Mar. 31, 2025 (Due: 08:00 Apr. 7, 2025)

1. Let $f(x)$ be an even (odd) function on $[-a, a]$. Show that any minimax polynomial approximation to $f(x)$ is also an even (odd) function.

2. Show that in each iterate of the Remez algorithm, the linear system has a unique solution.

3. Let $p(\lambda)$ be a real polynomial such that $\deg p(\lambda) \leq n$ and

$$\max_{-1 \leq \lambda \leq 1} |p(\lambda)| \leq 1.$$

Show that $|p(\mu)| \leq |T_n(\mu)|$ for any $\mu \in \mathbb{R} \setminus [-1, 1]$.

4. Find a few low-degree Padé approximants of e^x around $x = 0$. Visualize the approximation error.

5. Find

$$\min_{a,b,c \in \mathbb{R}} \max_{-1 \leq x \leq 1} |\mathrm{e}^x - ax^2 - bx - c|$$

by
(1) the Remez algorithm;
(2) solving the system of nonlinear equations in terms of the alternating set.

(optional) Compare the L^∞ errors for several different quadratic approximations to e^x on $[-1, 1]$: truncated Maclaurin series, the least squares approximation by Legendre polynomials, the least squares approximation by Chebyshev polynomials, and the minimax polynomial approximation.

6. (optional) Design a polynomial interpolation-based algorithm to solve the linear system in each iterate of the Remez algorithm.

7. (optional) Try to find a good approximation of the form $x + x^3 p(x^2)$ to the sine function on $[0, \pi/2]$. Estimate the approximation error.