Nelson Siegel Svensson Model (1994)

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NSS model is used to model the yield curve. NSS Model is an extension of Nelson Siegel (NS) Model.

1 Why was NSS Model Developed

NS Model could only capture **single** 'hump' in the yield curve, it was failing when we had more than one 'hump' (or 'dip') to model. NSS could capture as more than one 'humps', it struggled to capture **complex shape** of the yield curve (during economic stress, unconventional monetary policy, Covid19 period,...).

2 Mathematical framework

NSS formula is similar to NS+Term$_3$:

$$s_m = \beta_0+\beta_1\left(\frac{1-e^{-\lambda_1 m}}{\lambda_1 m}\right)+\beta_2\left(\frac{1-e^{-\lambda_1 m}}{\lambda_1 m}-e^{-\lambda_1 m}\right)+\beta_3\left(\frac{1-e^{-\lambda_2 m}}{\lambda_2 m}-e^{-\lambda_2 m}\right); \tag{1}$$

where $\beta_0, \beta_1, \beta_2$ represent the same things as the NS model, the difference is β_3. β_3 captures a second hump or dip in the yield curve. So, the first hump is captures by β_2 and the second hump is captures by β_3. In the same way as NS Model, λ_1, λ_2 represent the decay factors:

- λ_1 = control how quickly the curvature effect (β_2) will fade along the yield curve;
- λ_2 = control how quickly the second curvature effect (β_3) will fade along the yield curve;

finally, we have:

- β_0 = long term level;
- β_1 = slope (steepness of short term maturity);
- β_2 = first curvature;
- β_3 = second curvature;

3 Limitation of NSS Model

- **Overfitting**: in the NSS Model we include β_2 and λ_2 $\Rightarrow$ increase the risk of overfitting;
- **Parameter estimation**: in the NSS Model we have six parameters $\Rightarrow$ high computational demanding.

Feature	**NS Model**	**Nss Model**
Parameter	4 ($\beta_0, \beta_1, \beta_2, \lambda_1$)	6 ($\beta_0, \beta_1, \beta_2, \lambda_1, \lambda_2$)
Flexibility	Single hump or dip	can capture more complex shape
Use case	Typical yield curve	stressed market

Table 1: Caption

4 Calibration NSS Model with Market data using OLS

maturity (m)	Annualized yield	Modeled yield (NSS)	Error	Square Error
1 month	4.0%	4.15%	0.15%	0.0225%
2 month	4.1%	4.2%	0.1%	0.1 %
...	...	...	...	...
30 year	10%	12.0%	2.0%	4.0%
				Sum
				...

We have to calculate $\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2$ minimizing the sum of the square error, this method is called **OLS**,

- Error between actual yield and the model yield;
- Square the error;
- Sum the square error;
- Run optimization in such a way that sum of square error minimizes by changing $\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2$.