1) Исследовать функ-ю на условный экстремум

$U = 3 - 8x + 6y$, если $x^2 + y^2 = 36$

$L = 3 - 8x + 6y + \lambda(x^2 + y^2 - 36)$

$$\begin{cases} L'_x = -8x + 2\lambda x \\ L'_y = 6 + 2\lambda y \\ L'_\lambda = x^2 + y^2 - 36 \end{cases} \Rightarrow \quad x = \frac{4}{\lambda} \quad y = -\frac{3}{\lambda} \quad \lambda^2 = \frac{25}{36} \Rightarrow \lambda_1 = -\frac{5}{6} \quad \lambda_2 = \frac{5}{6}$$

$M_1 = \left(\frac{5}{6}; 4\frac{4}{5}; -3\frac{3}{5}\right)$ $\qquad M_2 = \left(-\frac{5}{6}; -4\frac{4}{5}; 3\frac{3}{5}\right)$

$L''_{xx} = 2\lambda \quad L''_{yy} = 2\lambda \quad L''_{\lambda\lambda} = 0$

$L''_{xy} = 0 \quad L''_{y\lambda} = 2y \quad L''_{x\lambda} = 2x$

$$\Delta \begin{pmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{pmatrix} = 0 + 0 + 0 - 4y^2 \cdot 2\lambda - 4x^2 \cdot 2\lambda - 0 = -8\lambda(x^2 + y^2)$$

$\Downarrow$

M_1 – min

M_2 – max

3) Найти производную функции $U = x^2 + y^2 + z^2$ по направлению вектора $\overline{c}(-9; 8; -12)$ в (·) $M_0(8; -12; 9)$

$u = x^2 + y^2 + z^2 \quad \frac{\partial u}{\partial x} = 2x = 16 \quad \frac{\partial u}{\partial y} = 2y = -24 \quad \frac{\partial u}{\partial z} = 2z = 18$

$|\overline{l}| = \sqrt{(-9)^2 + 8^2 + 12^2} = 17$

$$\frac{\partial u}{\partial l} = \frac{\partial u}{\partial x} \cdot \cos\alpha + \frac{\partial u}{\partial y} \cdot \cos\beta + \frac{\partial u}{\partial z} \cdot \cos\psi$$

$\cos\alpha = -\frac{9}{17} \quad \cos\beta = \frac{8}{17} \quad \cos\psi = \frac{12}{17}$

$$\left.\frac{\partial u}{\partial l}\right|_{M_0} = 16 \cdot \left(-\frac{9}{17}\right) + (-24) \cdot \frac{8}{17} + 18 \cdot \frac{12}{17} = \frac{-126 - 192 + 216}{17} = -6$$

4) $u = e^{x^2 + y^2 + z^2} \quad \vec{d}(4; -13; -16) \quad L(-16; 4; -13)$

$\frac{\partial u}{\partial x} = e^{441} \cdot 2x = e^{441} \cdot (-32) \quad \frac{\partial u}{\partial y} = e^{441} \cdot 2y = e^{441} \cdot 8 \quad \frac{\partial u}{\partial z} = e^{441} \cdot 2z = e^{441} \cdot (-26)$

$|\overline{l}| = \sqrt{441} = 21 \quad \cos\alpha = \frac{4}{21} \quad \cos\beta = -\frac{13}{21} \quad \cos\psi = -\frac{16}{21}$

$$\left.\frac{\partial u}{\partial l}\right|_{L} = e^{441} \cdot \left(\frac{-128 - 104 + 416}{21}\right) = e^{441} \cdot \frac{184}{21}$$