	H1	H2	H3		C1	C2	C3
Fcp[kW/C]	2.29	0.42	0.54	Fcp[kW/C]	0.93	1.56	0.45
Tin[C]	159	267	340	Tin[C]	26	100	60
Tout[C]	25	79	90	Tout[C]	148	260	178

1. Use GAMS to obtain minimum Utility.

The program's name is Minimum Utility. All I have done is changing the parament below.

```
 Sets    i        hot streams     /H1*H3/,
         j        cold streams    /C1*C3/;

Parameters
 HRAT
 fh(i)       heat capacity flowrate of hot stream
 fc(j)       heat capacity flowrate of cold stream
 thin(i)     supply temp. of hot stream
 thout(i)    target temp. of hot stream
 tcin(j)     supply temp. of cold stream
 tcout(j)    target temp. of cold stream     ;
*-------------------------------------------------------------------
*                            INPUT   DATA
*-------------------------------------------------------------------
* hot stream data:
 thin('H1')=159;   thout('H1')=25;      fh('H1')=2.29;
 thin('H2')=267;   thout('H2')=79;      fh('H2')=0.42;
 thin('H3')=340;   thout('H3')=90;      fh('H3')=0.54;
* cold stream data:
 tcin('C1')=26;   tcout('C1')=148;      fc('C1')=0.93;
 tcin('C2')=100;  tcout('C2')=260;      fc('C2')=1.56;
 tcin('C3')=60;   tcout('C3')=178;      fc('C3')=0.45;

 HRAT = 10 ;

*********************************************************************
*********************************Sets********************************
 Set                  int      temperature interval /int1*int12/;
 alias               (int, ints);
```

Except for the input desk, I set larger number of intervals, for the reason that, with the same calculation concept I get the result from excel that the sum of intervals is 11, which is larger than the original set data 10, which will cause a bad result when leaving the number unchanged. After trying different number larger than 10, I found that number whichever larger than 11 will be nice to get a right answer, specific to the situation given by the question. So I just set 12.

And the details of the solution from the program is shown below.

```
----     121 PARAMETER TINT

int1  340.000,    int2  270.000,    int3  267.000,    int4  188.000
int5  159.000,    int6  158.000,    int7  110.000,    int8   90.000
int9   79.000,    int10  70.000,    int11  36.000,    int12  25.000

----     121 VARIABLE UMIN.L                =       43.110

----     121 VARIABLE delta.L

int1    80.910,    int2    77.850,    int3    30.450,    int5     1.240
int6    16.120,    int7    53.520,    int8    68.150,    int9    76.340
int10 122.580,    int11 147.770,    int12 147.770

----     121 PARAMETER pinch_temperature    =      159.000  pinch temperature whe
                                                            re heat flow is zero

----     121 PARAMETER qint

int1    37.800,    int2    -3.060,    int3   -47.400,    int4   -30.450
int5     1.240,    int6    14.880,    int7    37.400,    int8    14.630
int9     8.190,    int10   46.240,    int11   25.190
```

In sum, the min hot utility is 43.11 kW, the min cold utility is 147.77 kW, the hot pinch is 159 C. Additionally, the situation of interval cut (based on the unchanged T of muliti-hot streams line) is just one-to-one correspondence the result given by the excel(based on the T minus 0.5*HRAT of muliti-hot streams line), which means I was given a right answer by the program.

2. Use Synheat to obtain a solution above and below the pinch as well as ignoring the pinch.

(0) Pick EMAT=HRAT=10 C.

(a) Conversion of units (cost of utilities)

As the cost is the center of question, let's just first make sure all the cost amount of standard unit.

Use $U = 0.1kW/(m^2°\text{C})$

Cost of Energy: $C_E = c_Q Qh_{min}$ $\quad c_Q = 30\ RMB/MMBTU$

Cost of cooling Water: $C_{cw} = c_w Qc_{min}$ $\quad c_w = 20\ RMB/1000\ Gallons$

Cost of Capital: $C_{ca} = \sum_{(i,j)}[c_f + d(A_{i,j})^e]$ $\quad c_f = 120000RMB \quad d = 3500RMB \quad e = 0.5$

Total Annualized Cost: $TAC = C_{ca}/n + C_E + C_{cw}$ $\quad n = 10\ years$

For $[Qh_{min}]$=kW and $[Qc_{min}]$=kW, I should make the conversion of units.

