

**Apr. 27, 2025 (Due: 08:00 May. 12, 2025)**

1. Show that the DFT matrix  $F_n$  diagonalizes the permutation matrix

$$J_n = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ 1 & & & & 0 \end{bmatrix},$$

i.e.,  $F_n^{-1}J_nF_n$  is diagonal.

2. For  $x \in \mathbb{C}^n$ , its Fourier transform  $X = F_n x$  is in general a complex vector. When is  $X$  real? Can you find a sufficient and necessary condition? What about 2-D FFT?

3. Implement the radix-2 FFT. A non-recursive implementation is preferred. Test your implementation with vectors of various lengths to make sure that the complexity is  $\Theta(n \log n)$ .

4. Use trigonometric polynomials up to degree  $d$  to interpolate the periodic square wave

$$f(x) = \begin{cases} 1, & x \in (2n, 2n+1) \\ -1, & x \in (2n-1, 2n) \\ 0, & x \text{ is an integer} \end{cases}$$

with equispaced interpolation nodes. Make plots for a few different values of  $d$ . What happens if  $d$  becomes large?

5. You are given an audio file `DTMF_dialing.ogg`, which contains 80 touch tones from a DTMF keyboard. Try to determine the keys corresponding to the tones according to the following table.

	1209 Hz	1336 Hz	1477 Hz	1633 Hz
697 Hz	1	2	3	A
770 Hz	4	5	6	B
852 Hz	7	8	9	C
941 Hz	*	0	#	D

6. (optional) Show that all eigenvalues of the unitary DFT matrix  $n^{-1/2}F_n$  belong to the set  $\{1, -1, i, -i\}$ .

7. (optional) Implement the radix-3 FFT. Make sure that your implementation has a complexity  $\Theta(n \log n)$ .

You may find the MATLAB/Octave function `fft` helpful for debugging purpose.