

Classical and Quantal Irregular Scatterings with Complex Target

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Abstract

One-dimensional scattering by a target with two internal degrees of freedom is investigated. The damping of resonance peaks and the associated appearance of the fluctuating background in the quantum inelastic scattering amplitudes are found. Examination of the analog classical system reveals a disorderly reaction function, which is then related to the quantum amplitude through a semiclassical argument.

I. INTRODUCTION

Already three decades ago, Ericson [1] recognized that the background fluctuations of the quantum scattering amplitudes carry as much physical information as the structure of resonance peaks. It is the recent advent of the theory of the non-integrable scatterings that has begun to unveil the intimate connection between the irregular scattering and the Ericson fluctuation, or the fluctuation of the elastic scattering amplitude when the incident bombarding energy is varied [2-4]. However, the continuous variation of the incident energy is hard to materialize experimentally in particle and nuclear physics. More often, the scattering experiments are performed with fixed energy accelerators, but with particle detectors that allow the detection of the scattering observables at various outgoing energies. There, one typically observes several large peaks corresponding to simple excitation modes and the fluctuating backgrounds in between. It is well known that the nature of the target dynamics determines the characteristics of these resonance peaks and the background fluctuations. Namely, when the mixing interactions among target states are present, one usually finds damping of the peaks and the resulting appearance of "noisy" background. Bearing the Ericson fluctuation in mind, it is natural to speculate on the possible relevance of the behavior of the inelastic amplitudes to the chaotic aspects of the system. In this note, we study a model system in which a one-dimensional particle is scattered by a target which has two dynamical degrees of freedom. In the quantum scattering, we find that the damping of the peaks indeed occurs when the target goes through complex motion. The study of the classical analog system reveals the existence of a novel type of irregular scattering in which the location of the maxima and minima of the reaction function becomes disorderly. We discuss a possible way to connect the behavior of classical and quantum systems in scattering.

II. MODEL AND TARGET DYNAMICS

We consider a projectile moving in one-dimensional space specified by the coordinate and its conjugate momentum (x, p) . The particle is scattered by a system (or the target) which is described by the two internal degrees of freedom specified by two sets of "angle" and conjugate "angular momentum" (ϑ, I) and (φ, J) . We assume that the target system is 2π -periodic, and treat the variables ϑ and φ as in the range of $[-\pi, \pi]$. The dynamics of the system of a projectile and a target is governed by the Hamiltonian

$$H = \frac{p^2}{2m} + \left[\frac{I^2}{2M} + \frac{J^2}{2N} + U(\vartheta, \varphi) \right] + V(x, \vartheta, \varphi) \quad (1)$$

where the terms inside the bracket, which we call target Hamiltonian H_I , describe the motion of the target with the internal coupling interaction

$$U(\vartheta, \varphi) = \frac{1}{2\pi} (1 - 3 \cos \vartheta + 2 \cos 2\vartheta) \frac{1}{2\pi} (1 - 3 \cos \varphi + 2 \cos 2\varphi) \quad (2)$$

The particle-target interaction V is given by

$$V(x, \vartheta, \varphi) = \lambda \cdot \frac{1}{\sqrt{\pi}} \exp(-x^2) \cos 2\vartheta \quad (3)$$

Here s and λ are the parameters which control the overall strength of target-internal and particle-target coupling interactions respectively. Throughout this work, we adopt such unit as to make $\hbar = 1$. The masses are fixed to be $m = 1$, $M = \sqrt{2}$ and $N = \sqrt{3}$.

We denote the eigenstates and eigenvalues of the target Hamiltonian H_I as $\{|\alpha\rangle\}$ and $\{\varepsilon_\alpha\}$. When the coupling is absent ($s = 0$), the target state is reduced to the "free rotation" in both angles ϑ and φ , and is specified by two integer quantum numbers m and n , namely, $|\alpha\rangle = |m, n\rangle$. The effect of the mixing interaction U on the global properties of the target states is conveniently characterized by the nearest neighbor spacing distribution of eigenvalues $P(s)$ [5,6]. In Figs. I (a)-(c), we plot $P(s)$ for the coupling strengths $s = 0, 1$ and 10 along with the Poisson distribution (dashed line) and the Wigner distribution (solid line) which are usually associated with regular and chaotic dynamics respectively. There are 200 lowest levels (between $\varepsilon = 0 - 75$) included in the calculation. We clearly observe the

gradual approach to the chaotic spectra with the increasing value of s . To strengthen the point, we also plot in Figs. I (d)-(f) the Poincare section (ϑ, I) , with the section of $\varphi = 0$ of the target Hamiltonian H_I , which is obtained from the solution of classical equations of motion using the same parameter sets as in Figs. I (a)-(c). Since the target Hamiltonian does not possess the mechanical similarity, the phase space structure depends on the target energy. In practice, however, this dependence is found to be very mild in the energy range we are interested in. Here, we take the target energy to be $\varepsilon = 10$. The transition from regular to chaotic target motion is better visible in these Figures.

