

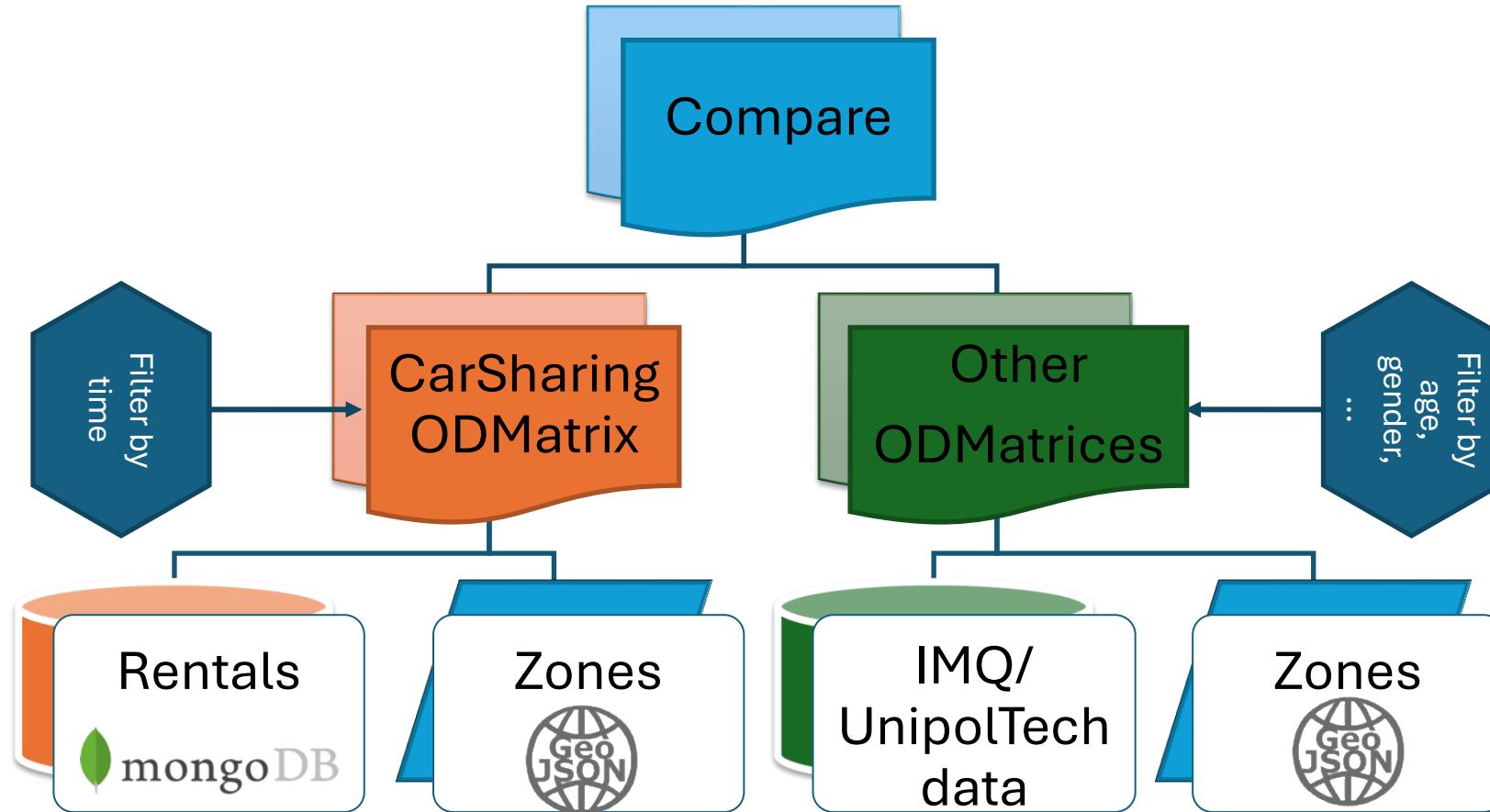
# LAB 2

# OD Matrices analytics

# Goal

- This laboratory builds on the data analyzed during the first lab
- We want to find out which are the most likely class of users of car sharing system
- For this, we compare the OD matrices obtained from different sources
- Use metadata to filter
  - Based on gender, age, goals of trip, etc.
- And obtain the most similar OD matrix as the one from car sharing users

# Overall view



# Datasets

# 3 Datasets

- Car Sharing rentals to obtain OD matrix of car sharing users
  - Car rentals for Enjoy and Car2Go
  - 2 months of data, with Origin/Destination (indexed for running geospatial queries)
  - Stored on MongoDB
- Open data from IMQ 2013 (Indagine Mobilità e Qualità)
  - <https://mtm.torino.it/it/dati-statistiche/indagini/matrici-od-imq-2013/>
- Data from UnipolTech collected in 2024
- **DO NOT SHARE carsharing and UnipolTech data**

