

Propensity scores for continuous exposures

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The story so far

Propensity score weighting

- 1 Fit a propensity model predicting exposure x , $x + z$ where z is all covariates
- 2 Calculate weights
- 3 Fit an outcome model estimating the effect of x on y weighted by the propensity score

Continuous exposures

- 1 Use a model like $\text{lm}(x \sim z)$ for the propensity score model
- 2 Scale weights to probability-like scale using `dnorm(true_value, fitted_value, estimated_sd)`
- 3 Apply the weights to the outcome model as normal!

Alternative: quantile binning

- 1** Bin the continuous exposure into quantiles and use categorical regression like a multinomial model to calculate probabilities.
- 2** Calculate the weights where the propensity score is the probability you fall into the quantile you

1. Fit a model for exposure ~ confounders

```
model <- lm(  
  exposure ~ confounder_1 + confounder_2,  
  data = df  
)
```

2. Calculate the weights with `dnorm()`

```
model %>%  
  augment(data = df) %>%  
  mutate(denominator = dnorm(  
    exposure,  
    mean = .fitted,  
    sd = mean(.sigma, na.rm = TRUE)  
  ))
```

Does change in smoking intensity (smkintensity82_71) affect weight gain among lighter smokers?

```
nhefs_light_smokers <- nhfs_complete %>%  
  filter(smokeintensity <= 25)
```


1. Fit a model for exposure ~ confounders

```
nhefs_denominator_model <- lm(  
  smkintensity82_71 ~ sex + race + age + I(age^2) +  
    education + smokeintensity + I(smokeintensity^2) +  
    smokeyrs + I(smokeyrs^2) + exercise + active +  
    wt71 + I(wt71^2),  
  data = nhefs_light_smokers  
)
```

2. Calculate the weights with dnorm()

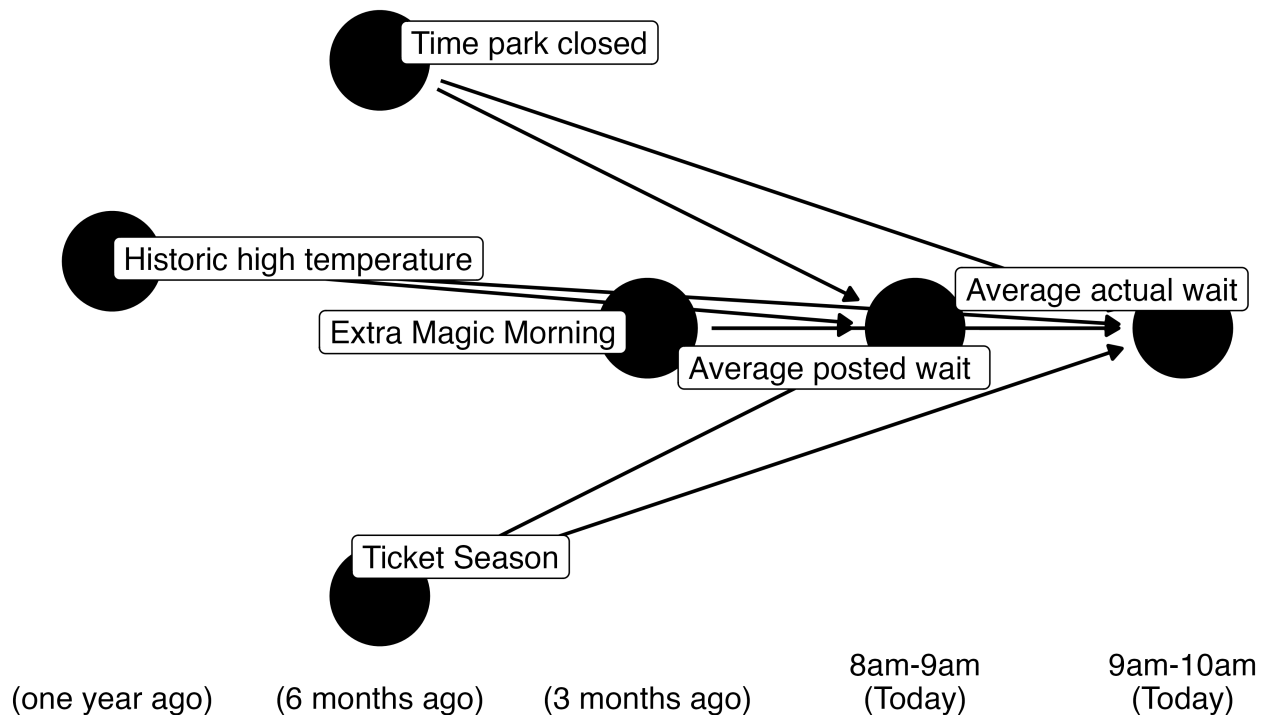
```
nhefs_denominators <- nhefs_denominator_model %>%  
  augment(data = nhefs_light_smokers) %>%  
  mutate(denominator = dnorm(  
    smkintensity82_71,  
    .fitted,  
    mean(.sigma, na.rm = TRUE)  
  )) %>%  
  select(id, denominator)
```