For hot utility, I write,

$$C_E = c_Q Qh_{min} = \frac{30\ RMB}{1\ MMBTU} \times Qh_{min}\ kW \times \frac{1MMBTU}{1.055e^{+6}\ kJ} = 2.843e^{-5} \times Qh_{min}\ RMB/s$$

For cold utility, I write,

$$C_{cw} = c_w Qc_{min} = \frac{20\ RMB}{1000\ Gallons} \times \frac{Qh_{min}}{\Delta T \times cp \times \rho} \frac{m^3}{s} \times \frac{1Gallons}{4.546e^{-3}\ m^3}$$

$$= \frac{20}{1000 \times (45-15) \times 4.187 \times 998.95 \times 4.546e^{-3}} Qc_{min}\ RMB/s = 3.506e^{-5} \times Qc_{min}\ RMB/s$$

As for the working time of one year, let's just pick 24 h/day*310 days.

So $C_E = 2.843e^{-5} \times Qh_{min}\ RMB/s = 762.5 \times Qh_{min}\ RMB/year$

$C_{cw} = 3.506e^{-5} \times Qc_{min}\ RMB/s = 939.1 \times Qc_{min}\ RMB/year$

Then the total cost is $TAC = \frac{C_{ca}}{n} + C_E + C_{cw}$

$$= \frac{\sum_{(i,j)}[120000 + 3500 \times (A_{i,j})^e]}{10} + 762.5 \times Qh_{min} + 939.1 \times Qc_{min}$$

$$= \sum_{(i,j)}[12000 + 350 \times (A_{i,j})^e] + 761.6 \times Qh_{min} + 939.1 \times Qc_{min}\ RMB/year$$

This form is just corresponding to the equation in GAMS code.

```
obj..cost =e= unitc*(sum((i,j,st),z(i,j,st))
            + sum(i,zcu(i)) + sum(j,zhu(j)))
            + acoeff*sum( (i,j,k),(q(i,j,k)*(1/U(i,j)  )
            / (((dt(i,j,k)*dt(i,j,k+1)*(dt(i,j,k) + dt(i,j,k+1))/2
            + 1e-6)**0.33333) + 1e-6) + 1e-6)**aexp)
            + hucoeff*(sum(j,(qh(j)*(1/Uhu(j)))
            / (((thuin - tcout(j))*dthu(j)*((thuin - tcout(j) + dthu(j))/2)
            + 1e-6)**0.33333) + 1e-6)**aexp)
            + cucoeff*sum(i,(qc(i)*(   1/Ucu(i)  )
            / (((thout(i)-tcuin)*dtcu(i)*(thout(i) - tcuin + dtcu(i))/2
            + 1e-6)**0.33333) + 1e-6)**aexp)
            + sum(j,qh(j)*hucost) + sum(i,qc(i)*cucost);
```

And for all the different cases, the sets for the parament in cost equation below will never change.

```
unitc = 12000; acoeff = 350;    aexp  = 0.5;
* Annualized cost of a hot utility exchanger = ur
hucoeff = 350;
* Annualized cost of a cold utility exchanger = u
cucoeff = 350;
* Annual cost of utilities ($/year)
* hucost and cucost: ($/(unit Power) for utilitie
hucost = 761.5900144;
cucost = 939.0917684;
* Value of EMAT (exchanger minimum approach tempe
tmapp  = 10;
```

Also, let's keep the Tin and Tout of hot and cold utilities unchanged in different cases.

```
thuin = 360; thuout = 340; hhu = 0.2;
tcuin = 15; tcuout = 45; hcu = 0.2;
```

Additionally, for the Points given for a correct transformation of the C_E & C_{cw} to $RMB/Kw-h$, I would like to write something more.

$C_E = c_Q Qh_{min} = \frac{30\ RMB}{1\ MMBTU} \times Qh_{min}\ kW \times \frac{1MMBTU}{1.055e^{+6}\ kJ} = 2.843e^{-5} \times Qh_{min}\ RMB/s$ for $[Qh_{min}]$=kW. If the unit of Qh_{min} is $Kw-h$, then $C_E = 2.843e^{-5} \frac{RMB}{kW{\times}s} \frac{3600s}{1h} \times Qh_{min}$

