PART 5 Biochemical Reaction Systems

This part is highly similar to Part I. Though the kinetic is a little bit different, it is still homogeneous. All you need is to pay attention to something unique in this part, like washout.

Chap 27 Enzyme Fermetation

一 M-M Kinetics

Enzyme Fermentation $A \xrightarrow{\text{E (Enzyme)}} R$ $\boxed{-r_A = r_R = k\dfrac{C_{E0}C_A}{C_M + C_A}}$ ← M-M Kinetic

k : Rate constant C_M : Michaelis constant C_{E0} : total enzyme

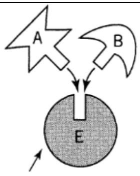
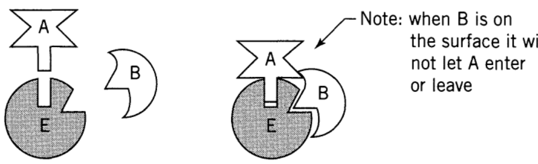
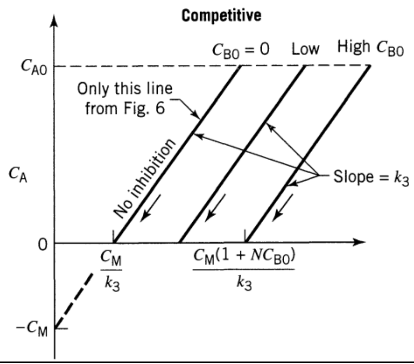
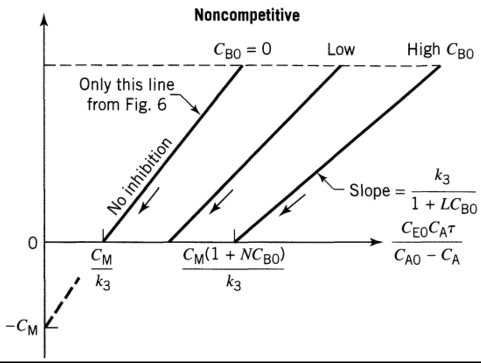
Analysis: ① When $C_A \gg C_M$ $-r_A = r_R \approx k\dfrac{C_{E0}C_A}{C_A} = kC_{E0}$ zero-order

② When $C_A \ll C_M$ $-r_A = r_R \approx k\dfrac{C_{E0}}{C_M}C_A$ first-order

Mechanism: $A + E \underset{2}{\overset{1}{\rightleftharpoons}} \underbrace{X}_{\text{intermediate}} \xrightarrow{3} R + E$ ... and ... $\underbrace{C_{E0}}_{\text{total enzyme}} = \overbrace{C_E}^{\text{free enzyme}} + \underbrace{C_X}_{\text{enzyme attached to reactant}}$

	Batch or Plug Flow Fermentor	Mixed Flow Fermentor
Integrated Form	$C_M \ln\dfrac{C_{A0}}{C_A} + (C_{A0} - C_A) = k_3 C_{E0} t$	$k_3 C_{E0}\tau = \dfrac{(C_{A0} - C_A)(C_M + C_A)}{C_A}$
Linear Form	$\dfrac{C_{A0} - C_A}{\ln\dfrac{C_{A0}}{C_A}} = -C_M + k_3 C_{E0}\dfrac{t}{\ln\dfrac{C_{A0}}{C_A}}$	$C_A = -C_M + \dfrac{k_3 C_{E0} C_A \tau}{C_{A0} - C_A}$

二 Inhibition

Competitive Inhibition	Noncompetitive Inhibition
	
$A + E \underset{2}{\overset{1}{\rightleftharpoons}} X \xrightarrow{3} R + E$ $B + E \underset{5}{\overset{4}{\rightleftharpoons}} Y$	$A + E \underset{2}{\overset{1}{\rightleftharpoons}} X \xrightarrow{3} R + E$ $B + E \underset{5}{\overset{4}{\rightleftharpoons}} Y$ $B + X \underset{7}{\overset{6}{\rightleftharpoons}} Z$
$r_R = \dfrac{k_3 C_{E0} C_A}{C_M(1 + NC_{B0}) + C_A}$	$r_R = \dfrac{\dfrac{k_3}{1 + LC_{B0}} C_{E0} C_A}{C_M\left(\dfrac{1 + NC_{B0}}{1 + LC_{B0}}\right) + C_A}$
$C_M = \dfrac{k_2 + k_3}{k_1}$ $N = \dfrac{k_4}{k_5}$ $L = \dfrac{k_6}{k_7}$	
	