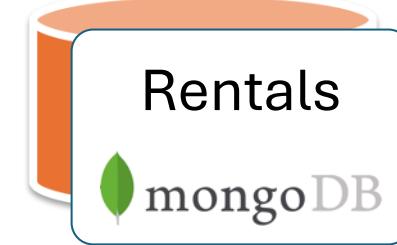


# Carsharing dataset

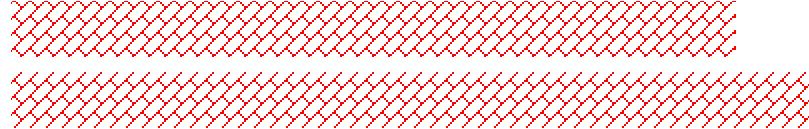
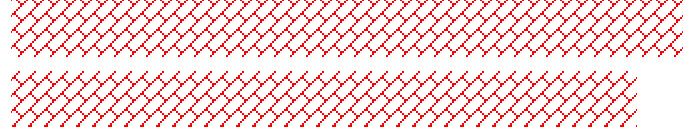
- Rentals are available on MongoDB
  - **ictts\_PermanentBookings**
  - **ictts\_enjoy\_PermanentBookings**
- There are 3 indexes
  - `init_loc` and `final_loc` with 2D coordinates to support geospatial queries
  - `init_time` for filtering over time

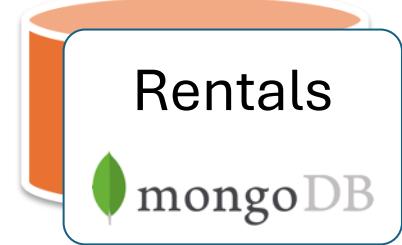
# Extracting data from rentals

- We can filter
  - By time
  - By position
- Useful **time operators** in \$aggregate pipeline
  - `$hours` – extract the hour in UTC from a ISODate
    - E.g., filter on rentals between midnight and midday
  - `$dayOfWeek` – Returns the day of the week for a date as a number
    - 1 (Sunday) and 7 (Saturday)
    - E.g., filter on rentals on Monday, Tuesday,..., Friday only



# Example

```
pb_coll.aggregate([
  { '$project': {
    
    'init_loc': 1,
    'final_loc': 1
  }},
  { '$match': {
    
  }},
  { '$count': 'tot'
])
```



*Transform the init\_date  
to get the hour and day*

*Filter on morning  
and weekdays (1=Sunday, ...)*

*Then count*

# Example

```
pb_coll.aggregate([
    { '$project': {
        'hour': { '$hour': '$init_date' },
        'day': { '$dayOfWeek': '$init_date' },
        'init_loc': 1,
        'final_loc': 1
    }},
    { '$match': {
        'hour': { '$gte': 0, '$lt': 12 },
        'day': { '$gte': 2, '$lt': 7 },
    }},
    { '$count': 'tot'}
])
```



*Transform the init\_date  
to get the hour and day*

*Filter on morning  
and weekdays (1=Sunday, ...)*

*Then count*

# Geospatial queries

- MongoDB's geospatial indexing allows you to efficiently execute spatial queries on a collection that contains geospatial shapes and points
- The geometry data in the location field must follow the **GeoJSON format**

```
<field>: { type: <GeoJSON type> , coordinates: <coordinates> }
```

- type can be
  - Point: {type:"Point", coordinates: [40,5]}
  - Linestring: {type:"LineString", coordinates: [ [40,5], [41,6] ] }
  - Polygon: {type:"Polygon", coordinates: [ [ [0,0], [3,6], [6,1], [0,0] ] ] }
  - ...

# Geospatial queries

- MongoDB support geopatial indexes that allow efficient queries
- A `2dsphere` index supports queries that calculate geometries on an earth-like sphere

## Possible queries

- `$geoWithin`: query for location data found within a GeoJSON polygon
  - Location data must be stored in GeoJSON format

```
<collection>.find( { <location field>.coordinates :  
  { '$geoWithin' :  
    { '$geometry' :  
      { 'type' : 'Polygon',  
        'coordinates' : [ <coordinates> ]  
    } } } })
```

# Geospatial queries

Other possible queries:

- `$geoIntersects` to select all indexed points and shapes that intersect with the polygon defined by the coordinates array
- `$near` operator or `$geoNear` return the points closest to the defined point and sorts the results by distance

```
<collection>.find( { <location field> :  
  { '$near' :  
    { '$geometry' :  
      { 'type' : 'Point' ,  
        'coordinates' : [ <longitude> , <latitude> ]  
      } ,  
      '$maxDistance' : <distance in meters>  
    } } } )
```

# Example of geospatial query

- Find all cars available in a region of 1000m from Politecnico di Torino entrance

```
pb_coll.find({  
    'init_loc': {  
        '$near': {  
            '$geometry': {  
                'type': 'Point',  
                'coordinates': [7.660, 45.0645]  
            },  
            '$maxDistance': 1000,  
        }  
    }  
})
```

Warning  
Cost of geoqueries is not negligible

# Extracting data from rentals



- Useful **geospatial operators** in \$aggregate pipeline
  - \$geoWithin: data that exists entirely within a specified shape documents with geospatial
  - The shape can be
    - GeoJSON Polygon
    - GeoJSON MultiPolygon (Polygon with “holes”)
  - The \$geoWithin operator uses the \$geometry operator to specify the GeoJSON object

```
res = pb_coll.aggregate([
  {'$match': {
    'init_loc': {
      '$geoWithin': {
        '$geometry': {
          'type': 'Polygon',
          'coordinates': [...]
        }
      }
    }
  }
})])
```