$= 0.1024 \frac{RMB}{kW-h} \times Qh_{min}$

$C_{cw} = c_w Qc_{min} = \frac{20\ RMB}{1000\ Gallons} \times \frac{Qh_{min}}{\Delta T{\times}cp{\times}\rho} \frac{m^3}{s} \times \frac{1Gallons}{4.546e^{-3}\ m^3} = 3.506e^{-5} \times Qc_{min}\ RMB/s$ for $[Qc_{min}]$=kW. If the unit of Qc_{min} is $Kw-h$, then $C_E = 3.506e^{-5} \frac{RMB}{kW{\times}s} \frac{3600s}{1h} \times Qc_{min}$

$= 0.1262 \frac{RMB}{kW-h} \times Qc_{min}$

(b) the optimal solution above the pinch

All the information I set is:

Number of stages = 2 with k=3; the T cut by pinch given by GAMS program minimum Utility; the number of cold utility must be 0.

```
* hot
thin('H1') = 267; thout('H1') = 159; fh('H1') = 0.42; hh('H1') = 0.2;
thin('H2') = 340; thout('H2') = 159; fh('H2') =0.54; hh('H2') = 0.2;

* cold
tcin('C1') = 149; tcout('C1') = 260; fc('C1') = 1.56; hc('C1') = 0.2;
tcin('C2') = 149; tcout('C2') = 178; fc('C2') = 0.45; hc('C2') = 0.2;
```

Without setting the limitation on the number of exchangers, the result is:

```
                2

H1.C1        1.000
H2.C1        1.000

----     259 VARIABLE zcu.L

                      ( ALL

----     259 VARIABLE zhu.L

C1 1.000,     C2 1.000
```

```
----     261 PARAMETER al

                2

H1.C1       26.959
H2.C1       25.822

----     270 PARAMETER acu

                      ( AL

----     270 PARAMETER ahu

C1 3.017,     C2 0.700
```

```
----     261 VARIABLE q.L    e

                2

H1.C1       45.360
H2.C1       97.740

----     261 VARIABLE qc.L

                      ( ALI

----     261 VARIABLE qh.L

C1 30.060,     C2 13.050
```

```
276 PARAMETER costheat              =    32869.276  cost of heating
    PARAMETER costcool              =        0.000  cost of cooling

281 PARAMETER invcost               =    52274.801  investment cost
    VARIABLE cost.L                 =    85144.077  hen and utility cost
```

Or after setting the limitation, which is to demand that the sum of exchangers and utilities number must be equal to Nmin = 4+1-1 = 4, the optimal solution it gives is:

```
              2

H1.C1       1.000
H2.C1       1.000

----     261 VARIABLE zcu.L

                           ( ALL

----     261 VARIABLE zhu.L

C1 1.000,    C2 1.000
```

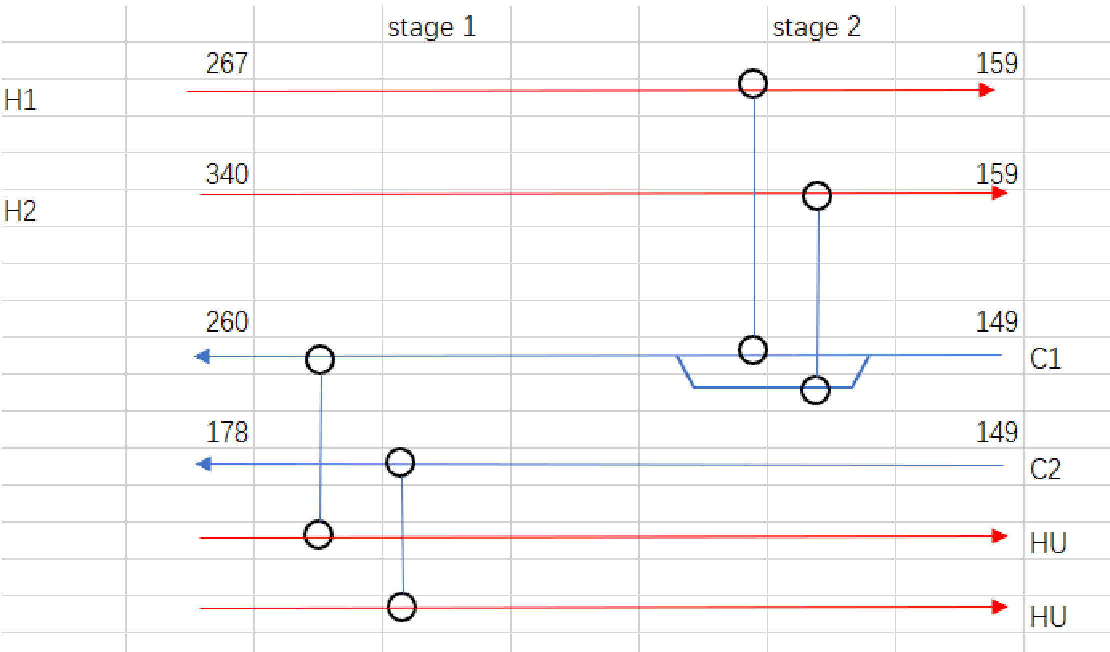
```
----     261 PARAMETER al

                  2

H1.C1       26.959
H2.C1       25.822

----     270 PARAMETER acu

                         ( ALL

----     270 PARAMETER ahu

C1 3.017,    C2 0.700
```

```
----     261 VARIABLE q.L   e

                   2

H1.C1       45.360
H2.C1       97.740

----     261 VARIABLE qc.L

                         ( ALL

----     261 VARIABLE qh.L

C1 30.060,    C2 13.050
```

```
276 PARAMETER costheat              =    32869.276  cost of heating
    PARAMETER costcool              =        0.000  cost of cooling

281 PARAMETER invcost               =    52274.801  investment cost
    VARIABLE cost.L                 =    85144.077  hen and utility cost
```

