#1

1, Claim: $\sum_{i=0}^{N-1} (r-1) * r^i = r^N - 1 \quad r \geq 0$

Proof: Since this proof work for most values of K we choose $r = 10$, since our induction will be over N number of digits.

First, for $N = K = 1$

$$\sum_{i=0}^{N-1} (r-1) * r^i = (10-1) * 10^0 = 9$$

$$* \text{ checking: } r^n - 1 = 10^1 - 1 = 9$$

- The formula is proven for a value $K = 1$.
- Now we evaluate for some values of $N = K$,

$$\sum_{i=1}^{N-1} [(10-1)(10^1)] = (9)(10^1) = 99$$

$$r^N - 1 = 10^2 - 1 = 99$$

* The formula holds for K.

* Now we prove for $N = K+1 = 3$,

$$\sum_{i=2}^{N-1} (10-1)(10^2) = (9)(10^2) = 999$$

$$r^n - 1 = 10^3 - 1 = 999$$

* Conclusion: By proving this rule for $K = 0$, K and $K+1$ we have proving the formula

$$\sum_{i=0}^{K+1} (r-1) * r^i = r^N - 1 \quad r \geq 0 \text{ by Induction}$$

$\square$

2. * In this proof we are going to prove that "for radix-r addition, the carry bit is always 0 or 1"

* By regarding our last proof, we proved that for any radix-r number expressed in N number of digits, the largest number that can be expressed in N digits is r^N-1.

* When we are adding numbers of the same radix-r our answer must come in the form of that same radix, so our answer can be only up to r^N-1 for N number of digits.

* Because of r^N-1 for N digits in $N=1$ digit we can express values up to r^1-1 per digit.

* Since each radix of size $N=1$ can have size r^1-1 that means when adding two digits the largest possible value could be $(r^1-1)+(r^1-1)$

$$(r^1-1)+(r^1-1)=r^2-2$$

* Now we find ourselve with a number larger than r^1-1, our one digit max now we need to use a carry bit to store the overflow.

* If there is overflow we subtract out r from $2(r^1-1)$, to move the overflow to the carry bit.

$$2(r^1-1)-r=r^1-2 \quad \text{carry bit}=1$$

* Only one r needs to be taken out to fit r^1-1, from largest values added, so carry bit ≤ 1. Since we only do this when $A+B>r^1-1$, the default is 0.