Experiment 4: Momentum and Impulse

Tian Ye

UID: 704931660

TA: Roli Esha

Lab Partner: Chris Ong

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Conservation of Momentum in a Glider Collision

T. Ye[1]

In Classical Physics, momentum is said to be conserved per the law of Conservation of Momentum. To test this principle, a glider with a photogate flag attached to the top of it was sent on an air track though a photogate so that it would collide with a force sensor. The photogate calculated the initial and final velocities of the glider while the force sensor measured the force readings over small time intervals. The impulse was then calculated using two distinct methods: by multiplying the change in velocity of the glider with the measured mass of the glider and by integrating the force applied on the glider with respect to time. The latter was accomplished via Riemann sum estimation. The experiment demonstrated that for the two methods the calculated value for the impulse exerted on the glider is indeed comparable

[1]Henry Samueli School of Engineering and Applied Sciences, Univ. California Los Angeles

1 Introduction

The premise of Experiment 4 is to test and observe conservation of momentum via colliding a glider with a photogate flag attached to the top of it with a force sensor. The glider passes through a photogate sensor once on the way towards the force sensor and once again after colliding with it, and the gate records the two different velocities of the glider.

We start by finding the impulse exerted on the glider in this experiment. The first method involves the usage of the two measured velocities and the measured mass of the glider to measure the impulse using the following equation:

$$\begin{aligned}\Delta p &= m\Delta v \\ \Delta p &= m(v_{\text{final}} - v_{\text{initial}})\end{aligned} \tag{1}$$

The second method involves using Riemann sums to solve for the integral of force with respect to time. The force is the force exerted on the force sensor by the glider, which by Newton's Third Law is also the force exerted on the glider by the force sensor. The force sensor then recorded the force exerted on it over very small time intervals, which can then be multiplied with each other to find impulse. Per Equation 4.3 in the Lab Manual, the impulse is as follows:

$$\begin{aligned}\Delta p &= \int_{t_1}^{t_n} F(t)\mathrm{d}t \\ \Delta p &\approx \Delta t \sum_{i=1}^{n} F(t_i)\end{aligned} \tag{2}$$

Word Count: 205

2 Methods

2.1 Calibration of Force Sensor

Before beginning any calculations and experimental testing, we first began by calibrating the force sensor. We began by massing various masses using a balancing scale, recording both their masses and their uncertainties. We then set up the force sensor by hanging it vertically so that the hook of the sensor was on the bottom and set the input into Analog Channel 1.

Entering the Capstone software, we set Analog Channel 1 as a User Defined Sensor and set up the display so that the recorded voltage would be output on screen. We then hung the various masses on the hook, making sure to tare the sensor following every reading to minimize systematic error.

By hanging various known masses on the force sensor and reading the various voltages output by the force sensor, we were consequently able to plot force against voltage, which is shown on the plot on the following page:

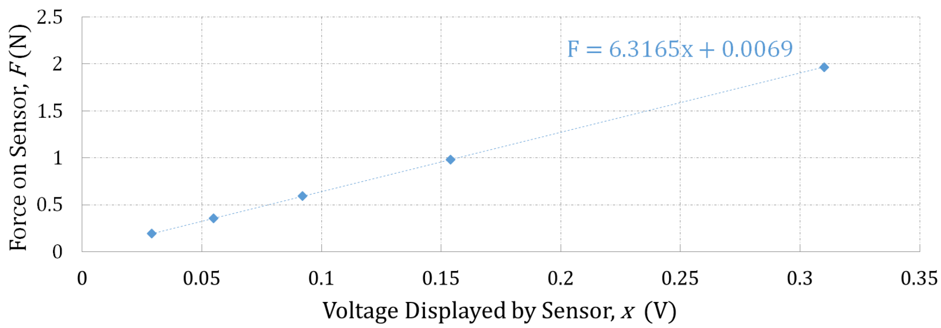

Figure 1: Calibration of force sensor. Each blue diamond represents the recorded voltage when a given mass is hung from the force sensor. The dotted blue line is a linear fit to the data, $F = \frac{N}{V}x + N$, with x being the recorded voltage, and having fit parameters of $\frac{N}{V} = (6.3165 \pm 0.0048)\ \frac{\mathrm{N}}{\mathrm{V}}$ and $N = (0.0067 \pm 0.0008)$ N. The uncertainty was provided by the Regression tool in Microsoft Excel. Note: the voltage recorded by the force sensor is positive for a compression force and negative for a tension force. Consequently, while the recorded voltages for the hanging masses were negative, for the purpose of this experiment we will want to use the compression force readings and therefore the positive value of $\frac{N}{V}$.