Just the same.

So the result is:



TAC (RMB/year)	Units	HU (kW)	Area (m^2)
85144.077	4	43.11	56.498

(c) the optimal solution below the pinch

All the information I set is:

Number of stages = 2 with k=3; the T cut by pinch given by GAMS program minimum Utility; the number of hot utility must be 0.

```
thin('H1') = 159; thout('H1') = 25; fh('H1') = 2.29; hh('H1') = 0.2;
thin('H2') = 159; thout('H2') = 79; fh('H2') = 0.42; hh('H2') = 0.2;
thin('H3') = 159; thout('H3') = 90; fh('H3') = 0.54; hh('H3') = 0.2;

* cold
tcin('C1') = 26; tcout('C1') = 148; fc('C1') = 0.93; hc('C1') = 0.2;
tcin('C2') = 100; tcout('C2') = 149; fc('C2') = 1.56; hc('C2') = 0.2;
tcin('C3') = 60; tcout('C3') = 149; fc('C3') = 0.45; hc('C3') = 0.2;
```

Without setting the limitation on the number of exchangers, the result is:

```
                1           2

H1.C1       1.000       1.000
H1.C2       1.000
H1.C3       1.000
H2.C1       1.000
H2.C3                   1.000
H3.C1                   1.000
H3.C2       1.000


----    261 VARIABLE zcu.L

H1 1.000


----    261 VARIABLE zhu.L

                    ( ALL
```

```
                 1           2
H1.C1       22.444      12.921
H1.C2       53.344
H1.C3       18.377
H2.C1       13.654
H2.C3                    6.698
H3.C1                    3.441
H3.C2       17.957


----    270 PARAMETER acu  are----

H1 64.297


----    270 PARAMETER ahu  are----

                    ( ALL
```

```
                 1           2

H1.C1       32.245      46.880
H1.C2       53.343
H1.C3       26.622
H2.C1       20.172
H2.C3                   13.428
H3.C1                   14.163
H3.C2       23.097


----    261 VARIABLE qc.L  energ

H1 147.770


----    261 VARIABLE qh.L  energ

                    ( ALL
```

```
----    276 PARAMETER costheat          =        0.000  cost of heating
            PARAMETER costcool          =   138769.591  cost of cooling

----    281 PARAMETER invcost           =   122118.966  investment cost
            VARIABLE cost.L             =   260888.556  hen and utility cost
```

Or after setting the limitation, which is to demand that the sum of exchangers and utilities number must be equal to Nmin = 6+1-1 = 6, the optimal solution it gives is none.

