Nelson Siegel Model (1987)

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Nelson Siegel model is a mathematical model which is used to define the **shape of yield curve**

1 Purpose of NS model

Fit the Yield Curve: NS model helps to accurately model the yield curve across different maturities. It helps to find the yield on maturities which is not in released by goverment.

2 Mathematical Formula NS Model

$$S_m = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda m}}{\lambda m} \right) + \beta_2 \left(\frac{1 - e^{-\lambda m}}{\lambda m} - e^{-\lambda m} \right); \tag{1}$$

where:

- S_m = Spot rate at maturity m (the spot rate observed in the market);
- $\beta_0, \beta_1, \beta_2, \lambda$ are the parameters of the model which we calibrate using market data;
- β_0 = the long term (maturities) of the yield curve;
- β_1 = the short term component (maturities) of the yield curve (the steepness of the yield curve at the start of the road);
- β_2 = the curvature of the yield curve, it captures the hump in the middle of the curve;
- λ = decay factor (it measures how quickly the slope and the curvature disappears as you move along the yield curve).

3 How does β_1 affects the short term maturity?

$$\text{Term}_1 = \beta_1 \left(\frac{1 - e^{-\lambda m}}{\lambda m} \right),$$

if m is very high $\Rightarrow$ Term_1 goes to $+\infty$, so β_1 affects only the short term maturity.

4 How does β_2 affects the short term maturity?

$$\text{Term}_2 = \beta_1 \left(\frac{1 - e^{-\lambda m}}{\lambda m} - e^{-\lambda m} \right),$$

- When m is really small ($m \simeq 0$):

  $$\frac{1 - e^{-\lambda m}}{\lambda m},$$

  let $x = \lambda m$, using Taylor series $\Rightarrow e^x \simeq 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ...$, we can take a look only at $e^x = 1 - x \Rightarrow x = 1 - e^{-\lambda x} \Rightarrow \lambda m = 1 - e^{-\lambda m} \Rightarrow \frac{1-e^{-\lambda m}}{\lambda m} = 1$. In the same way

  $$e^{-\lambda m} = \frac{1}{e^{\lambda m}}$$

  when $m \simeq 0 \Rightarrow \frac{1}{e^{\lambda m}} = \frac{1}{1} = 1$. Finally when m is small (short term maturity) $\Rightarrow$ Term $= 0$

- When m is really high $\Rightarrow \frac{1-e^{-\lambda m}}{\lambda m} = 0$ and $\frac{1}{e^{\lambda m}} = 0 \Rightarrow \text{Term}_2 = 0$.

5 Advantage of NS model

It helps of fit the yield curve, if so many data point are missing, than NS model help fill in more data point. It can be used to model different shape of yield curve.

Interpretability: easy to comprehend and explain.

6 Calibration of NS model

In the market we can observe:

maturity (m)	Annualized yield	Modeled yield (MY)	Error	Sqare Error
1 month	4.0%	4.15%	0.15%	0.0225%
2 month	4.1%	4.2%	0.1%	0.1 %
...	...	...	...	...
30 year	10%	12.0%	2.0%	4.0%
				Sum
				...

We have to calculate $\beta_0, \beta_1, \beta_2, \lambda$ minimizing the sum of the square error, this method is called **OLS**,

- Error between actual yield and the model yield;
- Square the error;
- Sum the square error;
- Run optimization in such a way that sum of square error minimizes by changing $\beta_0, \beta_1, \beta_2, \lambda$.