Applications of Fermi Golden Rule

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1 Transition Probabilities and Fermi Golden Rule

The famous Fermi Golden Rule describe the transition rate which depends on the strength of the coupling between the initial and fibal state of a system and upon the number of ways the transition can happen.

$$\begin{aligned} \mathscr{W}(t) &= \frac{d\mathscr{P}(t)}{dt} \\ &= \frac{2\pi}{\hbar} W_{fi}^2 \rho(E_{fi}) \end{aligned} \tag{1}$$

where the W_{fi} is matrix element for the interaction, $\rho(E_{fi})$ is density of the final state.

Lifetime: $\tau = 1/\mathscr{W}$.

2 Scattering and Decays from Fermi Golden Rule

Fermi's Theory of nuclear β-decay $\quad n \to p + e^- + \bar{\nu} \qquad (Z, N) \to (Z+1, Z-1) + e^- + \bar{\nu}$

The simplest form for thr matrix element describing nuclear β-decay is given by Fermi's ansatz:

$$W_{km} = \frac{G_z M}{V} \tag{2}$$

where G_z is Fermi constant, M is overlap of the initial/final nuclear wave function and V is the normalization volume. The density of possible states $dn/dE_0 = \rho(E_0)$. Typical value of E_0 is in the MeV range.
Main equations:

$$E_0 = E + cq \qquad 0 = P + q + p$$

where E is electron energy, q is neutrino momentum, P is nuclear momentum and p is electron mometum. The density of states is found from the product of the electron and neutrino phase space volumes.

$$dn = \frac{Vd^3\boldsymbol{p}}{(2\pi\hbar)^3} \cdot \frac{Vd^3\boldsymbol{q}}{(2\pi\hbar)^3} = \frac{1}{4\pi^4\hbar^6c^3} p^2(E-E_0)^2 dpdE \tag{3}$$

Back to the expression of transition rate,

$$d\mathscr{W} = \frac{G_f^2}{2\pi^3\hbar^7c^3} |M|^2 p^2 (E-E_0)^2 dp \tag{4}$$

when the electron can be treated as being relastitic($E = cp$).

$$\int p^2(E-E_0)^2 dp = \frac{Q_0^5}{30c^3} \tag{5}$$

where $Q_0 = E_0 - m_e c^2$. Then the transition rate can be calculated.

$$\mathscr{W} = \frac{G_f^2}{60\pi^3\hbar^7c^6} |M|^2 Q_0^5 \tag{6}$$

cross section

$$\mathscr{W} = d\sigma j_i \tag{7}$$

where σ is cross section and j_i is the flux of initial particals.