# Extracting the O/D matrix

```

orig_zone = ... # put here the arrays describing the zone
dest_zone = ... # put here the arrays describing the zone
pb_coll.aggregate([
    { '$match': {
        'init_loc': { '$geoWithin' :
            { '$geometry': {
                'type': 'MultiPolygon',
                'coordinates': orig_zone
            }
        }
    },
    'final_loc': { '$geoWithin' :
        { '$geometry': {
            'type': 'MultiPolygon',
            'coordinates': dest_zone
        }
    }
},
{ '$count': 'tot'
}
])

```



*Filter  
on init\_loc that falls  
within the orig\_zone*

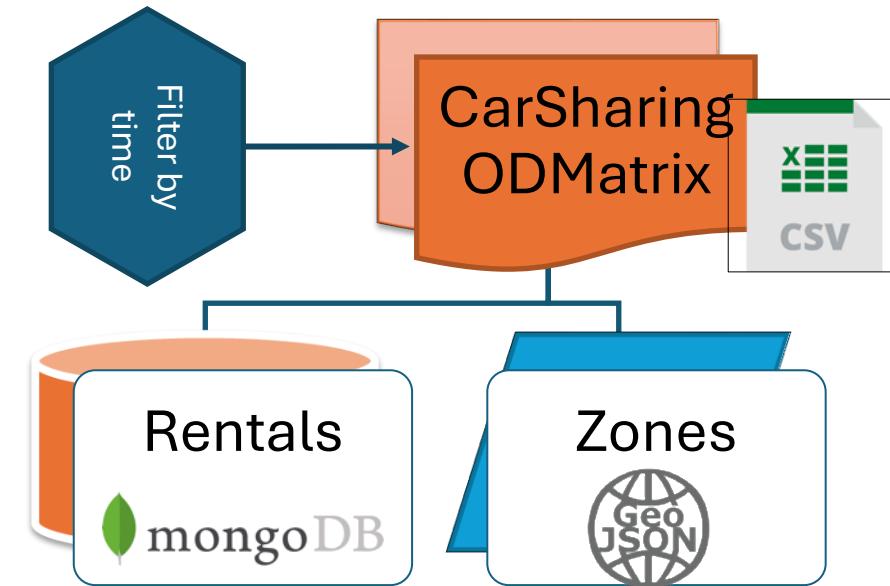
*and final\_loc that falls  
within the dest\_zone*

*Then count*

# Putting everything together

Prepare a script with the program to run

- For each orig\_zone
  - For each dest\_zone
    - Count rentals in ODMatrix
      - From orig\_zone
      - To dest\_zone
      - In a given timeslot
  - Derive different ODMatrixes
    - For Enjoy, Car2Go
    - Considering
      - All rentals
      - Rentals in weekends or weekdays only
      - Mornings and afternoons
      - ...
  - Save in a CSV file





# UnipolTech dataset

- Data collected by company – Anonymized
  - One week in September 2024 in Torino
- Data in 2 .csv files, link them by vehicle ID (`id_veicolo`)
- Information of users/vehicles (`Info_TO.csv`)
  - In total about 38k users
  - Gender, age, whether for commercial use
- Information of trips (`Trips_OD_TO.csv`)
  - Origin/Destination location (latitude and longitude)
  - Departure/arrival time
  - In total 545k trip records

# Dataset – IMQ

- IMQ 2013 contains data collected with phone interviews
  - 52,120 completed interviews
  - 105,099 trips reported
  - Trips from Monday to Friday
  - Covers 185 zones of Piedmont
    - **23 zones in Torino, we will use them**
- Interviews are stratified by
  - Gender (male/female)
  - Age (11-19, 20-49, 50-64, 65+)
  - Motivation (go to work, got to school, visiting parents, ...)

# Dataset – IMQ

- 183 zones
- Defined in a shape file
  - Resource: **Zone\_IMQ\_TO.rar**
- Available as geojson shape
  - Resource: **zoneBeauty.geojson**
- 23 zones in Torino
  - Resource: **TorinoZonescol.geojson**
  - Resource: **TorinoZonesArray.geojson**

Q001	TORINO - CENTRO
Q002	TORINO - S.SALVARIO
Q003	TORINO - CROCETTA
Q004	TORINO - S.PAOL0
Q005	TORINO - CENISIA
Q006	TORINO - S.DONATO
Q007	TORINO - AURORA
Q008	TORINO - VANCHIGLIA
Q009	TORINO - NIZZA-MILLEFONTI
Q010	TORINO - LINGOTTO
Q011	TORINO - S.RITA
Q012	TORINO - MIRAFIORI NORD
Q013	TORINO - POZZO STRADA
Q014	TORINO - PARELLA
Q015	TORINO - VALLETTE
Q016	TORINO - MADONNA CAMPAGNA
Q017	TORINO - B.TA VITTORIA
Q018	TORINO - B.RA MILANO
Q019	TORINO - FALCHERA
Q020	TORINO - REGIO PARCO
Q021	TORINO - MADONNA PILONE
Q022	TORINO - CAVORETTO
Q023	TORINO - MIRAFIORI SUD

