A Troubling Analysis of Reproducibility and Progress in Recommender Systems Algorithms Research - Online Appendix

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The design of algorithms that generate personalized ranked item lists is a central topic of research in the field of recommender systems. In the past few years, in particular, approaches based on deep learning (neural) techniques have become dominant in the literature. For all of them, substantial progress over the state-of-the-art is claimed. However, indications exist of certain problems in today's research practice, e.g., with respect to the choice and optimization of the baselines used for comparison, raising questions about the published claims. In order to obtain a better understanding of the actual progress, we have tried to reproduce recent results in the area of neural recommendation approaches based on collaborative filtering. The worrying outcome of the analysis of these recent works—all were published at prestigious scientific conferences between 2015 and 2018—is that 11 out of the 12 reproducible neural approaches can be outperformed by conceptually simple methods, e.g., based on the nearest-neighbors heuristics. None of the computationally complex neural methods was actually consistently better than already existing learning-based techniques, e.g., using matrix factorization or linear models. In our analysis, we discuss common problematic issues in today's research practice, which, despite the many papers that are published on the topic, has apparently led the field to a certain level of stagnation.[1]

CCS Concepts: • **Information systems** → **Recommender systems**; *Collaborative filtering*; • **General and reference** → Evaluation.

Additional Key Words and Phrases: Recommender Systems, Deep Learning, Evaluation; Reproducibility

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[1]This paper significantly extends or own previous work presented in [11, 14].

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Contents

A OVERVIEW

This is the additional material associated with our article [13]. This material contains the full results of our experiments of which, due to space reasons and for the sake of improving readability, only the most representative ones are reported in the paper. In Appendix B the complete list of all baselines is presented as long as a brief description and references for each of them. The following Appendices from D to O report the results of the evaluation of each deep learning algorithm, ordered by year of publication from 2015 to 2018. Lastly in Appendix P all hyperparameters for all baselines are listed with the relative search space.

The results for each deep learning algorithm we analysed are reported in a separate section. Each section is composed of three parts, a comparison of the recommendation accuracy of the algorithms, the list of all optimal hyperparameters, and a comparison of the computation time they required.

Recommendation accuracy. Compares the recommendation accuracy of all baselines and of the deep learning model in the evaluation scenario chosen by the original authors. Different tables will therefore report different metrics and cutoffs depending on the original paper. Values in bold refer to either the deep learning algorithm outperforming *all* baselines or any baseline outperforming the deep learning algorithm. In some cases the results for $EASE^R$ and SLIM BPR may be missing, this is due to the memory requirement exceeding instance capacity as the implementations we used did not optimize memory requirements.

Optimal hyperparameters. Reports the optimal hyperparameters for all baselines and datasets. Due to the stochastic nature of the Bayesian optimization and on how many local optima the model exhibits for that dataset, multiple optimization runs may yield equivalent results but different hyperparameters.

Computation time. Compares the computation time of all algorithms on a specific Amazon AWS instance.[2] The tables and are composed by three columns. The first column (*Train time*), reports the mean and standard deviation of the time required to fit the models during the Bayesian hyperparameter optimization. In case of machine learning models requiring the selection of the number of epochs via early-stopping, the time required by the validation steps is included as it constitutes an integral part of the training procedure. The last two columns report the time required by each evaluation of the model during the Bayesian hyperparameter optimization[3] (*Recommendation Time*) and the number of recommendation lists the algorithm is able to generate per second (*Recommendation [usr/s]*). For deep learning algorithms the train and evaluation time refer to the only hyperparameter configuration we report, therefore they are not associated to any standard deviation.

It should be noted that all algorithms implemented in our repository compute a score for each item but do not directly generate the recommended items list. The sorting of such items and generation of the recommended items list is done independently from the specific recommendation model. Due to the fixed cost of ranking the items based on their score, for each user, non personalized

[2]The computation time refers to the total instance time for one AWS instance p3.2xlarge, with 8 vCPU, 30GB RAM, and one Tesla V100-SXM2-16GB GPU.

[3]Note that the evaluation time refers to an evaluation performed on the test data. During the Bayesian optimization every time a new optimal set of hyperparameters is found, using the validation data, an additional evaluation is performed on the test data. No information from the test data is ever used. For this reason, it may happen that a baseline is not associated to a standard deviation in Recommendation Time, this means that the Bayesian optimization found an optimal solution which was not improved upon and therefore only one evaluation was performed.

algorithms, i.e., TopPop, will appear to generate the same number of recommendation per second as much more complex models.

Furthermore, the implementations of the baseline algorithms vary in terms of efficiency. Some use standard solvers (PureSVD, NMF, SLIM ElasticNet), others are written in Cython[4] and compiled (KNNs, MF BPR, FunkSVD, SLIM BPR), others are written in plain Python with vectorized operations ($P^3\alpha$, $RP^3\beta$, iALS), some are single-core others take advantage of multithreading. Similarly the deep learning models are implemented in Tensorflow or Keras and with varying degrees of efficiency. Due to this heterogeneity the computational time measurements should not be taken as exact measurements but rather as a qualitative comparison.

[4]https://cython.org/

B BASELINES

Over the last 25 years, a multitude of algorithms of different types were proposed. In order to obtain a picture that is as broad as possible, we selected algorithms of different families for inclusion in our measurements. An overview of all used baselines is given in Table 1 and the relative hyperparameter ranges are reported in Appendix P.

Table 1. Overview of Baseline Methods

Family	*Method*	*Description*
Non-personalized	TopPopular	Recommends the most popular items to everyone [10]
Nearest-Neighbor	UserKNN	User-based k-nearest neighbors [26]
	ItemKNN	Item-based k-nearest neighbors [27]
Graph-based	$P^3\alpha$	A graph-based method based on random walks [9]
	$RP^3\beta$	An extension of $P^3\alpha$ [23]
Content-Based and Hybrid	ItemKNN-CBF	ItemKNN with content-based similarity [20]
	ItemKNN-CFCBF	A simple item-based hybrid CBF/CF approach [21]
	UserKNN-CBF	UserKNN with content-based similarity
	UserKNN-CFCBF	A simple user-based hybrid CBF/CF approach
Non-Neural Machine Learning	iALS	Matrix factorization for implicit feedback data [15]
	PureSVD	A basic matrix factorization method [10]
	NFM	A basic non-negative matrix factorization method [8]
	FunkSVD	Matrix factorization for rating prediction [17]
	MF BPR	Matrix factorization optimized for ranking [25]
	SLIM ElasticNet	A scalable linear model [18, 22]
	SLIM BPR	A variation of SLIM optimizing ranking [4]
	$EASE^R$	A recent linear model, similar to auto-encoders [28]

B.0.1 Popularity-Based Ranking. Recommending the most popular items to everyone is a common strategy in practice. The method **TopPopular** implements this non-personalized recommendation approach. The popularity of an item is determined by its number of implicit or explicit ratings in the given dataset.

B.0.2 Nearest-Neighbor Methods. Nearest-neighbor techniques were used in the early GroupLens system [26] and first successful reports of collaborative filtering systems also used nearest-neighbor techniques [19]. We consider both *user-based* and *item-based* variants, **UserKNN** and **ItemKNN**.

Many variants of the basic nearest-neighbor prediction scheme were proposed over the years, see [7] for an early performance comparison. In this work, we therefore consider different variations of the nearest-neighbor techniques as well. For both UserKNN and ItemKNN, the following hyperparameters can be set and were optimized in our experiments, their ranges are reported in Appendix P.

- *Neighborhood Size*: This main parameter determines how many neighbors are considered for prediction.
- *Similarity Measure*: We made experiments with the Jaccard coefficient [24] as well as Cosine [27], Asymmetric Cosine [2], Dice-Sørensen [12] and Tversky [30] similarities. Some of these similarity measures also have their own parameters, as reported in Appendix P, which we optimized as well.
- *Shrinkage*: As proposed in [5], we used a parameter (the *shrink term*) to lower the similarity between items that have only few interactions in common. The shrinkage is applied to all similarities.
- *Feature Weighting*: Using feature weighting for ratings was proposed in [31]. In our experiments, we both tested configurations with no weighting and weighting with either the TF-IDF or the BM25 scheme.
- *Normalization*: This setting determines if we should consider the denominator in the similarity measure as normalization. Only some of the similarity measures have this parameter.

B.0.3 Graph-based Methods. Traditional nearest-neighbor models consider "direct" neighborhoods by computing similarities between pairs of objects. Graph-based models can help to overcome this possible limitation relying on a broader interpretation of neighborhoods. In our study, we consider two such graph-based methods called $P^3\alpha$ [9] and $RP^3\beta$ [23]. Both methods often lead to good recommendation quality at low computational cost. Interestingly, these two methods appear to be almost unknown in the community and seldom used as baselines, despite the fact that they are very simple, effective and have been published in top-tier venues.

- $\boldsymbol{P^3\alpha}$: This method implements a two-steps random walk from users to items and vice-versa, where the probabilities to jump between users and items are computed from the normalized ratings raised to the power of α. The method is equivalent to a KNN item-based CF algorithm, with the similarity matrix being computed as the dot-product of the probability vectors [9]. In addition to what described in the original algorithm, we normalize each row of the similarity matrix with its $l1$ norm. The hyperparameters of the algorithm include the size of the neighborhood and the value for α.
- $\boldsymbol{RP^3\beta}$: This is an improved version of $P^3\alpha$ proposed in [23]. In $RP^3\beta$, each similarity between two items is computed with $P^3\alpha$ and divided by the popularity of the items raised to the power of β. Again, we normalize each row of the similarity matrix with its $l1$ norm. If β is 0, $RP^3\beta$ is equivalent to $P^3\alpha$. The hyperparameters of the algorithm are the size of the neighborhood and the values for α and β.

B.0.4 Content-based and hybrid Methods. Some of the neural methods investigated in this paper include side information about items or users. We have therefore included two simple baselines that make usage of content information.

- **ItemKNN-CBF**, **UserKNN-CBF**: A neighborhood-based content-based-filtering (CBF) approach, where we compute the item (or user) similarities based on the items' (or user's) content features (attributes) [20]. We tested the same set of similarity measures described for the collaborative KNN methods (Jaccard coefficient, Cosine, Asymmetric Cosine, Dice-Sørensen and Tversky similarity). The hyperparameters are the same as for the ItemKNN and UserKNN methods.
- **ItemKNN-CFCBF**, **UserKNN-CFCBF**: A hybrid algorithm based on item-item (or user-user) similarities and described in [21]. The similarity between items is computed by first concatenating, for each item, the vector of implicit ratings (collaborative features) and the vector of item attributes (content features) and by later computing the similarity between

the concatenated vectors. In case of user-user similarities the algorithm operates in a similar way, concatenating the vector of implicit ratings of each user with the user's content feature vector. The hyperparameters and similarity measures are the same as for ItemKNN, plus a parameter w that controls the relative importance of the content features with respect to the collaborative features. When w is 0, this algorithm is equivalent to the pure collaborative versions, either ItemKNN or UserKNN.

B.0.5 Non-Neural Machine Learning Approaches. Countless machine learning models were proposed for *top-n* recommendation tasks in the literature. In our experiments, we included a number of comparably basic models from the literature as representatives of which methods were often considered the state-of-the-art in pre-neural times.

- *Matrix Factorization (MF) Techniques*: The application of matrix decomposition methods for collaborative filtering problems was investigated already in the early years of recommender systems [6], and became a de-facto standard after the Netflix prize competition (2006-2009). We made experiments with many variants, but will limit our discussion to two main techniques which proved to consistently lead to competitive results among the different MF techniques.
  - **iALS**: In their seminal work [15], Hu et al. proposed an *Alternating Least Squares* approach for implicit feedback datasets, which turns implicit feedback signals into confidence values. The authors also proposed a particular optimization method that has the advantage of scaling well on larger datasets. A number of hyperparameters can be tuned for the method, including the number of latent factors, the confidence scaling and the regularization factor.
  - **PureSVD**: This method corresponds to a basic matrix factorization approach as proposed in [10]. To implement PureSVD, we used a standard SVD decomposition method provided in the `scikit-learn` package for Python.[5] The only hyperparameter of this method is the number of latent factors.
  - **NMF**: This method performs a *Non Negative Matrix Factorization*, which is described in [8]. As opposed to PureSVD, NFM guarantees all latent factors to be positive. We used a standard NMF decomposition method provided in the `scikit-learn` package for Python.[6] The only hyperparameter of this method is the number of latent factors.
  - **FunkSVD**: This matrix factorization algorithm was proposed by Simon Funk in his well known online article[7] during the Netflix Prize. This method optimises rating prediction via MSE. The embeddings of users and items are regularises with a Frobenius norm. In order to ensure the suitability of FunkSVD for a *top-n* recommendation task we added a hyperparameter which ensures a certain quota of the samples used during training are randomly sampled among the unseen items and are associated with a rating of 0. Another hyperparameter controls whether the model should include the global bias, user bias and item bias. Other hyperparameters include the learning rate, the regularisation coefficients, and the number of latent factors.
  - **MF BPR**: This algorithm was presented in the well known article from Rendle et al. [25] as a matrix factorization model optimizing ranking accuracy via a BPR loss. MF BPR is a widely used baseline in the article we surveyed. This method, as opposed to FunkSVD, PureSVD and NFM, has been explicitly designed for implicit interactions. Furthermore, as opposed to iALS it is trained using gradient ascent. Hyperparameters of this method include the number of latent factor, the learning rate and the regularization coefficients.

[5]https://scikit-learn.org/stable/modules/generated/sklearn.utils.extmath.randomized_svd.html
[6]https://scikit-learn.org/stable/modules/generated/sklearn.decomposition.NMF.html
[7]http://sifter.org/~simon/journal/20061211.html

- *Sparse Linear Models (SLIM)*: SLIM was proposed as a well-performing regression-based method for *top-n* recommendation tasks in [22]. In our work, we use the more scalable variant proposed in [18] (**SLIM ElasticNet**) which learns the item similarity matrix one item at a time (e.g. one column w at a time) by solving a regression problem in such a way that the interactions for the target item y are learned by using all other interactions as training data. To implement *SLIM ElasticNet* we used a standard ElasticNet solver provided in the `scikit-learn` package for Python.[8] The hyperparameters of this method include the ratio of *l1* and *l2* regularizations as well as a regularization magnitude coefficient.
- *Sparse Linear Models BPR*: This algorithm is a variant of the previously mentioned SLIM ElacticNet which optimizes ranking accuracy rather than prediction error (**SLIM BPR**) [3, 4, 29]. The algorithm learns an item-item similarity matrix by optimizing the BPR loss function, described in [25], via gradient ascent. The hyperparameters of this method include the number of neighbours as described in the Nearest-Neighbor Methods, the regularization coefficients and whether the learned similarity matrix should be symmetric or not.
- ***EASE^R***: In a recent article [28] the author showed that an "embarrassingly shallow" linear model, which shares similarities with an auto-encoder, can produce highly-accurate recommendations that often outperform existing and much more complex techniques. A peculiarity of this model is the existence of a closed-form solution for the training objective which results in very fast training. The only hyperparameter is the choice of the regularization factor.
  This algorithm has been published in 2019 and, as such, the papers covered by our study could not include EASE^R as a baseline. However, we include EASE^R to investigate whether shallow auto-encoders are able to provide, on average, more accurate recommendations with respect to complex deep-learning architectures.

C EVALUATION METRICS

In order to assess the recommendation quality of the algorithms we evaluate in this study, we report two categories of metrics: accuracy metrics and beyond-accuracy metrics.

C.1 Accuracy metrics

Accuracy metrics are usually computed for a recommendation list of a certain length, i.e., cutoff. All the metrics reported are computed for each user independently and then averaged, so the equations reported in this section refer to a single user.

We define $rel(i)$ as a boolean vector having the same length as the recommendation list, its purpose is to model whether an item is *relevant*, i.e., is a correct recommendation, or not:

$$rel(i) = \begin{cases} 1 & \text{if the item in position } i \text{ is relevant} \\ 0 & \text{otherwise} \end{cases}$$

Furthermore, consider rel_t as the total number of relevant items the user has in the test data.

- **Precision (Prec)**: This metric measures the quota of correct recommendations received by a user over the full recommendation list.

$$Prec@k = \frac{1}{k} \sum_{i=0}^{k} rel(i)$$

[8] https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.ElasticNet.html

- **Recall (Rec)** This metric measures the quota of correct recommendations received by a user over the *true positives* in its test data.

$$Rec@k = \frac{1}{rel_t} \sum_{i=0}^{k} rel(i)$$

- **Hit Rate (HR):** This metric computes the number of correct recommendations a user received. If the data is split via leave-one-out its values are between 0 and 1. If the split is random holdout its value may exceed 1. Hit rate is a non normalized version of Precision, if the recommendation list length is constant for all users the two are equivalent. HR is among the most commonly used metrics in the algorithms we analyzed.

$$HR@k = \sum_{i=0}^{k} rel(i)$$

- **Normalized Discounted Cumulative Gain (NDCG):** As opposed to the previous ones, this metrics takes also into account the ranking of the correct recommendations. This metric was originally proposed to evaluate the effectiveness of information retrieval systems [16] and is among the most commonly used metrics in the algorithms we analyzed.
Assuming that the recommendations for user u are sorted according to the predicted relevance values in decreasing order, DCG is defined as follows:

$$DCG@k = \sum_{i=0}^{k} \frac{2^{r(i)} - 1}{\log_2(i+1)}$$

where $r(i)$ is the true relevance, as found in the test data, for the item ranked at position i. In case of datasets with explicit ratings, the relevance will be equal to the rating, in case of implicit interactions the relevance will be 1.
The cumulative gain for each user is normalized by computing the ideal DCG for that same user, denoted as $IDCG$. While the DCG considers all items in the recommendation list, the $IDCG$ is computed assigning to each item its true relevance (i.e., the one in the test data) and therefore obtaining the best possible ranking. The NDCG is then computed as follows:

$$NDCG@k = \frac{DCG@k}{IDCG@k}$$

- **Mean average precision (MAP)**: Is another metric used to measure ranking quality, though is less common than NDCG. MAP is computed based on the average precision (AP) at increasingly longer recommendation lists, up to the cutoff value, of each user. The AP of each user is defined as follows:

$$AP = \frac{1}{min(k, rel_t)} \sum_{i=0}^{k} Prec@i \cdot rel(i)$$

Finally, given the AP of each user, MAP will be defined as the global average:

$$MAP = \frac{1}{|U|} \sum_{u \in |U|} AP_u$$

C.2 Beyond accuracy metrics

The core purpose of a recommender system is to assist the user in exploring a catalog, this of course requires finding relevant recommendations but also to ensure the user is assisted in the discovery of new items. For some of the algorithms we analyzed, when the recommendation quality of the neural algorithm was similar to the TopPopular method, we also report beyond-accuracy metrics to assess whether the neural model was indeed able to differentiate its recommendations for different users.

Beyond-accuracy metrics measure how well the recommender is diversifying its recommendations for different users. Broadly speaking, diversity metrics can be classified in two different categories: *individual diversity* and *aggregate diversity*. While individual diversity only measures what is perceived by the user and is computed on each separate recommendation list, aggregate diversity considers the system as a whole and is measured taking into account the recommendations provided to all users [1, 32]. We will define $rec(i)$ as the number of times item i has been recommended, and rec_t as the total number of recommendations (i.e., cutoff value times the number of test users).

In this study, we will focus on the following measures:

- **Item Coverage:** This aggregate diversity metric represents the quota of items that were recommended at least once.

$$coverage = \frac{1}{|I|} \sum_{i \in I} rec(i) > 0 \tag{1}$$

  where $|I|$ is the cardinality of the item set and $rec(i) > 0$ is 1 if the item has been recommended at least once, 0 otherwise. Recommender systems exhibiting low coverage will be able to recommend only a low number of items, which can be a significant issue as the recommender fails in one of its most important tasks and constrains user exploration.
- **Shannon Entropy:** This aggregate diversity metric measures the distributional dispersion of the recommendation frequency, taking into account not only whether an item was recommended, but also how often. This metric is the Shannon entropy of how frequently each item has been recommended.

$$\text{Shannon} = -\sum_{i \in I} \frac{rec(i)}{rec_t} \cdot \ln \frac{rec(i)}{rec_t}$$

- **Gini Diversity:** This aggregate diversity metric is too a function of how frequently an item is recommended. It is derived from the definition of the Gini Index, but has its range flipped in such a way that a higher diversity recommender, therefore with balanced frequencies, will have a low Gini Index but a high Gini Diversity. This formulation is aimed at easing the comparison with other diversity metrics sharing a common behavior. Note that the item frequency in this case needs to be sorted by decreasing values. Function $s(i) = j$ given item i will provide its index j in the original non-sorted data.

$$\text{Gini} = \sum_{i=1}^{|I|} \frac{2i - |I| - 1}{|I|} \cdot \frac{rec(s(i))}{rec_t}$$

- **Herfindahl index (HHI):** This aggregate diversity metric originated from the economics sector and is a quadratic function of the item frequency. Due to its quadratic nature it is more sensitive to changes in the recommendation frequency of items that tend to be recommended often.

$$\text{HHI} = 1 - \frac{1}{rec_t^2} \sum_{i \in I} rec(i)^2$$

- **Mean Inter-list diversity (MIL):** This diversity metric compares all user's recommendation lists and measures how different they are [33]. It is among the metrics which are computed on the actual recommendations received by each user rather than on the global item count. This diversity considers the uniqueness of different user's recommendation lists and has a value between 0 and 1. The less likely any two users have been recommended the same items, hence the more diverse the recommendations are, the closer MIL will be to 1. Note that MIL was originally called *Personalization*, however we will not use this name due to the fact that the highest value for this metric (i.e., 1) is obtained by a non personalized Random recommender.

$$h(ua, ub) = 1 - \frac{q(ua, ub)}{c} \tag{2}$$

Equation 2 represents the *inter-list distance* for two users ua and ub, where $q(ua, ub)$ is the number of common items in their recommendation lists. Equation 3 shows how MIL is computed, as an average over all inter-list distances, excluding the diagonal.

$$MIL = \frac{1}{|U|^2 - |U|} \sum_{\substack{ua,ub \in U \\ ua \neq ub}} h(ua, ub) \tag{3}$$

Computing MIL requires to compute function $q(ua, ub)$ for all couples of users, which is quadratic in their number.

D KDD: COLLABORATIVE DEEP LEARNING

This algorithm is evaluated in the same experimental conditions and on the same data as *CVAE*. For the full results please refer to Section F.

E SIGIR: COLLABORATIVE MEMORY NETWORKS

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 2 and 3. The results of our evaluation can be seen in Table 4 (CiteULike), Table 5 (Epinions) and Table 6 (Pinterest). The corresponding optimal hyperparameters are reported in Table 10 (collaborative KNNs), Table 11 (non-neural machine learning and graph based) and Table 12 (CMN).

Lastly, the time required to train and evaluate the models is reported in Table 7 (CiteULike), Table 8 (Epinions) and Table 9 (Pinterest).

Table 2. Dataset characteristics.

Dataset	Interactions	Items	Users	Sparsity
Epinions	664.8 k	139.7 k	40.1 k	99.98%
CiteULike-a	204.9 k	16.9 k	5.5 k	99.78%
Pinterest	1.5 M	9.9 k	55.1 k	99.73%

Table 3. Dataset popularity bias characteristics.

	Max pop	Min pop	Avg pop	Gini Index	Shannon	Herfindahl
Citeulike	321.00	1.00	12.07	0.37	13.65	1.00
Pinterest	1636.00	1.00	147.60	0.45	12.77	1.00
Epinions	2026.00	1.00	4.76	0.69	15.11	1.00

Table 4. Experimental results for the CMN method for the Citeulike dataset.

	@ 5		@ 10	
	HR	NDCG	HR	NDCG
Random	0.0503	0.0293	0.0960	0.0439
TopPopular	0.1810	0.1226	0.2774	0.1537
UserKNN CF cosine	**0.8231**	**0.7027**	**0.8962**	**0.7265**
UserKNN CF dice	**0.8099**	**0.6839**	**0.8836**	**0.7079**
UserKNN CF jaccard	**0.8116**	**0.6880**	**0.8838**	**0.7115**
UserKNN CF asymmetric	**0.8226**	**0.7039**	**0.8959**	**0.7279**
UserKNN CF tversky	**0.8121**	**0.6892**	**0.8867**	**0.7135**
ItemKNN CF cosine	**0.8247**	**0.7045**	**0.8925**	**0.7267**
ItemKNN CF dice	**0.8089**	**0.6823**	**0.8863**	**0.7075**
ItemKNN CF jaccard	**0.8065**	**0.6793**	**0.8861**	**0.7053**
ItemKNN CF asymmetric	**0.8233**	**0.7041**	**0.8944**	**0.7274**
ItemKNN CF tversky	**0.8081**	**0.6796**	**0.8874**	**0.7055**
$P^3\alpha$	**0.8272**	**0.7144**	**0.8971**	**0.7370**
$RP^3\beta$	**0.8326**	**0.7227**	**0.9002**	**0.7447**
$EASE^R$	**0.8116**	**0.6973**	**0.8806**	**0.7197**
SLIM BPR	**0.8099**	**0.6916**	**0.8861**	**0.7164**
SLIM ElasticNet	**0.8265**	**0.7168**	**0.8908**	**0.7376**
MF BPR	0.7316	0.6053	0.8245	0.6356
MF FunkSVD	0.7860	0.6488	0.8672	0.6752
PureSVD	0.7233	0.6020	0.7954	0.6254
NMF	0.7161	0.5534	0.8245	0.5887
iALS	**0.8308**	**0.7085**	**0.9006**	**0.7313**
CMN	0.7874	0.6505	0.8746	0.6790

Table 5. Experimental results for the CMN method for the Epinions dataset.

	@ 5		@ 10	
	HR	NDCG	HR	NDCG
Random	0.0496	0.0293	0.0987	0.0449
TopPopular	**0.5492**	**0.4204**	**0.6672**	**0.4587**
UserKNN CF cosine	0.4282	0.3631	0.4764	0.3787
UserKNN CF dice	0.4108	0.3475	0.4589	0.3630
UserKNN CF jaccard	0.4108	0.3473	0.4589	0.3628
UserKNN CF asymmetric	0.4294	0.3642	0.4767	0.3795
UserKNN CF tversky	0.4207	0.3571	0.4700	0.3731
ItemKNN CF cosine	0.4309	0.3584	0.4854	0.3760
ItemKNN CF dice	0.4088	0.3426	0.4631	0.3601
ItemKNN CF jaccard	0.4088	0.3427	0.4631	0.3602
ItemKNN CF asymmetric	0.4149	0.3437	0.4761	0.3635
ItemKNN CF tversky	0.4179	0.3476	0.4757	0.3662
$P^3\alpha$	0.4008	0.3411	0.4389	0.3533
$RP^3\beta$	0.3928	0.3329	0.4341	0.3462
$EASE^R$	-	-	-	-
SLIM BPR	0.3988	0.3393	0.4422	0.3533
SLIM ElasticNet	0.4133	0.3471	0.4667	0.3643
MF BPR	0.4668	0.3662	**0.5594**	0.3962
MF FunkSVD	**0.5427**	**0.4196**	**0.6567**	**0.4566**
PureSVD	0.4073	0.3069	0.5045	0.3384
NMF	0.4055	0.3218	0.4951	0.3508
iALS	0.0519	0.0316	0.1003	0.0470
CMN	0.4699	0.3781	0.5399	0.4008

Table 6. Experimental results for the CMN method for the Pinterest dataset.

	@ 5		@ 10	
	HR	NDCG	HR	NDCG
Random	0.0499	0.0296	0.0984	0.0450
TopPopular	0.1665	0.1064	0.2740	0.1409
UserKNN CF cosine	**0.7017**	**0.5050**	0.8614	**0.5570**
UserKNN CF dice	**0.7026**	**0.5053**	0.8634	**0.5578**
UserKNN CF jaccard	**0.7034**	**0.5062**	0.8639	**0.5585**
UserKNN CF asymmetric	0.7005	**0.5037**	0.8630	**0.5567**
UserKNN CF tversky	**0.7024**	**0.5047**	0.8636	**0.5572**
ItemKNN CF cosine	**0.7132**	**0.5116**	**0.8781**	**0.5653**
ItemKNN CF dice	**0.7095**	**0.5091**	**0.8766**	**0.5635**
ItemKNN CF jaccard	**0.7094**	**0.5086**	**0.8764**	**0.5630**
ItemKNN CF asymmetric	**0.7126**	**0.5110**	**0.8776**	**0.5648**
ItemKNN CF tversky	**0.7095**	**0.5086**	**0.8761**	**0.5629**
$P^3\alpha$	0.6990	**0.5034**	0.8596	**0.5559**
$RP^3\beta$	**0.7147**	**0.5150**	**0.8772**	**0.5680**
$EASE^R$	**0.7072**	**0.5129**	0.8567	**0.5617**
SLIM BPR	**0.7120**	**0.5151**	**0.8733**	**0.5678**
SLIM ElasticNet	**0.7084**	**0.5107**	**0.8683**	**0.5628**
MF BPR	0.6924	0.4886	**0.8694**	0.5463
MF FunkSVD	**0.7088**	**0.5037**	**0.8686**	**0.5559**
PureSVD	0.6619	0.4721	0.8146	0.5219
NMF	0.6550	0.4618	0.8287	0.5183
iALS	**0.7219**	**0.5175**	**0.8677**	**0.5652**
CMN	0.7013	0.5005	0.8674	0.5547

Table 7. Computation time for the algorithms in the selected results for the CMN method on the Citeulike dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	4.83 [sec]	1150
TopPopular	0.01 [sec]	5.46 [sec]	1017
UserKNN CF cosine	0.52 ± 0.04 [sec]	9.76 ± 0.23 [sec]	566
UserKNN CF dice	0.52 ± 0.04 [sec]	9.41 ± 0.39 [sec]	575
UserKNN CF jaccard	0.52 ± 0.04 [sec]	9.69 ± 0.38 [sec]	572
UserKNN CF asymmetric	0.51 ± 0.04 [sec]	9.80 ± 0.07 [sec]	574
UserKNN CF tversky	0.50 ± 0.04 [sec]	9.58 ± 0.02 [sec]	580
ItemKNN CF cosine	3.10 ± 0.31 [sec]	9.75 ± 0.41 [sec]	564
ItemKNN CF dice	3.06 ± 0.21 [sec]	9.70 ± 0.38 [sec]	554
ItemKNN CF jaccard	3.06 ± 0.21 [sec]	9.87 ± 0.16 [sec]	575
ItemKNN CF asymmetric	3.24 ± 0.21 [sec]	9.73 ± 0.44 [sec]	553
ItemKNN CF tversky	3.02 ± 0.24 [sec]	9.70 ± 0.17 [sec]	581
$P^3\alpha$	13.78 ± 2.87 [sec]	9.56 ± 0.13 [sec]	583
$RP^3\beta$	15.82 ± 3.05 [sec]	9.51 ± 0.26 [sec]	576
$EASE^R$	95.50 [sec] / 1.59 ± 0.01 [min]	12.35 ± 0.06 [sec]	448
SLIM BPR	645.01 [sec] / 10.75 ± 4.22 [min]	10.16 ± 0.22 [sec]	538
SLIM ElasticNet	236.77 [sec] / 3.95 ± 1.56 [min]	9.79 ± 0.66 [sec]	559
MF BPR	776.37 [sec] / 12.94 ± 8.09 [min]	6.51 ± 1.04 [sec]	879
MF FunkSVD	1057.07 [sec] / 17.62 ± 12.82 [min]	6.12 ± 0.37 [sec]	881
PureSVD	1.23 ± 0.47 [sec]	7.32 ± 0.21 [sec]	744
NMF	153.39 [sec] / 2.56 ± 2.07 [min]	6.71 ± 0.50 [sec]	870
iALS	593.57 [sec] / 9.89 ± 4.71 [min]	5.92 ± 0.21 [sec]	911
CMN	6818.32 [sec] / 1.89 [hour]	20.18 [sec]	275

Table 8. Computation time for the algorithms in the selected results for the CMN method on the Epinions dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.01 [sec]	56.42 [sec]	712
TopPopular	0.02 [sec]	91.41 [sec] / 1.52 [min]	439
UserKNN CF cosine	12.81 ± 0.45 [sec]	120.93 [sec] / 2.02 ± 0.02 [min]	330
UserKNN CF dice	12.51 ± 0.39 [sec]	119.91 [sec] / 2.00 ± 0.03 [min]	329
UserKNN CF jaccard	12.51 ± 0.41 [sec]	120.24 [sec] / 2.00 ± 0.02 [min]	331
UserKNN CF asymmetric	13.04 ± 0.37 [sec]	121.49 [sec] / 2.02 ± 0.03 [min]	325
UserKNN CF tversky	12.66 ± 0.36 [sec]	121.45 [sec] / 2.02 ± 0.01 [min]	331
ItemKNN CF cosine	125.68 [sec] / 2.09 ± 0.14 [min]	128.99 [sec] / 2.15 ± 0.05 [min]	305
ItemKNN CF dice	122.99 [sec] / 2.05 ± 0.01 [min]	127.09 [sec] / 2.12 ± 0.04 [min]	311
ItemKNN CF jaccard	123.08 [sec] / 2.05 ± 0.01 [min]	128.41 [sec] / 2.14 ± 0.03 [min]	306
ItemKNN CF asymmetric	126.35 [sec] / 2.11 ± 0.02 [min]	129.97 [sec] / 2.17 ± 0.07 [min]	303
ItemKNN CF tversky	125.31 [sec] / 2.09 ± 0.01 [min]	127.61 [sec] / 2.13 ± 0.06 [min]	306
$P^3\alpha$	367.87 [sec] / 6.13 ± 0.19 [min]	116.08 [sec] / 1.93 ± 0.03 [min]	341
$RP^3\beta$	395.01 [sec] / 6.58 ± 0.20 [min]	116.68 [sec] / 1.94 ± 0.03 [min]	339
$EASE^R$	-	-	-
SLIM BPR	42149.10 [sec] / 11.71 ± 5.47 [hour]	124.94 [sec] / 2.08 ± 0.07 [min]	323
SLIM ElasticNet	14201.25 [sec] / 3.94 ± 1.31 [hour]	127.63 [sec] / 2.13 ± 0.14 [min]	310
MF BPR	10857.32 [sec] / 3.02 ± 1.65 [hour]	98.43 [sec] / 1.64 ± 0.28 [min]	440
MF FunkSVD	3409.08 [sec] / 56.82 ± 68.92 [min]	105.37 [sec] / 1.76 ± 0.19 [min]	327
PureSVD	2.36 ± 3.67 [sec]	88.22 [sec] / 1.47 ± 0.04 [min]	464
NMF	1754.00 [sec] / 29.23 ± 18.12 [min]	100.15 [sec] / 1.67 ± 0.18 [min]	448
iALS	4470.54 [sec] / 1.24 ± 0.79 [hour]	87.28 [sec] / 1.45 ± 0.00 [min]	459
CMN	33203.75 [sec] / 9.22 [hour]	292.74 [sec] / 4.88 [min]	137