Thus, our calibration for the Force Sensor is as follows:

$$F = (6.3165 \pm 0.0048)V + (0.0067 \pm 0.0008) \tag{3}$$

2.2 Experiment Setup

We then set up the track with a single photogate that measures the velocity of the glider and a single force sensor to measure the applied force on the glider when it impacts it. When we run the experiment, we push the glider down the track where it passes the photogate, recording the initial velocity. The glider then impacts the force sensor which records the voltage readings and bounces back, passing the photogate a second time which then records final velocity.

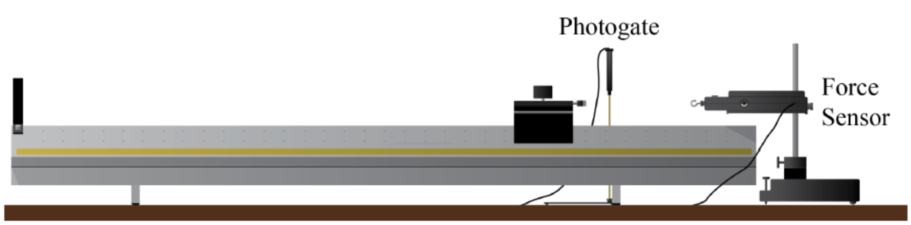

Figure 2: Visual representation of the glider, photogate, and force sensor system. Figure reproduced (with permission) from Fig. 4.1 by Campbell, W. C. *et al.*[1].

In order to set up the sensors and perform future calculations, we then measured the length of the flag on top of the glider and massed the glider itself, with the final values being (38 ± 0.5) mm and (202.5 ± 1.5) grams, respectively. To minimize systematic error, we measured the flag with various sections of the ruler, laying the flag on the ruler so that the tips of every increment would be visible over the flag. This helped in ensuring the flag was in line with the ruler.

Having plugged the photogate into Digital Channel 1, we enter Capstone again, this time selecting Timer Setup and using a Pre-Configured Timer with the setting of a photogate with a single flag. We set the photogate to measure the speed of the glider, and input the measured flag length, 0.038 m, into the software.

To record the data, we then set up a table within Capstone with three data columns: Time (s), User Defined (V), and Speed (m/s), and set the sample rate to 2.5 kHz and 4.0 kHz for our data runs. Making sure that the software was in Continuous Mode, we then hit Record and ran the experiment once for each person at each sample rate. We stopped each recording once the glider passed through the photogate a second time.

2.3 Systematic Errors

There are certain unavoidable systematic errors in this experiment, the most notable of which being the force of friction. As friction is present in all non-ideal systems, it will apply a force on the glider throughout the entirety of each experimental run, and consequently apply an impulse on the glider that is not measured. This can be accounted for by solving for the coefficient of kinetic friction between the glider and the track, and integrating the force of friction with respect to the time duration of the experimental run.

3 Data Analysis

3.1 Calculation of Impulse Using Equation 1

Referring back to Equation 1, we know that impulse can be calculated using $p = m\Delta v = m(v_f - v_i)$. Taking into account that our recorded velocities are in two different directions, we define the inital velocity as negative and final velocity as positive. Using the equation above, we find the following impulses:

Sample Rate (kHz)	v_i (m/s)	v_f (m/s)	Impulse (N·s)
2.5	-0.95	0.09	0.2106
4.0	-0.64	0.12	0.1539

Table 1: Mass of glider used in calculations, not shown, is 0.2025 kg. Note: the calculated values of impulse are incomplete as they lack uncertainty.

To solve for uncertainty of Δp, we then used Equation ii.14 from the Lab Manual to derive the following equation for uncertainty of Δp:

$$\delta p = \sqrt{(\Delta v \delta m)^2 + (m \delta \Delta v)^2} \tag{4}$$

While we have already have measured δm of the glider, we do not have a direct value for $\delta \Delta v$. However, we can define v as $v = \frac{x}{t}$, x being the length of the flag. Keeping mind that the fractional uncertainty of time is significantly lower than the fractional uncertainty of the flag length, we can define $\delta v = \frac{\delta x}{t}$.

Therefore, we find the following equation:

$$\delta p = \sqrt{\left(\Delta v \delta m\right)^2 + \left(m\sqrt{\delta v_f^2 + \delta v_i^2}\right)^2} \tag{5}$$

Inputting the various measured values into Equation 5, we find our final results for the impulses:

Sample Rate (kHz)	Δv (m/s)	Impulse (N·s)
2.5	1.04	0.211 ± 0.003
4.0	0.76	0.154 ± 0.002

Table 2: Mass of glider used in calculations, not shown, is (0.2025 ± 0.0005) kg. These are the final values for calculated impulse using Equation 1.