Chap 28　Microbial Fermentation

一　Microbial Fermentation

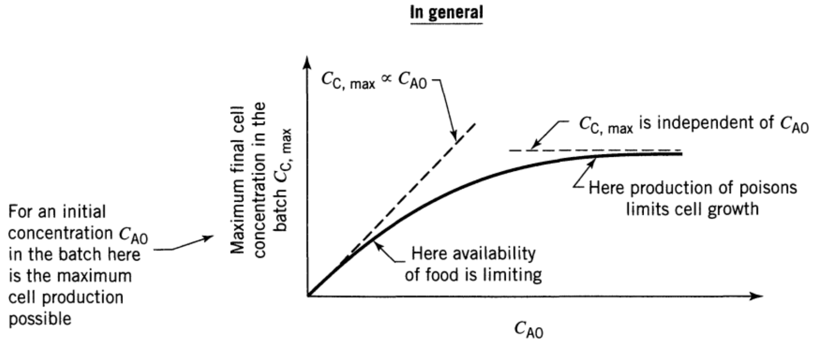
$$A \xrightarrow{C} C + R$$

A : food　C : Microbes　R : Waste material (can be product)

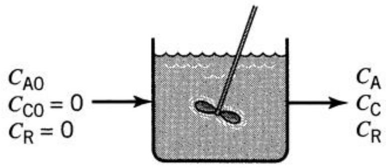
When cell grow in a friendly constant environment with food C_A :

Monod Equation :
$$\boxed{r_C = \frac{kC_A C_C}{C_A + C_M}}$$

In batch the cells will start dying after a exponential growth for lack of food or too much poisonous waste.



Thus we usually use MFR to operate at optimum.



二　Fractional Yield

Definition:
$$\left.\begin{aligned} (C/A) &= \varphi(C/A) = \frac{d(\text{C formed})}{d(\text{A used})} \\ (R/A) &= \varphi(R/A) = \frac{d(\text{R formed})}{d(\text{A used})} \\ (R/C) &= \varphi(R/C) = \frac{d(\text{R formed})}{d(\text{C used})} \end{aligned}\right\}$$

Relationship
$$\left.\begin{aligned} (R/A) &= (R/C) \cdot (C/A) \\ (A/C) &= 1/(C/A) \end{aligned}\right\} \qquad \left.\begin{aligned} r_C &= (-r_A)\,(C/A) \\ r_R &= (-r_A)\,(R/A) \\ r_R &= (r_C)\,(R/C) \end{aligned}\right\}$$

Suppose all φ values remain constant at all composition :

$$\left.\begin{aligned} C_C - C_{C0} &= (C/A)\,(C_{A0} - C_A) \quad \text{...or...} \quad C_C = C_{C0} + (C/A)\,(C_{A0} - C_A) \\ C_R - C_{R0} &= (R/A)\,(C_{A0} - C_A) \quad \text{...or...} \quad C_R = C_{R0} + (R/A)\,(C_{A0} - C_A) \\ C_R - C_{R0} &= (R/C)\,(C_C - C_{C0}) \quad \text{...or...} \quad C_R = C_{R0} + (R/C)\,(C_C - C_{C0}) \end{aligned}\right\}$$