Table 9. Computation time for the algorithms in the selected results for the CMN method on the Pinterest dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.02 [sec]	47.56 [sec]	1160
TopPopular	0.04 [sec]	52.75 [sec]	1046
UserKNN CF cosine	28.98 ± 1.55 [sec]	94.03 [sec] / 1.57 ± 0.03 [min]	586
UserKNN CF dice	29.42 ± 0.98 [sec]	94.33 [sec] / 1.57 ± 0.02 [min]	578
UserKNN CF jaccard	29.45 ± 1.21 [sec]	94.86 [sec] / 1.58 ± 0.01 [min]	582
UserKNN CF asymmetric	30.05 ± 1.40 [sec]	94.93 [sec] / 1.58 ± 0.06 [min]	567
UserKNN CF tversky	28.91 ± 1.58 [sec]	95.05 [sec] / 1.58 ± 0.02 [min]	571
ItemKNN CF cosine	1.82 ± 0.19 [sec]	92.88 [sec] / 1.55 ± 0.02 [min]	592
ItemKNN CF dice	1.76 ± 0.21 [sec]	91.06 [sec] / 1.52 ± 0.04 [min]	594
ItemKNN CF jaccard	1.77 ± 0.17 [sec]	91.28 [sec] / 1.52 ± 0.04 [min]	597
ItemKNN CF asymmetric	1.78 ± 0.17 [sec]	90.51 [sec] / 1.51 ± 0.05 [min]	593
ItemKNN CF tversky	1.74 ± 0.16 [sec]	90.53 [sec] / 1.51 ± 0.04 [min]	595
$P^3\alpha$	8.71 ± 2.25 [sec]	88.71 [sec] / 1.48 ± 0.02 [min]	627
$RP^3\beta$	9.23 ± 2.85 [sec]	90.04 [sec] / 1.50 ± 0.03 [min]	608
$EASE^R$	21.54 ± 0.10 [sec]	114.62 [sec] / 1.91 ± 0.00 [min]	481
SLIM BPR	3594.20 [sec] / 59.90 ± 28.93 [min]	91.58 [sec] / 1.53 ± 0.03 [min]	597
SLIM ElasticNet	433.57 [sec] / 7.23 ± 2.50 [min]	91.23 [sec] / 1.52 ± 0.04 [min]	595
MF BPR	6439.39 [sec] / 1.79 ± 1.12 [hour]	64.56 [sec] / 1.08 ± 0.18 [min]	755
MF FunkSVD	8220.55 [sec] / 2.28 ± 1.76 [hour]	58.83 ± 10.08 [sec]	1006
PureSVD	2.33 ± 1.89 [sec]	56.22 ± 0.27 [sec]	984
NMF	686.16 [sec] / 11.44 ± 9.74 [min]	72.56 [sec] / 1.21 ± 0.26 [min]	937
iALS	2694.24 [sec] / 44.90 ± 36.27 [min]	57.41 ± 1.73 [sec]	955
CMN	28100.23 [sec] / 7.81 [hour]	354.04 [sec] / 5.90 [min]	156

Table 10. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	CiteULike	Pinterest	Epinions
UserKNN CF cosine	topK	578	668	1000
	shrink	0	0	0
	similarity	cosine	cosine	cosine
	normalize	True	True	True
	feature weighting	BM25	none	TF-IDF
UserKNN CF dice	topK	627	818	1000
	shrink	0	0	0
	similarity	dice	dice	dice
	normalize	False	True	False
UserKNN CF jaccard	topK	637	807	1000
	shrink	0	0	0
	similarity	jaccard	jaccard	jaccard
	normalize	False	True	False
UserKNN CF asymmetric	topK	690	1000	1000
	shrink	1000	0	163
	similarity	asymmetric	asymmetric	asymmetric
	normalize	True	True	True
	asymmetric alpha	1.0291	0.4622	0.4379
	feature weighting	BM25	BM25	TF-IDF
UserKNN CF tversky	topK	533	940	935
	shrink	35	0	9
	similarity	tversky	tversky	tversky
	normalize	True	True	True
	tversky alpha	1.4634	2.0000	0.1591
	tversky beta	0.0885	0.0000	1.9682
ItemKNN CF cosine	topK	594	942	1000
	shrink	999	1000	448
	similarity	cosine	cosine	cosine
	normalize	True	True	False
	feature weighting	TF-IDF	BM25	TF-IDF
ItemKNN CF dice	topK	996	981	1000
	shrink	11	0	1000
	similarity	dice	dice	dice
	normalize	False	False	True
ItemKNN CF jaccard	topK	480	983	1000
	shrink	3	0	1000
	similarity	jaccard	jaccard	jaccard
	normalize	True	True	False
ItemKNN CF asymmetric	topK	1000	1000	1000
	shrink	649	845	850
	similarity	asymmetric	asymmetric	asymmetric
	normalize	True	True	True
	asymmetric alpha	0.2742	0.2281	1.5411
	feature weighting	TF-IDF	BM25	none
ItemKNN CF tversky	topK	421	1000	1000
	shrink	28	0	555
	similarity	tversky	tversky	tversky
	normalize	True	True	True
	tversky alpha	0.0103	1.9767	0.0000
	tversky beta	0.9612	2.0000	0.0000

Table 11. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	CiteULike	Pinterest	Epinions
$P^3\alpha$	topK	653	453	1000
	alpha	0.6310	1.1895	0.1164
	normalize similarity	False	True	False
$RP^3\beta$	topK	764	816	1000
	alpha	0.7110	1.1916	0.0000
	beta	0.2297	0.4365	0.0000
	normalize similarity	True	True	False
$EASE^R$	l2 norm	5.23E+02	1.32E+03	-
SLIM BPR	topK	803	726	1000
	epochs	165	235	370
	symmetric	False	True	False
	sgd mode	adam	adagrad	adagrad
	lambda i	1.00E-02	1.00E-05	1.00E-02
	lambda j	1.00E-02	3.06E-05	1.00E-02
	learning rate	1.00E-04	1.00E-01	1.00E-04
SLIM ElasticNet	topK	1000	705	1000
	l1 ratio	4.21E-05	1.55E-04	1.00E-05
	alpha	0.0265	0.0316	0.2911
MF BPR	sgd mode	adam	adagrad	adagrad
	epochs	1045	935	995
	num factors	175	146	200
	batch size	512	128	16
	positive reg	9.89E-03	7.72E-03	1.00E-02
	negative reg	7.25E-03	1.00E-02	1.00E-02
	learning rate	2.80E-03	4.63E-02	1.00E-01
MF FunkSVD	sgd mode	adam	adam	adam
	epochs	300	500	75
	use bias	True	False	True
	batch size	16	8	4
	num factors	55	37	1
	item reg	4.02E-05	1.00E-05	1.00E-05
	user reg	1.00E-02	1.00E-02	9.01E-03
	learning rate	2.44E-03	5.99E-04	1.58E-04
	negative quota	0.2792	0.0941	0.4998
PureSVD	num factors	320	77	1
NMF	num factors	122	77	45
	solver	mult. update	coord. descent	coord. descent
	init type	nndsvda	nndsvda	random
	beta loss	kullback-leibler	frobenius	frobenius
iALS	num factors	115	52	49
	confidence scaling	linear	linear	log
	alpha	15.4014	50.0000	9.8676
	epsilon	0.4163	0.0052	0.0013
	reg	1.00E-05	1.00E-05	6.20E-03
	epochs	60	90	100

Table 12. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	CiteULike	Pinterest	Epinions
CMN	epochs	50	5	45
	epochs gmf	100	100	100
	hops	3	3	3
	neg samples	4	4	4
	reg l2 cmn	1.00E-01	1.00E-01	1.00E-01
	reg l2 gmf	1.00E-04	1.00E-04	1.00E-04
	pretrain	True	True	True
	learning rate	1.00E-03	1.00E-03	1.00E-03
	verbose	False	False	False
	batch size	128	256	128
	embed size	50	50	40

F KDD: COLLABORATIVE VARIATIONAL AUTOENCODERS

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 13 and 14. The results of our evaluation can be seen in Table 15 (CiteULike-a, P=1), Table 16 (CiteULike-a, P=10), Table 17 (CiteULike-t, P=1), Table 18 (CiteULike-t, P=10). The corresponding optimal hyperparameters are reported in Table 23 (collaborative KNNs), Table 24 (non-neural machine learning and graph based), Table 25 (content-based KNNs), Table 26 (hybrid KNNs) and Table 27 (CVAE and CDL).

Lastly, the time required to train and evaluate the models is reported in Table 19 (CiteULike-a, P=1), Table 20 (CiteULike-a, P=10), Table 21 (CiteULike-t, P=1), Table 22 (CiteULike-t, P=10).

Table 13. Dataset characteristics.

Dataset	Interactions	Items	Users	Sparsity	Item features
CiteULike-a	204.9 k	16.9 k	5.5 k	99.78%	8.0 k
CiteULike-t	134.8 k	25.9 k	7.9 k	99.93%	20.0 k
NetflixPrize	15.3 M	9.2 k	407.2 k	99.59%	20.0 k

Table 14. Train data density for the different experimental settings of CDL.

Dataset	Experiment	Interactions	Density
CiteULike-a	P = 1	5.5 k	$5.8e-5$
CiteULike-a	P = 10	55.5 k	$5.8e-4$
CiteULike-t	P = 1	7.9 k	$3.8e-5$
CiteULike-t	P = 10	53.3 k	$2.5e-4$

Table 15. Experimental results for the CVAE method for the CiteULike-a P=1 dataset.

	REC@50	REC@100	REC@150	REC@200	REC@250	REC@300
Random	0.0027	0.0057	0.0084	0.0113	0.0142	0.0171
TopPopular	0.0253	0.0389	0.0486	0.0589	0.0651	0.0704
UserKNN CF cosine	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
UserKNN CF dice	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
UserKNN CF jaccard	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
UserKNN CF asymmetric	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
UserKNN CF tversky	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
ItemKNN CF cosine	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
ItemKNN CF dice	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
ItemKNN CF jaccard	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
ItemKNN CF asymmetric	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
ItemKNN CF tversky	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
$P^3\alpha$	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
$RP^3\beta$	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
$EASE^R$	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
SLIM BPR	0.0027	0.0052	0.0071	0.0102	0.0130	0.0155
SLIM ElasticNet	0.0026	0.0053	0.0069	0.0102	0.0127	0.0154
MF BPR	0.0046	0.0082	0.0119	0.0154	0.0188	0.0223
MF FunkSVD	0.0047	0.0087	0.0125	0.0161	0.0194	0.0227
PureSVD	0.0055	0.0111	0.0168	0.0226	0.0289	0.0356
NMF	0.0036	0.0067	0.0090	0.0121	0.0142	0.0167
iALS	0.0050	0.0102	0.0149	0.0190	0.0235	0.0279
ItemKNN CBF cosine	0.0242	0.0267	0.0284	0.0317	0.0341	0.0367
ItemKNN CBF dice	0.0210	0.0235	0.0253	0.0287	0.0310	0.0336
ItemKNN CBF jaccard	0.0253	0.0282	0.0301	0.0335	0.0360	0.0386
ItemKNN CBF asymmetric	0.0256	0.0295	0.0316	0.0350	0.0379	0.0405
ItemKNN CBF tversky	0.0173	0.0200	0.0217	0.0251	0.0275	0.0300
ItemKNN CFCBF cosine	0.0034	0.0061	0.0076	0.0110	0.0135	0.0161
ItemKNN CFCBF dice	0.0236	0.0262	0.0279	0.0313	0.0336	0.0362
ItemKNN CFCBF jaccard	0.0553	0.0614	0.0639	0.0670	0.0691	0.0717
ItemKNN CFCBF asymmetric	0.0029	0.0055	0.0071	0.0104	0.0130	0.0156
ItemKNN CFCBF tversky	0.0448	0.0512	0.0547	0.0583	0.0612	0.0639
CVAE	**0.0768**	**0.1171**	**0.1485**	**0.1744**	**0.1973**	**0.2168**
CDL	**0.0855**	**0.1208**	**0.1445**	**0.1623**	**0.1767**	**0.1901**

Table 16. Experimental results for the CVAE method for the CiteULike-a P=10 dataset.

	REC@50	REC@100	REC@150	REC@200	REC@250	REC@300
Random	0.0027	0.0057	0.0086	0.0112	0.0140	0.0172
TopPopular	0.0040	0.0078	0.0103	0.0204	0.0230	0.0258
UserKNN CF cosine	0.0769	0.1174	0.1443	0.1670	0.1859	0.2010
UserKNN CF dice	0.0788	0.1186	0.1463	0.1689	0.1875	0.2030
UserKNN CF jaccard	**0.0806**	0.1207	0.1480	0.1705	0.1887	0.2034
UserKNN CF asymmetric	0.0769	0.1173	0.1441	0.1671	0.1859	0.2013
UserKNN CF tversky	0.0799	0.1192	0.1466	0.1696	0.1880	0.2025
ItemKNN CF cosine	**0.0989**	0.1441	0.1752	0.1982	0.2156	0.2300
ItemKNN CF dice	**0.0945**	0.1373	0.1675	0.1912	0.2092	0.2233
ItemKNN CF jaccard	**0.0917**	0.1340	0.1642	0.1876	0.2062	0.2207
ItemKNN CF asymmetric	**0.0890**	0.1334	0.1631	0.1865	0.2065	0.2215
ItemKNN CF tversky	**0.0990**	0.1428	0.1736	0.1972	0.2143	0.2281
$P^3\alpha$	**0.0907**	0.1341	0.1636	0.1865	0.2055	0.2206
$RP^3\beta$	**0.0963**	0.1408	0.1692	0.1908	0.2090	0.2239
$EASE^R$	**0.0839**	0.1253	0.1546	0.1797	0.1988	0.2128
SLIM BPR	**0.0876**	0.1308	0.1583	0.1821	0.2005	0.2165
SLIM ElasticNet	**0.0869**	0.1281	0.1561	0.1789	0.1970	0.2115
MF BPR	0.0680	0.1011	0.1225	0.1402	0.1542	0.1663
MF FunkSVD	0.0483	0.0866	0.1157	0.1412	0.1636	0.1816
PureSVD	0.0715	0.1079	0.1313	0.1491	0.1636	0.1759
NMF	0.0628	0.1013	0.1285	0.1505	0.1679	0.1843
iALS	0.0779	0.1388	0.1834	0.2186	0.2472	0.2706
ItemKNN CBF cosine	**0.2235**	**0.3180**	**0.3829**	**0.4283**	**0.4651**	**0.4950**
ItemKNN CBF dice	**0.1734**	**0.2495**	**0.3035**	**0.3455**	**0.3798**	**0.4076**
ItemKNN CBF jaccard	**0.1752**	**0.2522**	**0.3045**	**0.3457**	**0.3794**	**0.4062**
ItemKNN CBF asymmetric	**0.2234**	**0.3186**	**0.3835**	**0.4288**	**0.4641**	**0.4945**
ItemKNN CBF tversky	**0.1748**	**0.2507**	**0.3040**	**0.3466**	**0.3814**	**0.4097**
ItemKNN CFCBF cosine	**0.1858**	**0.2816**	**0.3445**	**0.3930**	**0.4335**	**0.4642**
ItemKNN CFCBF dice	**0.1803**	**0.2600**	**0.3126**	**0.3558**	**0.3876**	**0.4126**
ItemKNN CFCBF jaccard	**0.1855**	**0.2650**	**0.3175**	**0.3598**	**0.3924**	**0.4181**
ItemKNN CFCBF asymmetric	**0.1712**	**0.2690**	**0.3355**	**0.3845**	**0.4237**	**0.4565**
ItemKNN CFCBF tversky	**0.1832**	**0.2618**	**0.3159**	**0.3577**	**0.3899**	**0.4162**
CVAE	0.0805	0.1569	0.2232	0.2760	0.3250	0.3687
CDL	0.0580	0.1108	0.1546	0.1946	0.2314	0.2640

Table 17. Experimental results for the CVAE method for the CiteULike-t P=1 dataset.

	REC@50	REC@100	REC@150	REC@200	REC@250	REC@300
Random	0.0026	0.0043	0.0065	0.0082	0.0101	0.0121
TopPopular	0.0134	0.0179	0.0247	0.0395	0.0456	0.0511
UserKNN CF cosine	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
UserKNN CF dice	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
UserKNN CF jaccard	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
UserKNN CF asymmetric	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
UserKNN CF tversky	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
ItemKNN CF cosine	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
ItemKNN CF dice	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
ItemKNN CF jaccard	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
ItemKNN CF asymmetric	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
ItemKNN CF tversky	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
$P^3\alpha$	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
$RP^3\beta$	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
$EASE^R$	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
SLIM BPR	0.0013	0.0038	0.0066	0.0084	0.0102	0.0122
SLIM ElasticNet	0.0012	0.0038	0.0065	0.0084	0.0104	0.0123
MF BPR	0.0054	0.0094	0.0125	0.0153	0.0176	0.0203
MF FunkSVD	0.0029	0.0057	0.0083	0.0107	0.0130	0.0151
PureSVD	0.0053	0.0094	0.0140	0.0193	0.0246	0.0283
NMF	0.0037	0.0054	0.0074	0.0089	0.0107	0.0129
iALS	0.0061	0.0100	0.0137	0.0173	0.0206	0.0245
ItemKNN CBF cosine	**0.0858**	**0.1248**	**0.1549**	**0.1790**	**0.2000**	**0.2180**
ItemKNN CBF dice	**0.1133**	**0.1566**	**0.1887**	**0.2122**	**0.2312**	**0.2478**
ItemKNN CBF jaccard	**0.1136**	**0.1567**	**0.1874**	**0.2116**	**0.2283**	**0.2433**
ItemKNN CBF asymmetric	**0.0916**	**0.1274**	**0.1493**	**0.1633**	**0.1743**	**0.1813**
ItemKNN CBF tversky	**0.1135**	**0.1566**	**0.1881**	**0.2125**	**0.2315**	**0.2490**
ItemKNN CFCBF cosine	**0.0944**	**0.1349**	**0.1647**	**0.1864**	**0.2059**	**0.2243**
ItemKNN CFCBF dice	**0.1129**	**0.1552**	**0.1867**	**0.2105**	**0.2300**	**0.2463**
ItemKNN CFCBF jaccard	**0.1133**	**0.1559**	**0.1848**	**0.2066**	**0.2218**	**0.2338**
ItemKNN CFCBF asymmetric	**0.0448**	0.0525	0.0554	0.0575	0.0590	0.0609
ItemKNN CFCBF tversky	**0.1133**	**0.1555**	**0.1855**	**0.2085**	**0.2249**	**0.2389**
CVAE	0.0430	0.0639	0.0803	0.0950	0.1076	0.1200
CDL	0.0351	0.0573	0.0715	0.0822	0.0915	0.0989

Table 18. Experimental results for the CVAE method for the CiteULike-t P=10 dataset.

	REC@50	REC@100	REC@150	REC@200	REC@250	REC@300
Random	0.0016	0.0040	0.0054	0.0069	0.0088	0.0106
TopPopular	0.0578	0.0862	0.1100	0.1257	0.1416	0.1568
UserKNN CF cosine	0.2141	0.2661	0.2964	0.3169	0.3320	0.3437
UserKNN CF dice	0.2138	0.2661	0.2958	0.3171	0.3325	0.3444
UserKNN CF jaccard	0.2139	0.2648	0.2954	0.3154	0.3308	0.3426
UserKNN CF asymmetric	0.2134	0.2656	0.2963	0.3170	0.3341	0.3458
UserKNN CF tversky	0.2120	0.2651	0.2955	0.3172	0.3336	0.3462
ItemKNN CF cosine	0.2133	0.2658	0.2964	0.3173	0.3342	0.3457
ItemKNN CF dice	0.2157	0.2681	0.2995	0.3206	0.3366	0.3492
ItemKNN CF jaccard	0.2167	0.2685	0.2994	0.3205	0.3366	0.3491
ItemKNN CF asymmetric	0.2027	0.2616	0.2958	0.3197	0.3381	0.3525
ItemKNN CF tversky	0.2015	0.2606	0.2949	0.3190	0.3372	0.3521
$P^3\alpha$	0.2276	0.2769	0.3069	0.3280	0.3450	0.3571
$RP^3\beta$	0.2073	0.2636	0.2975	0.3210	0.3398	0.3538
$EASE^R$	0.2130	0.2611	0.2886	0.3084	0.3253	0.3383
SLIM BPR	0.2187	0.2681	0.2988	0.3196	0.3383	0.3516
SLIM ElasticNet	0.2102	0.2612	0.2930	0.3129	0.3315	0.3446
MF BPR	0.1551	0.1990	0.2279	0.2482	0.2649	0.2824
MF FunkSVD	0.1231	0.1613	0.1857	0.2019	0.2155	0.2276
PureSVD	0.1329	0.1730	0.1994	0.2215	0.2393	0.2547
NMF	0.1082	0.1429	0.1771	0.2002	0.2199	0.2420
iALS	0.2338	0.3107	0.3566	0.3925	0.4175	0.4374
ItemKNN CBF cosine	0.1625	0.2237	0.2682	0.3001	0.3269	0.3493
ItemKNN CBF dice	0.1665	0.2323	0.2832	0.3206	0.3512	0.3756
ItemKNN CBF jaccard	0.1681	0.2342	0.2851	0.3210	0.3505	0.3761
ItemKNN CBF asymmetric	0.1630	0.2259	0.2689	0.3031	0.3314	0.3562
ItemKNN CBF tversky	0.1599	0.2291	0.2791	0.3170	0.3469	0.3727
ItemKNN CFCBF cosine	**0.2675**	**0.3490**	**0.3939**	0.4246	0.4519	0.4740
ItemKNN CFCBF dice	0.2166	0.2868	0.3361	0.3738	0.4024	0.4284
ItemKNN CFCBF jaccard	0.2172	0.2880	0.3363	0.3741	0.4026	0.4271
ItemKNN CFCBF asymmetric	**0.2412**	0.3160	0.3663	0.4051	0.4321	0.4548
ItemKNN CFCBF tversky	0.2178	0.2872	0.3383	0.3758	0.4053	0.4279
CVAE	0.2387	0.3274	0.3849	**0.4263**	**0.4606**	**0.4854**
CDL	0.2231	0.3019	0.3565	0.4031	0.4351	0.4618

Table 19. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-a P=1 dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	15.06 [sec]	369
TopPopular	0.02 [sec]	15.06 [sec]	369
UserKNN CF cosine	0.17 ± 0.02 [sec]	14.92 [sec]	372
UserKNN CF dice	0.17 ± 0.01 [sec]	15.25 [sec]	364
UserKNN CF jaccard	0.18 ± 0.01 [sec]	14.90 [sec]	372
UserKNN CF asymmetric	0.19 ± 0.00 [sec]	15.04 [sec]	369
UserKNN CF tversky	0.20 ± 0.01 [sec]	14.92 [sec]	372
ItemKNN CF cosine	1.10 ± 0.13 [sec]	14.92 [sec]	372
ItemKNN CF dice	1.16 ± 0.01 [sec]	14.97 [sec]	371
ItemKNN CF jaccard	1.20 ± 0.01 [sec]	14.82 [sec]	374
ItemKNN CF asymmetric	1.31 ± 0.02 [sec]	14.90 [sec]	372
ItemKNN CF tversky	1.42 ± 0.01 [sec]	14.93 [sec]	372
$P^3\alpha$	4.04 ± 0.09 [sec]	14.90 [sec]	372
$RP^3\beta$	4.11 ± 0.03 [sec]	14.86 [sec]	374
$EASE^R$	92.76 [sec] / 1.55 ± 0.01 [min]	34.46 [sec]	161
SLIM BPR	28.74 ± 8.04 [sec]	14.85 ± 0.14 [sec]	371
SLIM ElasticNet	574.75 [sec] / 9.58 ± 0.11 [min]	17.19 [sec]	323
MF BPR	47.23 ± 71.90 [sec]	16.61 [sec]	334
MF FunkSVD	37.78 ± 39.27 [sec]	17.44 ± 0.36 [sec]	314
PureSVD	0.74 ± 0.44 [sec]	15.48 [sec]	359
NMF	19.72 ± 25.84 [sec]	16.45 ± 0.15 [sec]	340
iALS	100.51 [sec] / 1.68 ± 0.89 [min]	16.76 ± 0.13 [sec]	333
ItemKNN CBF cosine	7.75 ± 0.62 [sec]	14.41 ± 0.67 [sec]	405
ItemKNN CBF dice	8.01 ± 0.62 [sec]	14.27 ± 0.72 [sec]	403
ItemKNN CBF jaccard	7.98 ± 0.59 [sec]	14.42 ± 0.63 [sec]	402
ItemKNN CBF asymmetric	7.94 ± 0.47 [sec]	14.28 ± 0.53 [sec]	401
ItemKNN CBF tversky	8.24 ± 0.59 [sec]	14.31 ± 0.52 [sec]	408
ItemKNN CFCBF cosine	7.86 ± 0.55 [sec]	14.40 ± 0.80 [sec]	411
ItemKNN CFCBF dice	7.99 ± 0.58 [sec]	14.54 ± 0.70 [sec]	404
ItemKNN CFCBF jaccard	8.03 ± 0.57 [sec]	14.55 ± 0.47 [sec]	394
ItemKNN CFCBF asymmetric	8.14 ± 0.59 [sec]	14.39 ± 0.65 [sec]	412
ItemKNN CFCBF tversky	8.22 ± 0.54 [sec]	14.70 ± 0.59 [sec]	396
CVAE	2151.48 [sec] / 35.86 [min]	23.07 [sec]	241
CDL	5461.42 [sec] / 1.52 [hour]	17.32 [sec]	321

Table 20. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-a P=10 dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.02 [sec]	14.45 [sec]	356
TopPopular	0.00 [sec]	13.93 [sec]	369
UserKNN CF cosine	0.25 ± 0.03 [sec]	14.19 ± 0.09 [sec]	363
UserKNN CF dice	0.26 ± 0.01 [sec]	14.12 ± 0.09 [sec]	364
UserKNN CF jaccard	0.26 ± 0.01 [sec]	14.29 ± 0.11 [sec]	358
UserKNN CF asymmetric	0.27 ± 0.01 [sec]	14.15 ± 0.07 [sec]	361
UserKNN CF tversky	0.29 ± 0.01 [sec]	14.08 ± 0.05 [sec]	365
ItemKNN CF cosine	1.38 ± 0.07 [sec]	14.16 ± 0.04 [sec]	362
ItemKNN CF dice	1.37 ± 0.02 [sec]	14.13 ± 0.04 [sec]	364
ItemKNN CF jaccard	1.40 ± 0.02 [sec]	14.10 ± 0.02 [sec]	364
ItemKNN CF asymmetric	1.49 ± 0.02 [sec]	14.13 ± 0.05 [sec]	362
ItemKNN CF tversky	1.63 ± 0.02 [sec]	14.18 ± 0.09 [sec]	359
$P^3\alpha$	4.36 ± 0.05 [sec]	14.26 ± 0.14 [sec]	364
$RP^3\beta$	4.54 ± 0.09 [sec]	14.24 ± 0.05 [sec]	362
$EASE^R$	93.04 [sec] / 1.55 ± 0.01 [min]	36.34 ± 0.12 [sec]	141
SLIM BPR	241.65 [sec] / 4.03 ± 1.30 [min]	14.46 ± 0.14 [sec]	356
SLIM ElasticNet	651.80 [sec] / 10.86 ± 0.57 [min]	13.86 ± 0.19 [sec]	367
MF BPR	717.46 [sec] / 11.96 ± 8.87 [min]	15.18 ± 0.48 [sec]	338
MF FunkSVD	546.76 [sec] / 9.11 ± 6.29 [min]	14.83 ± 0.33 [sec]	350
PureSVD	1.29 ± 0.54 [sec]	15.01 ± 0.57 [sec]	347
NMF	184.83 [sec] / 3.08 ± 3.41 [min]	15.62 ± 0.19 [sec]	328
iALS	325.86 [sec] / 5.43 ± 3.66 [min]	15.55 ± 0.18 [sec]	326
ItemKNN CBF cosine	8.25 ± 0.60 [sec]	14.59 ± 0.24 [sec]	346
ItemKNN CBF dice	8.36 ± 0.40 [sec]	14.33 ± 0.13 [sec]	358
ItemKNN CBF jaccard	8.33 ± 0.42 [sec]	14.55 ± 0.16 [sec]	353
ItemKNN CBF asymmetric	8.61 ± 0.52 [sec]	14.66 ± 0.26 [sec]	347
ItemKNN CBF tversky	8.66 ± 0.41 [sec]	14.41 ± 0.12 [sec]	355
ItemKNN CFCBF cosine	8.47 ± 0.54 [sec]	14.32 ± 0.32 [sec]	355
ItemKNN CFCBF dice	8.49 ± 0.41 [sec]	14.23 ± 0.20 [sec]	360
ItemKNN CFCBF jaccard	8.50 ± 0.41 [sec]	14.23 ± 0.10 [sec]	357
ItemKNN CFCBF asymmetric	8.59 ± 0.54 [sec]	14.22 ± 0.32 [sec]	356
ItemKNN CFCBF tversky	8.77 ± 0.45 [sec]	14.31 ± 0.20 [sec]	361
CVAE	4555.65 [sec] / 1.27 [hour]	21.33 [sec]	241
CDL	5443.56 [sec] / 1.51 [hour]	15.26 [sec]	337

Table 21. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-t P=1 dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	22.92 [sec]	347
TopPopular	0.00 [sec]	22.42 [sec]	355
UserKNN CF cosine	0.33 ± 0.03 [sec]	21.79 [sec]	365
UserKNN CF dice	0.34 ± 0.01 [sec]	21.92 [sec]	363
UserKNN CF jaccard	0.35 ± 0.01 [sec]	21.82 [sec]	364
UserKNN CF asymmetric	0.37 ± 0.01 [sec]	21.68 [sec]	367
UserKNN CF tversky	0.40 ± 0.01 [sec]	21.71 [sec]	366
ItemKNN CF cosine	2.42 ± 0.35 [sec]	21.79 [sec]	365
ItemKNN CF dice	2.59 ± 0.02 [sec]	21.84 [sec]	364
ItemKNN CF jaccard	2.68 ± 0.02 [sec]	21.71 [sec]	366
ItemKNN CF asymmetric	2.93 ± 0.03 [sec]	21.75 [sec]	365
ItemKNN CF tversky	3.21 ± 0.02 [sec]	21.87 [sec]	363
$P^3\alpha$	10.31 ± 0.07 [sec]	21.70 [sec]	366
$RP^3\beta$	10.26 ± 0.07 [sec]	21.73 [sec]	366
$EASE^R$	324.68 [sec] / 5.41 ± 0.02 [min]	47.38 [sec]	168
SLIM BPR	108.13 [sec] / 1.80 ± 0.55 [min]	22.03 ± 0.02 [sec]	361
SLIM ElasticNet	1223.53 [sec] / 20.39 ± 0.20 [min]	21.75 [sec]	365
MF BPR	158.20 [sec] / 2.64 ± 3.88 [min]	25.31 ± 0.10 [sec]	315
MF FunkSVD	101.31 [sec] / 1.69 ± 1.52 [min]	25.15 ± 0.07 [sec]	315
PureSVD	1.53 ± 0.65 [sec]	22.63 ± 0.22 [sec]	347
NMF	18.22 ± 45.02 [sec]	24.81 [sec]	320
iALS	118.47 [sec] / 1.97 ± 1.22 [min]	25.19 ± 0.12 [sec]	314
ItemKNN CBF cosine	5.86 ± 1.07 [sec]	20.60 [sec]	386
ItemKNN CBF dice	6.12 ± 0.97 [sec]	20.89 [sec]	381
ItemKNN CBF jaccard	6.18 ± 0.97 [sec]	20.93 [sec]	380
ItemKNN CBF asymmetric	6.36 ± 1.04 [sec]	20.34 [sec]	391
ItemKNN CBF tversky	6.70 ± 0.94 [sec]	21.09 [sec]	377
ItemKNN CFCBF cosine	5.84 ± 1.01 [sec]	20.89 [sec]	380
ItemKNN CFCBF dice	6.12 ± 0.94 [sec]	20.95 [sec]	379
ItemKNN CFCBF jaccard	6.17 ± 0.92 [sec]	20.62 [sec]	385
ItemKNN CFCBF asymmetric	6.38 ± 1.02 [sec]	19.40 [sec]	410
ItemKNN CFCBF tversky	6.71 ± 0.94 [sec]	20.67 [sec]	385
CVAE	3560.74 [sec] / 59.35 [min]	31.19 [sec]	255
CDL	22823.11 [sec] / 6.34 [hour]	24.51 [sec]	324

