1) Найти область определения функции:

$$z = \sqrt{1-x^3} + \ln(y^2-1)$$

$1-x^3 \geq 0 \qquad y^2-1>0$

$x \leq 1 \qquad y>1 \quad y<-1$

$x: [-\infty; 1] \qquad y: [-\infty; -1) \cup (1; +\infty]$

2) Найти производные первого порядка функции:

$$z = \left(1 + \frac{\ln x}{\ln y}\right)^{+3} = \left(1 + \ln x \cdot (\ln y)^{-1}\right)^3$$

$$z'_x = 3\left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \frac{1}{x \ln y} = \frac{3}{x \ln y}\left(1 + \frac{\ln x}{\ln y}\right)^2$$

$$\cancel{z'_y = 3\left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot (-1) \cdot (\ln y)^{-2} \cdot \frac{1}{y} =}$$

$$\cancel{= -\frac{3}{y \ln^2 y}\left(1 + \frac{\ln x}{\ln y}\right)^2}$$

$$z'_y = 3\left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \left(-\frac{\ln x}{(\ln y)^2}\right) \cdot \frac{1}{y} =$$

$$= \frac{-3}{y(\ln y)^2}\left(1 + \frac{\ln x}{\ln y}\right)^2$$

(3) $$z=\sqrt{2xy+\cos\frac{x}{y}}=\left(2xy+\cos\frac{x}{y}\right)^{\frac{1}{2}}$$

$$z'_x=\frac{1}{2}\left(2xy+\cos\frac{x}{y}\right)^{-\frac{1}{2}}\cdot\left(2y+\left(-\sin\frac{x}{y}\right)\cdot\frac{1}{y}\right)=$$

$$=\frac{2y-\frac{\sin\frac{x}{y}}{y}}{2\sqrt{2xy+\cos\frac{x}{y}}}$$

$$z'_y=\frac{1}{2}\left(2xy+\cos\frac{x}{y}\right)^{-\frac{1}{2}}\cdot\left(2x+\left(-\sin\frac{x}{y}\right)\cdot\left(-\frac{1}{y^2}\right)\right)=$$

$$=\frac{2x+\frac{\sin\frac{x}{y}}{y^2}}{2\sqrt{2xy+\cos\frac{x}{y}}}$$

$$dz=z'_x\,\Delta x+z'_y\,\Delta y=$$

$$=\frac{2y-\frac{\sin\frac{x}{y}}{y}}{2\sqrt{2xy+\cos\frac{x}{y}}}\cdot\Delta x+\frac{2x+\frac{\sin\frac{x}{y}}{y^2}}{2\sqrt{2xy+\cos\frac{x}{y}}}\cdot\Delta y=$$

$$=\frac{\left(2y-\frac{\sin\frac{x}{y}}{y}\right)\cdot\Delta x+\left(2x+\frac{\sin\frac{x}{y}}{y^2}\right)\cdot\Delta y}{2\sqrt{2xy+\cos\frac{x}{y}}}$$

$$dz(1;1)=\frac{\left(2-\frac{\sin 1}{1}\right)\cdot\Delta x+\left(2+\frac{\sin 1}{1}\right)\cdot\Delta y}{2\sqrt{2+\cos 1}}=$$

$$=\frac{2\Delta x-(\sin 1)\Delta x+2\Delta y+(\sin 1)\cdot\Delta y}{2\sqrt{2+\cos 1}}$$

4) Исследовать на экстремум функцию

$$z = x^2 + xy + y^2 - 6x - 9y$$

$$z'_x = 2x + y - 6$$

$$z'_y = x + 2y - 9$$

$$\begin{cases} 2x + y - 6 = 0 \\ x + 2y - 9 = 0 \end{cases}$$

$$\begin{cases} x = 9 - 2y \\ y = 6 - 2x = 6 - 2(9 - 2y) = 12 + 4y \Rightarrow 12 = 3y \Rightarrow y = 4 \end{cases}$$

$$\Downarrow$$

$$x = 1$$

Экстремум ф-и в (·) (1;4)

$z''_x = 2$ $\quad z''_{xy} = 1$

$z''_y = 2$ $\quad z''_{yx} = 1$

$$\Delta \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0 \Rightarrow (\cdot)(1;4) \text{ явл-ся}$$

экстремумом ф-и