Chap 29 Substrate-Limiting Microbial Fermentation

一 Kinetics and Max Rate

There is no slowing of rate so the kinetics would be monod equation

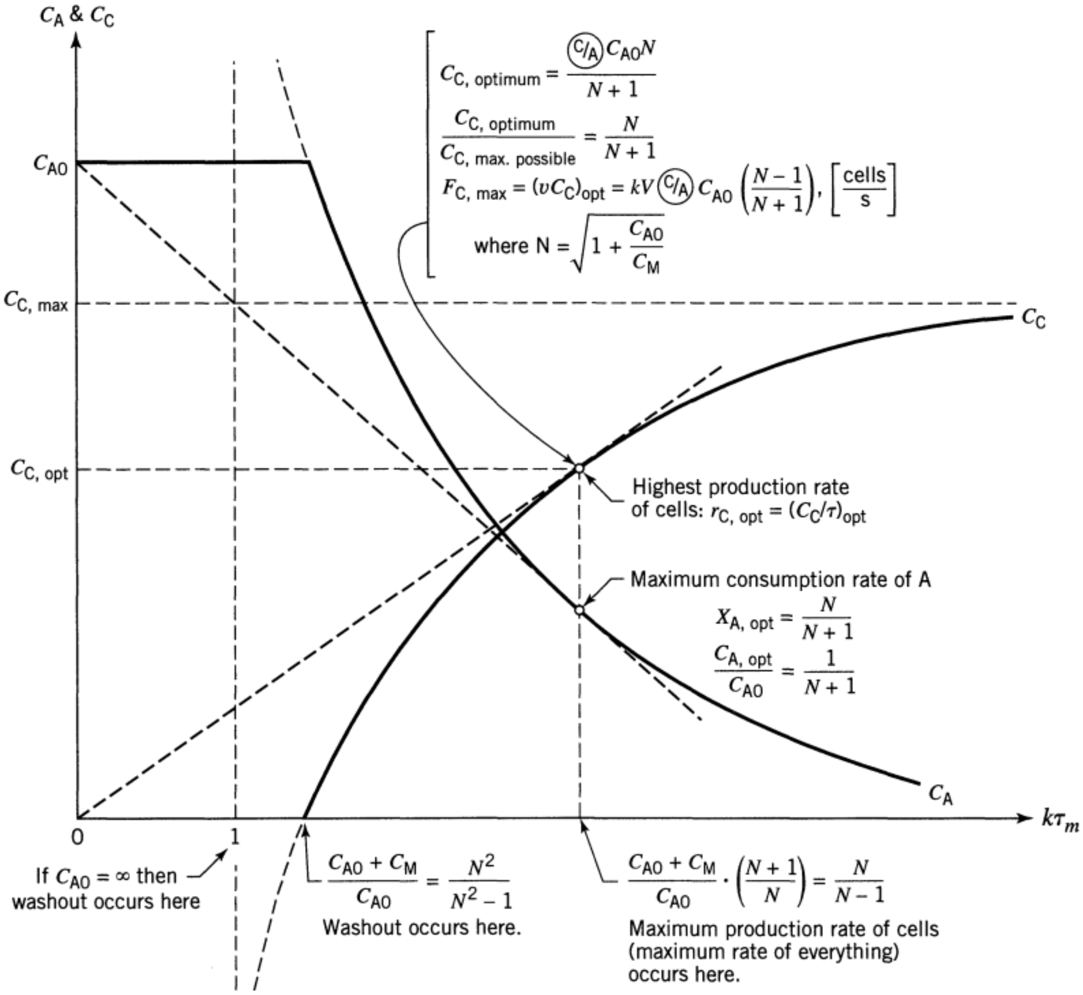
$$\boxed{r_C = \varphi(C/A)(-r_A) = \frac{kC_A C_C}{C_A + C_M}} \quad \boxed{C_C - C_{C0} = \varphi(C/A)(C_{A0} - C_A)}$$

When $C_A = \sqrt{C_M^2 + C_M(C_{A0} + \varphi(A/C)C_{C0})} - C_M$, r_C reaches to its maximum

二 Mixed Flow Fermentor ※

Assume : No cells entering in the feed → $C_{C0} = 0$

Performance Equation : $\boxed{k\tau_m = \frac{C_M + C_A}{C_A}}$ → $\boxed{C_A = \frac{C_M}{k\tau_m - 1}}$ $\boxed{C_C = \varphi(C/A)(C_{A0} - \frac{C_M}{k\tau_m - 1})}$



Note $\boxed{N = \sqrt{1 + \frac{C_{A0}}{C_M}}}$ Optimum operations $\boxed{\frac{C_A}{C_{A0}} = \frac{1}{N+1}}$ $\boxed{\frac{C_C}{C_{C,max}} = \frac{N}{N+1}}$ $\boxed{k\tau_{opt} = \frac{N}{N-1}}$

There must be $\boxed{k\tau_m > 1 + \frac{C_M}{C_{A0}}}$ otherwise washout would occur

