Mar. 10, 2025 (Due: 08:00 Mar. 17, 2025)

1. Interpolate the rational function $f(x) = (1 + 25x^2)^{-1}$ over $[-1, 1]$
(a) using polynomials with a few equispaced nodes;
(b) using polynomials with a few Chebyshev nodes;
(c) using Hermite spline with a few equispaced nodes.
Visualize the results and estimate the interpolation error.

2. Can a unique polynomial $p(x)$ of degree up to three always be found that satisfies the interpolation conditions $p(0)$, $p''(0)$, $p(1)$, and $p''(1)$? If so, construct the interpolating polynomial.

3. Similar to complete, natural, and periodic cubic splines, when the "not-a-knot" condition is used in cubic spline interpolation, the computational kernel is also to solve a sparse linear system. Try to derive the corresponding linear system.

4. When performing interpolation with a complete cubic spline, the choice of derivatives on the boundary is important. Suppose that Bob wants to interpolate the sine function $f(x) = \sin x$ at nine equispaced nodes over $[0, 2\pi]$, with $f'(0) = f'(2\pi) = 1$. Unfortunately, he made a typo on $f'(0)$ in his program and observed some strange results. Try to reproduce Bob's result with a few different values of $f'(0)$. For instance, $f'(0) = 0$, $f'(0) = -1$, etc.

5. The temperature in the human body is not a constant, but rather follows a daily rhythm driven by an internal biological clock. The following table lists 20 averaged values of temperature measurements taken from 70 English sailors in an experiment done in 1971.

Time (hour) t	Temperature (°C) $T(t)$
1	36.37
3	36.23
5	36.21
7	36.26
8	36.38
9	36.49
10	36.60
11	36.63
12	36.66
13	36.68
14	36.69
15	36.73
16	36.74
17	36.78
18	36.82
19	36.84
20	36.87
21	36.86
22	36.77
23	36.59

Interpolate the data with a periodic cubic spline and plot your solution for a two-day-period.

6. (optional) Launch an image processing program (e.g., `mspaint` on Windows). Open your left hand naturally, and put it on the computer screen. Then use the mouse to sketch the outline of your hand. (A few discrete points already suffice.) Reconstruct the outline of your hand with (piecewise) algebraic curves and visualize the result.