Table 22. Computation time for the algorithms in the selected results for the CVAE method on the CiteULike-t P=10 dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	7.69 [sec]	338
TopPopular	0.00 [sec]	7.51 [sec]	346
UserKNN CF cosine	0.40 ± 0.04 [sec]	7.48 ± 0.02 [sec]	348
UserKNN CF dice	0.41 ± 0.01 [sec]	7.52 ± 0.05 [sec]	346
UserKNN CF jaccard	0.42 ± 0.01 [sec]	7.51 ± 0.01 [sec]	346
UserKNN CF asymmetric	0.45 ± 0.01 [sec]	7.50 ± 0.11 [sec]	343
UserKNN CF tversky	0.48 ± 0.01 [sec]	7.49 ± 0.01 [sec]	346
ItemKNN CF cosine	2.17 ± 0.46 [sec]	7.51 ± 0.03 [sec]	346
ItemKNN CF dice	3.01 ± 0.06 [sec]	7.47 ± 0.02 [sec]	349
ItemKNN CF jaccard	3.10 ± 0.07 [sec]	7.40 ± 0.11 [sec]	347
ItemKNN CF asymmetric	3.35 ± 0.07 [sec]	7.53 ± 0.07 [sec]	341
ItemKNN CF tversky	3.65 ± 0.09 [sec]	7.47 ± 0.01 [sec]	348
$P^3\alpha$	10.63 ± 0.07 [sec]	7.58 [sec]	343
$RP^3\beta$	10.71 ± 0.10 [sec]	7.47 ± 0.08 [sec]	345
$EASE^R$	326.11 [sec] / 5.44 ± 0.02 [min]	21.18 ± 0.51 [sec]	120
SLIM BPR	529.68 [sec] / 8.83 ± 4.30 [min]	7.65 ± 0.10 [sec]	337
SLIM ElasticNet	1454.18 [sec] / 24.24 ± 1.19 [min]	7.46 ± 0.19 [sec]	339
MF BPR	1143.88 [sec] / 19.06 ± 13.39 [min]	8.27 ± 0.14 [sec]	308
MF FunkSVD	1004.38 [sec] / 16.74 ± 8.98 [min]	8.59 ± 0.09 [sec]	299
PureSVD	2.03 ± 0.80 [sec]	7.74 ± 0.01 [sec]	336
NMF	478.36 [sec] / 7.97 ± 10.31 [min]	8.57 ± 0.15 [sec]	300
iALS	570.10 [sec] / 9.50 ± 6.63 [min]	8.57 ± 0.02 [sec]	303
ItemKNN CBF cosine	6.24 ± 0.87 [sec]	7.58 ± 0.07 [sec]	345
ItemKNN CBF dice	6.34 ± 0.62 [sec]	7.62 ± 0.02 [sec]	340
ItemKNN CBF jaccard	6.43 ± 0.64 [sec]	7.62 ± 0.17 [sec]	339
ItemKNN CBF asymmetric	7.03 ± 0.77 [sec]	7.61 ± 0.20 [sec]	335
ItemKNN CBF tversky	7.04 ± 0.67 [sec]	7.69 ± 0.16 [sec]	333
ItemKNN CFCBF cosine	6.38 ± 0.76 [sec]	7.71 ± 0.22 [sec]	329
ItemKNN CFCBF dice	6.33 ± 0.65 [sec]	7.57 ± 0.11 [sec]	346
ItemKNN CFCBF jaccard	6.38 ± 0.65 [sec]	7.54 ± 0.04 [sec]	346
ItemKNN CFCBF asymmetric	7.31 ± 0.76 [sec]	7.65 ± 0.32 [sec]	317
ItemKNN CFCBF tversky	6.87 ± 0.60 [sec]	7.49 ± 0.05 [sec]	346
CVAE	9969.70 [sec] / 2.77 [hour]	11.68 [sec]	222
CDL	22343.75 [sec] / 6.21 [hour]	8.65 [sec]	300

Table 23. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	CiteULike-a P=1	CiteULike-a P=10	CiteULike-t P=1	CiteULike-t P=10
UserKNN CF cosine	topK	844	455	945	347
	shrink	998	490	229	1000
	similarity	cosine	cosine	cosine	cosine
	normalize	False	False	False	True
	feature weighting	BM25	TF-IDF	TF-IDF	none
UserKNN CF dice	topK	396	317	203	359
	shrink	551	1000	832	1000
	similarity	dice	dice	dice	dice
	normalize	False	True	False	False
UserKNN CF jaccard	topK	436	348	660	327
	shrink	918	1000	205	1000
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	False	False	True	True
UserKNN CF asymmetric	topK	748	483	584	777
	shrink	733	1000	709	314
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	1.9382	2.0000	1.5064	1.9573
	feature weighting	none	none	BM25	none
UserKNN CF tversky	topK	808	870	264	892
	shrink	917	967	705	981
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	1.3044	0.0119	0.2812	0.1122
	tversky beta	1.5023	1.9836	1.1578	0.0128
ItemKNN CF cosine	topK	484	423	139	419
	shrink	555	936	127	235
	similarity	cosine	cosine	cosine	cosine
	normalize	False	True	True	False
	feature weighting	none	TF-IDF	none	TF-IDF
ItemKNN CF dice	topK	472	540	610	760
	shrink	292	61	402	991
	similarity	dice	dice	dice	dice
	normalize	False	False	False	True
ItemKNN CF jaccard	topK	114	612	548	754
	shrink	784	93	679	378
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	True	True	False	True
ItemKNN CF asymmetric	topK	700	900	848	1000
	shrink	430	1000	797	0
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	1.5472	0.0000	0.1235	0.0000
	feature weighting	TF-IDF	TF-IDF	BM25	none
ItemKNN CF tversky	topK	831	728	317	749
	shrink	810	44	32	0
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	0.6361	1.4249	0.9626	0.0000
	tversky beta	1.4516	1.0858	1.8163	2.0000

Table 24. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	CiteULike-a P=1	CiteULike-a P=10	CiteULike-t P=1	CiteULike-t P=10
$P^3\alpha$	topK	688	662	314	961
	alpha	1.1735	0.5112	0.7234	0.3851
	normalize similarity	True	False	False	False
$RP^3\beta$	topK	54	710	537	1000
	alpha	1.1242	0.0000	0.4656	0.8688
	beta	1.3282	0.0000	0.2366	0.0000
	normalize similarity	True	True	True	True
$EASE^R$	l2 norm	7.93E+04	9.94E+06	4.84E+00	1.11E+02
SLIM BPR	topK	402	458	537	818
	epochs	45	160	50	135
	symmetric	False	False	False	False
	sgd mode	sgd	adagrad	adagrad	adagrad
	lambda i	2.35E-03	1.00E-05	1.44E-03	1.00E-02
	lambda j	7.93E-03	1.00E-05	1.09E-04	1.00E-02
	learning rate	2.65E-04	1.00E-04	1.58E-04	1.00E-04
SLIM ElasticNet	topK	795	450	644	949
	l1 ratio	1.72E-01	2.51E-04	1.23E-02	3.61E-05
	alpha	0.4724	0.0179	0.8273	1.0000
MF BPR	sgd mode	adam	adam	adagrad	adagrad
	epochs	25	660	20	1005
	num factors	90	200	73	200
	batch size	64	256	16	64
	positive reg	7.61E-04	1.00E-02	1.27E-04	1.00E-02
	negative reg	6.05E-04	9.89E-03	3.37E-04	1.00E-02
	learning rate	6.15E-04	4.56E-03	1.93E-02	1.00E-01
MF FunkSVD	sgd mode	adagrad	adam	adam	adagrad
	epochs	45	500	35	485
	use bias	False	False	False	True
	batch size	128	128	256	8
	num factors	48	28	77	200
	item reg	2.36E-05	3.45E-03	2.22E-04	1.00E-02
	user reg	6.40E-04	9.10E-04	2.28E-04	1.00E-02
	learning rate	8.37E-02	3.98E-03	9.13E-04	1.00E-01
	negative quota	0.1630	0.1334	0.4428	0.5000
PureSVD	num factors	284	350	350	350
NMF	num factors	245	188	127	301
	solver	mult. update	mult. update	mult. update	coord. descent
	init type	nndsvda	nndsvda	random	random
	beta loss	kullback-leibler	frobenius	kullback-leibler	frobenius
iALS	num factors	195	50	195	49
	confidence scaling	log	log	linear	log
	alpha	34.4360	50.0000	16.8297	50.0000
	epsilon	0.0055	0.0010	3.4947	0.0032
	reg	3.02E-04	1.00E-05	2.90E-04	1.00E-02
	epochs	5	60	5	105

Table 25. Hyperparameter values for our content based KNN baselines on all datasets.

Algorithm	Hyperparameter	CiteULike-a P=1	CiteULike-a P=10	CiteULike-t P=1	CiteULike-t P=10
ItemKNN CBF cosine	topK	5	849	692	563
	shrink	0	825	955	50
	similarity	cosine	cosine	cosine	cosine
	normalize	True	True	True	True
	feature weighting	none	BM25	BM25	TF-IDF
ItemKNN CBF dice	topK	5	571	626	636
	shrink	0	0	474	7
	similarity	dice	dice	dice	dice
	normalize	True	True	True	True
ItemKNN CBF jaccard	topK	13	527	429	543
	shrink	637	0	736	9
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	True	False	True	False
ItemKNN CBF asymmetric	topK	10	801	219	976
	shrink	14	1000	895	92
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	1.7300	0.4022	1.3276	0.5989
	feature weighting	TF-IDF	BM25	TF-IDF	TF-IDF
ItemKNN CBF tversky	topK	5	572	849	1000
	shrink	1000	0	491	0
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	0.0000	2.0000	1.4789	2.0000
	tversky beta	2.0000	2.0000	1.7951	2.0000

Table 26. Hyperparameter values for our hybrid KNN baselines on all datasets.

Algorithm	Hyperparameter	CiteULike-a P=1	CiteULike-a P=10	CiteULike-t P=1	CiteULike-t P=10
ItemKNN CFCBF cosine	topK	19	807	962	1000
	shrink	140	1000	151	1000
	similarity	cosine	cosine	cosine	cosine
	normalize	True	False	True	False
	feature weighting	BM25	BM25	TF-IDF	TF-IDF
	ICM weight	0.0101	1.1447	3.7971	1.5675
ItemKNN CFCBF dice	topK	7	492	918	332
	shrink	194	50	944	1000
	similarity	dice	dice	dice	dice
	normalize	False	False	False	False
	ICM weight	0.8195	72.9657	0.5285	0.0100
ItemKNN CFCBF jaccard	topK	34	554	418	347
	shrink	51	0	708	1000
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	True	True	False	True
	ICM weight	2.1492	0.0100	0.0121	100.0000
ItemKNN CFCBF asymmetric	topK	26	976	342	1000
	shrink	22	1000	252	1000
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	0.1370	0.0000	1.5682	0.0000
	feature weighting	BM25	TF-IDF	BM25	BM25
	ICM weight	0.0145	1.1144	0.0119	6.7370
ItemKNN CFCBF tversky	topK	59	585	530	283
	shrink	991	0	733	672
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	1.7260	1.3555	1.3336	0.0000
	tversky beta	0.6061	2.0000	0.8886	2.0000
	ICM weight	27.7050	100.0000	1.1171	0.0100

Table 27. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	CiteULike-a P=1	CiteULike-a P=10	CiteULike-t P=1	CiteULike-t P=10
CVAE	epochs	5	35	5	60
	learning rate vae	1.00E-02	1.00E-02	1.00E-02	1.00E-02
	learning rate cvae	1.00E-03	1.00E-03	1.00E-03	1.00E-03
	num factors	50	50	50	50
	dimensions vae	[200, 100]	[200, 100]	[200, 100]	[200, 100]
	epochs vae	[50, 50]	[50, 50]	[50, 50]	[50, 50]
	batch size	128	128	128	128
	lambda u	1.00E-01	1.00E-01	1.00E-01	1.00E-01
	lambda v	10	10	10	10
	lambda r	1	1	1	1
	a	1	1	1	1
	b	0.0100	0.0100	0.0100	0.0100
	M	300	300	300	300
CDL	para lv	10	10	10	10
	para lu	1	1	1	1
	para ln	1000.0000	1000.0000	1000.0000	1000.0000
	batch size	128	128	128	128
	epoch sdae	200	200	200	200
	epoch dae	200	200	200	200

G RECSYS: SPECTRAL COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 28 and 29. The results of our evaluation can be seen in Table 30 (Amazon Instant Video), Table 31 (Hetrec), Table 32 (Movielens 1M original split) and Table 33 (Movielens 1M our split). The results for beyond-accuracy metrics can be seen in Table 34 (Amazon Instant Video), Table 35 (Hetrec), Table 36 (Movielens 1M original split) and Table 37 (Movielens 1M our split). The corresponding optimal hyperparameters are reported in Table 42 (collaborative KNNs), Table 43 (non-neural machine learning and graph based) and 44 (SpectralCF original hyperparameters and ours).

Lastly, the time required to train and evaluate the models is reported in Table 38 (Amazon Instant Video), Table 39 (Hetrec), Table 40 (Movielens 1M original split) and Table 41 (Movielens 1M our split).

Table 28. Dataset characteristics.

Dataset	Interactions	Items	Users	Density
Movielens 1M	226 K	3706	6040	1.01%
Hetrec	71 K	10109	2113	0.33%
Amazon Instant Video	22 K	5860	3113	0.12%

Table 29. The statistics compare the popularity bias of the two splits of Movielens 1M we report the results of, the original one provided by the authors and the split generated by us following the description provided in the paper. The three rows refer to the statistics of the dataset as a whole and those of the train and test data split. In a truly random data split the statistics of train and test data should mirror closely those of the full dataset. It is possible to see that the original test data (values in bold) has very different statistical properties than the original full dataset, hinting at a possible error in the splitting procedure. *Kendall Tau* counts the number of pairwise disagreements between two ranking lists, its result is the percentage of item pairs whose ordering is discordant between the two splits. This metric highlight how inconsistent is the original test data with respect to the original train data.

	Max pop	Avg pop	Gini Index	Kendall Tau	Shannon
			Movielens original		
Full data	1963.00	61.08	0.78	1.00	9.99
Train data	1936.00	48.37	0.79	0.87	9.89
Test data	**1361.00**	12.71	**0.92**	**0.44**	**8.49**
			Movielens ours		
Full data	1963.00	58.08	0.79	1.00	9.99
Train data	1575.00	46.44	0.79	0.97	9.99
Test data	388.00	11.64	0.80	0.85	9.93

Table 30. Experimental results for the SpectralCF method for the Amazon Instant Video dataset.

	@ 20		@ 40		@ 60		@ 80		@ 100	
	REC	MAP	REC	MAP	REC	MAP	REC	MAP	REC	MAP
Random	0.0038	0.0004	0.0066	0.0005	0.0115	0.0006	0.0136	0.0006	0.0172	0.0007
TopPopular	**0.1134**	**0.0288**	**0.1687**	**0.0308**	0.2067	**0.0316**	**0.2448**	**0.0322**	**0.2716**	**0.0326**
UserKNN CF cosine	**0.2853**	**0.1200**	**0.3559**	**0.1228**	**0.3986**	**0.1238**	**0.4297**	**0.1243**	**0.4527**	**0.1246**
UserKNN CF dice	**0.2845**	**0.1198**	**0.3550**	**0.1225**	**0.3989**	**0.1236**	**0.4294**	**0.1240**	**0.4520**	**0.1243**
UserKNN CF jaccard	**0.2850**	**0.1198**	**0.3544**	**0.1225**	**0.3991**	**0.1235**	**0.4282**	**0.1240**	**0.4515**	**0.1243**
UserKNN CF asymmetric	**0.2850**	**0.1201**	**0.3552**	**0.1228**	**0.3991**	**0.1239**	**0.4297**	**0.1244**	**0.4526**	**0.1247**
UserKNN CF tversky	**0.2856**	**0.1196**	**0.3544**	**0.1223**	**0.3991**	**0.1233**	**0.4286**	**0.1238**	**0.4515**	**0.1241**
ItemKNN CF cosine	**0.2858**	**0.1209**	**0.3535**	**0.1236**	**0.3985**	**0.1246**	**0.4303**	**0.1252**	**0.4528**	**0.1255**
ItemKNN CF dice	**0.2968**	**0.1290**	**0.3628**	**0.1316**	**0.4010**	**0.1325**	**0.4262**	**0.1329**	**0.4487**	**0.1332**
ItemKNN CF jaccard	**0.2957**	**0.1296**	**0.3628**	**0.1322**	**0.3996**	**0.1331**	**0.4250**	**0.1335**	**0.4482**	**0.1338**
ItemKNN CF asymmetric	**0.3044**	**0.1309**	**0.3712**	**0.1336**	**0.4127**	**0.1346**	**0.4426**	**0.1351**	**0.4653**	**0.1354**
ItemKNN CF tversky	**0.2913**	**0.1195**	**0.3594**	**0.1221**	**0.4021**	**0.1231**	**0.4319**	**0.1236**	**0.4538**	**0.1239**
$P^3\alpha$	**0.3019**	**0.1276**	**0.3721**	**0.1304**	**0.4151**	**0.1314**	**0.4433**	**0.1319**	**0.4648**	**0.1322**
$RP^3\beta$	**0.3029**	**0.1354**	**0.3715**	**0.1382**	**0.4119**	**0.1391**	**0.4378**	**0.1396**	**0.4584**	**0.1398**
$EASE^R$	**0.2907**	**0.1215**	**0.3622**	**0.1243**	**0.3994**	**0.1251**	**0.4238**	**0.1255**	**0.4467**	**0.1258**
SLIM BPR	**0.2973**	**0.1326**	**0.3671**	**0.1353**	**0.3993**	**0.1361**	**0.4279**	**0.1366**	**0.4501**	**0.1369**
SLIM ElasticNet	**0.2882**	**0.1224**	**0.3576**	**0.1250**	**0.3966**	**0.1260**	**0.4256**	**0.1264**	**0.4482**	**0.1267**
MF BPR	**0.2320**	**0.0951**	**0.2925**	**0.0974**	**0.3340**	**0.0983**	**0.3678**	**0.0989**	**0.3921**	**0.0992**
MF FunkSVD	**0.1800**	**0.0484**	**0.2615**	**0.0516**	**0.3129**	**0.0527**	**0.3488**	**0.0533**	**0.3752**	**0.0536**
PureSVD	**0.1555**	**0.0533**	**0.2079**	**0.0553**	**0.2466**	**0.0561**	**0.2758**	**0.0566**	**0.3031**	**0.0569**
NMF	**0.1406**	**0.0569**	**0.1850**	**0.0586**	**0.2147**	**0.0592**	**0.2454**	**0.0597**	0.2654	**0.0600**
iALS	**0.2741**	**0.0951**	**0.3526**	**0.0981**	**0.4003**	**0.0993**	**0.4330**	**0.0998**	**0.4582**	**0.1002**
SpectralCF	0.1130	0.0246	0.1652	0.0266	0.2090	0.0275	0.2393	0.0280	0.2712	0.0283
SpectralCF article default	0.1063	0.0255	0.1542	0.0273	0.1969	0.0282	0.2332	0.0288	0.2670	0.0292

Table 31. Experimental results for the SpectralCF method for the Hetrec dataset.

	@ 20		@ 40		@ 60		@ 80		@ 100	
	REC	MAP	REC	MAP	REC	MAP	REC	MAP	REC	MAP
Random	0.0028	0.0003	0.0051	0.0004	0.0070	0.0004	0.0093	0.0004	0.0113	0.0005
TopPopular	**0.2044**	0.0639	**0.2918**	0.0684	**0.3484**	0.0710	**0.3927**	0.0726	**0.4291**	0.0737
UserKNN CF cosine	**0.2649**	**0.1081**	**0.3604**	**0.1132**	**0.4139**	**0.1159**	**0.4576**	**0.1177**	**0.4927**	**0.1189**
UserKNN CF dice	**0.2627**	**0.1081**	**0.3528**	**0.1132**	**0.4061**	**0.1158**	**0.4522**	**0.1176**	**0.4909**	**0.1189**
UserKNN CF jaccard	**0.2617**	**0.1077**	**0.3514**	**0.1127**	**0.4050**	**0.1153**	**0.4491**	**0.1170**	**0.4900**	**0.1184**
UserKNN CF asymmetric	**0.2667**	**0.1088**	**0.3606**	**0.1139**	**0.4142**	**0.1166**	**0.4571**	**0.1184**	**0.4934**	**0.1197**
UserKNN CF tversky	**0.2617**	**0.1079**	**0.3507**	**0.1129**	**0.4050**	**0.1156**	**0.4494**	**0.1173**	**0.4901**	**0.1187**
ItemKNN CF cosine	**0.2682**	**0.1092**	**0.3533**	**0.1136**	**0.4142**	**0.1165**	**0.4575**	**0.1182**	**0.4952**	**0.1196**
ItemKNN CF dice	**0.2549**	**0.1041**	**0.3464**	**0.1088**	**0.4061**	**0.1115**	**0.4504**	**0.1132**	**0.4860**	**0.1145**
ItemKNN CF jaccard	**0.2537**	**0.1051**	**0.3434**	**0.1098**	**0.4017**	**0.1125**	**0.4473**	**0.1143**	**0.4852**	**0.1156**
ItemKNN CF asymmetric	**0.2703**	**0.1003**	**0.3610**	**0.1058**	**0.4184**	**0.1087**	**0.4637**	**0.1107**	**0.4943**	**0.1121**
ItemKNN CF tversky	**0.2632**	**0.1038**	**0.3554**	**0.1091**	**0.4174**	**0.1121**	**0.4588**	**0.1139**	**0.4944**	**0.1153**
$P^3\alpha$	**0.2572**	**0.0981**	**0.3532**	**0.1037**	**0.4139**	**0.1067**	**0.4608**	**0.1085**	**0.5002**	**0.1099**
$RP^3\beta$	**0.2688**	**0.1058**	**0.3628**	**0.1114**	**0.4279**	**0.1146**	**0.4714**	**0.1165**	**0.5073**	**0.1178**
$EASE^R$	**0.2777**	**0.1203**	**0.3645**	**0.1250**	**0.4210**	**0.1279**	**0.4663**	**0.1299**	**0.4991**	**0.1312**
SLIM BPR	**0.2566**	**0.1035**	**0.3451**	**0.1086**	**0.4016**	**0.1114**	**0.4452**	**0.1130**	**0.4840**	**0.1144**
SLIM ElasticNet	**0.2791**	**0.1214**	**0.3634**	**0.1261**	**0.4226**	**0.1291**	**0.4637**	**0.1311**	**0.4976**	**0.1324**
MF BPR	0.1820	0.0586	0.2776	0.0638	0.3312	0.0662	0.3752	0.0678	0.4073	0.0689
MF FunkSVD	**0.2010**	0.0540	**0.2976**	0.0597	**0.3575**	0.0623	**0.3988**	0.0639	**0.4300**	0.0649
PureSVD	**0.2560**	**0.1087**	**0.3438**	**0.1130**	**0.3995**	**0.1158**	**0.4417**	**0.1175**	**0.4823**	**0.1190**
NMF	**0.2065**	**0.0865**	**0.2834**	**0.0903**	0.3388	**0.0931**	0.3790	**0.0947**	**0.4179**	**0.0960**
iALS	**0.2726**	**0.1104**	**0.3660**	**0.1163**	**0.4289**	**0.1195**	**0.4842**	**0.1217**	**0.5188**	**0.1232**
SpectralCF	0.1918	0.0660	0.2810	0.0707	0.3438	0.0735	0.3840	0.0749	0.4109	0.0757
SpectralCF article default	0.1450	0.0320	0.2261	0.0358	0.2849	0.0381	0.3366	0.0397	0.3866	0.0410

Table 32. Experimental results for the SpectralCF method for the Movielens 1M original dataset.

	@ 20		@ 40		@ 60		@ 80		@ 100	
	REC	MAP	REC	MAP	REC	MAP	REC	MAP	REC	MAP
Random	0.0062	0.0013	0.0114	0.0014	0.0163	0.0015	0.0213	0.0016	0.0267	0.0017
TopPopular	0.0382	0.0065	0.0969	0.0092	0.1207	0.0101	0.1651	0.0113	0.2286	0.0127
UserKNN CF cosine	0.0917	0.0193	0.1558	0.0226	0.2100	0.0246	0.2514	0.0258	0.2868	0.0267
UserKNN CF dice	0.0993	0.0214	0.1683	0.0250	0.2223	0.0271	0.2623	0.0283	0.2978	0.0292
UserKNN CF jaccard	0.0998	0.0211	0.1691	0.0247	0.2241	0.0268	0.2659	0.0280	0.3012	0.0289
UserKNN CF asymmetric	0.0982	0.0214	0.1644	0.0248	0.2165	0.0268	0.2578	0.0280	0.2943	0.0290
UserKNN CF tversky	0.0999	0.0208	0.1687	0.0244	0.2239	0.0265	0.2672	0.0278	0.3021	0.0287
ItemKNN CF cosine	0.0676	0.0143	0.1217	0.0170	0.1632	0.0185	0.2004	0.0195	0.2326	0.0203
ItemKNN CF dice	0.0673	0.0154	0.1226	0.0181	0.1679	0.0197	0.2071	0.0208	0.2443	0.0218
ItemKNN CF jaccard	0.0670	0.0146	0.1218	0.0173	0.1664	0.0189	0.2051	0.0200	0.2438	0.0209
ItemKNN CF asymmetric	0.0988	0.0215	0.1745	0.0256	0.2311	0.0279	0.2783	0.0296	0.3189	0.0308
ItemKNN CF tversky	0.0730	0.0147	0.1375	0.0179	0.1908	0.0199	0.2384	0.0213	0.2798	0.0224
$P^3\alpha$	0.1218	0.0256	0.2072	0.0306	0.2696	0.0335	0.3174	0.0353	**0.3583**	0.0367
$RP^3\beta$	0.0916	0.0192	0.1628	0.0230	0.2216	0.0254	0.2738	0.0271	0.3171	0.0284
$EASE^R$	0.0956	0.0212	0.1563	0.0243	0.1983	0.0259	0.2354	0.0270	0.2651	0.0278
SLIM BPR	0.1292	0.0283	0.2090	0.0329	0.2687	0.0355	0.3140	0.0371	**0.3525**	0.0384
SLIM ElasticNet	0.0915	0.0205	0.1521	0.0237	0.2014	0.0255	0.2392	0.0267	0.2692	0.0275
MF BPR	0.0863	0.0205	0.1532	0.0239	0.2062	0.0260	0.2518	0.0274	0.2874	0.0284
MF FunkSVD	0.1061	0.0262	0.1746	0.0301	0.2286	0.0323	0.2753	0.0339	0.3127	0.0351
PureSVD	0.0828	0.0172	0.1424	0.0204	0.1833	0.0219	0.2170	0.0229	0.2472	0.0236
NMF	0.0795	0.0199	0.1252	0.0223	0.1602	0.0236	0.1884	0.0245	0.2127	0.0251
iALS	0.0868	0.0188	0.1441	0.0218	0.1933	0.0235	0.2299	0.0245	0.2621	0.0252
SpectralCF	**0.1567**	**0.0621**	**0.2098**	**0.0642**	0.2470	**0.0658**	0.2899	**0.0675**	0.3337	**0.0689**
SpectralCF article default	**0.1849**	**0.0838**	**0.2376**	**0.0857**	**0.2808**	**0.0879**	**0.3248**	**0.0894**	0.3493	**0.0902**

Table 33. Experimental results for the SpectralCF method for the Movielens 1M ours dataset.

	@ 20		@ 40		@ 60		@ 80		@ 100	
	REC	MAP	REC	MAP	REC	MAP	REC	MAP	REC	MAP
Random	0.0055	0.0010	0.0117	0.0012	0.0178	0.0013	0.0234	0.0014	0.0280	0.0014
TopPopular	**0.1892**	**0.0584**	**0.2788**	**0.0636**	**0.3356**	**0.0666**	**0.3834**	**0.0687**	0.4226	**0.0702**
UserKNN CF cosine	**0.2978**	**0.1195**	**0.4108**	**0.1280**	**0.4866**	**0.1329**	**0.5444**	**0.1361**	**0.5868**	**0.1382**
UserKNN CF dice	**0.2960**	**0.1185**	**0.4111**	**0.1270**	**0.4872**	**0.1319**	**0.5443**	**0.1351**	**0.5873**	**0.1372**
UserKNN CF jaccard	**0.3001**	**0.1201**	**0.4134**	**0.1285**	**0.4901**	**0.1335**	**0.5457**	**0.1367**	**0.5884**	**0.1388**
UserKNN CF asymmetric	**0.2880**	**0.1156**	**0.4016**	**0.1236**	**0.4785**	**0.1285**	**0.5361**	**0.1316**	**0.5820**	**0.1338**
UserKNN CF tversky	**0.2903**	**0.1161**	**0.4061**	**0.1244**	**0.4818**	**0.1293**	**0.5394**	**0.1325**	**0.5842**	**0.1346**
ItemKNN CF cosine	**0.2929**	**0.1154**	**0.4067**	**0.1236**	**0.4843**	**0.1287**	**0.5401**	**0.1318**	**0.5829**	**0.1340**
ItemKNN CF dice	**0.2747**	**0.1053**	**0.3770**	**0.1125**	**0.4519**	**0.1170**	**0.5070**	**0.1200**	**0.5556**	**0.1222**
ItemKNN CF jaccard	**0.2731**	**0.1051**	**0.3783**	**0.1126**	**0.4539**	**0.1172**	**0.5099**	**0.1202**	**0.5582**	**0.1224**
ItemKNN CF asymmetric	**0.2876**	**0.1134**	**0.4000**	**0.1213**	**0.4768**	**0.1263**	**0.5367**	**0.1295**	**0.5820**	**0.1317**
ItemKNN CF tversky	**0.2760**	**0.1043**	**0.3877**	**0.1122**	**0.4659**	**0.1175**	**0.5250**	**0.1209**	**0.5713**	**0.1232**
$P^3\alpha$	**0.2939**	**0.1141**	**0.4150**	**0.1233**	**0.4900**	**0.1285**	**0.5463**	**0.1318**	**0.5903**	**0.1342**
$RP^3\beta$	**0.2737**	**0.1044**	**0.3879**	**0.1124**	**0.4664**	**0.1173**	**0.5234**	**0.1206**	**0.5726**	**0.1230**
$EASE^R$	**0.3085**	**0.1249**	**0.4255**	**0.1340**	**0.4986**	**0.1391**	**0.5559**	**0.1425**	**0.6010**	**0.1448**
SLIM BPR	**0.2886**	**0.1086**	**0.4048**	**0.1170**	**0.4813**	**0.1219**	**0.5362**	**0.1249**	**0.5782**	**0.1269**
SLIM ElasticNet	**0.3069**	**0.1265**	**0.4246**	**0.1356**	**0.5010**	**0.1410**	**0.5564**	**0.1443**	**0.6001**	**0.1466**
MF BPR	**0.2616**	**0.0956**	**0.3662**	**0.1028**	**0.4377**	**0.1071**	**0.4890**	**0.1097**	**0.5307**	**0.1116**
MF FunkSVD	**0.2684**	**0.0875**	**0.3890**	**0.0963**	**0.4663**	**0.1015**	**0.5252**	**0.1049**	**0.5720**	**0.1072**
PureSVD	**0.2595**	**0.1008**	**0.3638**	**0.1083**	**0.4378**	**0.1131**	**0.4913**	**0.1161**	**0.5347**	**0.1182**
NMF	**0.2384**	**0.0908**	**0.3351**	**0.0972**	**0.4032**	**0.1014**	**0.4568**	**0.1041**	**0.4981**	**0.1060**
iALS	**0.3033**	**0.1183**	**0.4201**	**0.1273**	**0.4933**	**0.1326**	**0.5493**	**0.1360**	**0.5925**	**0.1383**
SpectralCF	0.1813	0.0533	0.2643	0.0581	0.3274	0.0613	0.3823	0.0635	0.4261	0.0651
SpectralCF article default	0.1785	0.0540	0.2590	0.0586	0.3232	0.0614	0.3689	0.0632	0.4101	0.0646

Table 34. Experimental results for the SpectralCF method for the Amazon Instant Video dataset on beyond accuracy metrics.

	@ 50				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9915**	**0.9998**	**1.0000**	**0.8907**	**12.4892**
TopPopular	0.0340	0.9807	0.0104	0.0089	5.7129
UserKNN CF cosine	**0.8077**	**0.9961**	**0.7804**	**0.1230**	**9.3830**
UserKNN CF dice	**0.8102**	**0.9962**	**0.7812**	**0.1237**	**9.3986**
UserKNN CF jaccard	**0.8118**	**0.9962**	**0.7826**	**0.1247**	**9.4121**
UserKNN CF asymmetric	**0.8078**	**0.9962**	**0.7805**	**0.1224**	**9.3800**
UserKNN CF tversky	**0.8115**	**0.9962**	**0.7833**	**0.1250**	**9.4130**
ItemKNN CF cosine	**0.8078**	**0.9962**	**0.7797**	**0.1223**	**9.3794**
ItemKNN CF dice	**0.8542**	**0.9971**	**0.8176**	**0.1744**	**9.8970**
ItemKNN CF jaccard	**0.8664**	**0.9973**	**0.8183**	**0.1783**	**9.9675**
ItemKNN CF asymmetric	**0.8361**	**0.9967**	**0.8160**	**0.1521**	**9.6981**
ItemKNN CF tversky	**0.8169**	**0.9963**	**0.7881**	**0.1360**	**9.5093**
$P^3\alpha$	**0.8276**	**0.9965**	**0.8181**	**0.1486**	**9.6327**
$RP^3\beta$	**0.8685**	**0.9974**	**0.8340**	**0.1947**	**10.0877**
$EASE^R$	**0.8364**	**0.9967**	**0.8208**	**0.1635**	**9.7377**
SLIM BPR	**0.8513**	**0.9970**	**0.8420**	**0.1723**	**9.8906**
SLIM ElasticNet	**0.8339**	**0.9967**	**0.7630**	**0.1384**	**9.6056**
MF BPR	**0.8488**	**0.9970**	**0.7971**	**0.1458**	**9.7015**
MF FunkSVD	**0.8778**	**0.9975**	**0.3333**	**0.0814**	**9.3079**
PureSVD	**0.7981**	**0.9960**	0.1621	**0.0434**	**8.4209**
NMF	**0.8787**	**0.9976**	**0.2560**	**0.0740**	**9.1913**
iALS	**0.8992**	**0.9980**	**0.3904**	**0.0956**	**9.5477**
SpectralCF	0.3934	0.9879	0.2425	0.0270	7.1816
SpectralCF article default	0.0529	0.9811	0.0104	0.0089	5.7491

Table 35. Experimental results for the SpectralCF method for the Hetrec dataset on beyond accuracy metrics.