3.2 Calculation of Impulse Using Equation 2

Referring back to Equation 2, we know that impulse can be calculated by integrating force with respect to time. Using Equation 3, we convert the measured voltage to force and clean up our force data by subtracting the baseline background force from our data, presenting us with the two following plots:

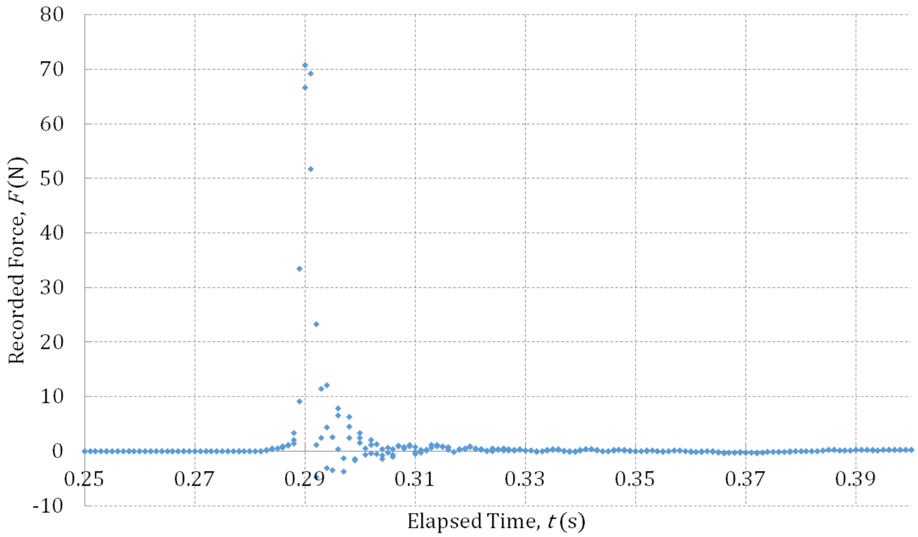

Figure 3: Impulse exerted by force sensor on glider at 2 kHz sample rate. Each blue diamond represents the calculated force at its respective point in time. Force was obtained via Equation 3.

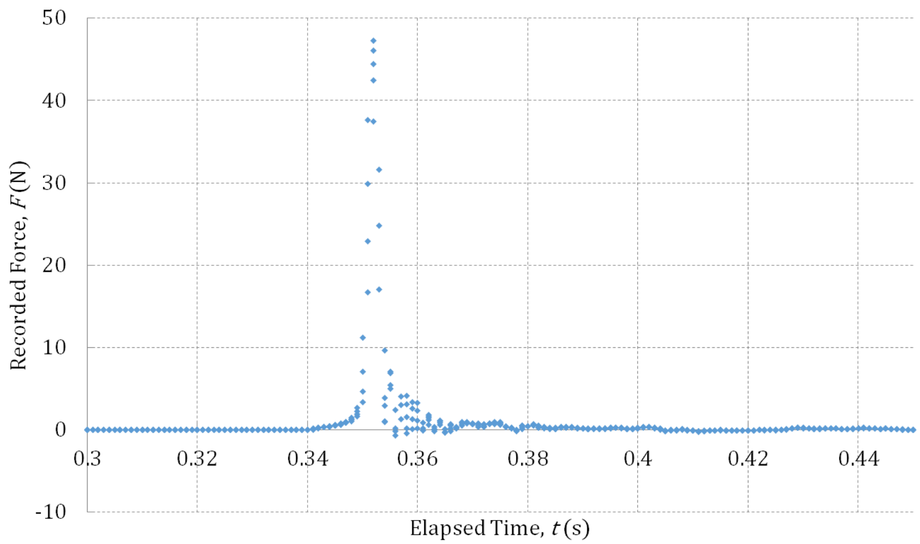

Figure 4: Impulse exerted by force sensor on glider at 4 kHz sample rate. Each blue diamond represents the calculated force at its respective point in time. Force was obtained via Equation 3.

Using the fact that $\int_{t_1}^{t_n} F(t)\mathrm{d}t \approx \Delta t \sum_{i=1}^{n} F(t_i)$, we can use Riemann sums to solve for the impulse exerted on the glider. Summing the data points within the "spike" of the plot and multiplying it by time step Δt, we are presented with the following values:

Sample Rate (kHz)	Δt (s)	Impulse (N·s)
2.5	0.00040	0.1930
4.0	0.00025	0.1486

Table 3: Calculated impulse using Riemann sums, per Equation 2. Note: the calculated values of impulse are incomplete as they lack uncertainty.