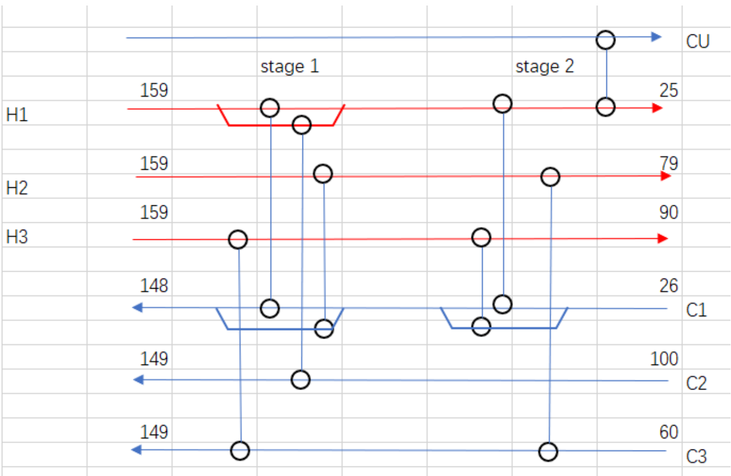
Try to increase the sum, I get a cheaper solution when set '8=e=sum(i,zcu(i))+sum(j,zhu(j))+sum((i,j,k),z(i,j,k))', the result is shown below.

```
                1          2

H1.C1       1.000      1.000
H1.C2       1.000
H2.C1       1.000
H2.C3                  1.000
H3.C1                  1.000
H3.C3       1.000


----     261 VARIABLE zcu.L

H1 1.000


----     261 VARIABLE zhu.L

                          ( ALL
```

```
----     261 PARAMETER al   area calcul

                1          2

H1.C1      22.004     11.916
H1.C2      75.967
H2.C1      13.123
H2.C3                  6.396
H3.C1                  2.594
H3.C3      18.851


----     270 PARAMETER acu   area coole

H1 64.297


----     270 PARAMETER ahu   area heate

                          ( ALL       0.(
```

```
                1          2

H1.C1      35.483     47.167
H1.C2      76.440
H2.C1      20.810
H2.C3                 12.790
H3.C1                 10.000
H3.C3      27.260


----     261 VARIABLE qc.L   ene:

H1 147.770


----     261 VARIABLE qh.L   ene:

                          ( ALL

 276 PARAMETER costheat           =        0.000  cost of heating
     PARAMETER costcool           =   138769.591  cost of cooling

 281 PARAMETER invcost            =   108951.770  investment cost
     VARIABLE cost.L              =   247721.361  hen and utility cost
```

By comparison, I pick the latter as the optimal solution. So the result is:



TAC (RMB/year)	Units	CU (kW)	Area (m^2)
247721.361	8	147.77	215.148

(d) the first and second and third optimal solution when ignoring the pinch

After trying to set different limitations, the conclusion I found is that, the program will calculate for finite times and give the cheapest solution among these, which is more likely to be far away from an expected optimal solution as the streams getting more complex. So I have to add more limitation on the numbers of the exchangers or the utilities or directly on the amount of cost, to get a relative optimal solution.

There are the general sets below.

```
*hot streams  (hh is the stream heat transfer coefficient)
thin('H1') = 159; thout('H1') = 25; fh('H1') = 2.29; hh('H1') = 0.2;
thin('H2') = 267; thout('H2') = 79; fh('H2') = 0.42; hh('H2') = 0.2;
thin('H3') = 340; thout('H3') = 90; fh('H3') = 0.54; hh('H3') = 0.2;
*cold streams (hc is the stream heat transfer coefficient)
tcin('C1') = 26; tcout('C1') = 148; fc('C1') = 0.93; hc('C1') = 0.2;
tcin('C2') = 100; tcout('C2') = 260; fc('C2') = 1.56; hc('C2') = 0.2;
tcin('C3') = 60; tcout('C3') = 178; fc('C3') = 0.45; hc('C3') = 0.2;
```

Now there are the best three solution I found within these days, for the situation of ignoring the pinch.

order	1st solution code is in untitled_8	2nd solution code is in untitled_8	3rd solution code is in untitled_7
Total cost	374720.007	378326.637	383795.181
Number of exchangers	6	6	8
Number of HU	1	1	1
Number of CU	1	1	1

