Experiment 1: Uniform Acceleration

Tian Ye

UID: 704931660

TA: Wen Li Wen

Lab Partner: Matthew Barba

Lab 8 Tuesday 6:00 PM

October 10th, 2017

1 Introduction

Experiment 1 centers about finding the magnitude of acceleration as gravity acts on a mass that is strung over a pulley, which in turn pulls a glider that is suspended over a nearly frictionless track.

1.1 Solving for Acceleration

To solve for acceleration of the glider, we first create free body diagrams for both the glider and the mass. The diagrams for both are displayed below:

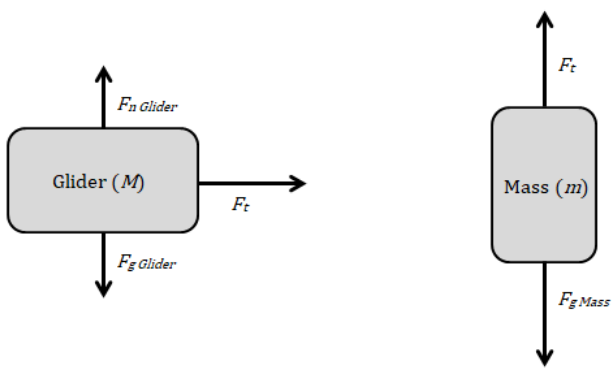

Figure 1: Free body diagrams for both the glider and the mass. Note that while steps were taken to minimize friction in the experiment, it is still present nonetheless. However, it was omitted from the diagram since its effect is negligible in this case.

From the diagram we can see that ΣF on the mass is the difference between the force of gravity, $F_{\text{g Mass}}$ which is given by mg, and the tension force, F_{t}. Similarly, for the glider we can see that ΣF is just the tension force F_{t} since $F_{\text{g Glider}}$ and $F_{\text{n Glider}}$ cancel each other out as they are equal in magnitude and opposite in direction. Using the known relationship that $\Sigma F = ma$, we can find the following:

$$\begin{aligned} F_{\text{t}} &= Ma \\ ma &= mg - F_{\text{t}} \\ &= mg - Ma \\ a &= \frac{mg}{m+M} \end{aligned} \tag{1}$$

With m and M referring to the masses of the mass and glider, respectively.

1.2 Solving for Uncertainty of Acceleration

Next, we need to find δa, using Equation ii.14 from the lab manual. Taking the partial derivative of a in terms of both m and M, we find the following:

$$\begin{aligned} \delta a_{\mathrm{m}} &= \frac{\partial a}{\partial m}\delta m \\ &= \frac{Mg}{(m+M)^2}\delta m \end{aligned} \qquad\qquad \begin{aligned} \delta a_{\mathrm{M}} &= \frac{\partial a}{\partial M}\delta M \\ &= -\frac{mg}{(m+M)^2}\delta M \end{aligned}$$

From this, we can derive the total uncertainty of a from using again Equation ii.14:

$$\begin{aligned} \delta a &= \sqrt{\delta a^2{}_{\mathrm{m}} + \delta a^2{}_{\mathrm{M}}} \\ &= \sqrt{\left(\frac{Mg}{(m+M)^2}\delta m\right)^2 + \left(-\frac{mg}{(m+M)^2}\delta M\right)^2} \\ &= \frac{g}{(m+M)^2}\sqrt{(M\delta m)^2 + (-m\delta M)^2} \end{aligned} \tag{2}$$

2 Data Analysis

There are two methods to find acceleration: predicted acceleration via equations and measured acceleration via the collected data points.

2.1 Predicted Acceleration

To find our predicted acceleration, we will use Equation 1, derived earlier, to find our acceleration using the measured values for mass of the glider and various hanging weights.

Mass of Weight	Uncertainty
2.5 grams	$\pm$ 0.5 grams
5.5 grams	$\pm$ 0.5 grams
10 grams	$\pm$ 1 gram
19.5 grams	$\pm$ 1.5 grams
22 grams	$\pm$ 2 grams
Mass of Glider	Uncertainty
224 grams	$\pm$ 2 grams

Table 1: The masses and their corresponding uncertainties that were used for this experiment. Mass was determined via balancing scale.