2. Calculate the weights with dnorm()

```
nhefs_denominators
```

```
## # A tibble: 1,162 × 2
##   id denominator
##   <int>         <dbl>
## 1     2      0.0265
## 2     3      0.0275
## 3     4      0.0314
## 4     5      0.0371
## 5     6      0.0262
## 6     7      0.0364
## 7     8      0.0381
## 8     9      0.0386
## 9    10      0.0129
## 10   13      0.0386
## # ... with 1,152 more rows
```

Do **posted** wait times at 8 am affect **actual** wait times at 9 am?



Your Turn 1

Fit a model using `lm()` with `avg_spostmin` as the outcome and the confounders identified in the DAG.

Use `augment()` to add model predictions to the data frame

In `dnorm()`, use `.fitted` as the mean and the mean of `.sigma` as the SD to calculate the propensity score for the denominator.

05:00

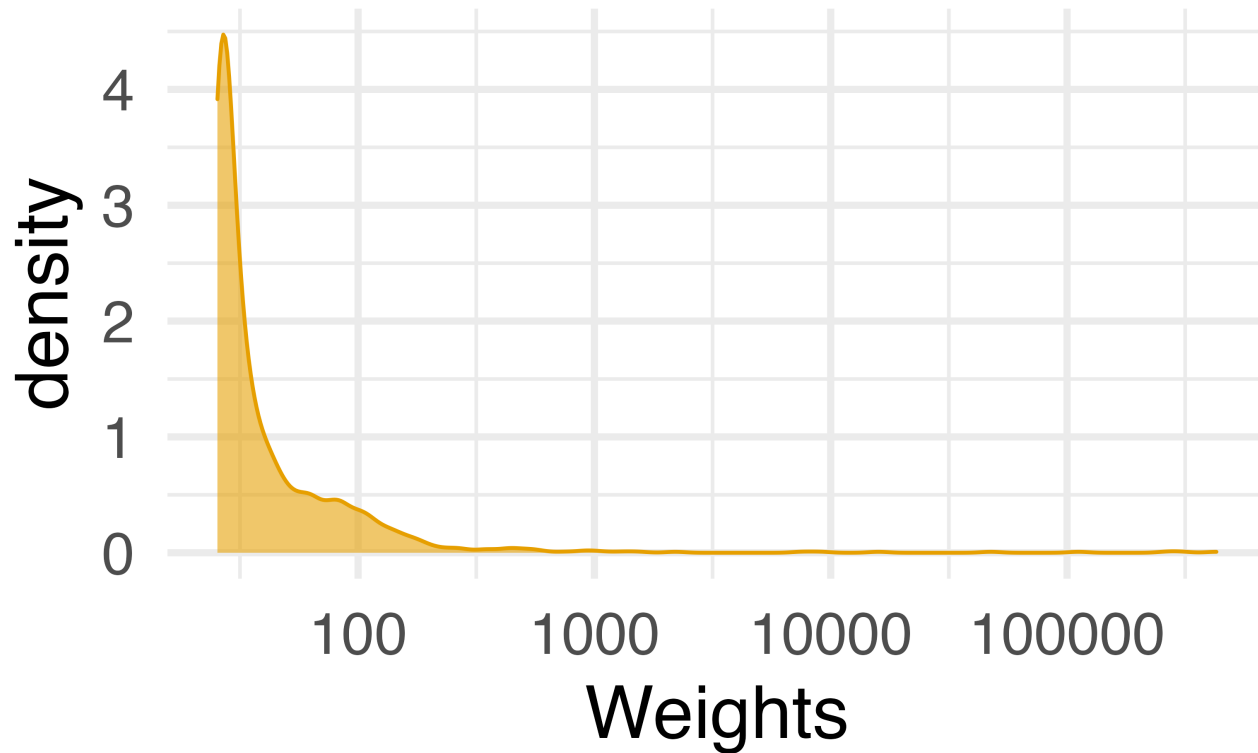
Your Turn 1

```
denominator_model <- lm(  
  avg_spostmin ~  
    close + extra_magic_morning +  
    weather_wdwhigh + wdw_ticket_season,  
  data = wait_times  
)
```