# IMQ data

- Phone interviews
  - From Tuesday to Saturday - 9.30 and 21.30
- Asked about movements done the day before
  - **Only obtained trips during weekdays**
  - Hour, departure place, arrival place, reason for the trip, modes, ...
  - <http://mtm.torino.it/it/dati-statistiche/imq-alle-fasi-finali>
  - Sampling specifications:
    - 3% in the Turin metropolitan area
    - At least 60 interviews per zone
- Results were available as open data
  - <http://mtm.torino.it/it/dati-statistiche/indagine-imq-2013/base-dati-imq-2013>  
[not available anymore]



# IMQ data

- Raw data in Microsoft Access from [http://mtm.torino.it/it/dati-statistiche/indagine-imq-2013/base-dati-imq-2013/IMQ2013\\_opendata.zip](http://mtm.torino.it/it/dati-statistiche/indagine-imq-2013/base-dati-imq-2013/IMQ2013_opendata.zip) [not available anymore]
- There are several tables
  - Spostamenti: information about the movements done in the previous day
  - Interviste: information about the person
  - Tab\_\*: tables explaining the mapping for each category
- We have extracted the subset of movements done in the Torino area
  - From zone Q\* to zone Q\*
  - 16,567 trips in total

# IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_PAR	ZONA_ARR	PROV_ARR	ORA_ARR
8139335	1	2	1	2	C006	5	Q005	1	02/02/17 10:00	Q023	1	02/02/17 10:20
8139641	1	2	2	4	C009	4	Q002	1	02/02/17 16:30	Q023	1	02/02/17 16:50
8139830	1	2	1	2	C004	4	Q019	1	02/02/17 14:30	Q019	1	02/02/17 14:50
8140616	1	2	1	2	C003	9	Q003	1	02/02/17 20:00	Q005	1	02/02/17 20:30
8140678	1	2	2	3	C011	4	Q009	1	02/02/17 09:00	Q009	1	02/02/17 09:10
8140745	1	2	2	3	C004	4	Q015	1	02/02/17 18:00	Q021	1	02/02/17 18:15
8140799	1	2	2	3	C006	4	Q010	1	02/02/17 10:45	Q020	1	02/02/17 11:00
8141314	1	2	2	3	C009	4	Q022	1	02/02/17 18:15	Q022	1	02/02/17 18:30
8141976	2	2	1	4	C003	9	Q001	1	02/02/17 15:40	Q007	1	02/02/17 16:10
8142800	1	2	2	2	C003	7	Q012	1	02/02/17 19:30	Q019	1	02/02/17 19:45
8143087	1	2	2	3	C003	4	Q006	1	02/02/17 15:00	Q006	1	02/02/17 15:10
8143375	1	2	1	4	C004	9	Q001	1	02/02/17 17:00	Q018	1	02/02/17 17:15
8146677	2	2	2	3	C023	7	Q008	1	02/02/17 19:20	Q002	1	02/02/17 19:40
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35

# IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_PAR	ZONA_ARR	PROV_ARR	ORA_ARR
8139335	1	2	1	2	C006	5	Q005	1	02/02/17 10:00	Q023	1	02/02/17 10:20
8139641	1	2	2	4	C009	4	Q002	1	02/02/17 16:30	Q023	1	02/02/17 16:50
8139830	1	2	1	2	Gender						1	02/02/17 14:50
8140616	1	2	1	2	1 Male						1	02/02/17 20:30
8140678	1	2	2	3	2 Female						1	02/02/17 09:10
8140745	1	2	2	3	C004	4	Q015	1	02/02/17 18:00	Q021	1	02/02/17 18:15
8140799	1	2	2	3	C006	4	Q010	1	02/02/17 10:45	Q020	1	02/02/17 11:00
8141314	1	2	2	3	C009	4	Q022	1	02/02/17 18:15	Q022	1	02/02/17 18:30
8141976	2	2	1	4	C003	9	Q001	1	02/02/17 15:40	Q007	1	02/02/17 16:10
8142800	1	2	2	2	C003	7	Q012	1	02/02/17 19:30	Q019	1	02/02/17 19:45
8143087	1	2	2	3	C003	4	Q006	1	02/02/17 15:00	Q006	1	02/02/17 15:10
8143375	1	2	1	4	C004	9	Q001	1	02/02/17 17:00	Q018	1	02/02/17 17:15
8146677	2	2	2	3	C023	7	Q008	1	02/02/17 19:20	Q002	1	02/02/17 19:40
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35

# IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_PAR	ZONA_ARR	PROV_ARR	ORA_ARR	
8139335	1	2	1	2	C006	5	Q005	1	02/02/17 10:00	Q023	1	02/02/17 10:20	
8139641	1	2	2	4	C009	4	Q002	1	02/02/17 16:30	Q023	1	02/02/17 16:50	
8139830	1	2	1	2	C009	Age						1	02/02/17 14:50
8140616	1	2	1	2	C009	1 From 11 to 19						1	02/02/17 20:30
8140678	1	2	2	3	C009	2 From 20 to 49						1	02/02/17 09:10
8140745	1	2	2	3	C009	3 From 50 to 64						1	02/02/17 18:15
8140799	1	2	2	3	C009	4 65+						1	02/02/17 11:00
8141314	1	2	2	3	C009	4	Q022	1	02/02/17 18:15	Q022	1	02/02/17 18:30	
8141976	2	2	1	4	C003	9	Q001	1	02/02/17 15:40	Q007	1	02/02/17 16:10	
8142800	1	2	2	2	C003	7	Q012	1	02/02/17 19:30	Q019	1	02/02/17 19:45	
8143087	1	2	2	3	C003	4	Q006	1	02/02/17 15:00	Q006	1	02/02/17 15:10	
8143375	1	2	1	4	C004	9	Q001	1	02/02/17 17:00	Q018	1	02/02/17 17:15	
8146677	2	2	2	3	C023	7	Q008	1	02/02/17 19:20	Q002	1	02/02/17 19:40	
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35	

# IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZON	Motivation							
8139335	1	2	1	2	C006	5	G	1	Go to work						
8139641	1	2	2	4	C009	4	G	2	Working reason						
8139830	1	2	1	2	C004	4	G	3	Study						
8140616	1	2	1	2	C003	9	G	4	Shopping						
8140678	1	2	2	3	C011	4	G	5	Bring someone						
8140745	1	2	2	3	C004	4	G	6	Cures or medical visits						
8140799	1	2	2	3	C006	4	G	7	Sport or leisure						
8141314	1	2	2	3	C009	4	G	8	Going back home						
8141976	2	2	1	4	C003	9	G	9	Visiting relatives or friends						
8142800	1	2	2	2	C003	7	G	10	Other						
8143087	1	2	2	3	C003	4	G	11	Going back home on the day of the interview						
8143375	1	2	1	4	C004	9	Q001	1	02/02/17 17:00	Q018	1	02/02/17 17:15			
8146677	2	2	2	3	C023	7	Q008	1	02/02/17 19:20	Q002	1	02/02/17 19:40			
8147093	1	2	1	1	C016	2	Q001	1	02/02/17 14:00	Q014	1	02/02/17 14:35			

# IMQ data sample

ID_INT	PROGR_USC	PROGR_SPOST	SESSO	FASCIA_ETA	ZONA_RES	SCOPO	ZONA_PAR	PROV_PAR	ORA_F	ZONE
8139335	1	2	1	2	C006	5	Q005	1	02/02/17	Q001 TORINO - CENTRO
										Q002 TORINO - S.SALVARIO
										Q003 TORINO - CROCETTA
										Q004 TORINO - S.PAOL
										Q005 TORINO - CENISIA
8139641	1	2	2	4	C009	4	Q002	1	02/02/17	Q006 TORINO - S.DONATO
										Q007 TORINO - AURORA
										Q008 TORINO - VANCHIGLIA
8140616	1	2	1	2	C003	9	Q003	1	02/02/17	Q009 TORINO - NIZZA-MILLEFONTI
										Q010 TORINO - LINGOTTO
										Q011 TORINO - S.RITA
8140745	1	2	2	3	C004	4	Q015	1	02/02/17	Q012 TORINO - MIRAFIORI NORD
										Q013 TORINO - POZZO STRADA
										Q014 TORINO - PARELLA
8141314	1	2	2	3	C009	4	Q022	1	02/02/17	Q015 TORINO - VALLETTE
										Q016 TORINO - MADONNA CAMPAGNA
8141976	2	2	1	4	C003	9	Q001	1	02/02/17	Q017 TORINO - B.TA VITTORIA
										Q018 TORINO - B.RA MILANO
8142800	1	2	2	2	C003	7	Q012	1	02/02/17	Q019 TORINO - FALCHERA
										Q020 TORINO - REGIO PARCO
8143087	1	2	2	3	C003	4	Q006	1	02/02/17	Q021 TORINO - MADONNA PILONE
										Q022 TORINO - CAVORETTO
8143375	1	2	1	4	C004	9	Q001	1	02/02/17	Q023 TORINO - MIRAFIORI SUD
8146677	2	2	2	3	C023	7	Q008	1	02/02/17	
8147093	1	2	1	1	C016	2	Q001	1	02/02/17	



