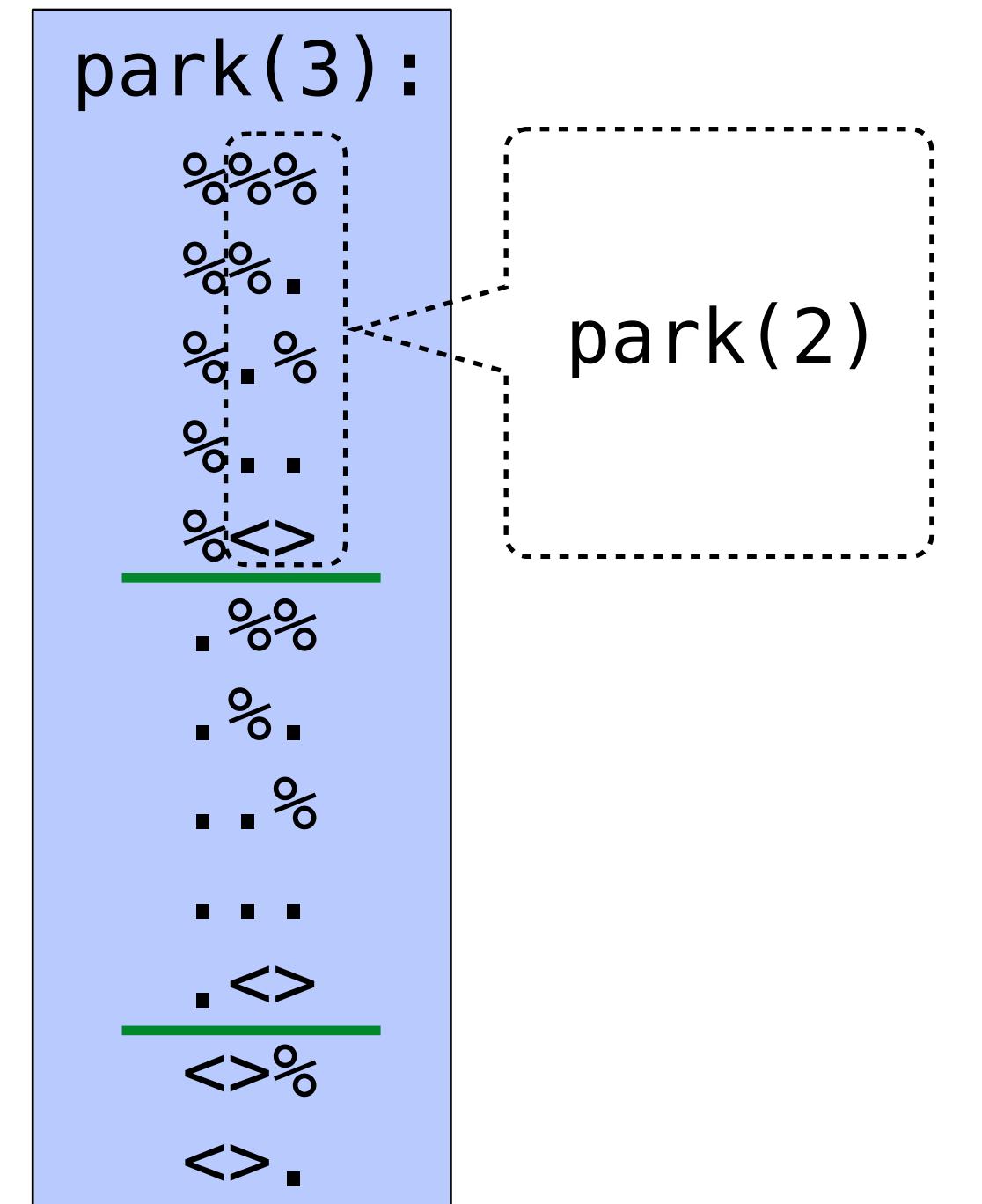
## Spring 2023 Midterm 2 Question 5(b) [modified a lot]

**Definition.** When parking vehicles in a row, a motorcycle takes up 1 parking spot and a car takes up 2 adjacent parking spots. A string of length  $n$  can represent  $n$  adjacent parking spots using `%` for a motorcycle, `<>` for a car, and `.` for an empty spot.

For example: `'.%%.<><>'` (Thanks to the Berkeley Math Circle for introducing this question.)

Implement `park`, which returns a list of all the ways, represented as strings, that vehicles can be parked in  $n$  adjacent parking spots for positive integer  $n$ . Spots can be empty.

```
def park(n):
    """Return the ways to park cars and motorcycles in n adjacent spots.
    >>> park(1)
    ['%', '.']
    >>> park(2)
    ['%%', '%.', '.%', '...', '<>']
    >>> len(park(4))  # some examples: '<><>', '.%%.', '%<>%', '%.<>'
    29
    ....
    if n < 0:
        return []
    elif n == 0:
        return ['']
    else:
        return ['%' + s for s in park(n-1)] + ['.' + s for s in park(n-1)] + ['<>' + s for s in park(n-2)]
```



motorcycle first

+

nothing first

+

car first

# Discussion 4

# Max Product

Write a function that takes in a list and returns the maximum product that can be formed using non-consecutive elements of the list. All numbers in the input list are greater than or equal to 1.

```
def max_product(s):  
    """Return the maximum product that can be  
    formed using non-consecutive elements of s.  
  
    >>> max_product([10, 3, 1, 9, 2]) # 10 * 9  
    90  
    >>> max_product([5, 10, 5, 10, 5]) # 5 * 5 * 5  
    125  
    >>> max_product([])  
    1  
    ....  
    if len(s) == 0:  
        return 1  
    elif len(s) == 1:  
        return s[0]  
    else:  
        return _____
```

What choices did we make?

- Use the 10
- Don't use the 1
- Use the 9

[10, 3, 1, 9, 2]

Use 10      Don't use 10

max\_product([1, 9, 2])      max\_product([3, 1, 9, 2])

max(10 \* max\_product([1, 9, 2]), max\_product([3, 1, 9, 2]))

max(s[0] \* max\_product(s[2:]), max\_product(s[1:]))

## Sum Fun

Implement `sums(n, m)`, which takes a total `n` and maximum `m`. It returns a list of all lists:

- that sum to `n`,
- that contain only positive numbers up to `m`, and
- in which no two adjacent numbers are the same.

```
>>> sums(5, 3)
[[1, 3, 1], [2, 1, 2], [2, 3], [3, 2]]
>>> sums(5, 5)
[[1, 3, 1], [1, 4], [2, 1, 2], [2, 3], [3, 2], [4, 1], [5]]
```

```
def sums(n, m): Start with a 1 Start with a 2 Start with a 5
  if n < 0:
    return []
  if n == 0:
    sums_to_zero = [] # The only way to sum to zero using positives
    return [sums_to_zero] # Return a list of all the ways to sum to zero
  result = []
  for k in range(1, m + 1):
    result = result + [k+rest] for rest in sums(n-k, m) if rest == [] or k != rest[0]
  return result
```

Choice: What should we start with?

# Data Abstraction

## Data Abstraction

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A small set of functions enforce an abstraction barrier between **representation** and **use**

- How data are represented (as some underlying list, dictionary, etc.)
- How data are manipulated (as whole values with named parts)

E.g., refer to the parts of a line (affine function) called `f`:

- `slope(f)` instead of `f[0]` or `f['slope']`
- `y_intercept(f)` instead of `f[1]` or `f['y_intercept']`

Why? Code becomes easier to read & revise.

(Demo)