Your Turn 1

```
denominators <- denominator_model %>%  
  augment(data = wait_times) %>%  
  mutate(  
    denominator = dnorm(  
      avg_spostmin, .fitted, mean(.sigma, na.rm = TRUE)  
    )  
  ) %>%  
  select(date, denominator)
```

Stabilizing extreme weights



Stabilizing extreme weights

- 1 Fit an intercept-only model (e.g. $\text{lm}(x \sim 1)$)
- 2 Calculate weights from this model
- 3 Divide these weights by the propensity score weights

Fit an intercept-only model

```
nhefs_numerator_model <- lm(  
  smkintensity82_71 ~ 1,  
  data = nhefs_light_smokers  
)
```

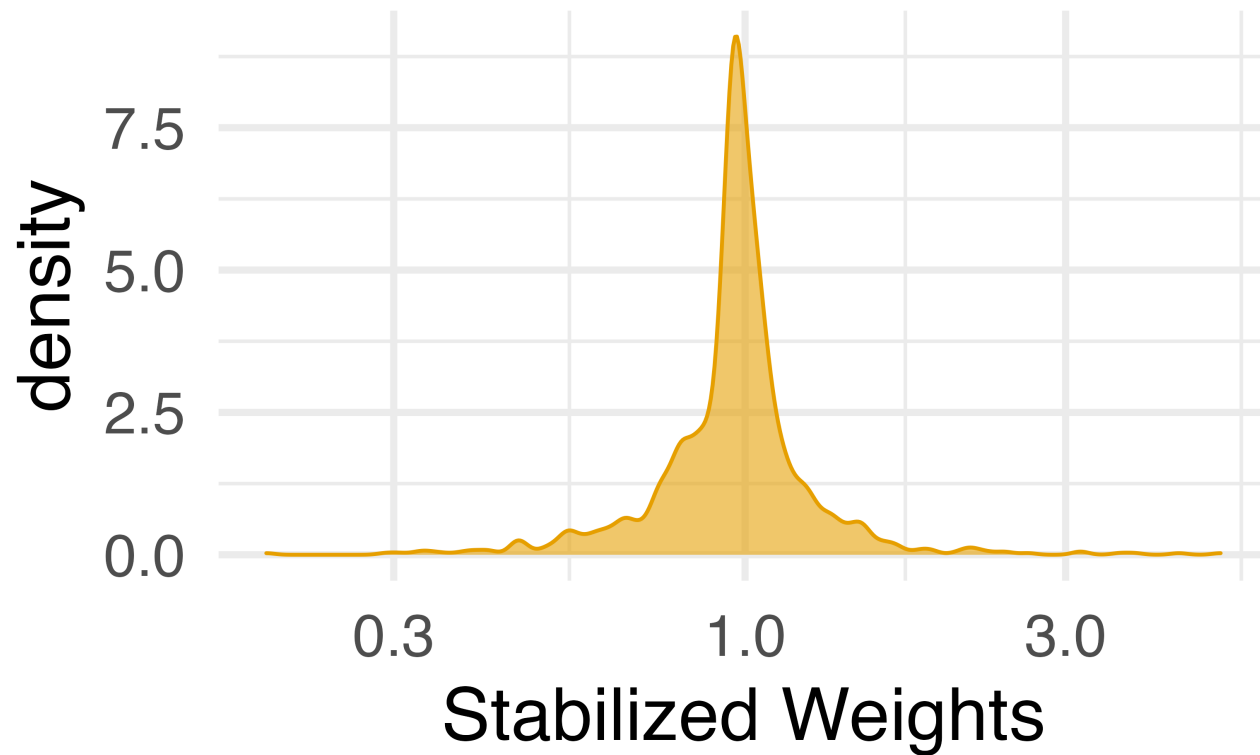
Calculate weights from this model

```
nhefs_numerators <- nhefs_numerator_model %>%  
  augment(data = nhefs_light_smokers) %>%  
  mutate(numerator = dnorm(  
    smkintensity82_71,  
    mean = .fitted,  
    sd = mean(.sigma, na.rm = TRUE))  
  ) %>%  
  select(id, numerator)
```