Plugging our values into Equation 1 and Equation 2 for acceleration and uncertainty of acceleration, respectively, we are presented with the following results:

Mass of Hanging Weight (kg)	Predicted Acceleration (m/s^2)
0.0025 $\pm$ 0.0005	0.108 $\pm$ 0.021
0.0055 $\pm$ 0.0005	0.235 $\pm$ 0.021
0.010 $\pm$ 0.001	0.419 $\pm$ 0.040
0.0195 $\pm$ 0.0015	0.785 $\pm$ 0.056
0.022 $\pm$ 0.002	0.876 $\pm$ 0.073

Table 2: Measured mass of weights in kilograms used in calculations with their respective uncertainties and their respective predicted acclerations with their respective uncertainties. Values were obtained by using Equation 1 and Equation 2

2.2 Measured Acceleration

To find our measured acceleration, we will use the data points collected earlier in the lab.

2.2.1 Lab Setup

The data was collected by rotating a "smart pulley" that recorded the time at which each full revolution occurred. The pulley is rotated by a string that connects the mass to the glider, as shown in the figure below:

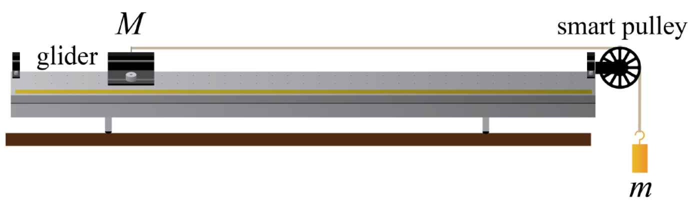

Figure 2: Visual representation of the glider, pulley, and mass system. Figure reproduced (with permission) from Fig. 1.1 by Campbell, W. C. *et al.*[1].

2.2.2 Data Collection

When the mass is dropped, the string that is pulling the glider also rotated the smart pulley, which then recorded two sets of data: the "blocks", or number of revolutions made by the pulley, and the time at which each block was measured.

2.2.3 Calculation and Analysis

In order to solve for velocity, and subsequently, acceleration from this data set, we had to first use the given constant $\kappa = (1.50 \pm 0.05)$ cm/block to solve for the displacement of the glider by multiplying the block number by κ . We ignore $\delta\kappa$ as it is statistical uncertainty, and consequently is reflected in the spread of the data points. Afterwards, to solve for velocity between each time interval, the following equation was used:

$$\bar{v} = \frac{x_{\mathrm{i}+1} - x_{\mathrm{i}}}{t_{\mathrm{i}+1} - t_{\mathrm{i}}}$$

Where x_{i} refers to displacement obtained by multiplying block number by κ, and t_{i} refers to change in time interval between each block measurement.

To solve for time interval, we averaged the time interval between each value, which also in turn gave us the same number of time data points as velocity:

$$\bar{t} = \frac{t_{\mathrm{i}+1} + t_{\mathrm{i}}}{2}$$

By plotting $\bar{v}$ against $\bar{t}$ in Microsoft Excel, we were able to find the approximate acceleration of the glider as $a = \frac{\Delta v}{\Delta t}$

Therefore the slope of the best fit lines generated by Microsoft Excel will be an estimate of the true acceleration of the glider for the various masses.