# Zone Description

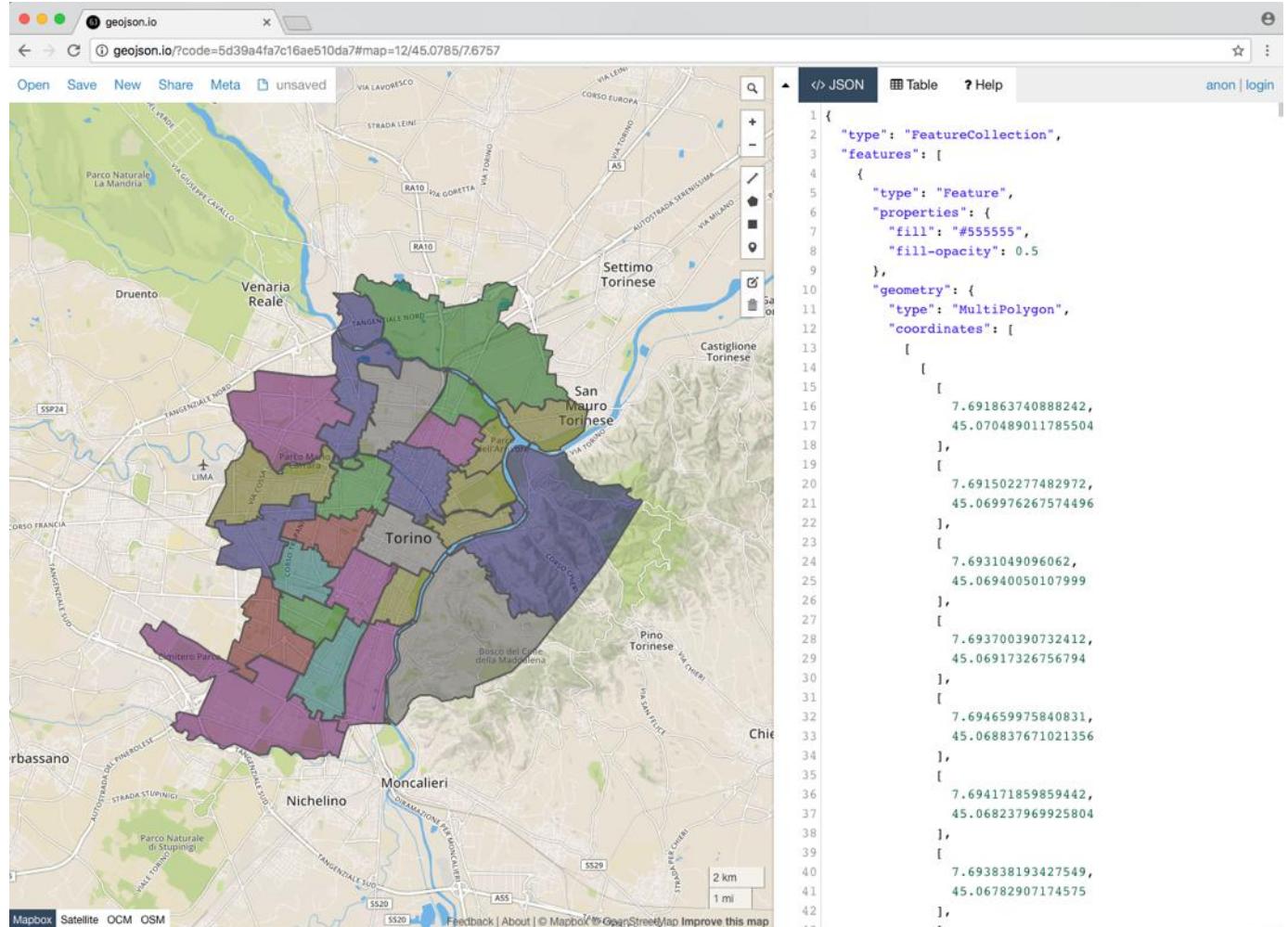
# Extracting the zones

- There are 23 zones in Torino
  - Q001 – Q023
- These are defined in
  - **TorinoZonesArray.geojson**
    - An array ready for use
  - **TorinoZonescol.geojson**
    - Original geojson shape
    - Play with it in <http://geojson.io/>

Q001	TORINO - CENTRO
Q002	TORINO - S.SALVARIO
Q003	TORINO - CROCETTA
Q004	TORINO - S.PAOLÒ
Q005	TORINO - CENISIA
Q006	TORINO - S.DONATO
Q007	TORINO - AURORA
Q008	TORINO - VANCHIGLIA
Q009	TORINO - NIZZA-MILLEFONTI
Q010	TORINO - LINGOTTO
Q011	TORINO - S.RITA
Q012	TORINO - MIRAFIORI NORD
Q013	TORINO - POZZO STRADA
Q014	TORINO - PARELLA
Q015	TORINO - VALLETTE
Q016	TORINO - MADONNA CAMPAGNA
Q017	TORINO - B.TA VITTORIA