	@ 50				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9950**	**0.9999**	**0.9999**	**0.8202**	**13.2277**
TopPopular	0.1661	0.9833	0.0113	0.0057	5.9971
UserKNN CF cosine	**0.4982**	**0.9900**	**0.1013**	**0.0107**	**7.0928**
UserKNN CF dice	**0.5171**	**0.9903**	**0.1285**	**0.0121**	**7.2141**
UserKNN CF jaccard	**0.5092**	**0.9902**	**0.1267**	**0.0118**	**7.1816**
UserKNN CF asymmetric	**0.5058**	**0.9901**	**0.1021**	**0.0109**	**7.1269**
UserKNN CF tversky	**0.5122**	**0.9902**	**0.1284**	**0.0120**	**7.1977**
ItemKNN CF cosine	**0.5381**	**0.9908**	**0.1502**	**0.0133**	**7.3124**
ItemKNN CF dice	**0.5750**	**0.9915**	**0.1891**	**0.0170**	**7.5607**
ItemKNN CF jaccard	**0.6061**	**0.9921**	**0.2707**	**0.0237**	**7.8102**
ItemKNN CF asymmetric	**0.6072**	**0.9921**	**0.1466**	**0.0164**	**7.6400**
ItemKNN CF tversky	**0.5485**	**0.9910**	**0.1763**	**0.0151**	**7.4221**
$P^3\alpha$	**0.4491**	**0.9890**	**0.1010**	**0.0097**	**6.9267**
$RP^3\beta$	**0.5381**	**0.9908**	**0.2086**	**0.0166**	**7.4314**
$EASE^R$	**0.6487**	**0.9930**	**0.1264**	**0.0163**	**7.7278**
SLIM BPR	**0.4518**	**0.9890**	**0.1490**	**0.0113**	**7.0062**
SLIM ElasticNet	**0.6657**	**0.9933**	**0.1419**	**0.0182**	**7.8591**
MF BPR	**0.5711**	**0.9914**	**0.2019**	**0.0160**	**7.4794**
MF FunkSVD	**0.6221**	**0.9924**	**0.1486**	**0.0219**	**7.9797**
PureSVD	**0.6808**	**0.9936**	**0.0542**	**0.0147**	**7.6309**
NMF	**0.7178**	**0.9943**	**0.0838**	**0.0176**	**7.9064**
iALS	**0.7224**	**0.9944**	**0.0799**	**0.0187**	**7.9915**
SpectralCF	0.1971	0.9839	0.0163	0.0058	6.1024
SpectralCF article default	0.2351	0.9847	0.0112	0.0059	6.1460

Table 36. Experimental results for the SpectralCF method for the Movielens 1M original dataset on beyond accuracy metrics.

	@ 50				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9865**	**0.9997**	**1.0000**	**0.9344**	**11.8455**
TopPopular	0.2035	0.9841	0.0367	0.0160	6.0744
UserKNN CF cosine	**0.6877**	**0.9938**	**0.3420**	**0.0519**	**7.9763**
UserKNN CF dice	**0.7012**	**0.9940**	**0.3806**	**0.0554**	**8.0611**
UserKNN CF jaccard	**0.6800**	**0.9936**	**0.3501**	**0.0505**	**7.9359**
UserKNN CF asymmetric	**0.7100**	**0.9942**	**0.4062**	**0.0574**	**8.1084**
UserKNN CF tversky	**0.6692**	**0.9934**	**0.3412**	**0.0484**	**7.8764**
ItemKNN CF cosine	**0.6152**	**0.9923**	**0.4704**	**0.0433**	**7.6568**
ItemKNN CF dice	**0.6527**	**0.9931**	**0.4996**	**0.0493**	**7.8369**
ItemKNN CF jaccard	**0.6395**	**0.9928**	**0.5009**	**0.0478**	**7.7830**
ItemKNN CF asymmetric	**0.6811**	**0.9936**	**0.2337**	**0.0428**	**7.7403**
ItemKNN CF tversky	**0.6024**	**0.9920**	**0.4507**	**0.0407**	**7.5887**
$P^3\alpha$	**0.7103**	**0.9942**	**0.3520**	**0.0535**	**8.0366**
$RP^3\beta$	**0.7150**	**0.9943**	**0.6896**	**0.0742**	**8.3048**
$EASE^R$	**0.7869**	**0.9957**	**0.2966**	**0.0703**	**8.4445**
SLIM BPR	**0.6920**	**0.9938**	**0.3765**	**0.0565**	**8.0698**
SLIM ElasticNet	**0.7685**	**0.9954**	**0.3093**	**0.0672**	**8.3712**
MF BPR	**0.6960**	**0.9939**	**0.3868**	**0.0577**	**8.0915**
MF FunkSVD	**0.7756**	**0.9955**	**0.1711**	**0.0590**	**8.1975**
PureSVD	**0.7769**	**0.9955**	**0.1447**	**0.0567**	**8.1242**
NMF	**0.8160**	**0.9963**	**0.2130**	**0.0754**	**8.5403**
iALS	**0.8394**	**0.9968**	**0.2753**	**0.0893**	**8.7898**
SpectralCF	0.2662	0.9853	0.0470	0.0170	6.2529
SpectralCF article default	0.1761	0.9835	0.0286	0.0153	6.0031

Table 37. Experimental results for the SpectralCF method for the Movielens 1M ours dataset on beyond accuracy metrics.

	@ 50				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9871**	**0.9997**	**1.0000**	**0.9348**	**11.9129**
TopPopular	0.1855	0.9837	0.0322	0.0150	6.0329
UserKNN CF cosine	**0.6879**	**0.9938**	**0.4160**	**0.0512**	**7.9970**
UserKNN CF dice	**0.6462**	**0.9929**	**0.3470**	**0.0431**	**7.7664**
UserKNN CF jaccard	**0.6718**	**0.9934**	**0.3830**	**0.0484**	**7.9176**
UserKNN CF asymmetric	**0.5823**	**0.9916**	**0.2450**	**0.0333**	**7.4237**
UserKNN CF tversky	**0.6079**	**0.9922**	**0.3096**	**0.0373**	**7.5686**
ItemKNN CF cosine	**0.6920**	**0.9938**	**0.6695**	**0.0610**	**8.1128**
ItemKNN CF dice	**0.6342**	**0.9927**	**0.5247**	**0.0458**	**7.7590**
ItemKNN CF jaccard	**0.6332**	**0.9927**	**0.5598**	**0.0475**	**7.7844**
ItemKNN CF asymmetric	**0.6214**	**0.9924**	**0.5621**	**0.0452**	**7.7170**
ItemKNN CF tversky	**0.6578**	**0.9932**	**0.5088**	**0.0495**	**7.8964**
$P^3\alpha$	**0.6824**	**0.9936**	**0.3449**	**0.0476**	**7.9227**
$RP^3\beta$	**0.6343**	**0.9927**	**0.6430**	**0.0492**	**7.7865**
$EASE^R$	**0.7258**	**0.9945**	**0.2777**	**0.0519**	**8.0749**
SLIM BPR	**0.6465**	**0.9929**	**0.3220**	**0.0424**	**7.7517**
SLIM ElasticNet	**0.7481**	**0.9950**	**0.3290**	**0.0599**	**8.2649**
MF BPR	**0.6152**	**0.9923**	**0.4088**	**0.0425**	**7.6973**
MF FunkSVD	**0.8022**	**0.9960**	**0.3006**	**0.0878**	**8.7693**
PureSVD	**0.7689**	**0.9954**	**0.1383**	**0.0520**	**8.0705**
NMF	**0.7306**	**0.9946**	**0.1069**	**0.0447**	**7.8312**
iALS	**0.8324**	**0.9966**	**0.2326**	**0.0772**	**8.6570**
SpectralCF	0.2217	0.9844	0.0425	0.0155	6.1331
SpectralCF article default	0.1925	0.9838	0.0289	0.0150	6.0420

Table 38. Computation time for the algorithms in the selected results for the SpectralCF method on the Amazon Instant Video dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	15.23 [sec]	204
TopPopular	0.00 [sec]	15.41 [sec]	202
UserKNN CF cosine	0.16 ± 0.01 [sec]	15.56 ± 0.04 [sec]	200
UserKNN CF dice	0.17 ± 0.00 [sec]	15.52 ± 0.03 [sec]	200
UserKNN CF jaccard	0.17 ± 0.00 [sec]	15.48 ± 0.02 [sec]	201
UserKNN CF asymmetric	0.17 ± 0.01 [sec]	15.41 ± 0.13 [sec]	201
UserKNN CF tversky	0.17 ± 0.00 [sec]	15.53 ± 0.03 [sec]	201
ItemKNN CF cosine	0.38 ± 0.04 [sec]	15.42 ± 0.09 [sec]	202
ItemKNN CF dice	0.40 ± 0.01 [sec]	15.53 ± 0.01 [sec]	201
ItemKNN CF jaccard	0.40 ± 0.01 [sec]	15.47 ± 0.05 [sec]	202
ItemKNN CF asymmetric	0.41 ± 0.01 [sec]	15.52 ± 0.02 [sec]	201
ItemKNN CF tversky	0.41 ± 0.01 [sec]	15.53 ± 0.04 [sec]	200
$P^3\alpha$	0.87 ± 0.03 [sec]	15.54 ± 0.07 [sec]	199
$RP^3\beta$	0.95 ± 0.04 [sec]	15.28 ± 0.17 [sec]	201
$EASE^R$	4.84 ± 0.06 [sec]	21.44 ± 0.02 [sec]	145
SLIM BPR	83.48 [sec] / 1.39 ± 0.59 [min]	15.66 ± 0.05 [sec]	199
SLIM ElasticNet	51.52 ± 3.17 [sec]	15.62 ± 0.16 [sec]	198
MF BPR	225.64 [sec] / 3.76 ± 2.96 [min]	15.77 ± 0.20 [sec]	194
MF FunkSVD	85.76 [sec] / 1.43 ± 1.50 [min]	15.58 ± 0.16 [sec]	198
PureSVD	0.15 ± 0.19 [sec]	15.38 ± 0.02 [sec]	202
NMF	28.00 ± 44.31 [sec]	15.64 ± 0.14 [sec]	199
iALS	193.65 [sec] / 3.23 ± 2.87 [min]	15.73 ± 0.09 [sec]	198
SpectralCF	1118.55 [sec] / 18.64 ± 2.65 [min]	10.42 ± 0.21 [sec]	295
SpectralCF article default	1335.22 [sec] / 22.25 [min]	10.27 [sec]	303

Table 39. Computation time for the algorithms in the selected results for the SpectralCF method on the Hetrec dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	10.21 [sec]	194
TopPopular	0.00 [sec]	11.17 [sec]	177
UserKNN CF cosine	0.22 ± 0.03 [sec]	11.47 ± 0.06 [sec]	172
UserKNN CF dice	0.23 ± 0.03 [sec]	11.38 ± 0.10 [sec]	174
UserKNN CF jaccard	0.23 ± 0.04 [sec]	11.41 ± 0.03 [sec]	174
UserKNN CF asymmetric	0.23 ± 0.04 [sec]	11.44 ± 0.04 [sec]	173
UserKNN CF tversky	0.23 ± 0.04 [sec]	11.45 ± 0.03 [sec]	174
ItemKNN CF cosine	1.13 ± 0.09 [sec]	11.52 ± 0.05 [sec]	172
ItemKNN CF dice	1.13 ± 0.06 [sec]	11.34 ± 0.19 [sec]	174
ItemKNN CF jaccard	1.14 ± 0.06 [sec]	11.42 ± 0.04 [sec]	174
ItemKNN CF asymmetric	1.13 ± 0.08 [sec]	11.29 ± 0.44 [sec]	172
ItemKNN CF tversky	1.15 ± 0.07 [sec]	11.40 ± 0.06 [sec]	174
$P^3\alpha$	4.78 ± 0.66 [sec]	11.40 ± 0.07 [sec]	174
$RP^3\beta$	4.55 ± 0.99 [sec]	11.07 ± 0.63 [sec]	172
$EASE^R$	21.50 ± 0.13 [sec]	15.00 ± 0.04 [sec]	132
SLIM BPR	130.28 [sec] / 2.17 ± 0.91 [min]	11.39 ± 0.07 [sec]	173
SLIM ElasticNet	306.32 [sec] / 5.11 ± 0.21 [min]	11.30 ± 0.57 [sec]	173
MF BPR	124.76 [sec] / 2.08 ± 2.29 [min]	11.36 ± 0.34 [sec]	171
MF FunkSVD	91.33 [sec] / 1.52 ± 1.67 [min]	11.59 ± 0.20 [sec]	170
PureSVD	0.18 ± 0.23 [sec]	11.08 ± 0.20 [sec]	177
NMF	36.78 ± 66.11 [sec]	11.50 ± 0.12 [sec]	172
iALS	187.90 [sec] / 3.13 ± 5.58 [min]	11.45 ± 0.03 [sec]	172
SpectralCF	1704.87 [sec] / 28.41 ± 4.33 [min]	7.80 ± 0.19 [sec]	249
SpectralCF article default	2197.20 [sec] / 36.62 [min]	7.42 [sec]	267

Table 40. Computation time for the algorithms in the selected results for the SpectralCF method on the Movielens 1M original dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	27.37 [sec]	200
TopPopular	0.01 [sec]	28.35 [sec]	193
UserKNN CF cosine	1.54 ± 0.20 [sec]	29.21 ± 0.45 [sec]	187
UserKNN CF dice	1.53 ± 0.22 [sec]	29.04 ± 0.19 [sec]	189
UserKNN CF jaccard	1.51 ± 0.21 [sec]	29.09 ± 0.18 [sec]	189
UserKNN CF asymmetric	1.58 ± 0.24 [sec]	29.19 ± 0.08 [sec]	188
UserKNN CF tversky	1.54 ± 0.25 [sec]	29.07 ± 0.39 [sec]	188
ItemKNN CF cosine	0.47 ± 0.07 [sec]	29.08 ± 0.33 [sec]	188
ItemKNN CF dice	0.41 ± 0.06 [sec]	28.73 ± 0.15 [sec]	191
ItemKNN CF jaccard	0.42 ± 0.06 [sec]	28.93 ± 0.21 [sec]	191
ItemKNN CF asymmetric	0.49 ± 0.05 [sec]	29.26 ± 0.50 [sec]	184
ItemKNN CF tversky	0.43 ± 0.07 [sec]	28.95 ± 0.11 [sec]	190
$P^3\alpha$	2.31 ± 0.82 [sec]	29.54 ± 0.23 [sec]	186
$RP^3\beta$	2.66 ± 0.90 [sec]	29.35 ± 0.97 [sec]	186
$EASE^R$	2.42 ± 0.25 [sec]	39.37 ± 0.25 [sec]	138
SLIM BPR	218.79 [sec] / 3.65 ± 1.84 [min]	29.69 ± 0.13 [sec]	185
SLIM ElasticNet	60.10 [sec] / 1.00 ± 0.16 [min]	28.88 ± 0.16 [sec]	190
MF BPR	431.40 [sec] / 7.19 ± 5.28 [min]	29.09 ± 0.09 [sec]	188
MF FunkSVD	370.25 [sec] / 6.17 ± 7.29 [min]	29.05 ± 0.52 [sec]	182
PureSVD	0.27 ± 0.33 [sec]	28.38 ± 0.24 [sec]	192
NMF	47.42 ± 66.95 [sec]	28.86 ± 0.43 [sec]	190
iALS	272.61 [sec] / 4.54 ± 5.91 [min]	28.65 ± 0.12 [sec]	191
SpectralCF	2429.47 [sec] / 40.49 ± 24.14 [min]	19.72 ± 0.28 [sec]	279
SpectralCF article default	2111.55 [sec] / 35.19 [min]	20.30 [sec]	270

Table 41. Computation time for the algorithms in the selected results for the SpectralCF method on the Movielens 1M ours dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	29.72 [sec]	201
TopPopular	0.01 [sec]	32.39 [sec]	184
UserKNN CF cosine	1.67 ± 0.23 [sec]	34.04 ± 0.03 [sec]	175
UserKNN CF dice	1.49 ± 0.21 [sec]	34.04 ± 0.10 [sec]	175
UserKNN CF jaccard	1.50 ± 0.23 [sec]	33.99 ± 0.41 [sec]	175
UserKNN CF asymmetric	1.59 ± 0.19 [sec]	34.16 ± 0.22 [sec]	174
UserKNN CF tversky	1.56 ± 0.21 [sec]	33.95 ± 0.74 [sec]	174
ItemKNN CF cosine	0.45 ± 0.07 [sec]	34.20 ± 0.87 [sec]	173
ItemKNN CF dice	0.46 ± 0.08 [sec]	33.75 ± 0.17 [sec]	176
ItemKNN CF jaccard	0.45 ± 0.07 [sec]	33.90 ± 0.19 [sec]	176
ItemKNN CF asymmetric	0.50 ± 0.07 [sec]	34.32 ± 0.18 [sec]	174
ItemKNN CF tversky	0.43 ± 0.09 [sec]	33.99 ± 0.31 [sec]	176
$P^3\alpha$	2.44 ± 0.93 [sec]	33.77 ± 0.27 [sec]	176
$RP^3\beta$	3.11 ± 1.03 [sec]	32.47 ± 1.99 [sec]	175
$EASE^R$	2.56 ± 0.02 [sec]	44.62 ± 0.13 [sec]	133
SLIM BPR	230.44 [sec] / 3.84 ± 2.74 [min]	34.23 ± 0.10 [sec]	174
SLIM ElasticNet	68.77 [sec] / 1.15 ± 0.24 [min]	34.91 ± 0.52 [sec]	169
MF BPR	358.82 [sec] / 5.98 ± 5.73 [min]	33.91 ± 0.41 [sec]	174
MF FunkSVD	328.94 [sec] / 5.48 ± 6.66 [min]	32.76 ± 1.40 [sec]	176
PureSVD	0.32 ± 0.40 [sec]	33.13 ± 0.47 [sec]	178
NMF	28.92 ± 61.24 [sec]	33.66 ± 0.76 [sec]	174
iALS	248.84 [sec] / 4.15 ± 4.96 [min]	33.20 ± 0.64 [sec]	176
SpectralCF	2243.74 [sec] / 37.40 ± 19.07 [min]	21.55 ± 0.33 [sec]	280
SpectralCF article default	2556.16 [sec] / 42.60 [min]	21.74 [sec]	274

Table 42. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Movielens 1M original	Hetrec	Amazon Instant Video
UserKNN CF cosine	topK	418	365	464	800
	shrink	402	0	0	346
	similarity	cosine	cosine	cosine	cosine
	normalize	True	True	True	False
	feature weighting	TF-IDF	TF-IDF	none	TF-IDF
UserKNN CF dice	topK	383	276	428	484
	shrink	0	1	1	940
	similarity	dice	dice	dice	dice
	normalize	False	True	True	False
UserKNN CF jaccard	topK	300	337	456	444
	shrink	0	0	0	303
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	False	True	False	True
UserKNN CF asymmetric	topK	734	369	441	855
	shrink	0	134	0	19
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	0.4193	0.6047	0.5026	0.7882
	feature weighting	TF-IDF	TF-IDF	TF-IDF	none
UserKNN CF tversky	topK	516	377	449	476
	shrink	0	0	0	806
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	1.2079	2.0000	2.0000	1.3499
	tversky beta	2.0000	2.0000	2.0000	1.7078
ItemKNN CF cosine	topK	197	615	322	998
	shrink	0	0	1000	21
	similarity	cosine	cosine	cosine	cosine
	normalize	True	True	True	False
	feature weighting	TF-IDF	TF-IDF	TF-IDF	TF-IDF
ItemKNN CF dice	topK	218	137	195	443
	shrink	2	0	33	172
	similarity	dice	dice	dice	dice
	normalize	True	False	False	False
ItemKNN CF jaccard	topK	158	135	222	290
	shrink	1	0	5	140
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	True	False	True	True
ItemKNN CF asymmetric	topK	269	1000	462	1000
	shrink	0	0	222	1000
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	0.3993	0.0466	0.0000	0.0000
	feature weighting	TF-IDF	TF-IDF	TF-IDF	TF-IDF
ItemKNN CF tversky	topK	48	143	142	1000
	shrink	77	0	23	1000
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	0.8429	0.4521	0.2786	0.0000
	tversky beta	1.7696	2.0000	1.3237	2.0000

Table 43. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Movielens 1M original	Hetrec	Amazon Instant Video
$P^3\alpha$	topK	350	332	901	1000
	alpha	0.6537	1.0075	0.7565	0.3705
	normalize similarity	True	True	False	False
$RP^3\beta$	topK	853	537	1000	442
	alpha	0.0000	0.7551	0.8163	0.6540
	beta	0.4098	0.5412	0.2099	0.0332
	normalize similarity	True	False	False	False
$EASE^R$	l2 norm	7.50E+02	2.92E+02	5.20E+02	1.02E+06
SLIM BPR	topK	329	1000	725	1000
	epochs	130	200	80	150
	symmetric	True	True	True	False
	sgd mode	sgd	adagrad	adagrad	adagrad
	lambda i	1.00E-02	1.00E-05	1.00E-05	1.00E-02
	lambda j	1.00E-02	1.00E-05	1.00E-05	1.00E-02
	learning rate	1.33E-02	1.00E-01	3.19E-04	1.00E-04
SLIM ElasticNet	topK	642	747	602	862
	l1 ratio	1.89E-05	7.37E-05	1.58E-05	6.11E-05
	alpha	0.0490	0.0371	0.1354	0.5507
MF BPR	sgd mode	adagrad	adagrad	adagrad	adagrad
	epochs	790	445	190	500
	num factors	200	200	200	200
	batch size	512	32	64	1
	positive reg	1.00E-02	1.00E-02	1.00E-02	1.00E-02
	negative reg	1.00E-05	1.00E-02	1.00E-02	1.00E-02
	learning rate	2.86E-02	1.00E-01	2.22E-02	1.00E-01
MF FunkSVD	sgd mode	adam	adam	adam	adam
	epochs	325	280	70	370
	use bias	True	False	True	False
	batch size	32	2	8	2
	num factors	19	8	98	13
	item reg	1.47E-04	1.28E-05	2.19E-05	6.87E-05
	user reg	1.88E-04	6.74E-04	6.92E-03	3.09E-04
	learning rate	2.45E-03	1.54E-03	4.73E-03	1.15E-02
	negative quota	0.2131	0.1045	0.4633	0.1323
PureSVD	num factors	16	15	9	33
NMF	num factors	9	20	22	37
	solver	coord. descent	mult. update	mult. update	mult. update
	init type	random	nndsvda	random	nndsvda
	beta loss	frobenius	kullback-leibler	frobenius	frobenius
iALS	num factors	24	22	11	26
	confidence scaling	log	log	linear	linear
	alpha	50.0000	1.7077	6.0056	50.0000
	epsilon	10.0000	0.0010	0.0010	0.0010
	reg	1.20E-04	1.00E-05	1.31E-03	1.00E-05
	epochs	20	75	45	50

Table 44. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Movielens 1M original	Hetrec	Amazon Instant Video
SpectralCF	batch size	2048	2048	512	1024
	embedding size	4	16	16	8
	decay	3.06E-02	3.20E-03	1.80E-03	2.78E-04
	learning rate	8.83E-04	7.00E-03	5.35E-03	9.68E-03
	k	2	3	2	3
	epochs	805	350	265	445
SpectralCF article default	epochs	600	410	185	425
	batch size	1024	1024	1024	1024
	embedding size	16	16	16	16
	decay	1.00E-03	1.00E-03	1.00E-03	1.00E-03
	k	3	3	3	3
	learning rate	1.00E-03	1.00E-03	1.00E-03	1.00E-03

H KDD: LEVERAGING META-PATH BASED CONTEXT FOR TOP-N RECOMMENDATION WITH A NEURAL CO-ATTENTION MODEL

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 45. The results of our evaluation can be seen in Table 46 (Movielens 100k). The corresponding optimal hyperparameters are reported in Table 48 (collaborative KNNs), Table 49 (non-neural machine learning and graph based), Table 50 (content-based KNNs), Table 51 (hybrid KNNs) and Table 52 (MCRec).

Lastly, the time required to train and evaluate the models is reported in Table 47 (Movielens 100k).

Table 45. Dataset characteristics.

Dataset	Interactions	Items	Users	Density
Movielens 100k	100 K	1682	943	6.30%
LastFM	92 K	17 k	1.8 k	0.27%
YelpBusiness	198 K	14 k	16.2 k	0.08%

Table 46. Experimental results for the MCRec method for the Movielens 100K dataset.

	@ 10		
	PREC	REC	NDCG
Random	0.0136	0.0070	0.0085
TopPopular	0.1907	0.1180	0.1361
UserKNN CF cosine	0.2807	0.1825	0.2260
UserKNN CF dice	**0.3442**	**0.2237**	**0.2692**
UserKNN CF jaccard	**0.3430**	**0.2225**	**0.2687**
UserKNN CF asymmetric	0.2814	0.1828	0.2264
UserKNN CF tversky	**0.3426**	**0.2227**	**0.2694**
ItemKNN CF cosine	**0.3293**	**0.2152**	**0.2571**
ItemKNN CF dice	**0.3211**	0.2040	0.2425
ItemKNN CF jaccard	**0.3177**	0.2043	0.2431
ItemKNN CF asymmetric	**0.3320**	**0.2171**	**0.2601**
ItemKNN CF tversky	**0.3283**	**0.2145**	**0.2562**
$P^3\alpha$	**0.3305**	0.2081	**0.2554**
$RP^3\beta$	**0.3435**	**0.2191**	**0.2588**
$EASE^R$	**0.3739**	**0.2430**	**0.2905**
SLIM BPR	**0.3127**	0.2040	0.2460
SLIM ElasticNet	**0.3770**	**0.2441**	**0.2957**
MF BPR	0.2816	0.1860	0.2195
MF FunkSVD	**0.3442**	**0.2203**	**0.2642**
PureSVD	**0.3545**	**0.2247**	**0.2719**
NMF	**0.3350**	**0.2139**	**0.2585**
iALS	**0.3596**	**0.2283**	**0.2759**
ItemKNN CBF cosine	0.0455	0.0185	0.0254
ItemKNN CBF dice	0.0135	0.0038	0.0054
ItemKNN CBF jaccard	0.0135	0.0038	0.0054
ItemKNN CBF asymmetric	0.0547	0.0243	0.0319
ItemKNN CBF tversky	0.0097	0.0031	0.0042
ItemKNN CFCBF cosine	**0.3398**	**0.2239**	**0.2646**
ItemKNN CFCBF dice	**0.3215**	0.2043	0.2403
ItemKNN CFCBF jaccard	**0.3200**	0.2057	0.2422
ItemKNN CFCBF asymmetric	**0.3390**	**0.2224**	**0.2662**
ItemKNN CFCBF tversky	**0.3127**	0.2023	0.2439
MCRec	0.3110	0.2113	0.2466

Table 47. Computation time for the algorithms in the selected results for the MCRec method on the Movielens 100K dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	0.37 [sec]	2513
TopPopular	0.00 [sec]	0.41 [sec]	2319
UserKNN CF cosine	0.10 ± 0.02 [sec]	0.86 ± 0.03 [sec]	1077
UserKNN CF dice	0.09 ± 0.02 [sec]	0.75 ± 0.02 [sec]	1283
UserKNN CF jaccard	0.08 ± 0.02 [sec]	0.78 ± 0.05 [sec]	1160
UserKNN CF asymmetric	0.10 ± 0.01 [sec]	0.85 ± 0.03 [sec]	1077
UserKNN CF tversky	0.09 ± 0.02 [sec]	0.76 ± 0.03 [sec]	1256
ItemKNN CF cosine	0.16 ± 0.03 [sec]	0.87 ± 0.02 [sec]	1077
ItemKNN CF dice	0.13 ± 0.03 [sec]	0.80 ± 0.05 [sec]	1256
ItemKNN CF jaccard	0.13 ± 0.03 [sec]	0.75 ± 0.02 [sec]	1256
ItemKNN CF asymmetric	0.15 ± 0.02 [sec]	0.84 ± 0.01 [sec]	1117
ItemKNN CF tversky	0.14 ± 0.04 [sec]	0.76 ± 0.04 [sec]	1256
$P^3\alpha$	0.88 ± 0.50 [sec]	0.73 ± 0.02 [sec]	1256
$RP^3\beta$	0.97 ± 0.50 [sec]	0.77 ± 0.02 [sec]	1231
$EASE^R$	0.49 ± 0.02 [sec]	1.36 ± 0.01 [sec]	688
SLIM BPR	38.71 ± 16.65 [sec]	1.11 ± 0.04 [sec]	839
SLIM ElasticNet	11.45 ± 5.35 [sec]	1.09 ± 0.01 [sec]	862
MF BPR	82.37 [sec] / 1.37 ± 1.20 [min]	1.56 ± 0.22 [sec]	548
MF FunkSVD	148.82 [sec] / 2.48 ± 3.72 [min]	1.69 ± 0.68 [sec]	877
PureSVD	0.05 ± 0.04 [sec]	0.60 ± 0.34 [sec]	1945
NMF	20.65 ± 27.78 [sec]	0.53 [sec]	1774
iALS	26.02 ± 33.39 [sec]	0.75 ± 0.44 [sec]	1774
ItemKNN CBF cosine	0.93 ± 0.61 [sec]	1.06 ± 0.05 [sec]	928
ItemKNN CBF dice	0.92 ± 0.61 [sec]	1.01 ± 0.04 [sec]	900
ItemKNN CBF jaccard	1.02 ± 0.59 [sec]	1.01 ± 0.04 [sec]	914
ItemKNN CBF asymmetric	0.93 ± 0.56 [sec]	1.06 ± 0.01 [sec]	887
ItemKNN CBF tversky	1.18 ± 1.25 [sec]	1.02 ± 0.02 [sec]	914
ItemKNN CFCBF cosine	0.29 ± 0.05 [sec]	1.13 ± 0.02 [sec]	838
ItemKNN CFCBF dice	0.26 ± 0.04 [sec]	1.09 ± 0.04 [sec]	901
ItemKNN CFCBF jaccard	0.26 ± 0.04 [sec]	1.10 ± 0.05 [sec]	900
ItemKNN CFCBF asymmetric	0.28 ± 0.06 [sec]	1.14 ± 0.04 [sec]	838
ItemKNN CFCBF tversky	0.26 ± 0.06 [sec]	1.12 ± 0.08 [sec]	888
MCRec	8496.61 [sec] / 2.36 [hour]	165.63 [sec] / 2.76 [min]	6

Table 48. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 100K
UserKNN CF cosine	topK	903
	shrink	2
	similarity	cosine
	normalize	True
	feature weighting	BM25
UserKNN CF dice	topK	129
	shrink	0
	similarity	dice
	normalize	True
UserKNN CF jaccard	topK	128
	shrink	0
	similarity	jaccard
	normalize	True
UserKNN CF asymmetric	topK	1000
	shrink	1000
	similarity	asymmetric
	normalize	True
	asymmetric alpha	2.0000
	feature weighting	BM25
UserKNN CF tversky	topK	125
	shrink	28
	similarity	tversky
	normalize	True
	tversky alpha	1.8829
	tversky beta	1.9666
ItemKNN CF cosine	topK	886
	shrink	403
	similarity	cosine
	normalize	True
	feature weighting	BM25
ItemKNN CF dice	topK	179
	shrink	0
	similarity	dice
	normalize	False
ItemKNN CF jaccard	topK	161
	shrink	0
	similarity	jaccard
	normalize	True
ItemKNN CF asymmetric	topK	468
	shrink	706
	similarity	asymmetric
	normalize	True
	asymmetric alpha	1.5629
	feature weighting	BM25
ItemKNN CF tversky	topK	122
	shrink	0
	similarity	tversky
	normalize	True
	tversky alpha	0.8648
	tversky beta	1.5755

Table 49. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Movielens 100K
$P^3\alpha$	topK	197
	alpha	0.0000
	normalize similarity	True
$RP^3\beta$	topK	324
	alpha	0.8593
	beta	0.6574
	normalize similarity	True
$EASE^R$	l2 norm	4.58E+02
SLIM BPR	topK	584
	epochs	120
	symmetric	True
	sgd mode	adam
	lambda i	1.00E-05
	lambda j	1.00E-05
	learning rate	1.00E-01
SLIM ElasticNet	topK	605
	l1 ratio	1.13E-04
	alpha	0.2225
MF BPR	sgd mode	adagrad
	epochs	355
	num factors	170
	batch size	256
	positive reg	2.16E-04
	negative reg	4.80E-05
	learning rate	3.97E-02
MF FunkSVD	sgd mode	adagrad
	epochs	135
	use bias	False
	batch size	2
	num factors	16
	item reg	1.00E-02
	user reg	1.00E-02
	learning rate	1.00E-01
	negative quota	0.0782
PureSVD	num factors	13
NMF	num factors	30
	solver	coord. descent
	init type	random
	beta loss	frobenius
iALS	num factors	15
	confidence scaling	log
	alpha	0.0010
	epsilon	0.0010
	reg	2.71E-04
	epochs	10

Table 50. Hyperparameter values for our content based KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 100K
ItemKNN CBF cosine	topK	113
	shrink	349
	similarity	cosine
	normalize	False
	feature weighting	BM25
ItemKNN CBF dice	topK	991
	shrink	1
	similarity	dice
	normalize	True
ItemKNN CBF jaccard	topK	991
	shrink	999
	similarity	jaccard
	normalize	False
ItemKNN CBF asymmetric	topK	403
	shrink	428
	similarity	asymmetric
	normalize	True
	asymmetric alpha	1.8470
	feature weighting	BM25
ItemKNN CBF tversky	topK	983
	shrink	3
	similarity	tversky
	normalize	True
	tversky alpha	0.0501
	tversky beta	0.2996

Table 51. Hyperparameter values for our hybrid KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 100K
ItemKNN CFCBF cosine	topK	355
	shrink	228
	similarity	cosine
	normalize	True
	feature weighting	BM25
	ICM weight	0.0757
ItemKNN CFCBF dice	topK	170
	shrink	0
	similarity	dice
	normalize	True
	ICM weight	0.0100
ItemKNN CFCBF jaccard	topK	164
	shrink	0
	similarity	jaccard
	normalize	True
	ICM weight	100.0000
ItemKNN CFCBF asymmetric	topK	311
	shrink	929
	similarity	asymmetric
	normalize	True
	asymmetric alpha	2.0000
	feature weighting	BM25
	ICM weight	0.0100
ItemKNN CFCBF tversky	topK	70
	shrink	537
	similarity	tversky
	normalize	True
	tversky alpha	0.3134
	tversky beta	1.4108
	ICM weight	0.8798

Table 52. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Movielens 100K
MCRec	epochs	130
	latent dim	128
	reg latent	0
	layers	[512, 256, 128, 64]
	reg layes	[0, 0, 0, 0]
	learning rate	1.00E-03
	batch size	256
	num negatives	4

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Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 53. The results of our evaluation can be seen in Table 55 (Movielens 1M) and Table 54 (Pinterest). The corresponding optimal hyperparameters are reported in Table 58 (collaborative KNNs), Table 59 (non-neural machine learning and graph based) and Table 60 (NCF).

