Vasicek Interest Rate Model (1977)

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1 Introduction

Vasicek Interest Rate Model is a stochastic interest rate model which tells how interest rates will change over time. Instead of assuming interest rate will follow a clear straight line, the model assumes that the interest rate can go up and down unpredictably.

What is the need of this model? It helps people understand or make better guess about the future interest rate (it can be used in Pricing bond, Pricing Interest rate derivative or in Risk Management)

2 Formula for Vasicek interest rate model

The Vasicek model can be described mathematically with the following equation,

$$dr_t = a(b - r_t)dt + \sigma dW_t,$$

where,

- a = speed of mean reversion (how fast the interest rate will return to the average);
- b = long term average to which the interest rate return;
- r_t = interest rate at time t;
- dt = the small change in time;
- $a(b - r_t)dt$ = expected change in interest rate (deterministic part);
- σ = volatility;
- dW_t = Wiener process (Brownian motion), it is used to represent the unpredictable fluctuations in the interest rate over time,
- dr_t = change in interest rate ($dr_t = r_t - r_{t-1}$).

3 Strength and limitations

3.1 Strength

1. **Forecast the future interest rate**: the concept of mean reversion aligns with how people think about interest rate returning to normal after periods of highs and low;
2. **Analytical Solution**: it's possible to derive close form solution for the Vasicek model.

3.2 Limitation

1. **Single Factor Model**:the Vasicek model depends only on the volatility of the market and is the only factor that affects the interest rate;
2. **Negative Interest Rate**: Vasicek model produces negative interest rate which is unrealistic (not always!);
3. **Constant volatility**: The model assumes constant volatility which doesn't reflect how real interest rate behave.

3.3 Vasicek Interest Rate Model is a short rate model

The short rate is a interest rate for a short period of time, typically over a span of day or overnight (the yield curve is on months or years).

4 Maximum Likelihood Estimator (MLE)

MLE is a method which is used to find the best fitting parameter for a mathematical model (based on observed data). MLE helps us understand which parameter makes the data we observed most likely (a, b, σ). MLE will try to estimate the value of a, b, σ in such a way that the interest rate coming out of Vasicek Model align with the market data (observed)

4.1 How to find the parameters of Vasicek Interest Rate Model using MLE

Calibrating the Vasicek model using MLE involves estimating the model parameter specifically (a, b, σ) from historical interest rate. The goal of MLE is to find the parameters that maximize the likelihood of observing the historical data given the Vasicek Model.

We need to follow the following steps,

1. **Discretize the Vasicek Model**: it means that we first need to convert the continuous time model into a discrete time approximation,

$$dr_t = a(b - r_t)dt + \sigma dW_t \Rightarrow \Delta r_t = a(b - r_t)\Delta t + \sigma\sqrt{\Delta t}\ \varepsilon_t,$$

where Δt is the time increment and ε_t is the normal random variable (it is our source of randomness);

2. **Likelihood function**: The difference between $(dr_t = r_t - r_{t-1})$ follows a normal distribution $\mathcal{N}(a(b - r_t)\Delta t, \sigma^2 \Delta t)$, we basically get the difference $A = r_t - r_{t-1}$ from the market and the model difference $V = \overline{r_t} - \overline{r_{t-1}}$ and we take $(A - M)$ that is normal distributed;

3. **Maximizing the likelihood**: We need to maximize $\ln(\mathcal{N}(A - M))$.