# Zones in Torino

- Check the map at <http://geojson.io/>



# Analyzing OD matrix

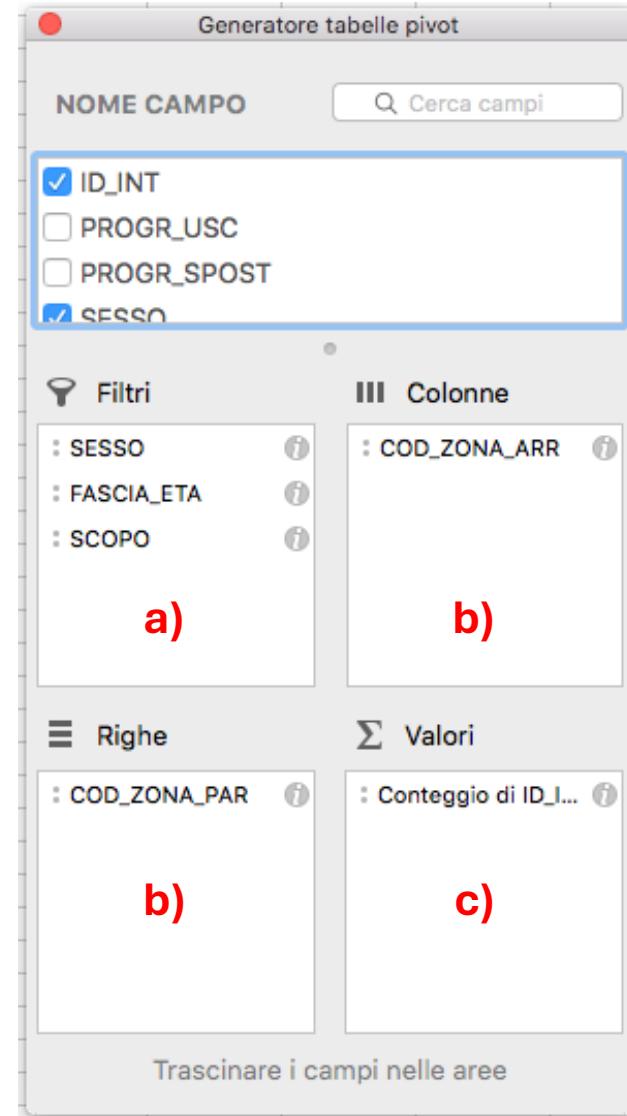
# Goal: extract OD matrix

- OD matrix[i][j]: fraction of trips from zone i to zone j
- The simplest way to extract the OD matrix is using **pivot tables**
  - Pivot tables are simple ways to summarize data
  - They allow to process and transform data
  - And filter data
- You can do it with pandas dataframe
  - [https://pandas.pydata.org/docs/reference/api/pandas.pivot\\_table.html](https://pandas.pydata.org/docs/reference/api/pandas.pivot_table.html)

# Create a pivot table with Excel

[Example in Excel – but any spreadsheet allow you to do it]

1. open a new spreadsheet and import the CVS file
2. Add a pivot table  
[menu->data->add pivot table]
3. Drag elements
  - a) In filters: sesso, scopo, eta, ...
  - b) Rows and columns: zona\_par, zona\_arr
  - c) Value: any field  
[we are going to "count" occurrences]