Lastly, the time required to train and evaluate the models is reported in Table 55 (Movielens 1M) and Table 54 (Pinterest).

Table 53. Dataset characteristics.

Dataset	Interactions	Items	Users	Sparsity
Movielens 1M	1.0 M	3.7 k	6.0 k	95.53%
Pinterest	1.5 M	9.9 k	55.1 k	99.73%

Table 54. Experimental results for the NeuMF method for the Pinterest dataset.

	@ 1		@ 5		@ 10	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0107	0.0107	0.0500	0.0298	0.0996	0.0456
TopPopular	0.0467	0.0467	0.1665	0.1064	0.2740	0.1409
UserKNN CF cosine	**0.2892**	**0.2892**	0.7006	**0.5036**	0.8632	0.5566
UserKNN CF dice	**0.2880**	**0.2880**	0.7039	**0.5047**	0.8649	0.5572
UserKNN CF jaccard	**0.2898**	**0.2898**	0.7038	**0.5056**	0.8655	**0.5583**
UserKNN CF asymmetric	**0.2877**	**0.2877**	0.7040	**0.5046**	0.8655	0.5573
UserKNN CF tversky	**0.2889**	**0.2889**	0.7039	**0.5052**	0.8660	**0.5580**
ItemKNN CF cosine	**0.2900**	**0.2900**	**0.7109**	**0.5090**	0.8762	**0.5628**
ItemKNN CF dice	**0.2917**	**0.2917**	0.7098	**0.5092**	0.8765	**0.5635**
ItemKNN CF jaccard	**0.2910**	**0.2910**	0.7093	**0.5086**	0.8763	**0.5631**
ItemKNN CF asymmetric	**0.2903**	**0.2903**	**0.7117**	**0.5096**	0.8766	**0.5633**
ItemKNN CF tversky	**0.2909**	**0.2909**	0.7093	**0.5086**	0.8760	**0.5629**
$P^3\alpha$	**0.2853**	**0.2853**	0.7022	0.5024	0.8700	0.5571
$RP^3\beta$	**0.2966**	**0.2966**	**0.7151**	**0.5149**	**0.8796**	**0.5685**
$EASE^R$	**0.2909**	**0.2909**	0.7070	**0.5077**	0.8684	**0.5604**
SLIM BPR	**0.2983**	**0.2983**	**0.7117**	**0.5138**	0.8736	**0.5666**
SLIM ElasticNet	**0.2913**	**0.2913**	0.7059	**0.5072**	0.8679	**0.5601**
MF BPR	0.2655	0.2655	0.6858	0.4833	0.8651	0.5418
MF FunkSVD	0.2601	0.2601	0.6890	0.4820	0.8658	0.5398
PureSVD	0.2630	0.2630	0.6628	0.4706	0.8268	0.5241
NMF	0.2307	0.2307	0.6445	0.4434	0.8343	0.5052
iALS	**0.2811**	**0.2811**	**0.7144**	**0.5061**	0.8761	**0.5590**
NeuMF	0.2801	0.2801	0.7101	0.5029	0.8777	0.5576

Table 55. Experimental results for the NeuMF method for the Movielens 1M dataset.

	@ 1		@ 5		@ 10	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0098	0.0098	0.0513	0.0303	0.0985	0.0454
TopPopular	0.1051	0.1051	0.3048	0.2064	0.4533	0.2542
UserKNN CF cosine	0.1825	0.1825	0.4925	0.3407	0.6606	0.3951
UserKNN CF dice	0.1911	0.1911	0.5053	0.3522	0.6700	0.4057
UserKNN CF jaccard	0.1906	0.1906	0.5045	0.3521	0.6725	0.4066
UserKNN CF asymmetric	0.1921	0.1921	0.5070	0.3546	0.6768	0.4100
UserKNN CF tversky	0.1921	0.1921	0.5073	0.3536	0.6684	0.4058
ItemKNN CF cosine	0.1825	0.1825	0.4942	0.3414	0.6694	0.3979
ItemKNN CF dice	0.1707	0.1707	0.4856	0.3323	0.6604	0.3887
ItemKNN CF jaccard	0.1692	0.1692	0.4772	0.3268	0.6533	0.3837
ItemKNN CF asymmetric	0.1843	0.1843	0.4906	0.3400	0.6627	0.3956
ItemKNN CF tversky	0.1735	0.1735	0.4856	0.3338	0.6546	0.3884
$P^3\alpha$	0.1791	0.1791	0.4846	0.3352	0.6460	0.3876
$RP^3\beta$	0.1836	0.1836	0.4935	0.3419	0.6758	0.4011
$EASE^R$	**0.2225**	**0.2225**	**0.5629**	**0.3986**	**0.7192**	**0.4494**
SLIM BPR	0.2013	0.2013	0.5320	0.3713	0.7002	0.4258
SLIM ElasticNet	**0.2207**	**0.2207**	**0.5576**	**0.3953**	**0.7162**	**0.4468**
MF BPR	0.1679	0.1679	0.4619	0.3186	0.6305	0.3730
MF FunkSVD	0.2008	0.2008	0.5202	0.3661	0.6844	0.4192
PureSVD	**0.2132**	**0.2132**	0.5339	0.3783	0.6937	0.4303
NMF	0.2056	0.2056	0.5171	0.3651	0.6844	0.4192
iALS	**0.2106**	**0.2106**	**0.5505**	**0.3862**	**0.7109**	**0.4382**
NeuMF	0.2088	0.2088	0.5411	0.3803	0.7093	0.4349

Table 56. Computation time for the algorithms in the selected results for the NeuMF method on the Pinterest dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.04 [sec]	337.91 [sec] / 5.63 [min]	163
TopPopular	0.08 [sec]	350.44 [sec] / 5.84 [min]	157
UserKNN CF cosine	59.52 ± 3.87 [sec]	442.63 [sec] / 7.38 ± 0.11 [min]	122
UserKNN CF dice	60.07 [sec] / 1.00 ± 0.05 [min]	440.65 [sec] / 7.34 ± 0.10 [min]	124
UserKNN CF jaccard	59.33 ± 3.16 [sec]	443.67 [sec] / 7.39 ± 0.05 [min]	124
UserKNN CF asymmetric	60.80 [sec] / 1.01 ± 0.04 [min]	445.33 [sec] / 7.42 ± 0.01 [min]	124
UserKNN CF tversky	60.89 [sec] / 1.01 ± 0.06 [min]	435.42 [sec] / 7.26 ± 0.23 [min]	124
ItemKNN CF cosine	3.73 ± 0.40 [sec]	435.26 [sec] / 7.25 ± 0.04 [min]	126
ItemKNN CF dice	3.56 ± 0.37 [sec]	433.80 [sec] / 7.23 ± 0.05 [min]	126
ItemKNN CF jaccard	3.70 ± 0.36 [sec]	435.32 [sec] / 7.26 ± 0.05 [min]	126
ItemKNN CF asymmetric	3.69 ± 0.35 [sec]	437.39 [sec] / 7.29 ± 0.05 [min]	126
ItemKNN CF tversky	3.64 ± 0.40 [sec]	436.99 [sec] / 7.28 ± 0.04 [min]	126
$P^3\alpha$	17.69 ± 4.33 [sec]	434.43 [sec] / 7.24 ± 0.02 [min]	127
$RP^3\beta$	17.95 ± 4.99 [sec]	433.36 [sec] / 7.22 ± 0.06 [min]	126
$EASE^R$	21.86 ± 0.21 [sec]	413.63 [sec] / 6.89 ± 0.02 [min]	133
SLIM BPR	4566.45 [sec] / 1.27 ± 0.55 [hour]	434.78 [sec] / 7.25 ± 0.05 [min]	127
SLIM ElasticNet	728.11 [sec] / 12.14 ± 5.73 [min]	428.35 [sec] / 7.14 ± 0.23 [min]	125
MF BPR	12620.32 [sec] / 3.51 ± 2.83 [hour]	461.88 [sec] / 7.70 ± 1.94 [min]	95
MF FunkSVD	8736.15 [sec] / 2.43 ± 1.89 [hour]	443.13 [sec] / 7.39 ± 1.77 [min]	150
PureSVD	6.58 ± 5.41 [sec]	430.77 [sec] / 7.18 ± 1.83 [min]	149
NMF	963.74 [sec] / 16.06 ± 22.37 [min]	543.13 [sec] / 9.05 ± 1.92 [min]	149
iALS	10812.68 [sec] / 3.00 ± 3.91 [hour]	372.05 [sec] / 6.20 ± 0.03 [min]	148
NeuMF	167670.36 [sec] / 1.94 [day]	6995.33 [sec] / 1.94 [hour]	8

Table 57. Computation time for the algorithms in the selected results for the NeuMF method on the Movielens 1M dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.04 [sec]	36.31 [sec]	166
TopPopular	0.06 [sec]	37.88 [sec]	159
UserKNN CF cosine	9.01 ± 0.26 [sec]	48.07 ± 1.08 [sec]	126
UserKNN CF dice	9.00 ± 0.31 [sec]	47.37 ± 1.63 [sec]	130
UserKNN CF jaccard	8.98 ± 0.29 [sec]	46.70 ± 1.38 [sec]	130
UserKNN CF asymmetric	9.06 ± 0.30 [sec]	47.01 ± 1.83 [sec]	129
UserKNN CF tversky	9.09 ± 0.39 [sec]	46.66 ± 2.00 [sec]	130
ItemKNN CF cosine	4.04 ± 0.17 [sec]	49.48 ± 4.20 [sec]	130
ItemKNN CF dice	4.07 ± 0.17 [sec]	47.25 ± 2.28 [sec]	133
ItemKNN CF jaccard	4.09 ± 0.19 [sec]	47.56 ± 3.43 [sec]	134
ItemKNN CF asymmetric	4.10 ± 0.15 [sec]	50.21 ± 1.92 [sec]	127
ItemKNN CF tversky	4.09 ± 0.18 [sec]	45.82 ± 1.18 [sec]	133
$P^3\alpha$	7.61 ± 2.17 [sec]	45.84 ± 0.56 [sec]	131
$RP^3\beta$	8.05 ± 2.55 [sec]	46.76 ± 0.42 [sec]	127
$EASE^R$	5.07 ± 0.11 [sec]	46.70 ± 0.39 [sec]	129
SLIM BPR	785.57 [sec] / 13.09 ± 7.36 [min]	49.96 ± 2.04 [sec]	118
SLIM ElasticNet	252.72 [sec] / 4.21 ± 2.51 [min]	46.78 ± 0.22 [sec]	129
MF BPR	937.36 [sec] / 15.62 ± 12.59 [min]	56.94 ± 9.66 [sec]	98
MF FunkSVD	3594.07 [sec] / 59.90 ± 47.65 [min]	41.99 ± 7.61 [sec]	154
PureSVD	2.84 ± 1.96 [sec]	44.36 ± 10.92 [sec]	153
NMF	721.37 [sec] / 12.02 ± 22.88 [min]	55.73 ± 12.22 [sec]	151
iALS	1022.66 [sec] / 17.04 ± 13.53 [min]	42.97 ± 8.91 [sec]	154
NeuMF	15050.89 [sec] / 4.18 [hour]	293.85 [sec] / 4.90 [min]	21

Table 58. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M	Pinterest
UserKNN CF cosine	normalize	True	True
	topK	516	1000
	feature weighting	BM25	BM25
	similarity	cosine	cosine
	shrink	0	0
UserKNN CF dice	normalize	True	True
	topK	246	991
	similarity	dice	dice
	shrink	0	138
UserKNN CF jaccard	normalize	True	True
	topK	259	972
	similarity	jaccard	jaccard
	shrink	0	0
UserKNN CF asymmetric	topK	306	1000
	feature weighting	TF-IDF	TF-IDF
	asymmetric alpha	0.2173	0.0000
	normalize	True	True
	similarity	asymmetric	asymmetric
	shrink	0	1000
UserKNN CF tversky	normalize	True	True
	topK	267	1000
	tversky alpha	0.6394	2.0000
	tversky beta	0.8051	1.9574
	similarity	tversky	tversky
	shrink	0	33
ItemKNN CF cosine	normalize	True	False
	topK	111	1000
	feature weighting	BM25	BM25
	similarity	cosine	cosine
	shrink	298	4
ItemKNN CF dice	normalize	True	True
	topK	61	1000
	similarity	dice	dice
	shrink	0	0
ItemKNN CF jaccard	normalize	False	False
	topK	62	997
	similarity	jaccard	jaccard
	shrink	19	1
ItemKNN CF asymmetric	topK	206	1000
	feature weighting	BM25	BM25
	asymmetric alpha	0.6914	0.0000
	normalize	True	True
	similarity	asymmetric	asymmetric
	shrink	1000	1000
ItemKNN CF tversky	normalize	True	True
	topK	83	1000
	tversky alpha	0.0000	2.0000
	tversky beta	2.0000	2.0000
	similarity	tversky	tversky
	shrink	561	0

Table 59. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M	Pinterest
$P^3\alpha$	alpha	1.2177	2.0000
	topK	1000	972
	normalize similarity	False	True
$RP^3\beta$	alpha	1.0807	0.8616
	topK	546	1000
	normalize similarity	True	True
	beta	0.7029	0.4255
$EASE^R$	l2 norm	1.32E+03	3.54E+03
SLIM BPR	learning rate	3.08E-02	1.00E-01
	sgd mode	adagrad	adagrad
	symmetric	True	True
	epochs	285	180
	lambda i	1.00E-02	1.00E-02
	topK	1000	916
	lambda j	4.51E-03	3.83E-05
SLIM ElasticNet	l1 ratio	1.19E-05	1.15E-04
	alpha	0.0788	0.0526
	topK	544	1000
MF BPR	positive reg	2.08E-05	1.00E-02
	num factors	200	200
	negative reg	1.00E-02	6.59E-03
	epochs	625	615
	batch size	8	2
	sgd mode	adagrad	adagrad
	learning rate	5.88E-02	6.74E-02
MF FunkSVD	sgd mode	adam	adagrad
	num factors	50	28
	negative quota	0.1651	0.4052
	user reg	7.35E-04	6.49E-04
	learning rate	4.38E-04	3.17E-02
	epochs	160	305
	batch size	512	1
	item reg	4.83E-03	1.56E-03
	use bias	True	True
PureSVD	num factors	52	50
NMF	init type	random	nndsvda
	beta loss	frobenius	kullback-leibler
	num factors	89	27
	solver	mult. update	mult. update
iALS	num factors	46	30
	epochs	10	45
	epsilon	10.0000	0.0010
	confidence scaling	log	linear
	reg	1.00E-05	1.00E-02
	alpha	50.0000	50.0000

Table 60. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Movielens 1M	Pinterest
NeuMF	epochs	10	5
	epochs gmf	10	45
	epochs mlp	10	10
	batch size	256	256
	num factors	64	16
	layers	[256, 128, 64]	[64, 32, 16]
	reg mf	0.00E+00	0.00E+00
	reg layers	[0, 0, 0]	[0, 0, 0]
	num negatives	4	4
	learning rate	1.00E-03	1.00E-03
	learning rate pretrain	1.00E-03	1.00E-03
	learner	sgd	sgd
	learner pretrain	adam	adam
	pretrain	True	True

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Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 61. The results of our evaluation can be seen in Table 62 (Movielens 20M) and Table 63 (Netflix Prize). The corresponding optimal hyperparameters are reported in Table 66 (collaborative KNNs)[9], Table 67 (graph based), Table 68 (non-neural machine learning) and Table 69 (Mult VAE).

Lastly, the time required to train and evaluate the models is reported in Table 64 (Movielens 20M) and Table 65 (Netflix Prize).

Table 61. Dataset characteristics.

Dataset	Interactions	Users	Items	Density	Held out users
Movielens 20M	10.0M	136k	20k	0.36	10k
Netflix Prize	56.9M	463k	17k	0.69	40k

Table 62. Experimental results for the MultVAE method for the Movielens 20M dataset.

	@ 20		@ 50		@ 100	
	REC	NDCG	REC	NDCG	REC	NDCG
Random	0.0010	0.0006	0.0023	0.0012	0.0047	0.0021
TopPopular	0.1441	0.1201	0.2320	0.1569	0.3296	0.1901
UserKNN CF cosine	-	-	-	-	-	-
UserKNN CF dice	-	-	-	-	-	-
UserKNN CF jaccard	-	-	-	-	-	-
UserKNN CF asymmetric	-	-	-	-	-	-
UserKNN CF tversky	-	-	-	-	-	-
ItemKNN CF cosine	0.2897	0.2434	0.4412	0.3054	0.5652	0.3492
ItemKNN CF dice	0.2689	0.2274	0.4095	0.2851	0.5316	0.3277
ItemKNN CF jaccard	0.2667	0.2284	0.4035	0.2844	0.5254	0.3268
ItemKNN CF asymmetric	0.2937	0.2444	0.4486	0.3087	0.5709	0.3527
ItemKNN CF tversky	0.2867	0.2395	0.4393	0.3030	0.5556	0.3458
$P^3\alpha$	0.2620	0.2168	0.4047	0.2742	0.5287	0.3182
$RP^3\beta$	0.3006	0.2501	0.4540	0.3133	0.5797	0.3583
$EASE^R$	0.3530	**0.3074**	0.5147	**0.3755**	0.6353	**0.4196**
SLIM BPR	0.3206	0.2646	0.4783	0.3291	0.6030	0.3731
SLIM ElasticNet	0.3356	0.2920	0.4893	0.3576	0.6110	0.4017
MF BPR	0.2379	0.1888	0.3867	0.2481	0.5100	0.2908
MF FunkSVD	0.2765	0.2254	0.4243	0.2864	0.5576	0.3331
PureSVD	0.2935	0.2514	0.4371	0.3117	0.5544	0.3538
NMF	0.2269	0.1960	0.3533	0.2480	0.4664	0.2875
iALS	0.2968	0.2496	0.4406	0.3090	0.5631	0.3521
Mult VAE	**0.3541**	0.2988	**0.5222**	0.3690	**0.6517**	0.4158

[9]The results for the UserKNN baseline are missing due to the user-holdout data split.

Table 63. Experimental results for the MultVAE method for the Netflix Prize dataset.

	@ 20		@ 50		@ 100	
	REC	NDCG	REC	NDCG	REC	NDCG
Random	0.0013	0.0011	0.0032	0.0020	0.0059	0.0032
TopPopular	0.0786	0.0762	0.1643	0.1159	0.2717	0.1570
UserKNN CF cosine	-	-	-	-	-	-
UserKNN CF dice	-	-	-	-	-	-
UserKNN CF jaccard	-	-	-	-	-	-
UserKNN CF asymmetric	-	-	-	-	-	-
UserKNN CF tversky	-	-	-	-	-	-
ItemKNN CF cosine	0.2091	0.1970	0.3387	0.2592	0.4598	0.3092
ItemKNN CF dice	0.1963	0.1862	0.3224	0.2479	0.4379	0.2983
ItemKNN CF jaccard	0.1997	0.1883	0.3248	0.2481	0.4450	0.2978
ItemKNN CF asymmetric	0.2119	0.1968	0.3466	0.2623	0.4764	0.3165
ItemKNN CF tversky	0.2075	0.1933	0.3420	0.2582	0.4708	0.3118
$P^3\alpha$	0.1960	0.1759	0.3325	0.2412	0.4633	0.2962
$RP^3\beta$	0.2210	0.2053	0.3633	0.2739	0.4932	0.3281
$EASE^R$	**0.2681**	**0.2591**	**0.4170**	**0.3334**	**0.5471**	**0.3890**
SLIM BPR	0.2394	0.2219	0.3767	0.2886	0.5004	0.3403
SLIM ElasticNet	0.2555	**0.2479**	0.4002	**0.3203**	0.5299	**0.3752**
MF BPR	0.1572	0.1403	0.2748	0.1952	0.3952	0.2431
MF FunkSVD	0.2300	0.2130	0.3609	0.2758	0.4803	0.3250
PureSVD	0.2271	0.2184	0.3593	0.2840	0.4784	0.3342
NMF	0.1844	0.1802	0.3035	0.2385	0.4172	0.2856
iALS	0.1956	0.1839	0.3138	0.2410	0.4216	0.2862
Mult VAE	0.2615	0.2423	0.4127	0.3167	0.5456	0.3730

Table 64. Computation time for the algorithms in the selected results for the MultVAE method on the Movielens 20M dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.23 [sec]	13.64 [sec]	733
TopPopular	0.38 [sec]	12.62 [sec]	792
UserKNN CF cosine	-	-	-
UserKNN CF dice	-	-	-
UserKNN CF jaccard	-	-	-
UserKNN CF asymmetric	-	-	-
UserKNN CF tversky	-	-	-
ItemKNN CF cosine	12.12 ± 0.53 [sec]	17.66 ± 2.93 [sec]	651
ItemKNN CF dice	11.85 ± 0.45 [sec]	13.84 ± 0.69 [sec]	741
ItemKNN CF jaccard	11.74 ± 0.46 [sec]	14.08 ± 0.67 [sec]	723
ItemKNN CF asymmetric	12.29 ± 0.71 [sec]	15.81 ± 1.82 [sec]	736
ItemKNN CF tversky	11.79 ± 0.58 [sec]	15.24 ± 2.53 [sec]	764
$P^3\alpha$	24.76 ± 4.74 [sec]	12.61 ± 0.31 [sec]	822
$RP^3\beta$	25.39 ± 5.11 [sec]	13.09 ± 0.68 [sec]	767
$EASE^R$	178.81 [sec] / 2.98 ± 0.01 [min]	66.41 [sec] / 1.11 ± 0.01 [min]	151
SLIM BPR	2315.60 [sec] / 38.59 ± 48.48 [min]	13.97 ± 1.00 [sec]	690
SLIM ElasticNet	6508.83 [sec] / 1.81 ± 1.26 [hour]	13.88 ± 0.70 [sec]	708
MF BPR	687.67 [sec] / 11.46 ± 14.12 [min]	47.37 ± 18.54 [sec]	226
MF FunkSVD	12684.59 [sec] / 3.52 ± 5.87 [hour]	67.63 [sec] / 1.13 [min]	148
PureSVD	9.14 ± 7.72 [sec]	27.80 ± 16.51 [sec]	593
NMF	2233.33 [sec] / 37.22 ± 48.55 [min]	18.84 [sec]	531
iALS	1892.81 [sec] / 31.55 ± 32.43 [min]	60.73 [sec] / 1.01 ± 0.50 [min]	134
Mult VAE	1296.97 [sec] / 21.62 [min]	18.72 [sec]	534

Table 65. Computation time for the algorithms in the selected results for the MultVAE method on the Netflix Prize dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	1.67 [sec]	60.99 [sec] / 1.02 [min]	656
TopPopular	2.95 [sec]	59.14 [sec]	676
UserKNN CF cosine	-	-	-
UserKNN CF dice	-	-	-
UserKNN CF jaccard	-	-	-
UserKNN CF asymmetric	-	-	-
UserKNN CF tversky	-	-	-
ItemKNN CF cosine	65.70 [sec] / 1.09 ± 0.03 [min]	84.91 [sec] / 1.42 ± 0.46 [min]	593
ItemKNN CF dice	64.29 [sec] / 1.07 ± 0.01 [min]	63.77 [sec] / 1.06 ± 0.08 [min]	682
ItemKNN CF jaccard	64.40 [sec] / 1.07 ± 0.01 [min]	61.38 [sec] / 1.02 [min]	652
ItemKNN CF asymmetric	65.41 [sec] / 1.09 ± 0.04 [min]	97.70 [sec] / 1.63 ± 0.59 [min]	654
ItemKNN CF tversky	64.82 [sec] / 1.08 ± 0.01 [min]	66.06 [sec] / 1.10 ± 0.12 [min]	657
$P^3\alpha$	74.12 [sec] / 1.24 ± 0.08 [min]	59.98 ± 0.57 [sec]	666
$RP^3\beta$	75.10 [sec] / 1.25 ± 0.09 [min]	58.00 ± 5.61 [sec]	648
$EASE^R$	219.69 [sec] / 3.66 ± 0.30 [min]	311.59 [sec] / 5.19 ± 0.03 [min]	129
SLIM BPR	5741.37 [sec] / 1.59 ± 1.80 [hour]	65.49 [sec] / 1.09 ± 0.09 [min]	600
SLIM ElasticNet	29589.53 [sec] / 8.22 ± 7.70 [hour]	69.53 [sec] / 1.16 ± 0.13 [min]	580
MF BPR	2846.25 [sec] / 47.44 ± 47.24 [min]	124.20 [sec] / 2.07 ± 0.28 [min]	279
MF FunkSVD	43962.64 [sec] / 12.21 ± 19.73 [hour]	108.71 [sec] / 1.81 ± 0.71 [min]	271
PureSVD	50.53 ± 36.49 [sec]	80.49 [sec] / 1.34 ± 0.02 [min]	496
NMF	24370.74 [sec] / 6.77 ± 2.81 [hour]	128.27 [sec] / 2.14 ± 0.80 [min]	425
iALS	5299.36 [sec] / 1.47 ± 0.93 [hour]	112.95 [sec] / 1.88 [min]	354
Mult VAE	4521.33 [sec] / 1.26 [hour]	81.53 [sec] / 1.36 [min]	491

Table 66. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 20M cold user	Netflix Prize cold user
UserKNN CF cosine	-	-	-
UserKNN CF dice	-	-	-
UserKNN CF jaccard	-	-	-
UserKNN CF asymmetric	-	-	-
UserKNN CF tversky	-	-	-
ItemKNN CF cosine	topK	278	140
	shrink	409	1000
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	BM25	BM25
ItemKNN CF dice	topK	107	9
	shrink	3	983
	similarity	dice	dice
	normalize	True	True
ItemKNN CF jaccard	topK	118	54
	shrink	214	544
	similarity	jaccard	jaccard
	normalize	True	True
ItemKNN CF asymmetric	topK	52	64
	shrink	0	360
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.9034	0.2002
	feature weighting	BM25	none
ItemKNN CF tversky	topK	25	54
	shrink	273	1000
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	0.0782	0.1997
	tversky beta	0.5191	0.9652

Table 67. Hyperparameter values for our graph based baselines on all datasets.

Algorithm	Hyperparameter	Movielens 20M cold user	Netflix Prize cold user
$P^3\alpha$	topK	523	534
	alpha	0.5575	1.6695
	normalize similarity	True	True
$RP^3\beta$	topK	399	185
	alpha	0.7242	1.3871
	beta	0.4945	0.4271
	normalize similarity	True	True

Table 68. Hyperparameter values for our non-neural machine learning baselines on all datasets.

Algorithm	Hyperparameter	Movielens 20M cold user	Netflix Prize cold user
EASE^R	l2 norm	6.79E+02	1.17E+03
SLIM BPR	topK	847	491
	epochs	630	240
	symmetric	True	True
	sgd mode	sgd	sgd
	lambda i	5.69E-05	5.87E-04
	lambda j	1.00E-05	1.00E-02
	learning rate	4.56E-03	7.53E-03
SLIM ElasticNet	topK	718	1000
	l1 ratio	6.74E-03	9.65E-04
	alpha	0.0010	0.0010
MF BPR	sgd mode	adam	adam
	epochs	1500	1500
	num factors	200	163
	batch size	64	8
	positive reg	1.00E-05	1.95E-05
	negative reg	9.44E-04	3.60E-04
	learning rate	7.16E-02	7.11E-02
	estimate model for cold users	itemKNN	itemKNN
	estimate model for cold users topK	135	433
MF FunkSVD	sgd mode	adam	adagrad
	epochs	500	500
	use bias	False	False
	batch size	128	128
	num factors	144	147
	item reg	1.61E-05	2.74E-04
	user reg	4.69E-05	1.05E-03
	learning rate	8.66E-04	1.00E-01
	negative quota	0.3693	0.0982
	estimate model for cold users	itemKNN	itemKNN
	estimate model for cold users topK	909	791
PureSVD	num factors	33	55
	estimate model for cold users	mean item factors	mean item factors
	estimate model for cold users topK	5	1000
NMF	num factors	67	142
	solver	mult. update	mult. update
	init type	nndsvda	random
	beta loss	frobenius	frobenius
	estimate model for cold users	mean item factors	mean item factors
	estimate model for cold users topK	758	560
iALS	num factors	60	146
	confidence scaling	linear	linear
	alpha	5.6517	0.0034
	epsilon	0.0010	0.0681
	reg	1.00E-05	1.65E-03
	estimate model for cold users	itemKNN	itemKNN
	estimate model for cold users topK	982	31

Table 69. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Movielens 20M cold user	Netflix Prize cold user
Mult VAE	epochs	95	80
	batch size	500	500
	total anneal steps	200000	200000
	p dims	-	-

K IJCAI: OUTER PRODUCT-BASED NEURAL COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 70. The results of our evaluation can be seen in Table 71 (Gowalla) and Table 72 (Yelp). The corresponding optimal hyperparameters are reported in Table 75 (collaborative KNNs), Table 76 (non-neural machine learning and graph based) and Table 77 (ConvNCF).

Lastly, the time required to train and evaluate the models is reported in Table 73 (Gowalla) and Table 74 (Yelp).

Table 70. Dataset characteristics.

Dataset	Interactions	Items	Users	Density
Yelp	69K	25815	25677	0.105
Gowalla	1249K	52400	54156	0.044

Table 71. Experimental results for the ConvNCF method for the Gowalla dataset.

	@ 5		@ 10		@ 20	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0049	0.0029	0.0099	0.0045	0.0205	0.0071
TopPopular	0.2188	0.1652	0.2910	0.1884	0.3803	0.2110
UserKNN CF cosine	**0.7131**	**0.5879**	**0.7939**	**0.6142**	0.8532	**0.6293**
UserKNN CF dice	**0.6848**	**0.5632**	0.7649	**0.5893**	0.8226	**0.6039**
UserKNN CF jaccard	**0.6786**	**0.5572**	0.7597	**0.5836**	0.8174	**0.5983**
UserKNN CF asymmetric	**0.6720**	**0.5486**	0.7555	**0.5758**	0.8156	**0.5911**
UserKNN CF tversky	**0.6769**	**0.5556**	0.7579	**0.5820**	0.8149	**0.5965**
ItemKNN CF cosine	**0.6806**	**0.5511**	0.7668	**0.5792**	0.8257	**0.5942**
ItemKNN CF dice	0.6605	0.5231	0.7592	0.5552	0.8280	0.5728
ItemKNN CF jaccard	**0.6890**	**0.5577**	0.7752	**0.5857**	0.8306	**0.5999**
ItemKNN CF asymmetric	**0.6953**	**0.5711**	0.7762	**0.5974**	0.8332	**0.6119**
ItemKNN CF tversky	**0.7047**	**0.5864**	0.7790	**0.6105**	0.8331	**0.6244**
$P^3\alpha$	**0.6926**	**0.5703**	0.7674	**0.5948**	0.8158	**0.6071**
$RP^3\beta$	**0.6836**	**0.5525**	0.7723	**0.5814**	0.8361	**0.5976**
$EASE^R$	-	-	-	-	-	-
SLIM BPR	-	-	-	-	-	-
SLIM ElasticNet	0.6365	**0.5284**	0.7083	0.5517	0.7608	0.5651
MF BPR	0.6376	0.4996	0.7416	0.5334	0.8234	0.5542
MF FunkSVD	0.6029	0.4592	0.7216	0.4979	0.8082	0.5199
PureSVD	0.5653	0.4482	0.6627	0.4798	0.7393	0.4993
NMF	0.5856	0.4607	0.6842	0.4927	0.7674	0.5138
iALS	0.6460	0.5081	0.7554	0.5436	0.8356	0.5641
ConvNCF	0.6702	0.5233	0.7799	0.5590	**0.8623**	0.5799

Table 72. Experimental results for the ConvNCF method for the Yelp dataset.

