Assessing Disparate Impacts of Personalized Interventions: Identifiability and Bounds

#### "What works for whom"

# Interventions and impact evaluation enact "data-driven decision-making" in the public sector and social services.

- Some criticism of fair ml: suggesting interventions over predictions, e.g. Barabas et al. [2017]
- Nonetheless, interventions under scarcity will be triaged based on predictions
- "True labels" are not observed in interventional data because of the "fundamental problem of causal inference"
- Interest in personalizing interventions based on predictions dating back to Behncke et al. [2007]: randomized caseworkers to predictions of job training program effects

#### Related work: Interventions and Fairness

- Methodology for predicting "what works":
   CATE estimation:
   Shalit et al. [2017], Wager and Athey [2017]
- Personalized allocations:
   Kube and Das [2019]
   homelessness services
- Quinn et al. [2019]: MAB trial for job training to refugees in Jordan



#### Personalized interventions

Data (RCT or observational), (X, A, T, Y), on individuals:

- Prognostic features X (for personalization)
- Sensitive attribute A
- ullet Binary treatment indicator  $\mathcal{T} \in \{0,1\}$
- Binary response outcome  $Y(0), Y(1) \in \{0, 1\}$  (benefit to the individual)
- Fundamental problem of causal inference: we only observe Y(T)
- Decision rules,  $Z(X, A) \in \{0, 1\}$

#### Personalized Allocations

#### Conditional average treatment effect $\tau$

$$au = au(X, A) = \mathbb{E}[Y(1) - Y(0) \mid X, A]$$
  
=  $\mathbb{P}(Y = 1 \mid T = 1, X, A) - \mathbb{P}(Y = 1 \mid T = 0, X, A),$ 

We assess disparities induced by Z, e.g. thresholds on  $\tau$ ,

$$Z = \mathbb{I}[\tau < \theta]$$

### True positive rate: responders

Enumerate all possible potential outcomes:

- Y(0) < Y(1): responders
- Y(0) > Y(1): anti-responders
- Y(0) = Y(1): non-responders.

True positive rate for interventions: of those *who would actually benefit from treatment*, how many were allocated treatment under *Z*?

TPR<sub>a</sub> = 
$$\mathbb{P}(Z = 1 \mid A = a, Y(1) > Y(0)),$$
  
TNR<sub>a</sub> =  $\mathbb{P}(Z = 0 \mid A = a, Y(1) \le Y(0)).$ 

#### Fairness for Personalized interventions

Denote

$$p_{yy'}(x) : \mathbb{P}(Y(0) = y, Y(1) = y' \mid X = x)$$

We want to estimate  $p_{01} = \mathbb{P}(Y(1) > Y(0) \mid X)$ , but crucially we can only estimate:

$$\mathbb{P}(Y \mid T = 1, X) = p_{11} + p_{01}$$
  
 $\mathbb{P}(Y \mid T = 0, X) = p_{10} + p_{11}$ 

**Proposition 1:** TPR, TNR are **not identifiable** from the data.

#### Fairness for Personalized interventions

#### Identification Assumption 1: Treatment monotonicity.

$$Y(1) \geq Y(0)$$

i.e.,  $p_{10} = 0$ : Job training cannot *hurt* one's ability to get a job.

**Proposition 2**: (Identification under monotonicity).

$$\begin{aligned} \mathsf{TPR}_{a} &= \frac{\mathbb{E}\left[\tau \mid A=a, Z=1\right] \mathbb{P}\left(Z=1 \mid A=a\right)}{\mathbb{E}\left[\tau \mid A=a\right]}, \\ \mathsf{TNR}_{a} &= \frac{\mathbb{E}\left[\left(1-\tau\right) \mid A=a, Z=0\right] \mathbb{P}\left(Z=0 \mid A=a\right)}{\mathbb{E}\left[\left(1-\tau\right) \mid A=a\right]} \end{aligned}$$

# Sensitivity Analysis

**Assumption 2:** *B*-relaxed monotone treatment response:

$$p_{10} \leq B$$

Anti-responder probability is uniformly less than B.

$$\rho_{\mathsf{a}}^{\mathsf{TPR}}(\mid \eta \mid) \coloneqq \frac{\mathbb{E}\left[\tau + \mid \eta \mid \mid A = \mathsf{a}, Z = 1\right] \mathbb{P}\left(Z = 1 \mid A = \mathsf{a}\right)}{\mathbb{E}\left[\tau + \mid \eta \mid \mid A = \mathsf{a}\right]}$$

Given an uncertainty set  $\mathcal{U}$  for  $p_{10}$ , we define the simultaneous identification region of the TPR and TNR for all groups  $a \in \mathcal{A}$  as:

$$\Theta = \{ \left( \rho_{\mathsf{a}}^{\mathsf{TPR}}(\eta), \rho_{\mathsf{a}}^{\mathsf{TNR}}(\eta) \right)_{\mathsf{a} \in A} \ : \ \eta \in \mathcal{U} \} \subseteq \mathbb{R}^{2 \times |\mathcal{A}|}.$$

**Proposition 4:** Support function access to the convex hull of the identified set.

Let  $r_a^z := \mathbb{P}(Z = z \mid A = a)$  and  $\tau_a^z := \mathbb{E}[\tau \mid A = a, Z = z]$ . For sets  $\mathcal{U}$  which are product sets over groups,

$$h_{\Theta}(\mu) := \sup_{\rho \in \Theta} \mu^{\top} \rho, \tag{1}$$

Eq. (1) can be reformulated as:

$$\begin{split} h_{\Theta}(\mu) &= \sum_{a \in \mathcal{A}} h_{\Theta_a}(\mu_a) \\ h_{\Theta_a}(\mu_a) &= \sup_{\omega_a, t_a} \begin{array}{c} \mu_a^{\mathsf{TPR}} r_a^1 \ \left( t_a \tau_a^1 + \mathbb{E} \left[ \omega_a(X) \mid \frac{A=a}{Z=1} \right] \right) + \\ \frac{\mu_a^{\mathsf{TNR}} r_a^0}{t_a - 1} \left( t_a \ \left( 1 - \tau_a^0 \right) + \mathbb{E} \left[ \omega_a(X) \mid \frac{A=a}{Z=0} \right] \right) \\ \mathrm{s.t.} \quad \omega_a(\cdot) \in t_a \ \mathcal{U}_a, \quad t_a \left( r_a^0 \tau_a^0 + r_a^1 \tau_a^1 \right) + \mathbb{E} \left[ \omega_a \mid A = a \right] = 1. \end{split}$$

#### Assessment on clinical data

**Proposition 5 (informal)**: Under Asn. 2, the sharp identification intervals for  $TPR_a$  and  $TNR_a$  (each) are closed-form intervals, respectively. Moreover, these intervals are tight;

$$(\underline{\rho}_{a}^{\mathsf{TPR}}(B), \underline{\rho}_{a}^{\mathsf{TNR}}(B)) \in \Theta_{B,a}$$

$$(\overline{\rho}_{a}^{\mathsf{TPR}}(B), \overline{\rho}_{a}^{\mathsf{TNR}}(B)) \in \Theta_{B,a},$$

i.e., the two extremes are simultaneously achievable.

# Job training case study

- Behaghel et al. [2014]: French job training program; n = 11k. Train  $\tau$  using generalized random forest [Wager and Athey, 2017].
- T=1 is assignment to public job training program; T=0 is control. ATE = 0.02 (significant)
- Y = 1 is employment at 6 months, A = 1 is age > 26.

# Job training case study