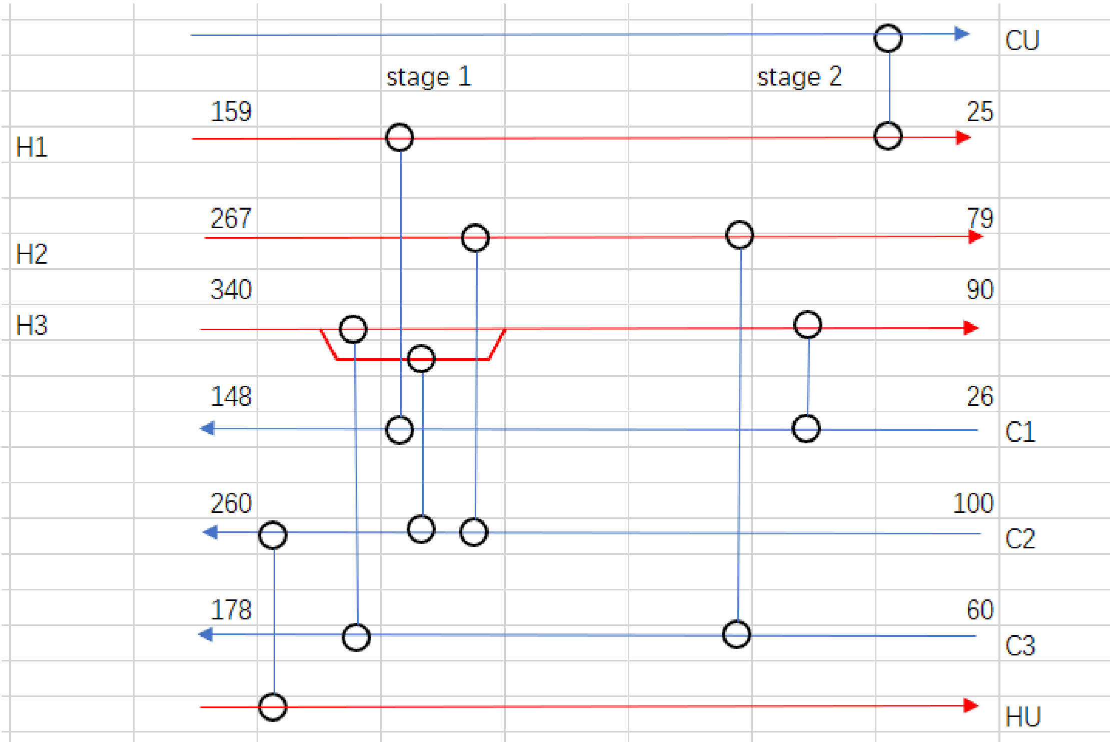
Additional limitation	sum((i,j,k),z(i,j,k))=e=6;
	sum(j, zhu(j))=g=1;
	sum(i,zcu(i))=g=1;
	cost=l=390000;
	one equation telling the program to exclude the solution of 2nd solution (*)
		sum((i,j,k),z(i,j,k))=e=6;
		sum(j, zhu(j))=g=1;
		sum(i,zcu(i))=g=1;
		cost=l=390000;
		0 additional equation to exclude another solution
			sum((i,j,k),z(i,j,k))=e=8;
			sum(j, zhu(j))=e=1;
			sum(i,zcu(i))=e=1;
			0 additional equation to exclude another solution

(*) the equation's detail are shown below.

```
*without limitation
*VARIABLE cost.L                =   378326.637347  hen and utility cost
Equation Min_units1;
Min_units1.. +z('H1','C1','1')+z('H2','C1','2')+z('H2','C2','1')+z('H3','C2','1')
             +z('H3','C3','1')+z('H3','C3','2')+zcu('H1')+zhu('C2')
             -(z('H1','C1','2')+z('H1','C2','1')+z('H1','C2','2')+z('H1','C3','1')+z('H1','C3','2')
             +z('H2','C1','1')+z('H2','C2','2')+z('H2','C3','1')+z('H2','C3','2')
             +z('H3','C1','1')+z('H3','C1','2')+z('H3','C2','2')+zhu('C1')+zcu('H2'))=l=7;
*           VARIABLE cost.L                =   374720.007  hen and utility cost
```

The results of the three solutions are shown below.