	@ 5		@ 10		@ 20	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0048	0.0030	0.0097	0.0045	0.0204	0.0072
TopPopular	0.0817	0.0538	0.1199	0.0661	0.1754	0.0800
UserKNN CF cosine	**0.2068**	**0.1355**	**0.3126**	**0.1695**	0.4401	**0.2017**
UserKNN CF dice	**0.1994**	**0.1306**	0.3014	**0.1634**	0.4271	0.1951
UserKNN CF jaccard	**0.2006**	**0.1311**	0.3023	**0.1638**	0.4286	0.1956
UserKNN CF asymmetric	**0.2185**	**0.1441**	**0.3275**	**0.1792**	**0.4553**	**0.2115**
UserKNN CF tversky	**0.2046**	**0.1346**	0.3049	**0.1669**	0.4320	**0.1990**
ItemKNN CF cosine	**0.2521**	**0.1686**	**0.3669**	**0.2056**	**0.4974**	**0.2385**
ItemKNN CF dice	**0.2329**	**0.1564**	**0.3396**	**0.1908**	**0.4665**	**0.2228**
ItemKNN CF jaccard	**0.2414**	**0.1634**	**0.3512**	**0.1988**	**0.4786**	**0.2309**
ItemKNN CF asymmetric	**0.2421**	**0.1598**	**0.3514**	**0.1950**	**0.4815**	**0.2278**
ItemKNN CF tversky	**0.2303**	**0.1546**	**0.3346**	**0.1884**	**0.4563**	**0.2192**
$P^3\alpha$	**0.2145**	**0.1394**	**0.3211**	**0.1738**	0.4442	**0.2049**
$RP^3\beta$	**0.2202**	**0.1431**	**0.3323**	**0.1793**	**0.4667**	**0.2132**
$EASE^R$	**0.2349**	**0.1557**	**0.3419**	**0.1902**	**0.4617**	**0.2205**
SLIM BPR	-	-	-	-	-	-
SLIM ElasticNet	**0.2330**	**0.1535**	**0.3475**	**0.1904**	**0.4799**	**0.2238**
MF BPR	0.1557	0.1024	0.2421	0.1302	0.3599	0.1598
MF FunkSVD	0.1728	0.1121	0.2621	0.1409	0.3727	0.1688
PureSVD	**0.2011**	**0.1307**	0.3002	**0.1626**	0.4238	0.1938
NMF	0.1816	0.1172	0.2825	0.1496	0.4090	0.1815
iALS	**0.2048**	**0.1348**	**0.3080**	**0.1680**	0.4319	**0.1993**
ConvNCF	0.1947	0.1250	0.3059	0.1608	0.4446	0.1957

Table 73. Computation time for the algorithms in the selected results for the ConvNCF method on the Gowalla dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.04 [sec]	94.26 [sec] / 1.57 [min]	575
TopPopular	0.08 [sec]	112.67 [sec] / 1.88 [min]	481
UserKNN CF cosine	24.25 ± 1.03 [sec]	161.09 [sec] / 2.68 ± 0.07 [min]	322
UserKNN CF dice	25.07 ± 0.93 [sec]	161.77 [sec] / 2.70 ± 0.03 [min]	332
UserKNN CF jaccard	25.07 ± 0.88 [sec]	161.03 [sec] / 2.68 ± 0.08 [min]	334
UserKNN CF asymmetric	25.75 ± 1.03 [sec]	161.07 [sec] / 2.68 ± 0.09 [min]	334
UserKNN CF tversky	25.62 ± 0.92 [sec]	162.28 [sec] / 2.70 ± 0.03 [min]	337
ItemKNN CF cosine	25.63 ± 1.65 [sec]	162.49 [sec] / 2.71 ± 0.03 [min]	338
ItemKNN CF dice	24.78 ± 1.52 [sec]	166.14 [sec] / 2.77 ± 0.09 [min]	339
ItemKNN CF jaccard	24.87 ± 1.55 [sec]	163.76 [sec] / 2.73 ± 0.07 [min]	341
ItemKNN CF asymmetric	25.52 ± 1.85 [sec]	166.42 [sec] / 2.77 ± 0.08 [min]	329
ItemKNN CF tversky	25.92 ± 1.72 [sec]	164.91 [sec] / 2.75 ± 0.07 [min]	324
$P^3\alpha$	90.96 [sec] / 1.52 ± 0.27 [min]	156.74 [sec] / 2.61 ± 0.04 [min]	348
$RP^3\beta$	103.55 [sec] / 1.73 ± 0.31 [min]	159.29 [sec] / 2.65 ± 0.01 [min]	342
$EASE^R$	-	-	-
SLIM BPR	24663.44 [sec] / 6.85 ± 4.36 [hour]	165.90 [sec] / 2.76 ± 0.11 [min]	316
SLIM ElasticNet	4531.84 [sec] / 1.26 ± 0.70 [hour]	158.98 [sec] / 2.65 ± 0.17 [min]	329
MF BPR	19828.95 [sec] / 5.51 ± 2.32 [hour]	128.32 [sec] / 2.14 ± 0.09 [min]	413
MF FunkSVD	14731.56 [sec] / 4.09 ± 2.15 [hour]	136.27 [sec] / 2.27 ± 0.13 [min]	383
PureSVD	11.12 ± 3.45 [sec]	321.24 [sec] / 5.35 ± 0.60 [min]	159
NMF	2368.87 [sec] / 39.48 ± 23.03 [min]	251.74 [sec] / 4.20 ± 0.68 [min]	195
iALS	2695.70 [sec] / 44.93 ± 26.66 [min]	129.40 [sec] / 2.16 ± 0.05 [min]	413
ConvNCF	44743.03 [sec] / 12.43 [hour]	233.89 [sec] / 3.90 [min]	232

Table 74. Computation time for the algorithms in the selected results for the ConvNCF method on the Yelp dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.02 [sec]	44.30 [sec]	580
TopPopular	0.03 [sec]	49.17 [sec]	522
UserKNN CF cosine	8.93 ± 0.65 [sec]	74.11 [sec] / 1.24 ± 0.01 [min]	348
UserKNN CF dice	9.26 ± 0.72 [sec]	74.30 [sec] / 1.24 ± 0.01 [min]	347
UserKNN CF jaccard	9.18 ± 0.69 [sec]	73.27 [sec] / 1.22 ± 0.01 [min]	348
UserKNN CF asymmetric	9.21 ± 0.96 [sec]	74.33 [sec] / 1.24 ± 0.02 [min]	346
UserKNN CF tversky	9.43 ± 0.70 [sec]	74.67 [sec] / 1.24 ± 0.02 [min]	350
ItemKNN CF cosine	8.04 ± 0.88 [sec]	75.74 [sec] / 1.26 ± 0.04 [min]	328
ItemKNN CF dice	7.95 ± 0.70 [sec]	73.03 [sec] / 1.22 ± 0.04 [min]	361
ItemKNN CF jaccard	7.95 ± 0.74 [sec]	74.48 [sec] / 1.24 ± 0.04 [min]	347
ItemKNN CF asymmetric	8.46 ± 0.79 [sec]	77.62 [sec] / 1.29 ± 0.02 [min]	336
ItemKNN CF tversky	8.13 ± 0.70 [sec]	73.72 [sec] / 1.23 ± 0.03 [min]	355
$P^3\alpha$	33.85 ± 5.97 [sec]	71.02 [sec] / 1.18 ± 0.01 [min]	363
$RP^3\beta$	37.25 ± 9.81 [sec]	71.48 [sec] / 1.19 ± 0.04 [min]	349
$EASE^R$	339.15 [sec] / 5.65 ± 0.01 [min]	77.75 [sec] / 1.30 ± 0.02 [min]	326
SLIM BPR	-	-	-
SLIM ElasticNet	846.95 [sec] / 14.12 ± 6.92 [min]	68.16 [sec] / 1.14 ± 0.07 [min]	361
MF BPR	3981.91 [sec] / 1.11 ± 0.88 [hour]	50.36 ± 1.71 [sec]	495
MF FunkSVD	3944.58 [sec] / 1.10 ± 0.85 [hour]	51.88 ± 1.46 [sec]	501
PureSVD	2.35 ± 1.79 [sec]	56.89 ± 4.49 [sec]	466
NMF	952.10 [sec] / 15.87 ± 6.43 [min]	75.16 [sec] / 1.25 ± 0.47 [min]	464
iALS	1778.07 [sec] / 29.63 ± 19.78 [min]	47.35 ± 1.33 [sec]	530
ConvNCF	11465.29 [sec] / 3.18 [hour]	102.80 [sec] / 1.71 [min]	250

Table 75. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Yelp	Gowalla
UserKNN CF cosine	topK	470	1000
	shrink	0	1000
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	none	TF-IDF
UserKNN CF dice	topK	494	513
	shrink	0	10
	similarity	dice	dice
	normalize	False	True
UserKNN CF jaccard	topK	553	455
	shrink	2	5
	similarity	jaccard	jaccard
	normalize	False	True
UserKNN CF asymmetric	topK	529	451
	shrink	721	173
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.1781	0.6950
	feature weighting	TF-IDF	TF-IDF
UserKNN CF tversky	topK	474	368
	shrink	67	0
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	1.9756	1.3012
	tversky beta	1.9345	2.0000
ItemKNN CF cosine	topK	1000	317
	shrink	387	1000
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	TF-IDF	TF-IDF
ItemKNN CF dice	topK	195	409
	shrink	10	20
	similarity	dice	dice
	normalize	False	True
ItemKNN CF jaccard	topK	479	302
	shrink	4	68
	similarity	jaccard	jaccard
	normalize	True	True
ItemKNN CF asymmetric	topK	918	712
	shrink	154	507
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.3530	0.2575
	feature weighting	TF-IDF	TF-IDF
ItemKNN CF tversky	topK	374	944
	shrink	0	16
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	0.7020	0.0558
	tversky beta	1.5460	1.9805

Table 76. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Yelp	Gowalla
$P^3\alpha$	topK	584	413
	alpha	0.3672	0.4424
	normalize similarity	True	True
$RP^3\beta$	topK	1000	640
	alpha	0.5548	0.5168
	beta	0.3389	0.2009
	normalize similarity	False	True
$EASE^R$	l2 norm	5.66E+02	-
SLIM ElasticNet	topK	1000	916
	l1 ratio	2.66E-05	4.30E-04
	alpha	0.0520	0.0010
MF BPR	sgd mode	adam	adam
	epochs	1420	1485
	num factors	200	200
	batch size	32	16
	positive reg	1.00E-02	1.00E-02
	negative reg	1.00E-02	1.00E-02
	learning rate	5.70E-04	1.69E-03
MF FunkSVD	sgd mode	adam	adam
	epochs	365	410
	use bias	True	True
	batch size	32	4
	num factors	103	192
	item reg	1.43E-05	1.02E-04
	user reg	9.96E-03	2.88E-04
	learning rate	2.16E-03	1.41E-03
	negative quota	0.0642	0.1492
PureSVD	num factors	70	350
NMF	num factors	53	286
	solver	mult. update	mult. update
	init type	random	nndsvda
	beta loss	kullback-leibler	kullback-leibler
iALS	num factors	145	200
	confidence scaling	log	log
	alpha	7.3331	50.0000
	epsilon	0.0270	0.1846
	reg	4.50E-03	1.00E-02
	epochs	60	5
SLIM BPR	topK	-	795
	epochs	-	570
	symmetric	-	False
	sgd mode	-	adagrad
	lambda i	-	4.42E-04
	lambda j	-	2.16E-04
	learning rate	-	2.98E-03

Table 77. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Yelp	Gowalla
ConvNCF	batch size	512	512
	epochs	145	445
	epochs MFBPR	480	490
	embedding size	64	64
	hidden size	128	128
	negative sample per positive	1	1
	negative instances per positive	4	4
	regularization users items	1.00E-02	1.00E-02
	regularization weights	10	10
	regularization filter weights	1	1
	learning rate embeddings	5.00E-02	5.00E-02
	learning rate CNN	5.00E-02	5.00E-02
	channel size	[32, 32, 32, 32, 32, 32]	[32, 32, 32, 32, 32, 32]
	dropout	0.0000	0.0000
	epoch verbose	1	1

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Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 78. The results of our evaluation can be seen in Table 79 (Movielens 1M), Table 80 (Movielens HetRec), Table 81 (Frappe) and Table 82 (FilmTrust). The corresponding optimal hyperparameters are reported in Table 91 (collaborative KNNs), Table 92 (non-neural machine learning and graph based) and Table 93 (NeuRec).

Lastly, the time required to train and evaluate the models is reported in Table 87 (Movielens 1M), Table 88 (Movielens HetRec), Table 89 (Frappe) and Table 90 (FilmTrust).

Table 78. Dataset characteristics.

Dataset	Interactions	Items	Users	Density
Movielens 1M	1M	3882	6039	4.25
Movielens HetRec	855K	10109	2113	4.01
Frappe	19K	4082	957	0.48
FilmTrust	35K	2071	1508	1.14

Table 79. Experimental results for the NeuRec method for the Movielens 1M dataset.

	@ 5					@ 10					@ 50				
	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR
Random	0.0090	0.0010	0.0044	0.0022	0.0213	0.0091	0.0024	0.0029	0.0035	0.0265	0.0091	0.0128	0.0016	0.0099	0.0353
TopPopular	0.2105	0.0402	0.1531	0.0689	0.3621	0.1832	0.0685	0.1168	0.0939	0.3793	0.1127	0.1985	0.0734	0.1732	0.3912
UserKNN CF cosine	**0.4075**	**0.1034**	**0.3298**	**0.1626**	**0.6335**	**0.3468**	**0.1667**	**0.2616**	**0.2158**	**0.6441**	**0.1972**	**0.4022**	**0.1910**	**0.3583**	**0.6482**
UserKNN CF dice	**0.4189**	**0.1055**	**0.3415**	**0.1658**	**0.6368**	**0.3583**	**0.1714**	**0.2738**	**0.2210**	**0.6475**	**0.2051**	**0.4116**	**0.2015**	**0.3672**	**0.6517**
UserKNN CF jaccard	**0.4179**	**0.1050**	**0.3406**	**0.1653**	**0.6365**	**0.3578**	**0.1705**	**0.2730**	**0.2204**	**0.6474**	**0.2038**	**0.4102**	**0.1996**	**0.3662**	**0.6515**
UserKNN CF asymmetric	**0.4212**	**0.1065**	**0.3441**	**0.1674**	**0.6399**	**0.3617**	**0.1726**	**0.2774**	**0.2230**	**0.6509**	**0.2069**	**0.4146**	**0.2047**	**0.3704**	**0.6550**
UserKNN CF tversky	**0.4042**	**0.1024**	**0.3265**	**0.1611**	**0.6277**	**0.3410**	**0.1647**	**0.2571**	**0.2132**	**0.6386**	**0.1940**	**0.3968**	**0.1873**	**0.3536**	**0.6428**
ItemKNN CF cosine	**0.4002**	**0.0987**	**0.3237**	**0.1561**	**0.6137**	**0.3432**	**0.1585**	**0.2600**	**0.2074**	**0.6247**	**0.1968**	**0.3831**	**0.1872**	**0.3452**	**0.6297**
ItemKNN CF dice	**0.3709**	**0.0854**	**0.2951**	**0.1383**	**0.5714**	**0.3215**	**0.1406**	**0.2367**	**0.1862**	**0.5845**	**0.1894**	**0.3612**	**0.1690**	**0.3211**	**0.5902**
ItemKNN CF jaccard	**0.3747**	**0.0875**	**0.2982**	**0.1401**	**0.5751**	**0.3219**	**0.1407**	**0.2376**	**0.1869**	**0.5870**	**0.1869**	**0.3559**	**0.1676**	**0.3187**	**0.5928**
ItemKNN CF asymmetric	**0.3995**	**0.0984**	**0.3244**	**0.1563**	**0.6179**	**0.3452**	**0.1590**	**0.2618**	**0.2084**	**0.6293**	**0.1978**	**0.3865**	**0.1886**	**0.3474**	**0.6341**
ItemKNN CF tversky	**0.3718**	**0.0867**	**0.2998**	**0.1414**	**0.5878**	**0.3116**	**0.1359**	**0.2343**	**0.1846**	**0.5985**	0.1750	**0.3300**	**0.1582**	**0.3037**	**0.6041**
$P^3\alpha$	**0.4041**	**0.1007**	**0.3286**	**0.1596**	**0.6250**	**0.3456**	**0.1627**	**0.2627**	**0.2121**	**0.6362**	**0.1988**	**0.3945**	**0.1919**	**0.3538**	**0.6410**
$RP^3\beta$	**0.4080**	**0.1007**	**0.3325**	**0.1602**	**0.6260**	**0.3508**	**0.1639**	**0.2676**	**0.2137**	**0.6374**	**0.2012**	**0.3938**	**0.1949**	**0.3551**	**0.6420**
$EASE^R$	**0.4488**	**0.1134**	**0.3717**	**0.1779**	**0.6620**	**0.3857**	**0.1820**	**0.3035**	**0.2364**	**0.6717**	**0.2259**	**0.4410**	**0.2305**	**0.3962**	**0.6754**
SLIM BPR	**0.3964**	**0.1034**	**0.3161**	**0.1606**	**0.6222**	**0.3358**	**0.1663**	**0.2494**	**0.2128**	**0.6335**	**0.1968**	**0.4048**	**0.1892**	**0.3568**	**0.6379**
SLIM ElasticNet	**0.4437**	**0.1106**	**0.3692**	**0.1749**	**0.6578**	**0.3813**	**0.1770**	**0.3003**	**0.2321**	**0.6679**	**0.2234**	**0.4333**	**0.2259**	**0.3902**	**0.6720**
MF BPR	**0.3576**	**0.0830**	**0.2812**	**0.1340**	**0.5628**	**0.3073**	**0.1384**	**0.2217**	**0.1807**	**0.5768**	**0.1828**	**0.3575**	**0.1593**	**0.3128**	**0.5825**
MF FunkSVD	**0.3936**	**0.0927**	**0.3154**	**0.1479**	**0.6000**	**0.3458**	**0.1555**	**0.2572**	**0.2014**	**0.6125**	**0.2090**	**0.4074**	**0.1926**	**0.3541**	**0.6176**
PureSVD	**0.4123**	**0.0987**	**0.3371**	**0.1586**	**0.6266**	**0.3575**	**0.1624**	**0.2722**	**0.2132**	**0.6380**	**0.2133**	**0.4089**	**0.2033**	**0.3651**	**0.6427**
NMF	**0.3811**	**0.0891**	**0.3017**	**0.1430**	**0.5817**	**0.3338**	**0.1499**	**0.2442**	**0.1948**	**0.5947**	**0.2070**	**0.4047**	**0.1872**	**0.3489**	**0.6005**
iALS	**0.4164**	**0.1036**	**0.3373**	**0.1635**	**0.6327**	**0.3628**	**0.1702**	**0.2743**	**0.2200**	**0.6443**	**0.2180**	**0.4265**	**0.2104**	**0.3774**	**0.6483**
INeuRec	0.3280	0.0663	0.2554	0.1110	0.5003	0.2839	0.1094	0.2027	0.1500	0.5129	0.1755	0.3048	0.1397	0.2719	0.5206
UNeuRec	0.2098	0.0395	0.1560	0.0684	0.3663	0.1856	0.0688	0.1199	0.0944	0.3852	0.1143	0.2002	0.0750	0.1743	0.3968

Table 80. Experimental results for the NeuRec method for the HetRec dataset.

			@ 5					@ 10					@ 50		
	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR
Random	0.0093	0.0005	0.0040	0.0011	0.0181	0.0093	0.0009	0.0027	0.0018	0.0233	0.0091	0.0048	0.0011	0.0051	0.0324
TopPopular	0.4556	0.0408	0.3850	0.0889	0.6264	0.4057	0.0712	0.3137	0.1237	0.6368	0.2632	0.2048	0.1768	0.2326	0.6406
UserKNN CF cosine	**0.5632**	**0.0605**	**0.4977**	**0.1237**	**0.7420**	**0.4988**	**0.1001**	**0.4145**	**0.1677**	**0.7486**	0.3131	**0.2685**	**0.2413**	**0.3005**	**0.7517**
UserKNN CF dice	**0.5714**	**0.0614**	**0.5079**	**0.1250**	**0.7465**	**0.5087**	**0.1015**	**0.4279**	**0.1698**	**0.7525**	**0.3237**	**0.2736**	**0.2538**	**0.3066**	**0.7560**
UserKNN CF jaccard	**0.5692**	**0.0617**	**0.5063**	**0.1250**	**0.7461**	**0.5088**	**0.1016**	**0.4276**	**0.1699**	**0.7520**	**0.3233**	**0.2729**	**0.2533**	**0.3062**	**0.7554**
UserKNN CF asymmetric	**0.5729**	**0.0619**	**0.5097**	**0.1251**	**0.7449**	**0.5151**	**0.1012**	**0.4346**	**0.1702**	**0.7504**	**0.3283**	**0.2750**	**0.2597**	**0.3086**	**0.7537**
UserKNN CF tversky	**0.5670**	**0.0612**	**0.5039**	**0.1245**	**0.7474**	**0.5044**	**0.1014**	**0.4216**	**0.1693**	**0.7538**	**0.3195**	**0.2713**	**0.2487**	**0.3044**	**0.7571**
ItemKNN CF cosine	0.5405	**0.0528**	0.4747	**0.1119**	**0.7096**	0.4750	**0.0873**	0.3922	**0.1519**	**0.7157**	0.2971	**0.2427**	0.2219	0.2759	**0.7196**
ItemKNN CF dice	0.5371	**0.0513**	0.4672	**0.1098**	0.6993	0.4741	**0.0867**	0.3880	**0.1504**	0.7062	0.2981	**0.2425**	0.2198	0.2754	0.7101
ItemKNN CF jaccard	0.5200	**0.0504**	0.4537	**0.1078**	0.6993	0.4560	0.0838	0.3711	0.1462	0.7063	0.2857	0.2316	0.2068	0.2647	0.7101
ItemKNN CF asymmetric	**0.5676**	**0.0572**	**0.5041**	**0.1210**	**0.7426**	**0.4996**	**0.0941**	**0.4165**	**0.1635**	**0.7494**	**0.3185**	**0.2437**	**0.2439**	**0.2900**	**0.7520**
ItemKNN CF tversky	0.5408	**0.0539**	0.4747	**0.1150**	**0.7234**	0.4791	**0.0898**	0.3923	**0.1563**	**0.7295**	0.3082	**0.2522**	0.2292	**0.2863**	**0.7332**
$P^3\alpha$	0.5032	**0.0519**	0.4351	0.1070	0.6831	0.4501	**0.0905**	0.3592	0.1484	0.6923	0.2861	**0.2481**	0.2057	0.2714	0.6963
$RP^3\beta$	**0.5464**	**0.0558**	0.4692	**0.1159**	**0.7110**	**0.4970**	**0.0950**	0.4013	**0.1607**	**0.7172**	0.2936	0.2246	0.2215	0.2727	**0.7201**
$EASE^R$	**0.6242**	**0.0657**	**0.5659**	**0.1349**	**0.7749**	**0.5573**	**0.1080**	**0.4842**	**0.1833**	**0.7796**	**0.3611**	**0.2936**	**0.3001**	**0.3332**	**0.7828**
SLIM BPR	0.5196	**0.0574**	0.4383	**0.1136**	0.6929	0.4709	**0.0989**	0.3701	**0.1581**	0.6999	0.3050	**0.2656**	0.2258	**0.2895**	0.7030
SLIM ElasticNet	**0.6283**	**0.0670**	**0.5732**	**0.1379**	**0.7879**	**0.5612**	**0.1103**	**0.4882**	**0.1874**	**0.7933**	**0.3549**	**0.2906**	**0.2958**	**0.3333**	**0.7959**
MF BPR	0.4204	0.0360	0.3493	0.0811	0.6028	0.3750	0.0659	0.2805	0.1143	0.6138	0.2533	0.1983	0.1608	0.2215	0.6189
MF FunkSVD	0.4882	0.0447	0.4088	0.0947	0.6446	0.4520	0.0820	0.3525	0.1364	0.6538	0.3145	**0.2541**	0.2240	0.2730	0.6585
PureSVD	**0.5977**	**0.0601**	**0.5364**	**0.1271**	**0.7524**	**0.5369**	**0.1010**	**0.4548**	**0.1746**	**0.7578**	**0.3521**	**0.2849**	**0.2784**	**0.3237**	**0.7609**
NMF	0.5432	**0.0570**	0.4686	**0.1180**	**0.7108**	**0.4892**	**0.0964**	0.3941	**0.1632**	**0.7165**	**0.3193**	**0.2675**	**0.2384**	**0.3023**	**0.7197**
iALS	**0.5900**	**0.0609**	**0.5253**	**0.1281**	**0.7542**	**0.5322**	**0.1039**	**0.4456**	**0.1770**	**0.7593**	**0.3464**	**0.2857**	**0.2715**	**0.3243**	**0.7620**
INeuRec	0.5435	0.0489	0.4797	0.1076	0.7021	0.4884	0.0844	0.4047	0.1493	0.7094	0.3151	0.2402	0.2371	0.2769	0.7129
UNeuRec	0.4467	0.0397	0.3785	0.0877	0.6278	0.3973	0.0693	0.3077	0.1216	0.6365	0.2599	0.2029	0.1731	0.2299	0.6413

Table 81. Experimental results for the NeuRec method for the Frappe dataset.

			@ 5					@ 10					@ 50		
	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR
Random	0.0017	0.0007	0.0009	0.0011	0.0047	0.0011	0.0018	0.0007	0.0016	0.0051	0.0013	0.0103	0.0010	0.0047	0.0073
TopPopular	0.1332	0.2010	0.1441	0.1842	0.3183	0.0920	0.2602	0.1411	0.2118	0.3302	0.0383	0.4325	0.1530	0.2721	0.3387
UserKNN CF cosine	0.1888	**0.2243**	0.1943	0.2213	0.3880	0.1339	0.2929	0.1845	0.2578	0.3980	**0.0480**	**0.4635**	0.1940	**0.3207**	0.4042
UserKNN CF dice	0.1810	0.2175	0.1899	0.2168	0.3821	0.1279	0.2873	0.1813	0.2532	0.3928	0.0467	**0.4541**	0.1906	**0.3149**	0.3991
UserKNN CF jaccard	0.1844	**0.2248**	0.1923	0.2203	0.3843	0.1291	0.2871	0.1823	0.2540	0.3937	0.0472	**0.4568**	0.1918	**0.3167**	0.4001
UserKNN CF asymmetric	0.1899	**0.2267**	0.1965	0.2232	0.3893	0.1349	0.2939	0.1873	0.2592	0.3979	0.0477	**0.4564**	0.1944	**0.3193**	0.4045
UserKNN CF tversky	0.1961	**0.2225**	0.2037	0.2231	0.3893	0.1401	**0.2965**	**0.1912**	**0.2619**	0.3996	**0.0490**	**0.4613**	**0.1987**	**0.3233**	0.4057
ItemKNN CF cosine	0.1947	0.2192	0.2032	**0.2243**	0.3980	0.1342	0.2780	0.1877	0.2564	0.4059	**0.0484**	**0.4524**	**0.1965**	**0.3198**	0.4120
ItemKNN CF dice	0.1779	0.2075	0.1856	0.2094	0.3743	0.1198	0.2643	0.1724	0.2390	0.3833	0.0453	0.4406	0.1800	0.3018	0.3911
ItemKNN CF jaccard	0.1785	0.2094	0.1855	0.2095	0.3726	0.1196	0.2669	0.1719	0.2389	0.3817	0.0452	0.4385	0.1792	0.3006	0.3891
ItemKNN CF asymmetric	0.1852	**0.2261**	0.1964	**0.2236**	0.3882	0.1263	0.2817	0.1839	0.2538	0.3970	0.0473	**0.4554**	0.1940	**0.3178**	0.4037
ItemKNN CF tversky	0.1777	0.2139	0.1840	0.2117	0.3762	0.1212	0.2698	0.1729	0.2414	0.3840	0.0462	0.4434	0.1814	0.3042	0.3912
$P^3\alpha$	0.1933	**0.2322**	0.1939	**0.2254**	0.3838	0.1447	**0.3050**	**0.1899**	**0.2664**	0.3932	**0.0511**	**0.4699**	**0.2029**	**0.3296**	0.3984
$RP^3\beta$	0.2059	**0.2349**	**0.2084**	**0.2341**	**0.4074**	**0.1486**	**0.3067**	**0.1981**	**0.2742**	**0.4163**	**0.0507**	**0.4631**	**0.2084**	**0.3341**	**0.4208**
$EASE^R$	**0.2254**	**0.2422**	**0.2278**	**0.2460**	**0.4189**	**0.1588**	**0.3129**	**0.2133**	**0.2856**	**0.4254**	**0.0523**	**0.4605**	**0.2217**	**0.3429**	**0.4297**
SLIM BPR	0.1791	**0.2231**	0.1860	0.2167	0.3773	0.1272	0.2812	0.1771	0.2493	0.3865	0.0456	**0.4464**	0.1860	0.3091	0.3931
SLIM ElasticNet	0.2014	**0.2299**	0.2023	**0.2285**	0.3943	0.1464	**0.3028**	**0.1938**	**0.2686**	0.4030	**0.0516**	**0.4665**	**0.2054**	**0.3312**	0.4076
MF BPR	0.1209	0.1429	0.1045	0.1333	0.2456	0.0933	0.2101	0.1036	0.1664	0.2604	0.0396	0.3914	0.1174	0.2316	0.2701
MF FunkSVD	0.1601	**0.2214**	0.1600	0.2016	0.3431	0.1205	**0.2973**	0.1597	0.2410	0.3569	0.0467	**0.4726**	0.1769	0.3069	0.3624
PureSVD	0.1282	0.1753	0.1362	0.1687	0.3064	0.0983	0.2386	0.1356	0.2017	0.3200	0.0375	0.3993	0.1443	0.2558	0.3263
NMF	0.1439	0.1008	0.1310	0.1197	0.2378	0.1047	0.1389	0.1118	0.1436	0.2434	0.0336	0.2282	0.1088	0.1784	0.2492
iALS	0.1774	0.1804	0.1768	0.1918	0.3503	0.1335	0.2534	0.1690	0.2313	0.3599	0.0454	0.4016	0.1750	0.2867	0.3657
INeuRec	0.2117	0.2196	0.2073	0.2235	0.4045	0.1479	0.2862	0.1891	0.2603	0.4116	0.0478	0.4305	0.1946	0.3133	0.4161
UNeuRec	0.1679	0.2206	0.1788	0.2100	0.3650	0.1194	0.2964	0.1731	0.2470	0.3780	0.0402	0.4448	0.1785	0.2992	0.3852

Table 82. Experimental results for the NeuRec method for the FilmTrust dataset.

	@ 5					@ 10					@ 50				
	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR	PREC	REC	MAP	NDCG	MRR
Random	0.0025	0.0018	0.0015	0.0019	0.0063	0.0026	0.0041	0.0012	0.0032	0.0081	0.0027	0.0217	0.0018	0.0095	0.0118
TopPopular	0.4200	**0.4126**	0.4393	**0.4203**	0.6145	0.3471	**0.6351**	**0.4597**	**0.5450**	0.6273	0.0916	**0.8614**	**0.4954**	**0.6314**	0.6316
UserKNN CF cosine	**0.4349**	**0.4357**	**0.4735**	**0.4498**	**0.6496**	**0.3545**	**0.6357**	**0.4909**	**0.5677**	**0.6581**	**0.0921**	0.8247	**0.5202**	**0.6429**	**0.6615**
UserKNN CF dice	**0.4341**	**0.4339**	**0.4667**	**0.4457**	**0.6452**	**0.3512**	0.6286	**0.4822**	**0.5604**	**0.6526**	0.0917	0.8241	**0.5135**	**0.6382**	**0.6561**
UserKNN CF jaccard	**0.4354**	**0.4400**	**0.4713**	**0.4510**	**0.6492**	**0.3560**	**0.6455**	**0.4907**	**0.5712**	**0.6579**	**0.0932**	0.8524	**0.5229**	**0.6517**	**0.6620**
UserKNN CF asymmetric	**0.4322**	**0.4292**	**0.4656**	**0.4431**	**0.6431**	**0.3517**	**0.6309**	**0.4825**	**0.5608**	**0.6515**	**0.0919**	0.8257	**0.5139**	**0.6386**	**0.6554**
UserKNN CF tversky	**0.4338**	**0.4335**	**0.4646**	**0.4436**	**0.6434**	**0.3519**	**0.6328**	**0.4805**	**0.5601**	**0.6515**	**0.0918**	0.8267	**0.5116**	**0.6372**	**0.6545**
ItemKNN CF cosine	**0.4268**	**0.4247**	**0.4551**	**0.4354**	**0.6332**	**0.3484**	**0.6303**	**0.4724**	**0.5540**	**0.6432**	0.0916	0.8478	**0.5072**	**0.6380**	**0.6470**
ItemKNN CF dice	**0.4264**	**0.4205**	**0.4559**	**0.4355**	**0.6388**	**0.3507**	**0.6338**	**0.4753**	**0.5577**	**0.6492**	0.0917	0.8473	**0.5083**	**0.6394**	**0.6527**
ItemKNN CF jaccard	**0.4259**	**0.4202**	**0.4561**	**0.4358**	**0.6407**	**0.3511**	**0.6344**	**0.4758**	**0.5585**	**0.6512**	0.0917	0.8473	**0.5086**	**0.6398**	**0.6548**
ItemKNN CF asymmetric	**0.4286**	**0.4238**	**0.4564**	**0.4360**	**0.6348**	**0.3490**	**0.6303**	**0.4734**	**0.5548**	**0.6447**	**0.0923**	0.8516	**0.5081**	**0.6400**	**0.6487**
ItemKNN CF tversky	**0.4262**	**0.4208**	**0.4566**	**0.4365**	**0.6413**	**0.3509**	**0.6342**	**0.4763**	**0.5587**	**0.6513**	0.0917	0.8473	**0.5092**	**0.6402**	**0.6549**
$P^3\alpha$	**0.4240**	**0.4199**	**0.4526**	**0.4321**	**0.6351**	**0.3500**	**0.6343**	**0.4719**	**0.5550**	**0.6467**	**0.0938**	**0.8605**	**0.5080**	**0.6430**	**0.6511**
$RP^3\beta$	**0.4373**	**0.4365**	**0.4709**	**0.4492**	**0.6537**	**0.3575**	**0.6436**	**0.4880**	**0.5701**	**0.6631**	**0.0950**	**0.8647**	**0.5228**	**0.6562**	**0.6674**
$EASE^R$	**0.4458**	**0.4498**	**0.4825**	**0.4604**	**0.6620**	**0.3597**	**0.6590**	**0.4990**	**0.5805**	**0.6717**	**0.0938**	**0.8598**	**0.5302**	**0.6591**	**0.6752**
SLIM BPR	**0.4327**	**0.4351**	**0.4643**	**0.4455**	**0.6465**	**0.3522**	**0.6399**	**0.4825**	**0.5647**	**0.6549**	**0.0922**	0.8529	**0.5166**	**0.6472**	**0.6589**
SLIM ElasticNet	**0.4418**	**0.4417**	**0.4803**	**0.4572**	**0.6600**	**0.3583**	**0.6566**	**0.4983**	**0.5796**	**0.6708**	**0.0944**	**0.8631**	**0.5309**	**0.6604**	**0.6742**
MF BPR	0.4115	0.4047	0.4309	0.4114	0.5979	0.3433	0.6156	0.4519	0.5330	0.6088	0.0902	0.8433	0.4877	0.6193	0.6138
MF FunkSVD	0.4112	0.4004	0.4148	0.3972	0.5781	0.3452	0.6265	0.4378	0.5245	0.5917	0.0906	0.8486	0.4731	0.6095	0.5957
PureSVD	**0.4292**	**0.4255**	**0.4563**	**0.4366**	**0.6366**	**0.3478**	0.6255	**0.4724**	**0.5526**	**0.6453**	0.0912	0.8301	**0.5041**	**0.6322**	**0.6490**
NMF	0.2721	0.2407	0.2769	0.2584	0.4131	0.1983	0.3332	0.2443	0.3123	0.4234	0.0684	0.5484	0.2744	0.3968	0.4315
iALS	0.4038	0.3855	0.4240	0.4028	0.6021	0.3342	0.5920	0.4400	0.5201	0.6137	0.0889	0.8124	0.4720	0.6043	0.6193
INeuRec	0.4221	0.4089	0.4398	0.4196	0.6151	0.3466	0.6187	0.4577	0.5398	0.6261	0.0918	0.8556	0.4935	0.6285	0.6310
UNeuRec	0.4174	0.4062	0.4384	0.4181	0.6157	0.3472	0.6291	0.4596	0.5436	0.6286	0.0912	0.8570	0.4952	0.6304	0.6337

Table 83. Experimental results for the NeuRec method for the Movielens 1M dataset on beyond accuracy metrics.

