

Mar. 24, 2025 (Due: 08:00 Mar. 31, 2025)

1. Find

$$\min_{a,b,c \in \mathbb{R}} \int_0^{\pi/2} |\sin x - ax^2 - bx - c|^2 dx$$

without programming.

2. Find

$$\min_{a,b \in \mathbb{R}} \max_{-1 \leq x \leq 2} |x^3 + ax + b|$$

without programming. Give an elementary proof of the correctness of your solution.

3. Suppose that there is a bimodal function of the form

$$y = \alpha_1 g(\beta_1(x - \gamma_1)) + \alpha_2 g(\beta_2(x - \gamma_2)),$$

where $g(x) = \exp(-x^2)$. Can you find out the parameters from the following *noisy* data set sampled from this model?

x_i	y_i
-4.00000	0.00001
-3.50000	0.00726
-3.00000	0.25811
-2.50000	1.87629
-2.00000	1.55654
-1.50000	0.17209
-1.00000	0.00899
-0.50000	0.05511
0.00000	0.24564
0.50000	0.60455
1.00000	0.89370
1.50000	1.03315
2.00000	0.51633
2.50000	0.18032
3.00000	0.04287
3.50000	0.00360
4.00000	0.00045

(optional) Generate noisy data for high-dimensional models and try fitting the data using Gauss–Newton or Levenberg–Marquardt algorithms.

(optional) Try some data fitting software (e.g., gnuplot) and compare the result produced by the software with your own solution.

4. In the homework on March 10, you have been asked to interpolate temperature in human body using a cubic spline. In fact, interpolation is not a good idea because the data are noisy. Try to fit the data using a periodic cubic spline with equispaced nodes $\{1, 4, 7, \dots, 19, 22, 25\}$. Plot your solution for a two-day-period, and compare it with the interpolating spline.

5. Write a program to compute the roots of a few low-degree Legendre polynomials.

6. (optional) Use a few points to interpolate the bivariate function $z = \exp(-x^2 - y^2)$ over $[-1, 1] \times [-1, 1]$. Compare the bad approach using complex polynomial interpolation (see the homework on March 3) with other approaches discussed last week.

7. (optional) Hermite polynomials can be defined through one of the following ways:

(1) $H_n(x) = (-1)^n e^{x^2} (e^{-x^2})^{(n)}.$

(2) $H_0(x) = 1$, $H_1(x) = 2x$, and $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$ for $n \geq 1$.

(3) $\sum_{n=0}^{\infty} H_n(x) t^n / n! = e^{2xt - t^2}.$

Show that these definitions are equivalent.