1st solution:



TAC (RMB/year)	Units	HU (kW)	CU (kW)	Area (m^2)
374720.007	8	99.540	204.200	157.466

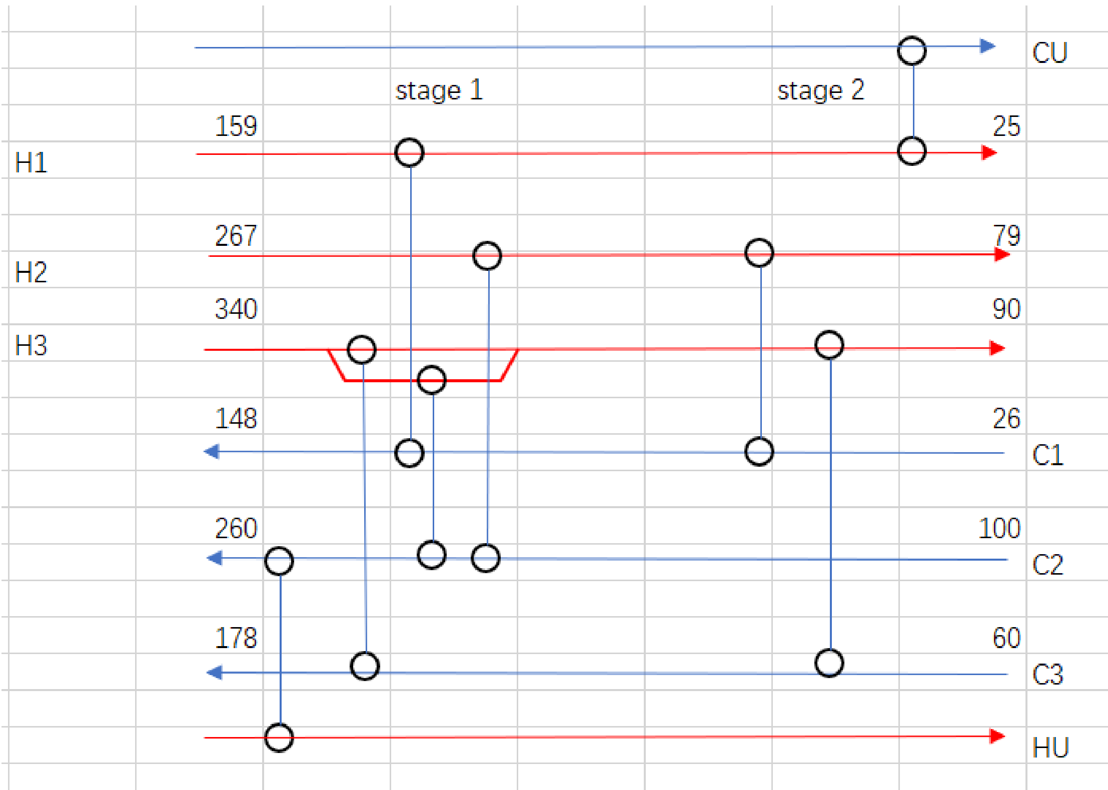
```
301 PARAMETER costheat              =     75894.405  cost of heating
    PARAMETER costcool              =    191762.539  cost of cooling

306 PARAMETER invcost               =    107063.064  investment cost
    VARIABLE  cost.L                =    374720.007  hen and utility cost
```

```
                1           2

H1.C1       1.000
H2.C2       1.000
H2.C3                   1.000
H3.C1                   1.000
H3.C2       1.000
H3.C3       1.000


----    286 VARIABLE zcu.L

H1 1.000


----    286 VARIABLE zhu.L

C2 1.000
```

```
                1           2

H1.C1      30.846
H2.C2      16.588
H2.C3                   7.964
H3.C1                   1.586
H3.C2      18.173
H3.C3       6.301


----    295 PARAMETER acu  area coole

H1 67.752


----    295 PARAMETER ahu  area heate

C2 8.256
```

```
                 1           2

H1.C1      102.660
H2.C2       62.831
H2.C3                   16.129
H3.C1                   10.800
H3.C2       87.229
H3.C3       36.971


----     286 VARIABLE qc.L  energy

H1 204.200


----     286 VARIABLE qh.L  energy

C2 99.540
```

2nd solution:



TAC (RMB/year)	Units	HU (kW)	CU (kW)	Area (m^2)
378326.637	8	101.760	206.420	154.562