	@ 10				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9974**	**0.9997**	**1.0000**	**0.8530**	**11.8725**
TopPopular	0.4946	0.9495	0.0180	0.0050	4.5858
UserKNN CF cosine	0.9116	0.9911	0.2380	0.0409	7.6345
UserKNN CF dice	0.9261	0.9926	**0.2537**	0.0494	7.9118
UserKNN CF jaccard	0.9193	0.9919	0.2311	0.0444	7.7648
UserKNN CF asymmetric	0.9288	0.9929	**0.2602**	0.0511	7.9644
UserKNN CF tversky	0.9062	0.9906	0.2396	0.0391	7.5544
ItemKNN CF cosine	0.9296	0.9929	**0.2687**	0.0507	7.9535
ItemKNN CF dice	**0.9399**	**0.9940**	**0.2692**	**0.0571**	**8.1447**
ItemKNN CF jaccard	0.9280	0.9928	**0.2540**	0.0493	7.9144
ItemKNN CF asymmetric	0.9283	0.9928	**0.2664**	0.0500	7.9312
ItemKNN CF tversky	0.8564	0.9856	0.1837	0.0262	6.9452
$P^3\alpha$	0.9080	0.9908	0.1741	0.0371	7.5290
$RP^3\beta$	0.9121	0.9912	0.1927	0.0426	7.6928
$EASE^R$	**0.9398**	**0.9940**	**0.2486**	**0.0582**	**8.1688**
SLIM BPR	0.8748	0.9875	0.1914	0.0313	7.2022
SLIM ElasticNet	0.9296	0.9929	0.2354	0.0508	7.9582
MF BPR	0.9148	0.9915	**0.2589**	0.0483	7.8170
MF FunkSVD	**0.9596**	**0.9959**	0.2259	**0.0731**	**8.5521**
PureSVD	**0.9422**	**0.9942**	0.1968	**0.0523**	**8.0678**
NMF	**0.9589**	**0.9959**	**0.2607**	**0.0792**	**8.6403**
iALS	**0.9535**	**0.9953**	**0.2463**	**0.0682**	**8.4346**
INeuRec	0.9377	0.9938	0.2414	0.0515	8.0217
UNeuRec	0.5048	0.9505	0.0185	0.0052	4.6377

Table 84. Experimental results for the NeuRec method for the HetRec dataset on beyond accuracy metrics.

	@ 10				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9990**	**0.9999**	**0.8764**	**0.6220**	**12.9112**
TopPopular	0.6868	0.9686	0.0083	0.0030	5.2470
UserKNN CF cosine	0.8445	0.9844	0.0599	0.0086	6.6973
UserKNN CF dice	0.8730	0.9873	0.0667	0.0110	7.0740
UserKNN CF jaccard	0.8698	0.9869	0.0648	0.0107	7.0284
UserKNN CF asymmetric	0.8870	0.9887	**0.0849**	0.0136	7.3136
UserKNN CF tversky	0.8625	0.9862	0.0625	0.0100	6.9246
ItemKNN CF cosine	0.8068	0.9806	0.0499	0.0060	6.2156
ItemKNN CF dice	0.8473	0.9847	0.0712	0.0090	6.6976
ItemKNN CF jaccard	0.7826	0.9782	0.0493	0.0054	6.0134
ItemKNN CF asymmetric	**0.9365**	**0.9936**	0.0790	**0.0192**	**7.9766**
ItemKNN CF tversky	0.8434	0.9843	0.0537	0.0076	6.5684
$P^3\alpha$	0.7553	0.9755	0.0275	0.0042	5.7454
$RP^3\beta$	0.8555	0.9855	0.0615	0.0099	6.9156
$EASE^R$	**0.9030**	**0.9903**	0.0697	0.0140	**7.4639**
SLIM BPR	0.8049	0.9805	0.0406	0.0063	6.3283
SLIM ElasticNet	0.8867	0.9886	0.0623	0.0112	7.1582
MF BPR	0.6716	0.9671	0.0085	0.0028	5.1830
MF FunkSVD	**0.9470**	**0.9947**	0.0769	**0.0214**	**8.1640**
PureSVD	**0.9373**	**0.9937**	**0.0932**	**0.0219**	**8.1156**
NMF	**0.9512**	**0.9951**	**0.1510**	**0.0311**	**8.5347**
iALS	**0.9439**	**0.9943**	**0.1029**	**0.0244**	**8.2731**
INeuRec	0.8932	0.9893	0.0814	0.0147	7.4323
UNeuRec	0.6874	0.9687	0.0081	0.0030	5.2636

Table 85. Experimental results for the NeuRec method for the Frappe dataset on beyond accuracy metrics.

	@ 10				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9976**	**0.9996**	**0.8297**	**0.5933**	**11.5283**
TopPopular	0.3355	0.9335	0.0071	0.0036	4.0546
UserKNN CF cosine	0.6740	0.9673	0.0593	0.0099	5.6087
UserKNN CF dice	0.6658	0.9665	0.0676	0.0101	5.5909
UserKNN CF jaccard	0.6623	0.9661	0.0676	0.0100	5.5767
UserKNN CF asymmetric	0.6132	0.9612	0.0426	0.0078	5.2856
UserKNN CF tversky	0.6973	0.9696	0.0637	0.0116	5.8127
ItemKNN CF cosine	0.7049	0.9704	0.0664	0.0114	5.8001
ItemKNN CF dice	0.5875	0.9587	0.0473	0.0073	5.1743
ItemKNN CF jaccard	0.5800	0.9579	0.0470	0.0072	5.1371
ItemKNN CF asymmetric	0.6024	0.9602	0.0465	0.0077	5.2453
ItemKNN CF tversky	0.5776	0.9577	0.0539	0.0074	5.1480
$P^3\alpha$	0.6518	0.9651	0.0725	0.0109	5.6336
$RP^3\beta$	0.6959	0.9695	**0.1183**	0.0169	6.0198
$EASE^R$	0.7804	0.9779	**0.0821**	0.0169	6.3612
SLIM BPR	0.6468	0.9646	0.0759	0.0103	5.5485
SLIM ElasticNet	0.6958	0.9695	0.0639	0.0112	5.7767
MF BPR	**0.8290**	**0.9828**	**0.1470**	**0.0266**	**6.8390**
MF FunkSVD	0.5941	0.9593	0.0644	0.0095	5.4215
PureSVD	0.3881	0.9388	0.0159	0.0042	4.3441
NMF	**0.9069**	**0.9906**	**0.1083**	**0.0330**	**7.4387**
iALS	0.8099	0.9809	0.0706	0.0157	6.3573
INeuRec	0.8181	0.9817	0.0818	0.0211	6.6866
UNeuRec	0.4172	0.9417	0.0115	0.0043	4.4205

Table 86. Experimental results for the NeuRec method for the FilmTrust dataset on beyond accuracy metrics.

	@ 10				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9952**	**0.9994**	**0.9971**	**0.7800**	**10.9000**
TopPopular	0.6318	0.9631	0.0275	0.0129	5.0890
UserKNN CF cosine	**0.7653**	**0.9765**	**0.1632**	**0.0236**	**5.9188**
UserKNN CF dice	**0.7560**	**0.9755**	**0.1482**	**0.0219**	**5.8248**
UserKNN CF jaccard	**0.7332**	**0.9733**	**0.1318**	**0.0199**	**5.7041**
UserKNN CF asymmetric	**0.7461**	**0.9745**	**0.1294**	**0.0205**	**5.7352**
UserKNN CF tversky	**0.7564**	**0.9756**	**0.1468**	**0.0220**	**5.8320**
ItemKNN CF cosine	0.6649	0.9664	0.0584	0.0148	5.2886
ItemKNN CF dice	**0.6764**	**0.9676**	**0.0763**	**0.0155**	**5.3589**
ItemKNN CF jaccard	**0.6767**	**0.9676**	**0.0787**	**0.0155**	**5.3636**
ItemKNN CF asymmetric	0.6646	0.9664	0.0526	0.0147	5.2798
ItemKNN CF tversky	**0.6753**	**0.9675**	**0.0724**	**0.0153**	**5.3481**
$P^3\alpha$	**0.7020**	**0.9701**	**0.1241**	**0.0178**	**5.5226**
$RP^3\beta$	**0.7148**	**0.9714**	**0.2849**	**0.0292**	**5.8254**
$EASE^R$	**0.7303**	**0.9730**	**0.0966**	**0.0184**	**5.6183**
SLIM BPR	**0.6962**	**0.9696**	**0.1053**	**0.0169**	**5.4571**
SLIM ElasticNet	**0.7418**	**0.9741**	**0.0980**	**0.0188**	**5.6488**
MF BPR	**0.6944**	**0.9694**	**0.1719**	**0.0192**	**5.5164**
MF FunkSVD	0.6467	0.9646	0.0328	0.0139	5.1954
PureSVD	**0.7665**	**0.9766**	0.0502	**0.0196**	**5.7145**
NMF	**0.9242**	**0.9923**	**0.4389**	**0.1234**	**8.1504**
iALS	**0.7856**	**0.9785**	0.0478	**0.0206**	**5.7426**
INeuRec	0.6654	0.9665	0.0715	0.0149	5.3145
UNeuRec	0.6348	0.9634	0.0285	0.0131	5.1080

Table 87. Computation time for the algorithms in the selected results for the NeuRec method on the Movielens 1M dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.02 [sec]	52.96 [sec]	114
TopPopular	0.03 [sec]	51.76 [sec]	117
UserKNN CF cosine	3.50 ± 0.15 [sec]	54.05 ± 0.34 [sec]	112
UserKNN CF dice	3.32 ± 0.24 [sec]	53.36 ± 0.64 [sec]	115
UserKNN CF jaccard	3.30 ± 0.22 [sec]	53.55 ± 0.68 [sec]	114
UserKNN CF asymmetric	3.35 ± 0.25 [sec]	53.54 ± 0.31 [sec]	113
UserKNN CF tversky	3.38 ± 0.27 [sec]	53.32 ± 0.90 [sec]	112
ItemKNN CF cosine	1.58 ± 0.14 [sec]	54.81 ± 1.12 [sec]	111
ItemKNN CF dice	1.53 ± 0.12 [sec]	54.57 ± 1.19 [sec]	114
ItemKNN CF jaccard	1.51 ± 0.13 [sec]	53.87 ± 1.13 [sec]	114
ItemKNN CF asymmetric	1.52 ± 0.17 [sec]	53.16 ± 1.05 [sec]	112
ItemKNN CF tversky	1.52 ± 0.15 [sec]	53.93 ± 0.54 [sec]	113
$P^3\alpha$	3.88 ± 1.58 [sec]	52.34 ± 0.77 [sec]	116
$RP^3\beta$	4.49 ± 1.98 [sec]	51.69 ± 0.88 [sec]	117
$EASE^R$	4.39 ± 0.02 [sec]	75.17 [sec] / 1.25 ± 0.00 [min]	80
SLIM BPR	540.48 [sec] / 9.01 ± 9.45 [min]	53.73 ± 1.12 [sec]	110
SLIM ElasticNet	157.83 [sec] / 2.63 ± 1.60 [min]	51.36 ± 2.17 [sec]	112
MF BPR	403.10 [sec] / 6.72 ± 5.21 [min]	51.98 ± 0.87 [sec]	114
MF FunkSVD	1471.34 [sec] / 24.52 ± 36.83 [min]	51.81 ± 0.17 [sec]	116
PureSVD	0.70 ± 0.66 [sec]	52.60 ± 0.15 [sec]	114
NMF	295.91 [sec] / 4.93 ± 3.69 [min]	52.88 ± 0.17 [sec]	114
iALS	350.79 [sec] / 5.85 ± 5.04 [min]	52.84 ± 0.05 [sec]	114
INeuRec	71409.67 [sec] / 19.84 [hour]	42.31 [sec]	143
UNeuRec	57989.25 [sec] / 16.11 [hour]	42.22 [sec]	143

Table 88. Computation time for the algorithms in the selected results for the NeuRec method on the HetRec dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.02 [sec]	41.87 [sec]	50
TopPopular	0.03 [sec]	40.99 [sec]	52
UserKNN CF cosine	1.12 ± 0.05 [sec]	42.23 ± 1.16 [sec]	49
UserKNN CF dice	1.15 ± 0.07 [sec]	42.47 ± 0.44 [sec]	50
UserKNN CF jaccard	1.15 ± 0.07 [sec]	42.41 ± 0.49 [sec]	50
UserKNN CF asymmetric	1.16 ± 0.08 [sec]	42.10 ± 1.37 [sec]	50
UserKNN CF tversky	1.16 ± 0.08 [sec]	42.46 ± 0.46 [sec]	50
ItemKNN CF cosine	4.34 ± 0.52 [sec]	45.85 ± 1.56 [sec]	47
ItemKNN CF dice	4.43 ± 0.59 [sec]	43.91 ± 1.37 [sec]	49
ItemKNN CF jaccard	4.38 ± 0.61 [sec]	43.30 ± 0.70 [sec]	49
ItemKNN CF asymmetric	4.14 ± 0.55 [sec]	43.05 ± 3.72 [sec]	52
ItemKNN CF tversky	4.37 ± 0.54 [sec]	43.20 ± 2.21 [sec]	51
$P^3\alpha$	16.00 ± 5.00 [sec]	41.18 ± 0.26 [sec]	51
$RP^3\beta$	13.93 ± 6.45 [sec]	41.57 ± 0.99 [sec]	52
$EASE^R$	30.04 ± 0.06 [sec]	61.51 [sec] / 1.03 ± 0.00 [min]	34
SLIM BPR	265.23 [sec] / 4.42 ± 3.41 [min]	43.10 ± 0.64 [sec]	48
SLIM ElasticNet	310.27 [sec] / 5.17 ± 2.36 [min]	42.49 ± 1.28 [sec]	49
MF BPR	80.12 [sec] / 1.34 ± 1.07 [min]	42.17 ± 0.28 [sec]	50
MF FunkSVD	208.95 [sec] / 3.48 ± 5.74 [min]	42.47 ± 0.20 [sec]	50
PureSVD	0.59 ± 0.40 [sec]	42.15 ± 0.04 [sec]	50
NMF	326.21 [sec] / 5.44 ± 4.32 [min]	42.71 ± 0.84 [sec]	50
iALS	328.16 [sec] / 5.47 ± 5.00 [min]	42.58 ± 0.05 [sec]	50
INeuRec	50152.19 [sec] / 13.93 [hour]	35.26 [sec]	60
UNeuRec	74545.39 [sec] / 20.71 [hour]	39.53 [sec]	53

Table 89. Computation time for the algorithms in the selected results for the NeuRec method on the Frappe dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	5.78 [sec]	124
TopPopular	0.00 [sec]	4.94 [sec]	145
UserKNN CF cosine	0.06 ± 0.01 [sec]	5.07 ± 0.03 [sec]	141
UserKNN CF dice	0.06 ± 0.01 [sec]	5.13 ± 0.03 [sec]	141
UserKNN CF jaccard	0.06 ± 0.01 [sec]	5.09 ± 0.03 [sec]	141
UserKNN CF asymmetric	0.06 ± 0.01 [sec]	5.09 ± 0.01 [sec]	140
UserKNN CF tversky	0.06 ± 0.01 [sec]	5.07 ± 0.03 [sec]	141
ItemKNN CF cosine	0.24 ± 0.01 [sec]	5.00 ± 0.02 [sec]	143
ItemKNN CF dice	0.24 ± 0.01 [sec]	4.98 ± 0.01 [sec]	143
ItemKNN CF jaccard	0.24 ± 0.01 [sec]	4.98 ± 0.01 [sec]	144
ItemKNN CF asymmetric	0.24 ± 0.01 [sec]	4.99 ± 0.02 [sec]	143
ItemKNN CF tversky	0.25 ± 0.01 [sec]	4.99 ± 0.00 [sec]	144
$P^3\alpha$	0.84 ± 0.13 [sec]	4.93 ± 0.03 [sec]	145
$RP^3\beta$	0.88 ± 0.19 [sec]	4.96 ± 0.03 [sec]	144
$EASE^R$	2.68 ± 0.20 [sec]	8.41 ± 0.01 [sec]	85
SLIM BPR	22.86 ± 9.88 [sec]	4.96 ± 0.05 [sec]	144
SLIM ElasticNet	26.27 ± 3.82 [sec]	5.01 ± 0.01 [sec]	143
MF BPR	21.62 ± 41.61 [sec]	5.01 ± 0.03 [sec]	143
MF FunkSVD	39.35 ± 32.47 [sec]	5.05 ± 0.03 [sec]	141
PureSVD	0.06 ± 0.09 [sec]	4.93 ± 0.02 [sec]	145
NMF	24.63 ± 19.89 [sec]	5.01 ± 0.04 [sec]	144
iALS	138.27 [sec] / 2.30 ± 2.19 [min]	5.17 ± 0.18 [sec]	142
INeuRec	825.96 [sec] / 13.77 [min]	4.13 [sec]	174
UNeuRec	356.02 [sec] / 5.93 [min]	4.12 [sec]	174

Table 90. Computation time for the algorithms in the selected results for the NeuRec method on the FilmTrust dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	6.43 [sec]	197
TopPopular	0.00 [sec]	6.21 [sec]	204
UserKNN CF cosine	0.19 ± 0.03 [sec]	6.41 ± 0.07 [sec]	196
UserKNN CF dice	0.20 ± 0.04 [sec]	6.47 ± 0.05 [sec]	195
UserKNN CF jaccard	0.20 ± 0.03 [sec]	6.46 ± 0.02 [sec]	197
UserKNN CF asymmetric	0.19 ± 0.04 [sec]	6.47 ± 0.06 [sec]	196
UserKNN CF tversky	0.20 ± 0.04 [sec]	6.49 ± 0.01 [sec]	195
ItemKNN CF cosine	0.10 ± 0.01 [sec]	6.33 ± 0.05 [sec]	200
ItemKNN CF dice	0.11 ± 0.01 [sec]	6.35 ± 0.00 [sec]	199
ItemKNN CF jaccard	0.10 ± 0.01 [sec]	6.33 ± 0.02 [sec]	200
ItemKNN CF asymmetric	0.11 ± 0.01 [sec]	6.33 ± 0.05 [sec]	199
ItemKNN CF tversky	0.11 ± 0.01 [sec]	6.36 ± 0.00 [sec]	199
$P^3\alpha$	0.55 ± 0.11 [sec]	6.31 ± 0.04 [sec]	202
$RP^3\beta$	0.58 ± 0.13 [sec]	6.31 ± 0.03 [sec]	201
$EASE^R$	0.48 ± 0.03 [sec]	9.60 ± 0.01 [sec]	132
SLIM BPR	14.96 ± 9.29 [sec]	6.34 ± 0.06 [sec]	199
SLIM ElasticNet	9.53 ± 1.99 [sec]	6.31 ± 0.04 [sec]	201
MF BPR	38.98 ± 45.37 [sec]	6.23 ± 0.14 [sec]	200
MF FunkSVD	31.97 ± 39.84 [sec]	6.26 ± 0.02 [sec]	202
PureSVD	0.05 ± 0.06 [sec]	6.15 ± 0.14 [sec]	202
NMF	32.20 ± 35.82 [sec]	6.32 ± 0.17 [sec]	197
iALS	58.77 ± 37.14 [sec]	6.25 ± 0.05 [sec]	201
INeuRec	424.72 [sec] / 7.08 [min]	5.07 [sec]	250
UNeuRec	230.86 [sec] / 3.85 [min]	5.11 [sec]	248

Table 91. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Frappe	Filmtrust	Movielens 1M	Hetrec
UserKNN CF cosine	topK	235	490	565	609
	shrink	3	921	749	0
	similarity	cosine	cosine	cosine	cosine
	normalize	True	True	True	True
	feature weighting	TF-IDF	TF-IDF	TF-IDF	none
UserKNN CF dice	topK	224	593	182	277
	shrink	0	336	4	1
	similarity	dice	dice	dice	dice
	normalize	False	True	False	False
UserKNN CF jaccard	topK	237	510	244	290
	shrink	0	5	7	0
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	False	False	True	False
UserKNN CF asymmetric	topK	398	594	193	279
	shrink	0	1000	348	306
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	0.0000	2.0000	0.0744	0.3358
	feature weighting	TF-IDF	TF-IDF	TF-IDF	TF-IDF
UserKNN CF tversky	topK	309	598	580	358
	shrink	0	32	169	18
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	0.0000	0.0113	2.0000	1.9153
	tversky beta	2.0000	0.2846	2.0000	1.5318
ItemKNN CF cosine	topK	1000	320	307	264
	shrink	524	497	1	1000
	similarity	cosine	cosine	cosine	cosine
	normalize	True	False	True	True
	feature weighting	TF-IDF	TF-IDF	TF-IDF	TF-IDF
ItemKNN CF dice	topK	973	676	99	103
	shrink	273	746	82	226
	similarity	dice	dice	dice	dice
	normalize	True	True	True	True
ItemKNN CF jaccard	topK	763	301	112	87
	shrink	304	734	59	974
	similarity	jaccard	jaccard	jaccard	jaccard
	normalize	True	True	True	True
ItemKNN CF asymmetric	topK	779	1000	258	5
	shrink	1000	1000	0	1000
	similarity	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True
	asymmetric alpha	0.2985	0.0000	0.4750	0.0000
	feature weighting	TF-IDF	TF-IDF	TF-IDF	TF-IDF
ItemKNN CF tversky	topK	258	1000	331	30
	shrink	196	859	0	394
	similarity	tversky	tversky	tversky	tversky
	normalize	True	True	True	True
	tversky alpha	0.3885	0.9712	0.9659	0.7213
	tversky beta	1.6358	0.6517	2.0000	1.9521

Table 92. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Frappe	Filmtrust	Movielens 1M	Hetrec
$P^3\alpha$	topK	347	270	162	857
	alpha	0.6960	1.5154	1.4534	1.8765
	normalize similarity	False	True	True	True
$RP^3\beta$	topK	640	263	109	5
	alpha	0.6964	1.0001	1.1222	0.6525
	beta	0.2177	0.4673	0.1090	0.3863
	normalize similarity	False	True	True	False
$EASE^R$	l2 norm	4.75E+01	3.89E+02	8.87E+02	1.89E+03
SLIM BPR	topK	1000	1000	1000	1000
	epochs	25	35	195	160
	symmetric	True	True	True	True
	sgd mode	adagrad	adagrad	sgd	sgd
	lambda i	1.00E-02	1.00E-02	1.00E-02	1.00E-02
	lambda j	1.00E-02	1.00E-05	1.00E-02	1.00E-02
	learning rate	1.00E-04	3.60E-02	8.77E-03	1.55E-02
SLIM ElasticNet	topK	671	458	659	344
	l1 ratio	1.13E-05	1.00E-05	1.19E-03	3.19E-03
	alpha	0.3182	0.1082	0.1394	0.5223
MF BPR	sgd mode	adagrad	adam	adagrad	adagrad
	epochs	95	135	1005	400
	num factors	200	104	200	1
	batch size	256	2	16	8
	positive reg	3.97E-03	4.85E-05	2.45E-03	1.00E-05
	negative reg	1.00E-02	1.00E-05	1.00E-02	1.00E-02
	learning rate	8.84E-02	4.67E-04	3.37E-02	4.08E-02
MF FunkSVD	sgd mode	adagrad	adam	adam	adam
	epochs	170	70	420	65
	use bias	True	False	False	True
	batch size	1	8	128	1024
	num factors	200	1	32	63
	item reg	1.00E-05	1.00E-02	1.05E-05	1.22E-04
	user reg	1.00E-02	1.00E-05	7.21E-04	9.95E-04
	learning rate	1.00E-01	1.31E-03	1.95E-04	3.29E-04
	negative quota	0.5000	0.1059	0.1103	0.1179
PureSVD	num factors	1	5	30	43
NMF	num factors	42	208	62	139
	solver	coord. descent	mult. update	mult. update	coord. descent
	init type	nndsvda	nndsvda	random	nndsvda
	beta loss	frobenius	frobenius	kullback-leibler	frobenius
iALS	num factors	22	10	35	61
	confidence scaling	log	linear	log	linear
	alpha	0.0045	0.3831	0.8507	0.0094
	epsilon	0.1795	0.0984	0.3278	9.7751
	reg	1.45E-03	1.38E-03	1.41E-03	1.50E-05
	epochs	5	15	60	60

Table 93. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Frappe	Filmtrust	Movielens 1M	Hetrec
INeuRec	num neurons	300	150	300	300
	num factors	50	40	50	50
	dropout percentage	0.0300	0.0000	0.0300	0.0300
	learning rate	1.00E-04	5.00E-05	1.00E-04	1.00E-04
	regularization rate	1.00E-02	1.00E-01	1.00E-01	1.00E-01
	epochs	50	25	5	10
	batch size	1024	1024	1024	1024
	display epoch	-	-	-	-
	display step	-	-	-	-
	verbose	True	True	True	True
UNeuRec	num neurons	300	150	300	300
	num factors	50	40	50	50
	dropout percentage	0.0300	0.0000	0.0300	0.0300
	learning rate	1.00E-04	5.00E-05	1.00E-04	1.00E-04
	regularization rate	1.00E-02	1.00E-01	1.00E-01	1.00E-01
	epochs	5	5	50	145
	batch size	1024	1024	1024	1024
	display epoch	-	-	-	-
	display step	-	-	-	-
	verbose	True	True	True	True

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Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 94. The results of our evaluation can be seen in Table 95 (Amazon Music original), Table 96 (Amazon Music ours), Table 97 (Amazon Movie), Table 98 (Movielens 100k) and Table 99 (Movielens 1M). The corresponding optimal hyperparameters are reported in Table 105 (collaborative KNNs), Table 106 (non-neural machine learning and graph based) and Table 107 (NeuRec).

Lastly, the time required to train and evaluate the models is reported in Table 100 (Amazon Music original), Table 101 (Amazon Music ours), Table 102 (Amazon Movie), Table 103 (Movielens 100k) and Table 104 (Movielens 1M).

Table 94. Dataset characteristics.

Dataset	Interactions	Items	Users	Density
Amazon Music original	46.5K	18813	844	0.293
Amazon Music ours	37K	23184	844	0.189
Amazon Movie	878K	83512	15067	0.070
Movielens 100k	100K	1682	943	6.305
Movielens 1M	1M	3706	6040	4.468

Table 95. Experimental results for the DMF method for the Amazon Music original dataset.

	@ 10	
	HR	NDCG
Random	0.0972	0.0466
TopPopular	0.4372	0.2489
UserKNN CF cosine	**0.5509**	**0.3955**
UserKNN CF dice	**0.5462**	**0.3873**
UserKNN CF jaccard	**0.5498**	**0.3874**
UserKNN CF asymmetric	**0.5509**	**0.3950**
UserKNN CF tversky	0.5427	**0.3870**
ItemKNN CF cosine	**0.5450**	**0.3944**
ItemKNN CF dice	0.5308	0.3804
ItemKNN CF jaccard	0.5284	0.3790
ItemKNN CF asymmetric	0.5415	**0.3907**
ItemKNN CF tversky	0.5284	0.3781
$P^3\alpha$	**0.5509**	**0.3975**
$RP^3\beta$	**0.5498**	**0.3972**
$EASE^R$	0.5213	**0.3881**
SLIM BPR	0.5403	0.3771
SLIM ElasticNet	0.5355	**0.3922**
MF BPR	0.4443	0.3139
MF FunkSVD	0.4479	0.3220
PureSVD	0.4502	0.3386
NMF	0.5130	0.3557
iALS	**0.5533**	**0.4033**
DMF NCE	0.3863	0.2773
DMF BCE	0.5427	0.3850

Table 96. Experimental results for the DMF method for the Amazon Music ours dataset.

	@ 10	
	HR	NDCG
Random	0.1126	0.0526
TopPopular	0.5308	0.3037
UserKNN CF cosine	**0.6694**	0.4798
UserKNN CF dice	0.6576	0.4740
UserKNN CF jaccard	0.6564	0.4731
UserKNN CF asymmetric	**0.6730**	0.4813
UserKNN CF tversky	0.6517	0.4754
ItemKNN CF cosine	0.6647	**0.4880**
ItemKNN CF dice	0.6540	0.4698
ItemKNN CF jaccard	0.6576	0.4721
ItemKNN CF asymmetric	0.6647	**0.4853**
ItemKNN CF tversky	0.6481	0.4665
$P^3\alpha$	0.6588	**0.4823**
$RP^3\beta$	**0.6754**	**0.4912**
$EASE^R$	0.6600	**0.4836**
SLIM BPR	**0.6694**	0.4720
SLIM ElasticNet	0.6469	0.4744
MF BPR	0.5367	0.3689
MF FunkSVD	0.5474	0.3870
PureSVD	0.5912	0.4190
NMF	0.6540	0.4486
iALS	0.6600	**0.4880**
DMF NCE	0.4799	0.3371
DMF BCE	0.6659	0.4815

Table 97. Experimental results for the DMF method for the Amazon Movie dataset.

	@ 10	
	HR	NDCG
Random	0.1007	0.0457
TopPopular	0.5794	0.3489
UserKNN CF cosine	0.7327	0.5132
UserKNN CF dice	0.7064	0.4963
UserKNN CF jaccard	0.7066	0.4968
UserKNN CF asymmetric	0.7325	0.5132
UserKNN CF tversky	0.7033	0.4952
ItemKNN CF cosine	0.6983	0.4967
ItemKNN CF dice	0.6947	0.4868
ItemKNN CF jaccard	0.6972	0.4902
ItemKNN CF asymmetric	0.6986	0.4914
ItemKNN CF tversky	0.6660	0.4789
$P^3\alpha$	0.6972	0.5028
$RP^3\beta$	0.7107	0.5078
$EASE^R$	-	-
SLIM BPR	-	-
SLIM ElasticNet	0.6981	0.5005
MF BPR	0.6422	0.4161
MF FunkSVD	0.5972	0.4091
PureSVD	0.6021	0.4156
NMF	0.6252	0.4217
iALS	0.7352	0.5230
DMF NCE	0.6832	0.4677
DMF BCE	**0.7818**	**0.5417**

Table 98. Experimental results for the DMF method for the Movielens 100k dataset.

	@ 10	
	HR	NDCG
Random	0.0914	0.0416
TopPopular	0.4145	0.2342
UserKNN CF cosine	0.5834	0.3400
UserKNN CF dice	0.5834	0.3325
UserKNN CF jaccard	0.5760	0.3335
UserKNN CF asymmetric	0.5994	0.3492
UserKNN CF tversky	0.5802	0.3382
ItemKNN CF cosine	0.5781	0.3363
ItemKNN CF dice	0.5962	0.3484
ItemKNN CF jaccard	0.5834	0.3422
ItemKNN CF asymmetric	0.5855	0.3416
ItemKNN CF tversky	0.6026	0.3506
$P^3\alpha$	0.5717	0.3421
$RP^3\beta$	0.5685	0.3270
$EASE^R$	**0.6089**	0.3571
SLIM BPR	**0.6206**	0.3578
SLIM ElasticNet	**0.6238**	**0.3765**
MF BPR	0.5951	0.3365
MF FunkSVD	0.5707	0.3354
PureSVD	0.5877	0.3555
NMF	0.5855	0.3515
iALS	**0.6142**	**0.3691**
DMF NCE	0.5930	0.3410
DMF BCE	0.6026	0.3623

Table 99. Experimental results for the DMF method for the Movielens 1M dataset.