# Result

You can get different ODmatrixes  
by selecting different filters

FASCIA\_ETA (più elementi) ▾

SCOPO (più elementi) ▾ →

SESSO (Tutto) ▾

ORA\_ARR (Tutto) ▾

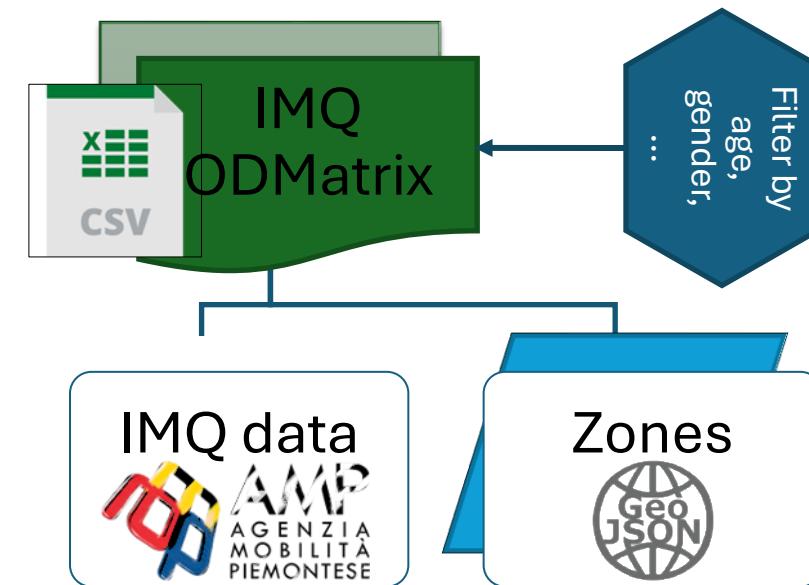
Conteggio di PR Etichette di col ▾

Etichette di r ▾ Q001

	Q002	Q003	Q004	Q005	Q006	Q007	Q008	Q009	Q010	Q011	Q012	Q013	Q014	Q015	Q016	Q017	Q018	Q019	Q020	Q021	Q022	Q023	Totale complessivo	
Q001	102	11	25	1	18	15	10	19	11	3	6	9	12	3	3	4	3	1	1	9	8	8	293	
Q002	37	27	12	1	5	1	6	6	16	2	1	4	3	3	1	2	1	1	1	3	2	4	5	144
Q003	35	7	41	7	11	8	5	4	9	9	6	2	2	1	2	3	2	2	1	2	2	1	157	
Q004	18	8	7	11	16	8	4	3	6	2	8	8	7	3	1	1	2	1	1	1	5	5	120	
Q005	28	7	17	6	33	14	4	4	5	2	5	6	21	8	3	5	1	5	5	5	5	5	179	
Q006	50	7	14	3	14	70	12	15	13	8	2	4	6	13	6	7	14	3	2	2	4	5	274	
Q007	44	5	11	2	10	18	36	15	5	3	3	7	4	8	2	9	8	7	17	5	2	3	2	226
Q008	44	6	1	5	3	3	12	32	3	2	3	2	1	2	1	4	3	3	5	4	7	5	150	
Q009	27	14	6	2	5	2	10	4	46	14	2	1	1	4	1	2	4	2	1	1	1	2	2	154
Q010	21	5	11	2	3	3	4	6	14	29	7	7	7	2	3	5	1	3	1	1	1	15	150	
Q011	15	8	8	7	6	4	7	2	4	8	27	14	2	1	3	4	1	1	1	4	1	126		
Q012	19	4	8	3	10	11	4	2	15	14	13	27	9	2	3	2	1	2	2	2	16	169		
Q013	34	4	15	10	9	11	3	8	8	4	4	7	46	28	5	4	3	4	2	1	1	6	217	
Q014	28	6	6	4	6	23	5	3	4	4	5	6	15	41	1	2	6	3	1	2	1	2	171	
Q015	17	3	3	2	1	11	1	6	6	3	1	2	2	7	20	9	4	5	1	1	1	1	104	
Q016	30	2	9	4	9	15	6	7	6	7	2	4	5	12	22	17	5	10	4	1	8	1	185	
Q017	35	6	7	2	5	18	11	6	3	8	4	2	3	11	8	12	57	3	7	3	1	5	217	
Q018	25	6	5	2	3	10	25	7	2	2	1	1	2	3	8	9	35	25	16	2	3	5	199	
Q019	19	4	5	1	1	4	6	2	1	1	1	1	1	2	2	6	10	25	5	1	1	1	97	
Q020	13	2	4			2	3	5	3					1	2	2	2	5	14		2	60		
Q021	17	1		3	5	4	5	15	1	4	1	1	1	1	2	3	1	11	6	2	2	84		
Q022	16	12	1	1	4	2	6	3	1	1	1	1	1	3	1			1	2	16	4	76		
Q023	26	4	6	2	5	3	1	7	12	22	2	6	1	1		2	1	1	2	2	3	27	136	
<b>Totale complessivo</b>	<b>700</b>	<b>159</b>	<b>222</b>	<b>80</b>	<b>179</b>	<b>262</b>	<b>182</b>	<b>184</b>	<b>196</b>	<b>159</b>	<b>99</b>	<b>117</b>	<b>146</b>	<b>158</b>	<b>77</b>	<b>100</b>	<b>160</b>	<b>78</b>	<b>118</b>	<b>69</b>	<b>46</b>	<b>67</b>	<b>130</b>	<b>3688</b>

# Results

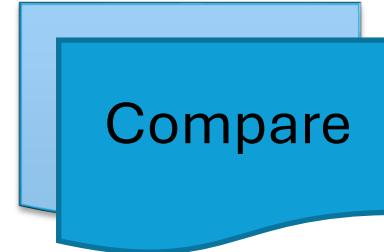
- You can compute different OD matrixes
  - For different age
  - For different sex
  - For different trip reason
  - ...



# Last step: evaluate similarity

- Given matrices  $\mathbf{A} = [a_{ij}]$  and  $\mathbf{B} = [b_{ij}]$  similarity can be defined as the sum of “distance” between each element

$$d_1(\mathbf{A}, \mathbf{B}) = \sum_{i=1}^n \sum_{j=1}^n |a_{ij} - b_{ij}|$$



Compare

$$d_2(\mathbf{A}, \mathbf{B}) = \sqrt{\sum_{i=1}^n \sum_{j=1}^n (a_{ij} - b_{ij})^2}$$

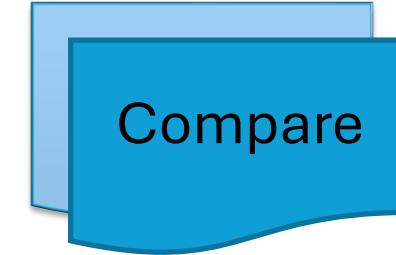
$$d_\infty(\mathbf{A}, \mathbf{B}) = \max_{1 \leq i \leq n} \max_{1 \leq j \leq n} |a_{ij} - b_{ij}|$$

$$d_m(\mathbf{A}, \mathbf{B}) = \max \{ \|(\mathbf{A} - \mathbf{B})\mathbf{x}\| : \mathbf{x} \in \mathbb{R}^n, \|\mathbf{x}\| = 1 \}$$

# Last step: compare similarity

To better appreciate the distance computation, compare

- Two uniform random matrices
  - $a_{ij} = \text{rnd}(0,1)$  and  $b_{ij} = \text{rnd}(0,1)$
- Two OD matrices extracted from the same dataset
  - From car sharing data: A = OD(first week), B = OD(second week)
  - From car sharing data: A = OD(enjoy), B = OD(Car2Go)
  - From IMQ: A = OD(all), B = OD(males), ...
- A uniform random matrix and a matrix from the data



# Warning

1. Normalize elements, e.g.,  $\sum_{ij}(a_{ij})=1$
2. Compute distance of each element  
 $d_{ij}=(a_{ij} - b_{ij})^n$  similarity
3. Sum all distances  
 $D_n(A,B)=\sum_{ij}(d_{ij})$