↑ the microbes in fermentor start to decrease

Chap 30　Product-Limiting Microbial Fermentation

一　Kinetics

In many cases, microbes' waste products like alcohol inhibit their growth

So the monod equation need to be extended :

$$r_C = \varphi(C/R) r_R = k(1 - \frac{C_R}{C_R^*})^n \frac{C_A C_C}{C_A + C_M}$$

C_R^* : When C_R reaches it, reactions stop

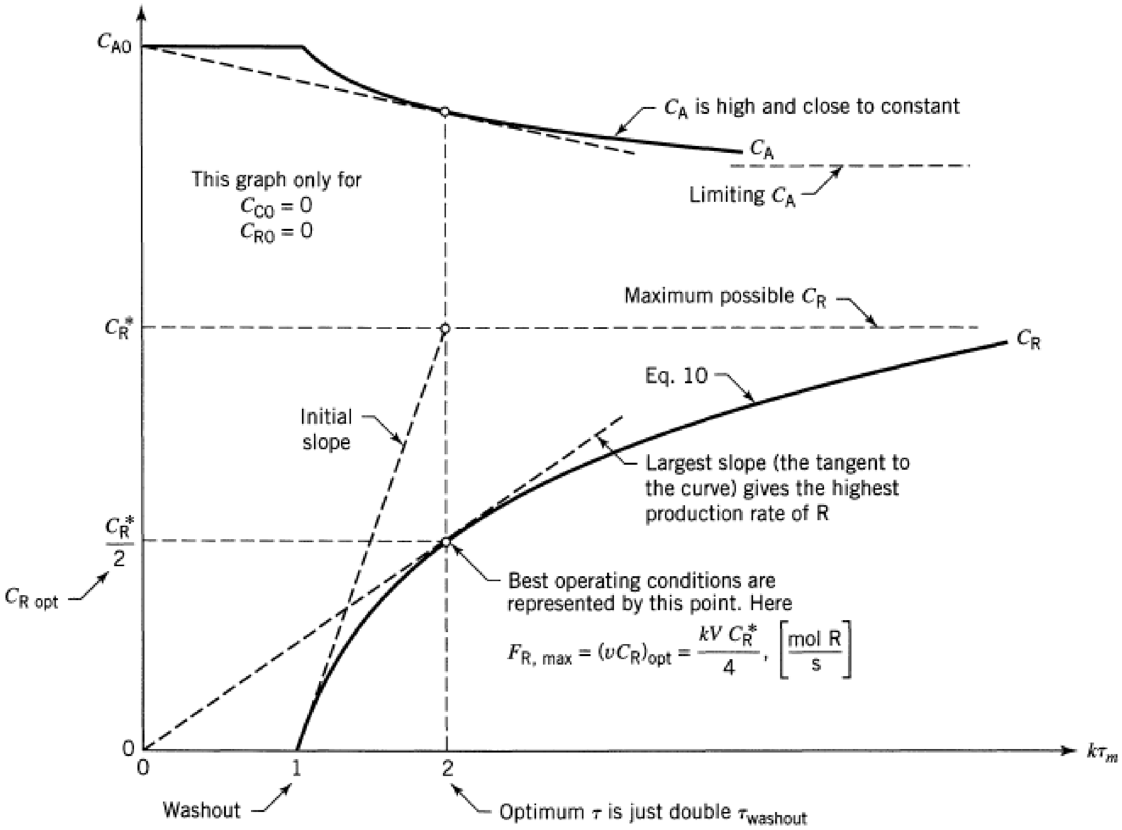
We focus on special case of sufficient food $C_A \gg C_M$, $C_{R0} = C_{C0} = 0$ and $n = 1$

$$r_C = \varphi(C/R) r_R = k(1 - \frac{C_R}{C_R^*}) C_C \qquad r_R = k(1 - \frac{C_R}{C_R^*}) C_R$$

So r_R reach maximum when $\boxed{C_{R,\mathrm{maxrate}} = \frac{1}{2} C_R^*}$

二　MFR For n = 1 ※

Performance Equation : $\boxed{k\tau_m = \frac{C_R^*}{C_R^* - C_R}}$



- Washout occurs when $k\tau_m = 1$
- Maximum production rate obtained at $k\tau_m = 2$ when $C_R = C_R^* / 2$
- Maximum production $F_{R,\max} = \varphi(R/C) F_{C,\max} = kVC_R^* / 4$