III. QUANTUM SCATTERING

The quantum scattering is most efficiently described with the transition matrix formalism [7]. The quantum state of the system is specified by the direct product of the particle state $|p\rangle$ and the target eigenstate $|\alpha\rangle$. We refer to the quantum states with internal wave function $|\alpha\rangle$ as belonging to "channel" α . The inelastic transition between channel α and β occurs through the operation of the interaction $V_{\beta\alpha}(p', p) \equiv \langle p|(\beta|V|\alpha)|p\rangle$. The transition matrix (T-matrix) from channel α to β is obtained from the coupled-channel Lippmann-Schwinger equation

$$T_{\beta\alpha}(p_\beta, p_\alpha; \omega) = V_{\beta\alpha}(p_\beta, p_\alpha) + \sum_\gamma \int \frac{dp}{2\pi} V_{\beta\gamma}(p_\beta, p) G_\gamma^{(0)}(p; \omega) T_{\gamma\alpha}(p, p_\alpha; \omega) \quad (4)$$

where the free particle propagator in the channel α is given by $G_\alpha^{(0)}(p; \omega) = (\omega - p^2/2m - \varepsilon_\alpha + i0)^{-1}$. The scattering matrix (S-matrix) between the initial channel α with particle incident momentum (meaning the asymptotic initial momentum) p_α and the final channel β with particle outgoing momentum p_β is given by

$$S_{\beta\alpha}(p_\beta, p_\alpha; \omega) = \delta_{\beta\alpha} + \frac{m}{i\sqrt{|p_\beta p_\alpha|}} \cdot T_{\beta\alpha}(p_\beta, p_\alpha; \omega) \quad (5)$$

where the on-shell condition $\omega = p_\beta^2/2m + \varepsilon_\beta = p_\alpha^2/2m + \varepsilon_\alpha$ is imposed on the momenta p_α and p_β . All the scattering observables are obtainable from the S-matrix elements.

We look at the behavior of the S-matrix elements with the change of the final state β , while keeping the initial state α to be the ground state and also keeping the incident energy $p_\alpha^2/2m$ constant. In Fig. II, the absolute value of the S-matrix is plotted as a function of energy difference between the final and initial target states ($\varepsilon_\beta - \varepsilon_\alpha$). The incident momentum is set to be $p_\alpha = 6.5$ (or $p_\alpha^2/2m = 21.125$) which is capable of exciting about 60 target levels. The overall strength of the particle-target interaction is chosen to be $\lambda = 50$. This choice makes the strength of the interaction comparable to the kinetic energy of the incoming particle $p_\alpha^2/2m$, and gives the sufficient strength to the inelastic amplitudes. The Figs. II (a) to (c) show the results of the scattering by the target states with different mixing properties. Each graph corresponds to the one with same alphabet of Fig. I, namely; (a) no mixing with $s = 0$, (b) weak chaotic mixing with $s = 1$ and (c) strong chaotic mixing with $s = 10$. Only scattering amplitudes to the positive value of p_b is shown. The backward scattering amplitudes (found to be generally small with the model interaction V) show similar characteristics. Because of the selection rule of the matrix elements of the interaction V , the scattering from integrable target shows a simple pattern of exciting very few states which form the sharp peaks in the Fig. II (a). With the complex target states (Figs. II (b) and (c)), the peaks of the scattering amplitude begin to loose their strength to various other states with strength seemingly at random. To understand this behavior, one needs to look at the matrix element $(\beta | T | \alpha) = \sum_{n,m,n,m} (\beta | n, m) (n, m | T | n, m) (n, m | \alpha)$. When the coupling s increases, each target state $|\alpha\rangle$ becomes a complex mixture of unperturbed states $|n, m\rangle$ and the summation by n and m will cause generally incoherent interferences, resulting in the white-noise like behavior of $(\beta | T | \alpha)$. This mechanism by itself is frequently encountered in atomic and nuclear physics as the partitioning of excitation strength of simple states to more complex configurations. We now ask a question: Is there any the property of underlying classical dynamics that is relevant to the quantum phenomenon of the spreading of resonance peaks?

IV. CLASSICAL SCATTERING

It is nowadays widely recognized⁶ that the deeper understanding of quantum phenomena is achieved by the examination of the classical trajectories $\{x(t), p(t), \vartheta(t), I(t), \varphi(t), J(t)\}$ which is the solution of the classical equations of motion obtained from the Hamiltonian eq. (1). The scattering process is described by the reaction functions,⁸ or the functional relation between the "outgoing" variables such as $p_{out} = p(T)$ and $x_{out} = x(T) - p(T)T/m$ (where T is sufficiently larger than the reaction time) and the "incoming" variables $p_{int} = p(0)$, $x_{in} = x(0)$, $I_{in} = I(0)$, $J_{in} = J(0)$ etc.. Here, we look at the particular reaction function