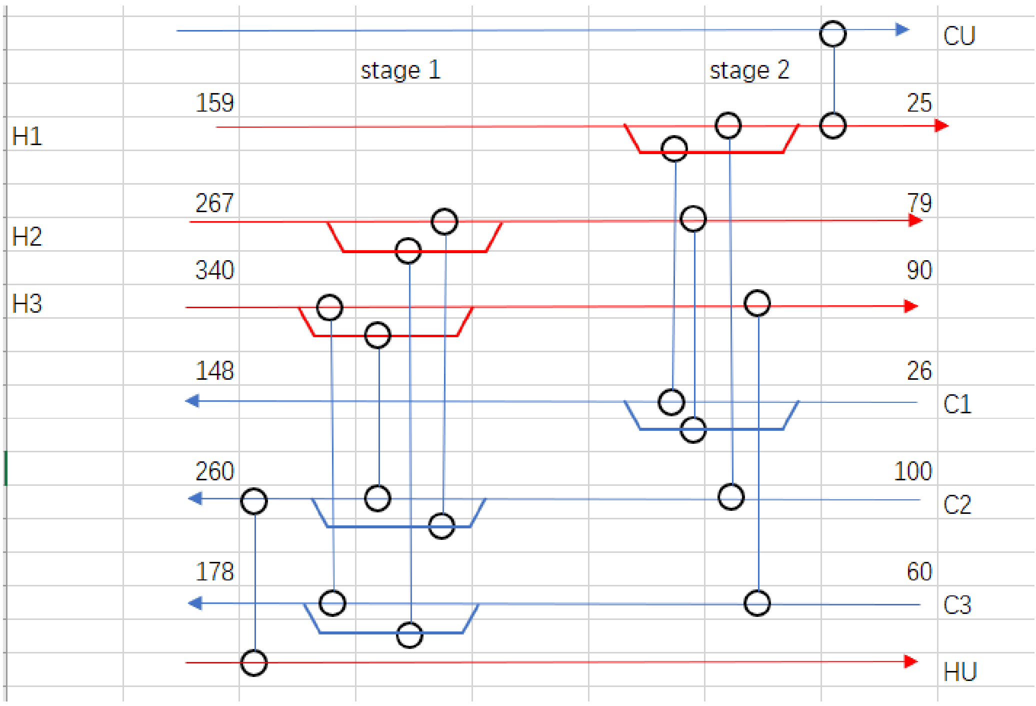
```
 301 PARAMETER costheat             =    77587.047  cost of heating
     PARAMETER costcool             =   193847.323  cost of cooling

 306 PARAMETER invcost              =   106892.268  investment cost
     VARIABLE cost.L                =   378326.637  hen and utility cost
```

```
                 1            2

H1.C1        1.000
H2.C1                     1.000
H2.C2        1.000
H3.C2        1.000
H3.C3        1.000        1.000


----     286 VARIABLE zcu.L

H1 1.000


----     286 VARIABLE zhu.L

C2 1.000
```

```
                 1            2

H1.C1       30.533
H2.C1                     2.131
H2.C2       21.294
H3.C2       12.940
H3.C3        5.120        6.253


----     295 PARAMETER acu   area cool

H1 67.895


----     295 PARAMETER ahu   area heat

C2 8.396
```

```
                 1            2

H1.C1      100.440
H2.C1                    13.020
H2.C2       65.940
H3.C2       81.900
H3.C3       36.356       16.744


----     286 VARIABLE qc.L   ener

H1 206.420


----     286 VARIABLE qh.L   ener

C2 101.760
```

3rd solution:



TAC (RMB/year)	Units	HU (kW)	CU (kW)	Area (m^2)
383795.181	10	89.990	194.650	164.208

```
371 PARAMETER costheat            =    68612.994  cost of heating
    PARAMETER costcool            =   182794.213  cost of cooling

376 PARAMETER invcost             =   132387.974  investment cost
    VARIABLE cost.L               =   383795.181  hen and utility cost
```

```
                 1            2

H1.C1                     1.000
H1.C2                     1.000
H2.C1                     1.000
H2.C2        1.000
H2.C3        1.000
H3.C2        1.000
H3.C3        1.000        1.000


----     356 VARIABLE zcu.L

H1 1.000


----     356 VARIABLE zhu.L

C2 1.000
```

```
                 1            2

H1.C1                    18.822
H1.C2                    25.401
H2.C1                     9.834
H2.C2        3.465
H2.C3        1.387
H3.C2       14.770
H3.C3        1.570       14.186


----     365 PARAMETER acu  area

H1 67.136


----     365 PARAMETER ahu  area

C2 7.637
```

```
                 1            2

H1.C1                    66.390
H1.C2                    45.820
H2.C1                    47.070
H2.C2       21.890
H2.C3       10.000
H3.C2       91.900
H3.C3       10.000       33.100


----     356 VARIABLE qc.L   ene:

H1 194.650


----     356 VARIABLE qh.L   ene:

C2 89.990
```