To solve for uncertainty of Δp, we use the fact that the ratio of the fractional uncertainty of the calibration constant is the same as the fractional uncertainty of the impulse, per the lab manual. Therefore, we can use the values from Equation 3 and the following equation to solve for δp:

$$\delta p = \frac{\Delta p(\delta x)}{x} \tag{6}$$

Where x and δx refer to $\frac{N}{V}$ and $\delta \frac{N}{V}$ from Equation 3, respectively. Using Equation 6, we can then solve for the δp, shown on the following page:

Sample Rate (kHz)	Δt (s)	Impulse (N·s)
2.5	0.00040	0.1930 ± 0.0001
4.0	0.00025	0.1486 ± 0.0001

Table 4: Calculated impulse using Riemann sums with uncertainties, per Equation 2 and Equation 6.

4 Conclusion

When we compare our final values for Δp using our two methods, we are presented with the following:

Sample Rate (kHz)	Equation 1 Impulse (N·s)	Equation 2 Impulse (N·s)
2.5	0.211 ± 0.003	0.1930 ± 0.0001
4.0	0.154 ± 0.002	0.1486 ± 0.0001

Table 5: Calculated impulse with their respective uncertainties using Equation 1 ($\Delta p = mv$) and Equation 2 ($\Delta p \approx \Delta t \sum_{i=1}^{n} F(t_i)$) for the two trials. Note how Equation 1 produces a larger magnitude of impulse.

Upon viewing the values, we see that the two methods produce comparable results, as expected. The fact that Equation 1 produces a larger magnitude of impulse is can be attributed to the force of friction. Over the course of the entire experiment, friction is exerting a force, and consequently, an impulse on the glider. Therefore, the impulse the force sensor exerts on the glider cannot be attributed to the entire change in momentum; Equation 2 does not account for change in momentum due to the impulse from the force of friction.

Therefore the fact that Equation 1 produces a greater magnitude of impulse than Equation 2 is logical: the impulse exerted by the force sensor cannot account for the entirety of the change in momentum of the glider.

5 Extra Credit

Via colliding two gliders and measuring their velocities, we find the following:

Bumper Type	Initial Speed of Gliders (1 | 2) (m/s)	Final Speed of Glider (1 | 2) (m/s)
White	0.71 | 0.68	0.16 | 0.02
Black	0.47 | 0.39	0.08 | 0.03
None	0.64 | 0.64	0.01 | 0.04
C_r of White	C_r of Black	C_r of None
0.13	0.13	0.04

Table 6: Coefficient of restitution, C_r, for multiple runs of the glider. The initial and final velocities of all the combinations of the gliders are displayed above.

As the values of C_r are all rather close to zero, we can conclude that the collision between the gliders is relatively inelastic. As expected, the collision with no bumpers is the most inelastic, and the collision with bumpers are relatively elastic in comparison. Having found C_r, we can then solve for ΔK:

Bumper Type	K_0 (N·m)	K_f (N·m)	ΔK (N·m)	% Loss of K
White	0.0976	0.0026	-0.0950	97.3%
Black	0.0377	0.0007	-0.0370	98.1%
None	0.0827	0.0002	-0.0825	99.8%

Table 7: Kinetic energy for multiple runs of the glider. Mass of Glider 1 and 2, not shown, are 0.2025 kg and 0.2015 kg, respectively. For the test involving removal of bumpers, 0.0055 kg of mass was removed from each glider.

It is easily visible from Table 7 that the majority of kinetic energy is lost in any form of collision, with only a fraction of the energy remaining in the system regardless of bumper type. The majority of the energy is most likely converted to vibration and sound energy, with some lost to heat and friction as well. We can now also solve for Δp of the gliders.

Bumper Type	p_0 (N·s)	p_f (N·s)	Δp (N·s)	% Loss of p
White	0.2808	0.0364	-0.2444	87.0%
Black	0.1738	0.0222	-0.1516	87.2%
None	0.2586	0.0101	-0.2485	96.1%

Table 8: Momentum for multiple runs of the glider. Mass of Glider 1 and 2, not shown, are 0.2025 kg and 0.2015 kg, respectively. For the test involving removal of bumpers, 0.0055 kg of mass was removed from each glider.

Table 8 demonstrates the lack of conservation of momentum between the two gliders, indicating that momentum is transferred to elsewhere in the system - even potentially to the Earth itself.

References

[1] Campbell, W. C. *et al.* Physics 4AL: Mechanics Lab Manual (ver. April 3, 2017). (Univ. California Los Angeles, Los Angeles, California).