Divide these weights by the propensity score weights

```
nhefs_light_smokers <- dhefs_light_smokers %>%  
  left_join(nhefs_numerators, by = "id") %>%  
  left_join(nhefs_denominators, by = "id") %>%  
  mutate(swts = numerator / denominator)
```

Stabilizing extreme weights



Your Turn 2

Fit an intercept-only model of posted weight times to use as the numerator model

Calculate the numerator weights using `dnorm()` as above.

Finally, calculate the stabilized weights, `swts`, using the numerator and denominator weights

05:00

Your Turn 2

```
numerator_model <- lm(  
  avg_spostmin ~ 1,  
  data = wait_times  
)
```

Your Turn 2

```
numerators <- numerator_model %>%  
  augment(data = wait_times) %>%  
  mutate(  
    numerator = dnorm(  
      avg_spostmin, .fitted, mean(.sigma, na.rm = TRUE)  
    )  
  ) %>%  
  select(date, numerator)  
  
wait_times_wts <- wait_times %>%  
  left_join(numerators, by = "date") %>%  
  left_join(denominators, by = "date") %>%  
  mutate(swts = numerator / denominator)
```


Fitting the outcome model

- 1 Use the stabilized weights in the outcome model. Nothing new here!

```
lm(
  wt82_71 ~ smkintensity82_71,
  weights = swts,
  data = nhefs_light_smokers
) %>%
  tidy() %>%
  filter(term == "smkintensity82_71") %>%
  mutate(estimate = estimate * -10)
```

```
## # A tibble: 1 × 5
```

	term	estimate	std.error	statistic	p.value
	<chr>	<dbl>	<dbl>	<dbl>	<dbl>
1	smkintensity82_71	0.960	0.0210	-4.58	0.00000519

Your Turn 3

Estimate the relationship between posted wait times and actual wait times using the stabilized weights we just created.

03:00

Your Turn 3

```
lm(
  avg_sactmin ~ avg_spostmin,
  weights = swts,
  data = wait_times_wts
) %>%
  tidy() %>%
  filter(term == "avg_spostmin") %>%
  mutate(estimate = estimate * 10)
```

```
## # A tibble: 1 × 5
##   term          estimate std.error statistic p.value
##   <chr>          <dbl>    <dbl>    <dbl>    <dbl>
## 1 avg_spostmin    -2.63    0.0807    -3.26  0.00162
```