

1) Исследовать функ-ю на условной экстремум

$$U = 3 - 8x + 6y, \text{ если } x^2 + y^2 = 36$$

$$L = 3 - 8x + 6y + \lambda (x^2 + y^2 - 36)$$

$$\begin{cases} L'_x = -8 + 2\lambda x \\ L'_y = 6 + 2\lambda y \\ L'_\lambda = x^2 + y^2 - 36 \end{cases} \Rightarrow \begin{cases} x = \frac{4}{\lambda} \\ y = -\frac{3}{\lambda} \\ \lambda^2 = \frac{25}{36} \Rightarrow \lambda_1 = -\frac{5}{6} \\ \lambda_2 = \frac{5}{6} \end{cases}$$

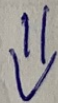
$$M_1 = \left(\frac{5}{6}, -4\frac{4}{5}, -3\frac{3}{5} \right)$$

$$M_2 = \left(-\frac{5}{6}, -4\frac{4}{5}, 3\frac{3}{5} \right)$$

$$L''_{xx} = 2\lambda \quad L''_{yy} = 2\lambda \quad L''_{\lambda\lambda} = 0$$

$$L''_{xy} = 0 \quad L''_{y\lambda} = 2y \quad L''_{x\lambda} = 2x$$

$$\Delta \begin{pmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{pmatrix} = 0 + 0 + 0 - 4y^2 \cdot 2\lambda - 4x^2 \cdot 2\lambda - 0 = -8\lambda(x^2 + y^2)$$



$$M_1 - \min$$

$$M_2 - \max$$

3) Найти производную функции $U = x^2 + y^2 + z^2$ по направлению вектора $\vec{c}(-9; 8; -12)$ в $(\cdot) M_0(8; -12; 9)$

$$U = x^2 + y^2 + z^2 \quad \frac{\partial U}{\partial x} = 2x = 16 \quad \frac{\partial U}{\partial y} = 2y = -24 \quad \frac{\partial U}{\partial z} = 2z = 18$$

$$|\vec{c}| = \sqrt{(-9)^2 + 8^2 + 12^2} = 17$$

$$\frac{\partial U}{\partial \ell} = \frac{\partial U}{\partial x} \cdot \cos \alpha + \frac{\partial U}{\partial y} \cdot \cos \beta + \frac{\partial U}{\partial z} \cdot \cos \gamma$$

$$\cos \alpha = -\frac{9}{17} \quad \cos \beta = \frac{8}{17} \quad \cos \gamma = \frac{12}{17}$$

$$\left. \frac{\partial U}{\partial \ell} \right|_{M_0} = 16 \cdot \left(-\frac{9}{17}\right) + (-24) \cdot \frac{8}{17} + 18 \cdot \frac{12}{17} = \frac{-126 - 192 + 216}{17} = -6$$

4) $U = e^{x^2 + y^2 + z^2}$ $\vec{d}(4; -13; -16)$ $L(-16; 4; -13)$

$$\frac{\partial U}{\partial x} = e^{441} \cdot 2x = e^{441} \cdot (-32) \quad \frac{\partial U}{\partial y} = e^{441} \cdot 2y = e^{441} \cdot 8 \quad \frac{\partial U}{\partial z} = e^{441} \cdot 2z = e^{441} \cdot (-26)$$

$$|\vec{d}| = \sqrt{441} = 21 \quad \cos \alpha = \frac{4}{21} \quad \cos \beta = -\frac{13}{21} \quad \cos \gamma = -\frac{16}{21}$$

$$\left. \frac{\partial U}{\partial \ell} \right|_L = e^{441} \cdot \left(\frac{-128 - 104 + 416}{21} \right) = e^{441} \cdot \frac{184}{21}$$