$$p_{out} = p_{out}(x_{in}) \quad (6)$$

while keeping all other incoming variables constant. In Fig. III we display the typical case. The initial momentum is taken to be $p_{in} = 6.5$ as before. The initial values of the other dynamical variables are taken to be $\vartheta_{in} = \varphi_{in} = 0$ and $I_{in} = J_{in} = 1.0$ in this example. The choice of the strength parameter s in each of Figs. III (a)-(c) corresponds to the same alphabet in previous figures. In all cases, the reaction functions display singular structure (dotty areas) which is the reflection of the non-integrability of the model system as a whole. The irregular structure in case of no internal coupling (Fig. III (a)) is essentially identical to the one previously encountered in the study of Blummel and Smilansky [2]. It is related to the instability of the "trapping" orbits. It is worth noting that the irregular area appears in regular interval in x_{in} reflecting the periodicity (or quasi-periodicity) of the target motion. When the target-internal coupling is turned on (Figs. III (b) and (c)), the nature of the reaction function seems to acquire qualitatively different irregularity. Not only the trapping areas occur at irregular locations but the smooth areas get disorderly shape deformation. It is understood as the result of the chaotic motion of the target which makes the value of the target variables practically unpredictable at the time of the particle hitting the target. Thus, one can conclude that the disorderly reaction function is a generic property of the scatterings by complex targets.

Semiclassical consideration should relate the classical and quantum scatterings. We extend the argument by Smilansky [8] to the current model with two target degrees of freedom. We prepare an ensemble of initial conditions of the target motion with constant energy. This can be done by bringing various ϑ_{in} , I_{in} , φ_{in} and J_{in} while keeping internal energy constant. When the motion of the target is ergodic or quasi-periodic, another way of achieving this ensemble is preparing different x_{in} distributed uniformly (with density one) over certain region beyond the interaction range. This is understood by noticing that advancing the projectile position x_{in} is equivalent to letting target variables move backward in time. Then, the outgoing value p_{out} will be distributed with a probability density $P(p_{out})$ which is given by

$$P(p_{out})dp_{out} = \sum_s [dx_{in}]_{(s)} \quad (7)$$

where the summation (s) is over the initial values for x_{in} satisfying $p_{out} = p_{out}(x_{in})$. Semiclassically, the square of the scattering matrix should approach the classical probability density. Therefore, we have the classical approximation to the S-matrix

$$|S^{cl}(p_{out}, p_{in})| = \sqrt{P(p_{out})} = \sqrt{\sum_s \left[\frac{dp_{out}}{dx_{in}} \right]_{(s)}^{-1}} \quad (8)$$

This expression shows that the classical S-matrix diverges at the extrema of the reaction function $p_{out}(x_{in})$. Presuming that the quantum corrections amend the divergence and make it a finite peak instead, we can relate our findings in classical and quantum scatterings: Mostly smooth reaction function with several repeating minima and maxima will result in the simple quantum amplitude with few distinct peaks, while the reaction function with disorderly located extrema should make the fluctuating background in the scattering amplitude. To demonstrate these arguments, we construct the classical S-matrix, eq. (8) from the reaction function shown in Fig. III. As in the full quantum treatment, $|S^{cl}|$ is plotted in Fig. IV as the function of energy transfer $\varepsilon_\beta - \varepsilon_\alpha = (p_{in}^2 - p_{out}^2)/2m$. Each of the graph (a)-(c) corresponds to the same alphabet of both Figs. II and III. Although no quantitative agreement is observed (nor is it expected at the value of $\hbar = 1$), one notices remarkable

qualitative similarities between classical and quantal results. Specifically, the key feature of our interest is already present in classical S-matrix; that is, the partitioning of the strength of large peaks to more numerous smaller peaks as the motion of the target becomes chaotic. It is possible to improve the classical approximation of the S-matrix by incorporating the action integral and Maslov index to include quantum phases in the manner of Miller [9]:

$$S^{sc}(p_{out}, p_{in}) = \sum_s \sqrt{\left[\frac{dp_{out}}{dx_{in}} \right]_{(s)}^{-1}} \cdot \exp \left\{ iS(p_{out}, p_{in})_{(s)} - i\frac{\nu(s)\pi}{2} \right\} \quad (9)$$

The effect of the inclusion of the phase factor can be seen in the Fig. V. Since the expression eq. (9) does not guarantee the unitarity of the S-matrix, we have numerically renormalized the results to make the total transition probability unity. With this admittedly ad hoc procedure, we can still observe the general improvement toward the full quantum results, while leaving the essential points made with classical approximation intact.

V. CONCLUSION

We have studied the phenomenon of damping (or the spreading) of the peaks in the scattering off the complex target from a semiclassical point of view.

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FIGURES

FIG. 1. Dynamics of the target Hamiltonian H_I . Quantum level statistics $P(s)$ and the classical Poincare sections $\varphi = 0$ at the energy $\varepsilon = 10$

FIG. 2. The absolute value of inelastic S-matrix at $p_\alpha = 6.5$ with $\lambda = 50$ as a function of excitation energy.

FIG. 3. Classical reaction function $p_{out}(x_{in})$ at $p_{in} = 6.5$.

FIG. 4. Classical approximation to the quantum S-matrix, S^{cl} .

FIG. 5. Semiclassical approximation to the quantum S-matrix, S^{sc} .