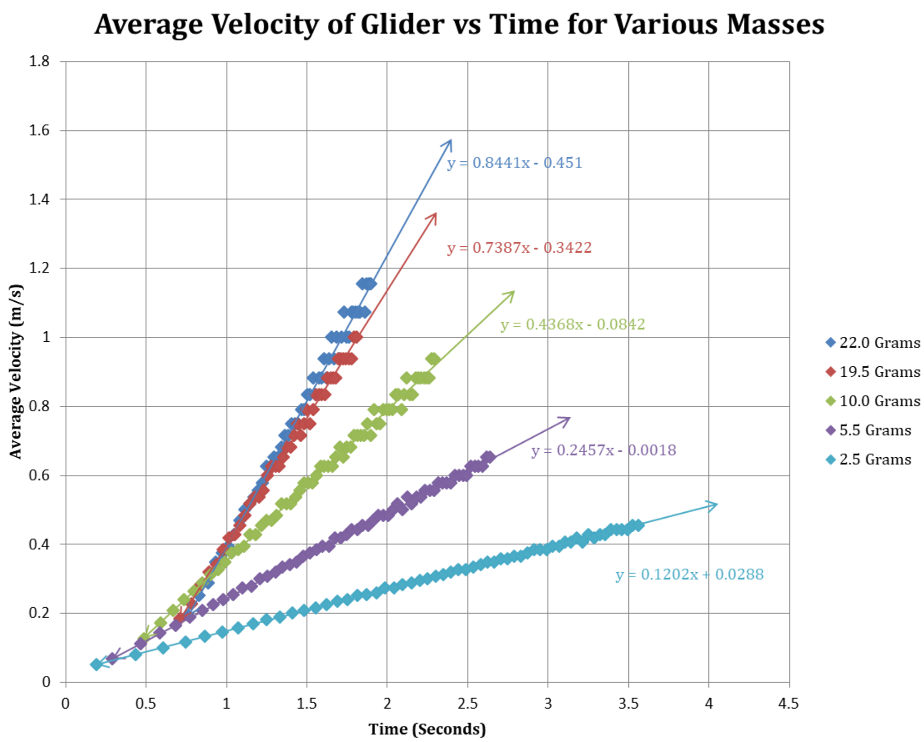

Figure 3: Velocity vs Time graph of the glider for various masses. Each mass has its own color for its linear fit line, equation, and data points. The same glider was used for all tests. The slope of these lines represents the measured acceleration.

To solve for uncertainty of each slope, we then use Microsoft Excel's data analysis function; the results of which for each mass are recorded in the table below:

Mass of Hanging Weight (kg)	Measured Acceleration (m/s^2)
0.0025 ± 0.0005	0.1202 ± 0.0004
0.0055 ± 0.0005	0.246 ± 0.001
0.010 ± 0.001	0.437 ± 0.004
0.0195 ± 0.0015	0.739 ± 0.007
0.022 ± 0.002	0.84 ± 0.01

Table 3: Measured mass of weights in kilograms used in Excel with their respective uncertainties and their respective measured acclerations with their respective uncertainties. The same glider was used for all tests. Values were obtained by using the Regression tool in Microsoft Excel

3 Conclusion

The goal of Experiment 1 was to find several different accelerations of a glider as it was pulled along a frictionless track by a mass that is connected to the glider and hanging over a pulley. The results for the experiment were reached via two different methods: via calculating acceleration via equations and free body diagrams or by solving for measured velocity and time and plotting them against one another in order to obtain the slope. A table listing all the results is shown below:

Mass of Hanging Weight (kg)	Predicted Acceleration (m/s^2)	Measured Acceleration (m/s^2)
0.0025 ± 0.0005	0.108 ± 0.021	0.1202 ± 0.0004
0.0055 ± 0.0005	0.235 ± 0.021	0.246 ± 0.001
0.010 ± 0.001	0.419 ± 0.040	0.437 ± 0.004
0.0195 ± 0.0015	0.785 ± 0.056	0.739 ± 0.007
0.022 ± 0.002	0.876 ± 0.073	0.84 ± 0.01

Table 4: Combined data for Table 2 and Table 3

The reason for the discrepancy between the predicted acceleration and measured acceleration can be attributed to various sources of error, the most notable of which being friction. Neither air resistance nor kinetic friction were accounted for in the free body diagrams and the calculations associated with them.
Another source of error that most likely explains why the predicted acceleration is lower in magnitude than the measured acceleration for certain data values is that the track is not perfectly level. Any object of substantial length bends downward slightly under its own weight. As the track was not positioned nearly high enough above the ground so that the mass would accelerate the glider for the entire length, but rather for only approximately 3/4 of the length, the glider would have been traveling primarily down the part of the track that curves slightly downwards, causing the glider to also be accelerated by a component of gravity as well.
Finally, the fact that neither the string nor the pulley are ideal could also have led some error in the data measurement, such as the larger spread of data as the velocity increased over time.

