Dielectric function and Reflectivity

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1 Complex refractive index

The refraction of light occurs when light advances into optically different media. The refraction of light is determined from the refractive index n and, classically, n is defined by

$$n = c/s \tag{1}$$

in which, c is speed of light in vacuum and s is the one in media.

When there is no light absorption in media, the propagation number K is

$$K = \frac{2\pi n}{\lambda} \tag{2}$$

where λ is the wavelength in vacuum. The electric field becomes

$$E = E_{t0}exp[i(\omega t - Kx + \delta)] = E_{t0}exp[i(\omega t - \frac{2\pi n}{\lambda}x + \delta)] \tag{3}$$

where E_{t0} corresponds to E_0 in the transparent medium and δ is a phase difference when the light crosses the interface. So the quantity n represents how much wavelength changes in media. If there are media that show strong light absorption, and such a phenomenon cannot be expressed only with n. We introduce a complex refractive index N.

$$N = n - ik \tag{4}$$

The electric field becomes

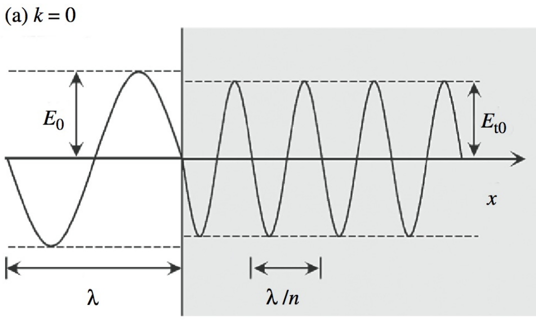
$$E = E_{t0}exp[i(\omega t - \frac{2\pi N}{\lambda}x + \delta)] = E_{t0}exp(-\frac{2\pi k}{\lambda})exp[i(\omega t - \frac{2\pi n}{\lambda}x + \delta)] \tag{5}$$

Obviously, $exp(2\pi k/\lambda)$ represents absorption. (In oscillator model, the damping may be caused by interaction between next oscillators.) Comes to energy(square). There is an absorption coefficient $\alpha = 4\pi k/\lambda$

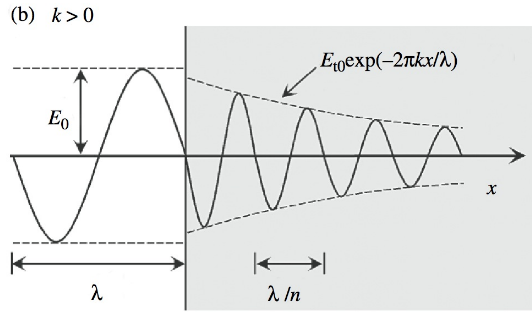
Refractive index and Band gap The imaginary part of the index describes wave decay. We can calculate stimulated absorption results in a wave to decay in a medium (optical loss).

To begin with, we introduce the density of state $\rho(E)$, which means the number of states between E and $E + dE$ over VdE. In the k-space each electronic state occupies a volume of $\frac{4\pi^3}{V}$, where V is the volume of the system.

$$\begin{aligned} E_c(k_c) &= E_c + \frac{\hbar k_c^2}{2m_e} \\ E_v(k_v) &= E_v + \frac{\hbar k_v^2}{2m_h} \\ \hbar\omega \sim E_c(k_e) - E_v(k_e) &= E_{gap} + \frac{\hbar k_e^2}{m_e} \end{aligned} \tag{6}$$



(a) No absorption



(b) Absorption

Figure 1: Electric field in media

where c means conductor band, v means valenced band e means electrons, h means holes.

$$\rho(E) = \rho(k) \cdot \frac{dk}{VdE} = \frac{4\pi k^2}{4\pi^3/V} \frac{dk}{VdE} = \frac{m_e}{2\pi^2\hbar^3}\sqrt{m_e(E - E_c)} \tag{7}$$

When the incident photon has an energy $E = \hbar\omega$,

$$\rho(\hbar\omega) = \frac{m_e}{2\pi^2\hbar^3}\sqrt{m_e(\hbar\omega - E_{gap})} \tag{8}$$

Make a quantum correction(electrons are Fermions), the absorption coefficient $\alpha(\omega)$

$$\alpha(\omega)\hbar\omega = \rho(\hbar\omega) f_c(E_c)(1 - f_v E_v) \tag{9}$$

As we already know $\alpha = 4\pi k/\lambda$, k can be calculated now.

2 Dielectric constant

According to the Maxwell equation, the speed of light in vacuum can be expressed as

$$c = \sqrt{\mu_0 \epsilon_0} \tag{10}$$

And the one in media is

$$s = \sqrt{\mu\epsilon} \tag{11}$$

For a material that is not magnetic the permeability is μ_0, so that, according to the definition of refractive index

$$N = \frac{c}{s} = \sqrt{\epsilon/\epsilon_0} = \sqrt{\epsilon_p} \tag{12}$$

Negative dielectric constant A simple model for the dynamics of the displacement x of the bound electron is as follows

$$m\ddot{x} = eE - kx - m\gamma\dot{x} \tag{13}$$

spring-like restoring force to the binding of the electron to nucleus, ω_0 represents bounds between electrons. friction-type force proportional to the velocity of the electron, the frictional term γ arises from collisions that tend to slow down the electron.

$$\epsilon(\omega) = \epsilon_1(\omega) - i\epsilon_2(\omega)$$

$$\epsilon_1 = \epsilon_0 + \frac{\epsilon_0\omega_p^2(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad \epsilon_2 = \frac{\epsilon_0\omega_p^2\omega\gamma}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2} \tag{14}$$

This is Lorentz model. There are different conditions with three kinds of materials.

- Dielectrics, $\omega_0 \neq 0, \quad \gamma \neq 0$.
- Conductors, $\omega_0 = 0, \quad \gamma \neq 0$.
- Plasmas, $\omega_0 = 0, \quad \gamma = 0$.

For dielectrics, around the resonant frequency ω_0, the real part ϵ_1 behaves in an anomalous manner, that is, it drops rapidly with frequency to values less than ϵ_0 and the material exhibits strong absorption.

For conductors and plasmas, when ω is low enough, the real part ϵ_1 can be negative.