

# **Assessing Disparate Impacts of Personalized Interventions: Identifiability and Bounds**

## “What works for whom”

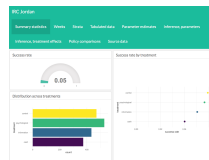
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**Interventions and impact evaluation enact “data-driven decision-making” in the public sector and social services.**

- Some criticism of fair ml: suggesting interventions over predictions, e.g. Barabas et al. [2017]
- Nonetheless, interventions under scarcity will be triaged based on predictions
- “True labels” are **not observed in interventional data** because of the “fundamental problem of causal inference”
- Interest in personalizing interventions based on predictions dating back to Behncke et al. [2007]: randomized caseworkers to predictions of job training program effects

## Related work: Interventions and Fairness

- Methodology for predicting “what works”:  
CATE estimation:  
Shalit et al. [2017], Wager and Athey [2017]
- Personalized allocations:  
Kube and Das [2019]  
homelessness services
- Quinn et al. [2019]: MAB trial for job training to refugees in Jordan



## Personalized interventions

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Data (RCT or observational),  $(X, A, T, Y)$ , on individuals:

- Prognostic features  $X$  (for personalization)
- Sensitive attribute  $A$
- Binary treatment indicator  $T \in \{0, 1\}$
- Binary response outcome  $Y(0), Y(1) \in \{0, 1\}$   
(benefit to the individual)
- Fundamental problem of causal inference:  
we only observe  $Y(T)$
- Decision rules,  $Z(X, A) \in \{0, 1\}$

# Personalized Allocations

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**Conditional average treatment effect**  $\tau$

$$\begin{aligned}\tau &= \tau(X, A) = \mathbb{E}[Y(1) - Y(0) \mid X, A] \\ &= \mathbb{P}(Y = 1 \mid T = 1, X, A) - \mathbb{P}(Y = 1 \mid T = 0, X, A),\end{aligned}$$

We assess disparities induced by  $Z$ , e.g. thresholds on  $\tau$ ,

$$Z = \mathbb{I}[\tau < \theta]$$

## True positive rate: responders

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Enumerate all possible potential outcomes:

- $Y(0) < Y(1)$ : responders
- $Y(0) > Y(1)$ : anti-responders
- $Y(0) = Y(1)$ : non-responders.

True positive rate for interventions:

of those *who would actually benefit from treatment*,

how many were allocated treatment under  $Z$ ?

$$\text{TPR}_a = \mathbb{P}(Z = 1 \mid A = a, Y(1) > Y(0)),$$

$$\text{TNR}_a = \mathbb{P}(Z = 0 \mid A = a, Y(1) \leq Y(0)).$$

# Fairness for Personalized interventions

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Denote

$$p_{yy'}(x) : \mathbb{P}(Y(0) = y, Y(1) = y' \mid X = x)$$

We want to estimate  $p_{01} = \mathbb{P}(Y(1) > Y(0) \mid X)$ ,  
but crucially we can only estimate:

$$\mathbb{P}(Y \mid T = 1, X) = p_{11} + p_{01}$$

$$\mathbb{P}(Y \mid T = 0, X) = p_{10} + p_{11}$$

**Proposition 1:** TPR, TNR are **not identifiable** from the data.

# Fairness for Personalized interventions

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## Identification Assumption 1: Treatment monotonicity.

$$Y(1) \geq Y(0)$$

i.e.,  $p_{10} = 0$ : Job training cannot *hurt* one's ability to get a job.

**Proposition 2:** (Identification under monotonicity).

$$\begin{aligned} \text{TPR}_a &= \frac{\mathbb{E}[\tau \mid A = a, Z = 1] \mathbb{P}(Z = 1 \mid A = a)}{\mathbb{E}[\tau \mid A = a]}, \\ \text{TNR}_a &= \frac{\mathbb{E}[(1 - \tau) \mid A = a, Z = 0] \mathbb{P}(Z = 0 \mid A = a)}{\mathbb{E}[(1 - \tau) \mid A = a]}. \end{aligned}$$



# Sensitivity Analysis

**Assumption 2:**  $B$ -relaxed monotone treatment response:

$$p_{10} \leq B$$

Anti-responder probability is uniformly less than  $B$ .

$$\rho_a^{\text{TPR}}(\eta) := \frac{\mathbb{E} \left[ \tau + \eta \mid A = a, Z = 1 \right] \mathbb{P}(Z = 1 \mid A = a)}{\mathbb{E} \left[ \tau + \eta \mid A = a \right]}$$

Given an uncertainty set  $\mathcal{U}$  for  $p_{10}$ , we define the simultaneous identification region of the TPR and TNR for all groups  $a \in \mathcal{A}$  as:

$$\Theta = \left\{ \left( \rho_a^{\text{TPR}}(\eta), \rho_a^{\text{TNR}}(\eta) \right)_{a \in \mathcal{A}} : \eta \in \mathcal{U} \right\} \subseteq \mathbb{R}^{2 \times |\mathcal{A}|}.$$

**Proposition 4:** Support function access to the convex hull of the identified set.

Let  $r_a^z := \mathbb{P}(Z = z \mid A = a)$  and  $\tau_a^z := \mathbb{E}[\tau \mid A = a, Z = z]$ . For sets  $\mathcal{U}$  which are product sets over groups,

$$h_{\Theta}(\mu) := \sup_{\rho \in \Theta} \mu^{\top} \rho, \quad (1)$$

Eq. (1) can be reformulated as:

$$\begin{aligned} h_{\Theta}(\mu) &= \sum_{a \in \mathcal{A}} h_{\Theta_a}(\mu_a) \\ h_{\Theta_a}(\mu_a) &= \sup_{\omega_a, t_a} \begin{aligned} &\mu_a^{\text{TPR}} r_a^1 (t_a \tau_a^1 + \mathbb{E}[\omega_a(X) \mid \frac{A=a}{Z=1}]) + \\ &\frac{\mu_a^{\text{TPR}} r_a^0}{t_a - 1} (t_a (1 - \tau_a^0) + \mathbb{E}[\omega_a(X) \mid \frac{A=a}{Z=0}]) \end{aligned} \\ &\text{s.t. } \omega_a(\cdot) \in t_a \mathcal{U}_a, \quad t_a (r_a^0 \tau_a^0 + r_a^1 \tau_a^1) + \mathbb{E}[\omega_a \mid A = a] = 1. \end{aligned}$$

## Assessment on clinical data

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**Proposition 5 (informal):** Under Asn. 2, the sharp identification intervals for  $\text{TPR}_a$  and  $\text{TNR}_a$  (each) are closed-form intervals, respectively. Moreover, these intervals are tight;

$$\begin{aligned}(\underline{\rho}_a^{\text{TPR}}(B), \underline{\rho}_a^{\text{TNR}}(B)) &\in \Theta_{B,a} \\ (\bar{\rho}_a^{\text{TPR}}(B), \bar{\rho}_a^{\text{TNR}}(B)) &\in \Theta_{B,a},\end{aligned}$$

i.e., the two extremes are simultaneously achievable.

## Job training case study

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- Behaghel et al. [2014]: French job training program;  $n = 11k$ .  
Train  $\tau$  using generalized random forest [Wager and Athey, 2017].
- $T = 1$  is assignment to public job training program;  
 $T = 0$  is control.  
ATE = 0.02 (significant)
- $Y = 1$  is employment at 6 months,  $A = 1$  is age  $> 26$ .

# Job training case study