4 Extra Credit

For this particular analysis, the 2.5 gram test data was used as it was the most linear of all the data.

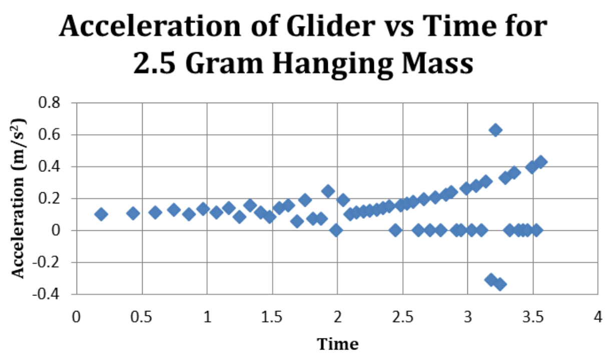

Figure 4: Acceleration vs Time graph of the glider for 2.5 gram mass.

4.1 Analysis

A quick glance at the graph of acceleration plotted against time shows that while the data begins relatively constant, it quickly begins to fluctuate significantly around the 2 second mark, the fluctuation increasing drastically as it approaches the end of the recorded data.

There is a small fluctuation initially, which is to be expected, and the acceleration is relatively constant since as shown in Equation 1, the acceleration is dependent on nonvariable constants g, m, and M. However, at around the two second mark, the acceleration begins to deviate. This is due to the manner in which the data is recorded.

The ”smart pulley” records data as number of revolutions completed and the point of time at which said revolution occurred. Thus, as velocity increases, the time frame between each ”block” decreases rapidly, and thus even small deviations and inconsistencies are magnified significantly when the derivative of the positional data is taken twice. By taking $\frac{\Delta x}{\Delta t}$ and subsequently $\frac{\Delta v}{\Delta t}$, we are in essence using Reimann sums to calculate acceleration.

One possible source of inconsistency is the fact that air resistance is a frictional force proportional to velocity. Consequently, as velocity increases, the force of air resistance against the acceleration due to tension in the string can be thought of as a form of oscillation, with the force of air resistance slowing the glider down until the force of tension is greater, whereby the glider is then accelerated forward again.

4.2 Conclusion

When the acceleration data is averaged, we find that $\bar{a} = 0.12$ m/s^2 and that the sample standard deviation is 0.15, with a standard error of $\pm$ 0.02. Comparing these values to the measured acceleration, we find that the while both values for acceleration are similar, the standard error of this method, $\pm$ 0.02, is significantly greater than the standard error of the measured acceleration, $\pm$ 0.0004. Therefore, it can be concluded that generally, it is more accurate to calculate acceleration via fitting a linear fit line to a velocity vs time graph than by differentiating the data a second time.

5 Bibliography

1. Campbell, W. C. et al. Physics 4AL: Mechanics Lab Manual (ver. April 3, 2017). (Univ. California Los Angeles, Los Angeles, California).

Experiment 1: Mini Report

Tian Ye

UID: 704931660

TA: Wen Li Wen

Lab 8 Tuesday 6:00 PM

October 10th, 2017

Introduction

While it is not completely evident upon first glance, modern day accelerometers are ubiquitous. From being found in motion-sensing games, HDD protection inside laptops, airbag deployment,[1] to more niche fields such as avionics,[2] animal studies,[3] and health monitoring,[4,5] the ubiquity of accelerometers in many of today's technologies serves as a primary driving point for its development.

Mechanics of Accelerometers

While modern accelerometers have evolved greatly in comparison to their older cousins thanks to the advancement in the microelectromechanical systems (MEMS),[2,4] the core premise of the modern accelerometer remains the same. Accelerometers operate by measuring linear acceleration of an object along its axes, typically the dimensions of x, y, and z.[1]

One type of the modern day accelerometer, the piezoresistive type, operates via the measurement of elongation or compression of a series of stress beams connected to a mass. Pizoresistors are placed at the end of the stress beams, where the maximum stress regions are located. As the mass moves, the beams compress, causing the resistance of the pizoresistors to change.[2] From this point it is a matter of conversion of a voltage output to acceleration, a task similar to what was undertaken in Experiment 0.

While the text above only refers to a single type of modern accelerometers, the common trend of miniaturization and integration in technology can be found across nearly all types of accelerometers as well. Thanks to the advancement of modern material sciences, accelerometers have become relatively easy to fabricate and extremely miniaturized, even to the point of being embedded into fabric as an e-textile system.[2,4] Advances in the field of material science also indicate the future possibility of printing a full circuit board on fabric.[1]

Another feature of accelerometers is that different accelerometers measure acceleration with different precision corresponding to the application of each accelerometer. In avionics, accelerometers are able to measure change in acceleration of $\pm$ 10g, with a maximum acceleration being withstood being $\pm$ 50g,[2] while in other fields regarding acceleration of a much smaller magnitude, such as that of an animal, the accuracy being of the magnitude of $\pm$ 0.06g.[3] The versatility of the precision of accelerometers allows it to be employed in a variety of fields for many different purposes - a key reason for its pervasiveness.

With increased miniaturization of accelerometers, the application of it, while still being very much relevant in its traditional fields of avionics and automobiles, has begun shifting towards the field of monitoring as well. The field that accelerometer usage is currently shifting towards is in the field of health monitoring for both children[5] and those with health disabilities and/or need of rehabilitation.[4]

Application

One of the primary obstacles facing American society today in regards to children is the high obesity rate, and accelerometers of the triaxial nature are well suited to measuring activity that does not necessarily contain movement in the vertical plane, as do traditional accelerometers. This allows the measurement of physical activity besides those that involve movement in the vertical plane, such as cycling and swimming.[5]

In the field of healthcare, accelerometers are able to measure a particular patient's health data and relay the information to hospitals, families, and clinicians via the usage of accelerometers integrated into clothing. This in turn allows caregivers to implement intervention measures as need be, and can potentially in the future lead to remote monitoring of patients' health.[4]

From these applications it is possible to extrapolate future applications of accelerometers as they become more and more miniaturized and sensitive, capable of being integrated into everyday clothing and measuring the slightest of change in body systems and reporting it to emergency personnel, if need be.[4]

Conclusion

The application of accelerometers, however, need not be isolated to only their current fields in the future. The fact that it is nearly universal now only serves as a statement to their future ubiquity, and consequently, the importance of their development for the future.

648

References

[1] Gerhard, D. Three Degrees of Gs: How an Airbag Deployment Sensor Transformed Video Games, Exercise, and Dance. *M/C Journal*, 16:6, 2013.

[2] Sharma, A., Mukhiya, R., Kumar, S. S., Pant, B. D. Design and Simulation of Bulk Micromachined Accelerometer for Avionics Application. Arya College of Engineering and IT, CSIR-Central Electronics Engineering Research Institute (CEERI). Available online at https://link.springer.com/content/pdf/10.1007%2F978-3-642-42024-5_12.pdf (2013).

[3] Shepard, E. L. C. *et al*. Identification of Animal Movement Patterns Using Tri-Axial Accelerometry. Preprint at http://www.int-res.com/articles/esr2008/theme/Tracking/TMVpp1.pdf (2008).

[4] Patel, S., Park, H., Bonato, P., Chan, L., Rodgers, M. A Review of Wearable Sensors and Systems with Application in Rehabilitation. *Journal of NeuroEngineering and Rehabilitation* https://doi.org/10.1186/1743-0003-9-21 (2011).

[5] Robertson, W. *et al*. Utility of Accelerometers to Measure Physical Activity in Children Attending an Obesity Treatment Intervention. *Journal of Obesity* http://dx.doi.org/10.1155/2011/398918 (2010).