	@ 10	
	HR	NDCG
Random	0.1052	0.0472
TopPopular	0.4418	0.2475
UserKNN CF cosine	0.6293	0.3766
UserKNN CF dice	0.6324	0.3822
UserKNN CF jaccard	0.6323	0.3828
UserKNN CF asymmetric	0.6324	0.3779
UserKNN CF tversky	0.6362	0.3840
ItemKNN CF cosine	0.6347	0.3808
ItemKNN CF dice	0.6293	0.3692
ItemKNN CF jaccard	0.6255	0.3682
ItemKNN CF asymmetric	0.6190	0.3704
ItemKNN CF tversky	0.6326	0.3757
$P^3\alpha$	0.6097	0.3639
$RP^3\beta$	0.6304	0.3726
$EASE^R$	0.6693	**0.4100**
SLIM BPR	0.6719	**0.4068**
SLIM ElasticNet	**0.6825**	**0.4209**
MF BPR	0.6323	0.3729
MF FunkSVD	0.6499	0.3912
PureSVD	0.6570	0.4015
NMF	0.6422	0.3862
iALS	**0.6947**	**0.4257**
DMF NCE	0.6266	0.3768
DMF BCE	0.6731	0.4033

Table 100. Computation time for the algorithms in the selected results for the DMF method on the Amazon Music original dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	1.36 [sec]	621
TopPopular	0.00 [sec]	1.53 [sec]	551
UserKNN CF cosine	0.05 ± 0.02 [sec]	2.28 ± 0.01 [sec]	368
UserKNN CF dice	0.05 ± 0.00 [sec]	2.26 ± 0.03 [sec]	378
UserKNN CF jaccard	0.05 ± 0.00 [sec]	2.27 ± 0.02 [sec]	376
UserKNN CF asymmetric	0.05 ± 0.00 [sec]	2.26 ± 0.04 [sec]	370
UserKNN CF tversky	0.05 ± 0.00 [sec]	2.27 ± 0.03 [sec]	380
ItemKNN CF cosine	3.33 ± 0.41 [sec]	2.36 ± 0.01 [sec]	357
ItemKNN CF dice	3.45 ± 0.15 [sec]	2.28 ± 0.01 [sec]	369
ItemKNN CF jaccard	3.46 ± 0.13 [sec]	2.31 ± 0.04 [sec]	360
ItemKNN CF asymmetric	3.55 ± 0.15 [sec]	2.32 ± 0.05 [sec]	358
ItemKNN CF tversky	3.53 ± 0.17 [sec]	2.29 ± 0.04 [sec]	370
$P^3\alpha$	14.26 ± 1.70 [sec]	2.31 ± 0.02 [sec]	363
$RP^3\beta$	14.36 ± 2.21 [sec]	2.30 ± 0.06 [sec]	362
$EASE^R$	127.78 [sec] / 2.13 ± 0.03 [min]	2.64 ± 0.04 [sec]	313
SLIM BPR	304.44 [sec] / 5.07 ± 2.50 [min]	2.18 ± 0.01 [sec]	387
SLIM ElasticNet	410.27 [sec] / 6.84 ± 5.29 [min]	2.14 ± 0.03 [sec]	388
MF BPR	77.68 [sec] / 1.29 ± 0.90 [min]	1.58 ± 0.03 [sec]	527
MF FunkSVD	212.46 [sec] / 3.54 ± 3.48 [min]	1.65 ± 0.05 [sec]	497
PureSVD	0.39 ± 0.34 [sec]	1.60 ± 0.03 [sec]	532
NMF	50.60 ± 46.44 [sec]	1.77 ± 0.10 [sec]	508
iALS	441.76 [sec] / 7.36 ± 5.91 [min]	1.57 ± 0.01 [sec]	535
DMF NCE	1549.96 [sec] / 25.83 [min]	10.86 [sec]	78
DMF BCE	2902.64 [sec] / 48.38 [min]	8.04 [sec]	105

Table 101. Computation time for the algorithms in the selected results for the DMF method on the Amazon Music ours dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	1.29 [sec]	653
TopPopular	0.00 [sec]	1.69 [sec]	498
UserKNN CF cosine	0.05 ± 0.00 [sec]	2.37 ± 0.01 [sec]	355
UserKNN CF dice	0.04 ± 0.00 [sec]	2.34 ± 0.02 [sec]	361
UserKNN CF jaccard	0.04 ± 0.00 [sec]	2.35 ± 0.01 [sec]	361
UserKNN CF asymmetric	0.04 ± 0.00 [sec]	2.36 ± 0.03 [sec]	354
UserKNN CF tversky	0.04 ± 0.00 [sec]	2.35 ± 0.03 [sec]	363
ItemKNN CF cosine	3.65 ± 0.65 [sec]	2.41 ± 0.03 [sec]	347
ItemKNN CF dice	4.01 ± 0.08 [sec]	2.40 ± 0.02 [sec]	351
ItemKNN CF jaccard	4.01 ± 0.06 [sec]	2.40 ± 0.01 [sec]	353
ItemKNN CF asymmetric	4.07 ± 0.09 [sec]	2.41 ± 0.01 [sec]	349
ItemKNN CF tversky	4.13 ± 0.07 [sec]	2.39 ± 0.05 [sec]	351
$P^3\alpha$	13.21 ± 0.96 [sec]	2.38 ± 0.02 [sec]	357
$RP^3\beta$	14.29 ± 1.10 [sec]	2.41 ± 0.02 [sec]	349
$EASE^R$	234.11 [sec] / 3.90 ± 0.02 [min]	2.71 ± 0.01 [sec]	310
SLIM BPR	342.16 [sec] / 5.70 ± 3.42 [min]	2.41 ± 0.02 [sec]	347
SLIM ElasticNet	1400.37 [sec] / 23.34 ± 2.14 [min]	2.43 ± 0.01 [sec]	345
MF BPR	90.35 [sec] / 1.51 ± 1.43 [min]	1.73 ± 0.02 [sec]	484
MF FunkSVD	146.22 [sec] / 2.44 ± 1.95 [min]	1.75 ± 0.01 [sec]	481
PureSVD	0.41 ± 0.48 [sec]	1.76 ± 0.07 [sec]	492
NMF	67.64 [sec] / 1.13 ± 0.96 [min]	1.91 ± 0.10 [sec]	453
iALS	184.95 [sec] / 3.08 ± 2.43 [min]	1.72 ± 0.00 [sec]	490
DMF NCE	952.84 [sec] / 15.88 [min]	11.40 [sec]	74
DMF BCE	2183.37 [sec] / 36.39 [min]	8.43 [sec]	100

Table 102. Computation time for the algorithms in the selected results for the DMF method on the Amazon Movie dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.03 [sec]	28.58 [sec]	527
TopPopular	0.06 [sec]	43.04 [sec]	350
UserKNN CF cosine	4.67 ± 0.53 [sec]	63.92 [sec] / 1.07 ± 0.05 [min]	225
UserKNN CF dice	4.57 ± 0.52 [sec]	63.27 [sec] / 1.05 ± 0.06 [min]	228
UserKNN CF jaccard	4.59 ± 0.48 [sec]	64.06 [sec] / 1.07 ± 0.06 [min]	228
UserKNN CF asymmetric	4.72 ± 0.49 [sec]	66.17 [sec] / 1.10 ± 0.02 [min]	225
UserKNN CF tversky	4.63 ± 0.40 [sec]	63.04 [sec] / 1.05 ± 0.05 [min]	232
ItemKNN CF cosine	55.00 ± 5.26 [sec]	70.23 [sec] / 1.17 ± 0.08 [min]	203
ItemKNN CF dice	56.26 ± 1.87 [sec]	68.45 [sec] / 1.14 ± 0.08 [min]	216
ItemKNN CF jaccard	56.34 ± 1.87 [sec]	69.12 [sec] / 1.15 ± 0.05 [min]	226
ItemKNN CF asymmetric	56.82 ± 1.89 [sec]	71.95 [sec] / 1.20 ± 0.04 [min]	206
ItemKNN CF tversky	57.14 ± 2.12 [sec]	62.86 [sec] / 1.05 ± 0.05 [min]	239
$P^3\alpha$	196.62 [sec] / 3.28 ± 0.32 [min]	59.93 ± 1.25 [sec]	249
$RP^3\beta$	214.41 [sec] / 3.57 ± 0.31 [min]	60.62 [sec] / 1.01 ± 0.04 [min]	239
$EASE^R$	-	-	-
SLIM BPR	-	-	-
SLIM ElasticNet	17335.50 [sec] / 4.82 ± 1.00 [hour]	64.46 [sec] / 1.07 ± 0.11 [min]	211
MF BPR	5985.66 [sec] / 1.66 ± 0.93 [hour]	43.42 ± 0.22 [sec]	345
MF FunkSVD	6606.42 [sec] / 1.84 ± 1.22 [hour]	47.53 ± 2.25 [sec]	309
PureSVD	4.39 ± 3.54 [sec]	45.12 ± 1.44 [sec]	343
NMF	2030.98 [sec] / 33.85 ± 20.16 [min]	45.26 [sec]	333
iALS	3384.98 [sec] / 56.42 ± 45.83 [min]	43.14 ± 0.15 [sec]	350
DMF NCE	70909.11 [sec] / 19.70 [hour]	833.82 [sec] / 13.90 [min]	18
DMF BCE	72580.90 [sec] / 20.16 [hour]	380.28 [sec] / 6.34 [min]	40

Table 103. Computation time for the algorithms in the selected results for the DMF method on the Movielens 100k dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	1.33 [sec]	706
TopPopular	0.00 [sec]	1.39 [sec]	676
UserKNN CF cosine	0.14 ± 0.02 [sec]	2.09 ± 0.04 [sec]	448
UserKNN CF dice	0.14 ± 0.02 [sec]	2.13 ± 0.05 [sec]	449
UserKNN CF jaccard	0.15 ± 0.02 [sec]	2.13 ± 0.07 [sec]	454
UserKNN CF asymmetric	0.15 ± 0.02 [sec]	2.15 ± 0.08 [sec]	445
UserKNN CF tversky	0.15 ± 0.02 [sec]	2.09 ± 0.03 [sec]	452
ItemKNN CF cosine	0.20 ± 0.04 [sec]	2.14 ± 0.05 [sec]	429
ItemKNN CF dice	0.18 ± 0.04 [sec]	2.18 ± 0.11 [sec]	460
ItemKNN CF jaccard	0.19 ± 0.04 [sec]	2.16 ± 0.14 [sec]	458
ItemKNN CF asymmetric	0.20 ± 0.04 [sec]	2.20 ± 0.09 [sec]	454
ItemKNN CF tversky	0.18 ± 0.04 [sec]	2.14 ± 0.10 [sec]	454
$P^3\alpha$	0.99 ± 0.63 [sec]	2.08 ± 0.05 [sec]	460
$RP^3\beta$	1.21 ± 0.70 [sec]	2.12 ± 0.04 [sec]	446
$EASE^R$	0.41 ± 0.01 [sec]	2.58 ± 0.03 [sec]	362
SLIM BPR	64.02 [sec] / 1.07 ± 0.69 [min]	2.15 ± 0.07 [sec]	440
SLIM ElasticNet	9.40 ± 4.87 [sec]	2.08 ± 0.01 [sec]	454
MF BPR	44.66 ± 46.61 [sec]	1.45 ± 0.04 [sec]	631
MF FunkSVD	156.60 [sec] / 2.61 ± 2.88 [min]	1.46 ± 0.01 [sec]	644
PureSVD	0.06 ± 0.05 [sec]	1.43 ± 0.01 [sec]	661
NMF	64.98 [sec] / 1.08 ± 1.03 [min]	1.45 ± 0.02 [sec]	646
iALS	39.52 ± 27.96 [sec]	1.44 ± 0.00 [sec]	655
DMF NCE	13775.00 [sec] / 3.83 [hour]	24.09 [sec]	39
DMF BCE	23861.05 [sec] / 6.63 [hour]	22.85 [sec]	41

Table 104. Computation time for the algorithms in the selected results for the DMF method on the Movielens 1M dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.03 [sec]	8.68 [sec]	695
TopPopular	0.05 [sec]	9.07 [sec]	665
UserKNN CF cosine	4.72 ± 0.25 [sec]	14.59 ± 1.40 [sec]	380
UserKNN CF dice	4.43 ± 0.21 [sec]	14.60 ± 0.55 [sec]	426
UserKNN CF jaccard	4.43 ± 0.22 [sec]	14.97 ± 0.70 [sec]	427
UserKNN CF asymmetric	4.51 ± 0.20 [sec]	15.31 ± 0.62 [sec]	388
UserKNN CF tversky	4.55 ± 0.22 [sec]	14.97 ± 0.36 [sec]	410
ItemKNN CF cosine	2.06 ± 0.11 [sec]	15.87 ± 1.14 [sec]	410
ItemKNN CF dice	2.02 ± 0.11 [sec]	14.59 ± 1.18 [sec]	437
ItemKNN CF jaccard	2.03 ± 0.11 [sec]	14.40 ± 0.66 [sec]	441
ItemKNN CF asymmetric	2.10 ± 0.11 [sec]	15.84 ± 1.07 [sec]	402
ItemKNN CF tversky	2.03 ± 0.13 [sec]	14.24 ± 0.36 [sec]	435
$P^3\alpha$	4.79 ± 1.34 [sec]	14.13 ± 0.30 [sec]	417
$RP^3\beta$	4.80 ± 1.32 [sec]	14.26 ± 0.51 [sec]	426
$EASE^R$	4.83 ± 0.10 [sec]	18.11 ± 0.09 [sec]	332
SLIM BPR	1067.49 [sec] / 17.79 ± 18.34 [min]	15.51 ± 0.33 [sec]	383
SLIM ElasticNet	204.56 [sec] / 3.41 ± 3.33 [min]	14.18 ± 0.01 [sec]	425
MF BPR	450.68 [sec] / 7.51 ± 6.28 [min]	9.52 ± 0.12 [sec]	624
MF FunkSVD	2080.92 [sec] / 34.68 ± 26.08 [min]	9.49 ± 0.11 [sec]	638
PureSVD	0.83 ± 0.64 [sec]	9.56 ± 0.11 [sec]	635
NMF	355.21 [sec] / 5.92 ± 13.25 [min]	9.56 ± 0.02 [sec]	630
iALS	273.69 [sec] / 4.56 ± 2.68 [min]	9.52 ± 0.02 [sec]	631
DMF NCE	671545.15 [sec] / 7.77 [day]	548.79 [sec] / 9.15 [min]	11
DMF BCE	351928.51 [sec] / 4.07 [day]	482.96 [sec] / 8.05 [min]	12

Table 105. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Amazon Music original	Amazon Music ours	Movielens 100K	Amazon Movie	Movielens 1M
UserKNN CF cosine	topK	883	614	159	1000	770
	shrink	990	15	0	0	0
	similarity	cosine	cosine	cosine	cosine	cosine
	normalize	True	True	True	True	True
	feature weighting	BM25	BM25	BM25	BM25	BM25
UserKNN CF dice	topK	163	202	148	955	177
	shrink	0	6	5	11	0
	similarity	dice	dice	dice	dice	dice
	normalize	True	False	False	True	True
UserKNN CF jaccard	topK	195	204	100	999	178
	shrink	1	0	3	23	0
	similarity	jaccard	jaccard	jaccard	jaccard	jaccard
	normalize	False	True	True	True	False
UserKNN CF asymmetric	topK	651	786	169	1000	613
	shrink	846	1000	0	0	0
	similarity	asymmetric	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True	True
	asymmetric alpha	0.4426	0.7110	0.1994	0.4960	0.3317
	feature weighting	BM25	BM25	BM25	BM25	BM25
UserKNN CF tversky	topK	152	147	107	858	337
	shrink	0	92	0	31	0
	similarity	tversky	tversky	tversky	tversky	tversky
	normalize	True	True	True	True	True
	tversky alpha	2.0000	1.9995	1.5901	1.0846	2.0000
	tversky beta	2.0000	1.3750	2.0000	1.8286	2.0000
ItemKNN CF cosine	topK	929	1000	303	1000	191
	shrink	285	893	0	1000	1
	similarity	cosine	cosine	cosine	cosine	cosine
	normalize	False	True	True	False	True
	feature weighting	BM25	TF-IDF	BM25	BM25	BM25
ItemKNN CF dice	topK	232	500	12	845	66
	shrink	99	0	37	18	4
	similarity	dice	dice	dice	dice	dice
	normalize	False	False	False	True	True
ItemKNN CF jaccard	topK	726	422	28	582	63
	shrink	9	11	2	42	5
	similarity	jaccard	jaccard	jaccard	jaccard	jaccard
	normalize	True	False	False	False	True
ItemKNN CF asymmetric	topK	778	669	24	978	303
	shrink	641	1000	1000	859	0
	similarity	asymmetric	asymmetric	asymmetric	asymmetric	asymmetric
	normalize	True	True	True	True	True
	asymmetric alpha	0.2886	0.3042	0.2911	0.0000	0.3402
	feature weighting	TF-IDF	TF-IDF	TF-IDF	none	BM25
ItemKNN CF tversky	topK	207	626	118	360	58
	shrink	949	95	0	71	143
	similarity	tversky	tversky	tversky	tversky	tversky
	normalize	True	True	True	True	True
	tversky alpha	2.0000	1.2255	0.0876	0.1388	0.0000
	tversky beta	2.0000	2.0000	0.6605	0.9260	0.3728

Table 106. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Amazon Music original	Amazon Music ours	Movielens 100K	Amazon Movie	Movielens 1M
$P^3\alpha$	topK	871	453	93	1000	1000
	alpha	0.8151	0.4565	0.0000	0.5195	1.2781
	normalize similarity	True	True	True	False	False
$RP^3\beta$	topK	864	900	311	1000	405
	alpha	0.8665	0.9432	0.3966	0.4285	0.9764
	beta	0.1478	0.2462	0.5232	0.2689	0.5807
	normalize similarity	True	True	True	False	False
$EASE^R$	l2 norm	5.11E+03	6.40E+03	1.05E+04	-	5.67E+04
SLIM BPR	topK	789	676	241	-	765
	epochs	175	255	260	-	345
	symmetric	True	True	True	-	True
	sgd mode	adam	adagrad	adagrad	-	adagrad
	lambda i	6.07E-04	2.02E-05	2.85E-03	-	1.00E-02
	lambda j	2.71E-03	5.73E-04	1.23E-04	-	1.00E-05
	learning rate	1.00E-01	7.89E-03	2.91E-02	-	4.12E-02
SLIM ElasticNet	topK	1000	728	96	1000	694
	l1 ratio	8.28E-05	4.68E-05	2.76E-03	4.41E-04	1.86E-04
	alpha	1.0000	1.0000	0.9354	1.0000	0.6571
MF BPR	sgd mode	adagrad	adagrad	adam	adagrad	sgd
	epochs	420	495	365	1490	810
	num factors	183	200	200	200	200
	batch size	128	1	256	256	2
	positive reg	1.00E-02	1.00E-05	1.00E-05	1.00E-02	3.82E-05
	negative reg	1.00E-05	1.14E-03	1.00E-05	1.00E-02	1.00E-02
	learning rate	5.72E-02	7.38E-02	2.36E-03	6.68E-02	8.09E-02
MF FunkSVD	sgd mode	adam	sgd	sgd	adam	sgd
	epochs	450	230	130	245	400
	use bias	True	False	True	True	False
	batch size	16	2	8	32	1
	num factors	200	190	70	177	41
	item reg	3.33E-05	1.00E-02	5.26E-04	2.69E-05	2.21E-04
	user reg	1.00E-02	1.00E-02	7.28E-03	1.00E-02	1.54E-03
	learning rate	3.64E-04	2.43E-02	1.48E-02	1.32E-03	1.15E-03
	negative quota	0.0432	0.3709	0.1257	0.0409	0.1628
PureSVD	num factors	81	17	24	77	57
NMF	num factors	121	164	40	99	64
	solver	mult. update	mult. update	coord. descent	mult. update	coord. descent
	init type	random	random	nndsvda	nndsvda	nndsvda
	beta loss	frobenius	frobenius	frobenius	kullback-leibler	frobenius
iALS	num factors	31	28	25	52	63
	confidence scaling	log	log	log	log	log
	alpha	3.2850	50.0000	0.0150	2.8548	0.3345
	epsilon	0.0157	9.4295	9.3913	0.0010	0.0010
	reg	1.00E-02	1.00E-02	1.00E-05	1.00E-05	1.00E-05
	epochs	65	110	20	60	20

Table 107. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Amazon Music original	Amazon Music ours	Movielens 100K	Amazon Movie	Movielens 1M
DMF NCE	epochs	10	5	75	10	120
	learning rate	1.00E-04	1.00E-04	1.00E-04	1.00E-04	1.00E-04
	batch size	256	256	256	256	256
	num negatives	7	7	7	7	7
	last layer size	128	128	64	64	64
DMF BCE	epochs	75	80	165	35	65
	learning rate	1.00E-04	1.00E-04	1.00E-04	1.00E-04	1.00E-04
	batch size	256	256	256	256	256
	num negatives	7	7	7	7	7
	last layer size	128	128	64	64	64
	max rating	1.0000	1.0000	1.0000	1.0000	1.0000

N IJCAI: COUPLEDCF: LEARNING EXPLICIT AND IMPLICIT USER-ITEM COUPLINGS IN RECOMMENDATION FOR DEEP COLLABORATIVE FILTERING

Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 108. The results of our evaluation can be seen in Table 109 (Movielens 1M) and Table 110 (Tafeng). The corresponding optimal hyperparameters are reported in Table 115 (collaborative KNNs), Table 116 (non-neural machine learning and graph based), Table 117 (content-based KNNs), Table 118 (item-based hybrid KNNs), Table 119 (user-based hybrid KNNs), Table 120 (content-based KNNs for Tafeng), Table 121 (hybrid KNNs for Tafeng) and Table 122 (CoupledCF).

Lastly, the time required to train and evaluate the models is reported in Table 113 (Movielens 1M) and Table 114 (Tafeng).

Table 108. Dataset characteristics.

Dataset	Interactions	Items	Users	Density
Movielens 1M	1M	3953	6041	4.18
Tafeng	743K	23813	32267	0.097

Table 109. Experimental results for the CoupledCF method for the Movielens 1M dataset.

	@ 1		@ 5		@ 10	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0096	0.0096	0.0500	0.0295	0.0993	0.0451
TopPopular	0.1593	0.1593	0.4217	0.2936	0.5813	0.3451
UserKNN CF cosine	0.3540	0.3540	0.6884	0.5324	0.8060	0.5704
UserKNN CF dice	**0.3556**	**0.3556**	0.6829	0.5305	0.8012	0.5689
UserKNN CF jaccard	0.3546	0.3546	0.6858	0.5315	0.8045	0.5699
UserKNN CF asymmetric	0.3546	0.3546	0.6914	0.5343	0.8114	0.5735
UserKNN CF tversky	0.3515	0.3515	0.6820	0.5277	0.8007	0.5663
ItemKNN CF cosine	0.3305	0.3305	0.6682	0.5080	0.7940	0.5488
ItemKNN CF dice	0.3149	0.3149	0.6540	0.4930	0.7861	0.5360
ItemKNN CF jaccard	0.3089	0.3089	0.6513	0.4886	0.7856	0.5323
ItemKNN CF asymmetric	0.3333	0.3333	0.6654	0.5082	0.7925	0.5495
ItemKNN CF tversky	0.3273	0.3273	0.6556	0.5011	0.7810	0.5419
$P^3\alpha$	0.3316	0.3316	0.6543	0.5031	0.7687	0.5402
$RP^3\beta$	0.3464	0.3464	0.6743	0.5198	0.7959	0.5591
$EASE^R$	**0.4003**	**0.4003**	**0.7258**	**0.5738**	**0.8343**	**0.6093**
SLIM BPR	0.3515	0.3515	0.6843	0.5281	0.7983	0.5651
SLIM ElasticNet	**0.3906**	**0.3906**	**0.7116**	**0.5625**	**0.8315**	**0.6014**
MF BPR	0.3151	0.3151	0.6550	0.4945	0.7838	0.5365
MF FunkSVD	**0.3646**	**0.3646**	0.7017	**0.5434**	0.8151	**0.5802**
PureSVD	**0.3735**	**0.3735**	**0.7088**	**0.5522**	0.8132	**0.5861**
NMF	0.3508	0.3508	0.6879	0.5291	0.7995	0.5656
iALS	**0.3816**	**0.3816**	**0.7121**	**0.5581**	0.8200	**0.5933**
ItemKNN CBF cosine	0.0889	0.0889	0.2545	0.1735	0.3775	0.2129
ItemKNN CBF dice	0.0864	0.0864	0.2535	0.1714	0.3725	0.2097
ItemKNN CBF jaccard	0.0879	0.0879	0.2518	0.1711	0.3786	0.2117
ItemKNN CBF asymmetric	0.0884	0.0884	0.2586	0.1752	0.3780	0.2137
ItemKNN CBF tversky	0.0892	0.0892	0.2518	0.1716	0.3795	0.2124
UserKNN CBF cosine	0.1719	0.1719	0.4432	0.3114	0.6050	0.3635
UserKNN CBF dice	0.1719	0.1719	0.4432	0.3114	0.6048	0.3634
UserKNN CBF jaccard	0.1714	0.1714	0.4427	0.3108	0.6065	0.3636
UserKNN CBF asymmetric	0.1724	0.1724	0.4427	0.3113	0.6048	0.3635
UserKNN CBF tversky	0.1714	0.1714	0.4427	0.3108	0.6065	0.3636
ItemKNN CFCBF cosine	0.3328	0.3328	0.6694	0.5107	0.7985	0.5526
ItemKNN CFCBF dice	0.3136	0.3136	0.6553	0.4927	0.7879	0.5358
ItemKNN CFCBF jaccard	0.3096	0.3096	0.6497	0.4881	0.7868	0.5326
ItemKNN CFCBF asymmetric	0.3540	0.3540	0.6740	0.5227	0.7947	0.5620
ItemKNN CFCBF tversky	0.3325	0.3325	0.6581	0.5055	0.7811	0.5455
UserKNN CFCBF cosine	0.3497	0.3497	0.6806	0.5252	0.8013	0.5645
UserKNN CFCBF dice	**0.3555**	**0.3555**	0.6869	0.5328	0.8008	0.5698
UserKNN CFCBF jaccard	0.3533	0.3533	0.6879	0.5321	0.8050	0.5701
UserKNN CFCBF asymmetric	0.3507	0.3507	0.6805	0.5259	0.8045	0.5662
UserKNN CFCBF tversky	0.3522	0.3522	0.6767	0.5254	0.8000	0.5654
DeepCF	0.3550	0.3550	0.7017	0.5388	0.8272	0.5794
CoupledCF	0.3522	0.3522	0.7018	0.5374	0.8247	0.5775

Table 110. Experimental results for the CoupledCF method for the Tafeng dataset.

	@ 1		@ 5		@ 10	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0576	0.0576	0.0963	0.0764	0.1469	0.0925
TopPopular	**0.2654**	**0.2654**	0.5194	0.3965	0.6549	0.4402
UserKNN CF cosine	**0.3215**	**0.3215**	**0.5412**	**0.4369**	0.6415	**0.4693**
UserKNN CF dice	**0.3193**	**0.3193**	**0.5351**	**0.4323**	0.6353	**0.4648**
UserKNN CF jaccard	**0.3194**	**0.3194**	**0.5353**	**0.4324**	0.6355	**0.4648**
UserKNN CF asymmetric	**0.3217**	**0.3217**	**0.5395**	**0.4362**	0.6399	**0.4686**
UserKNN CF tversky	**0.3192**	**0.3192**	**0.5380**	**0.4338**	0.6382	**0.4662**
ItemKNN CF cosine	**0.3314**	**0.3314**	**0.5424**	**0.4427**	0.6376	**0.4735**
ItemKNN CF dice	**0.3217**	**0.3217**	**0.5409**	**0.4368**	0.6334	**0.4668**
ItemKNN CF jaccard	**0.3229**	**0.3229**	**0.5448**	**0.4391**	0.6441	**0.4713**
ItemKNN CF asymmetric	**0.3322**	**0.3322**	**0.5445**	**0.4442**	0.6356	**0.4736**
ItemKNN CF tversky	**0.3184**	**0.3184**	**0.5421**	**0.4356**	0.6426	**0.4683**
$P^3\alpha$	**0.3245**	**0.3245**	**0.5503**	**0.4437**	0.6404	**0.4730**
$RP^3\beta$	**0.3202**	**0.3202**	**0.5525**	**0.4424**	0.6470	**0.4732**
$EASE^R$	**0.3272**	**0.3272**	**0.5452**	**0.4417**	0.6435	**0.4736**
SLIM BPR	**0.3171**	**0.3171**	**0.5454**	**0.4368**	0.6457	**0.4693**
SLIM ElasticNet	**0.3233**	**0.3233**	**0.5438**	**0.4389**	0.6476	**0.4726**
MF BPR	0.2556	0.2556	0.5017	0.3827	0.6315	0.4247
MF FunkSVD	**0.2676**	**0.2676**	0.5196	0.3980	0.6541	0.4414
PureSVD	0.2462	0.2462	0.4889	0.3714	0.6260	0.4156
NMF	0.2556	0.2556	0.4761	0.3706	0.5765	0.4031
iALS	**0.2920**	**0.2920**	0.5219	**0.4126**	0.6293	**0.4473**
ItemKNN CBF cosine	0.0557	0.0557	0.0959	0.0753	0.1414	0.0897
ItemKNN CBF dice	0.0556	0.0556	0.0921	0.0734	0.1379	0.0879
ItemKNN CBF jaccard	0.0555	0.0555	0.0922	0.0734	0.1381	0.0880
ItemKNN CBF asymmetric	0.0589	0.0589	0.0958	0.0769	0.1467	0.0931
ItemKNN CBF tversky	0.0555	0.0555	0.0922	0.0734	0.1381	0.0880
UserKNN CBF cosine	0.2462	0.2462	0.4651	0.3598	0.5794	0.3967
UserKNN CBF dice	0.2414	0.2414	0.4550	0.3519	0.5645	0.3873
UserKNN CBF jaccard	0.2409	0.2409	0.4540	0.3512	0.5632	0.3866
UserKNN CBF asymmetric	0.2464	0.2464	0.4654	0.3600	0.5798	0.3970
UserKNN CBF tversky	0.2418	0.2418	0.4560	0.3526	0.5655	0.3880
ItemKNN CFCBF cosine	**0.3314**	**0.3314**	**0.5424**	**0.4427**	0.6376	**0.4735**
ItemKNN CFCBF dice	**0.3092**	**0.3092**	**0.5323**	**0.4258**	0.6331	**0.4586**
ItemKNN CFCBF jaccard	**0.3085**	**0.3085**	**0.5343**	**0.4266**	0.6345	**0.4591**
ItemKNN CFCBF asymmetric	**0.3331**	**0.3331**	**0.5434**	**0.4442**	0.6314	**0.4727**
ItemKNN CFCBF tversky	**0.3091**	**0.3091**	**0.5345**	**0.4273**	0.6280	**0.4578**
UserKNN CFCBF cosine	**0.3443**	**0.3443**	**0.5888**	**0.4726**	**0.6947**	**0.5069**
UserKNN CFCBF dice	**0.3153**	**0.3153**	**0.5448**	**0.4356**	0.6507	**0.4699**
UserKNN CFCBF jaccard	**0.3157**	**0.3157**	**0.5454**	**0.4361**	0.6523	**0.4707**
UserKNN CFCBF asymmetric	**0.3424**	**0.3424**	**0.5882**	**0.4713**	**0.6937**	**0.5055**
UserKNN CFCBF tversky	**0.3152**	**0.3152**	**0.5404**	**0.4333**	0.6455	**0.4674**
DeepCF	0.2647	0.2647	0.5244	0.3995	0.6583	0.4428
CoupledCF	0.2641	0.2641	0.5175	0.3948	0.6499	0.4377

Table 111. Experimental results for the CoupledCF method for the Movielens 1M dataset on beyond accuracy metrics.

	@ 5				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9987**	**0.9997**	**0.9992**	**0.7970**	**11.8516**
TopPopular	0.9846	0.9969	0.1682	0.0738	8.5531
UserKNN CF cosine	0.9933	0.9986	0.5113	0.1991	10.0351
UserKNN CF dice	0.9938	0.9987	0.5457	0.2144	10.1387
UserKNN CF jaccard	0.9936	0.9987	0.5338	0.2078	10.0951
UserKNN CF asymmetric	0.9933	0.9986	0.5004	0.1967	10.0170
UserKNN CF tversky	0.9931	0.9986	0.5009	0.1928	9.9905
ItemKNN CF cosine	0.9934	0.9986	0.4946	0.1932	10.0005
ItemKNN CF dice	0.9924	0.9984	0.4667	0.1721	9.8321
ItemKNN CF jaccard	0.9921	0.9984	0.4548	0.1651	9.7744
ItemKNN CF asymmetric	0.9944	0.9989	0.5176	0.2199	10.1866
ItemKNN CF tversky	0.9935	0.9987	0.4404	0.1858	9.9434
$P^3\alpha$	0.9920	0.9984	0.3564	0.1504	9.6368
$RP^3\beta$	0.9933	0.9986	0.5226	0.1929	10.0000
$EASE^R$	0.9944	0.9988	0.5087	0.2211	10.1884
SLIM BPR	0.9928	0.9985	0.4690	0.1805	9.8990
SLIM ElasticNet	0.9941	0.9988	0.5120	0.2138	10.1425
MF BPR	0.9938	0.9987	0.4943	0.2038	10.0747
MF FunkSVD	0.9950	0.9990	0.5011	0.2393	10.2918
PureSVD	**0.9952**	**0.9990**	0.5252	0.2482	10.3504
NMF	**0.9958**	**0.9991**	**0.6393**	**0.2932**	**10.5859**
iALS	**0.9956**	**0.9991**	0.5596	**0.2658**	**10.4474**
ItemKNN CBF cosine	0.9921	0.9984	0.4521	0.1671	9.7882
ItemKNN CBF dice	0.9944	0.9989	0.5917	0.2430	10.3083
ItemKNN CBF jaccard	0.9944	0.9988	0.5980	0.2396	10.2927
ItemKNN CBF asymmetric	0.9942	0.9988	0.5464	0.2226	10.1981
ItemKNN CBF tversky	0.9944	0.9988	0.5995	0.2401	10.2977
UserKNN CBF cosine	0.9873	0.9974	0.2398	0.0918	8.9217
UserKNN CBF dice	0.9873	0.9974	0.2408	0.0919	8.9232
UserKNN CBF jaccard	0.9873	0.9974	0.2411	0.0918	8.9213
UserKNN CBF asymmetric	0.9873	0.9974	0.2406	0.0920	8.9245
UserKNN CBF tversky	0.9873	0.9974	0.2411	0.0918	8.9213
ItemKNN CFCBF cosine	0.9946	0.9989	0.4867	0.2201	10.1808
ItemKNN CFCBF dice	0.9922	0.9984	0.4609	0.1682	9.8002
ItemKNN CFCBF jaccard	0.9919	0.9983	0.4480	0.1608	9.7372
ItemKNN CFCBF asymmetric	0.9939	0.9988	0.5092	0.2071	10.1001
ItemKNN CFCBF tversky	0.9940	0.9988	0.4688	0.2001	10.0500
UserKNN CFCBF cosine	0.9926	0.9985	0.4703	0.1795	9.8874
UserKNN CFCBF dice	0.9937	0.9987	0.5454	0.2133	10.1305
UserKNN CFCBF jaccard	0.9935	0.9987	0.5237	0.2065	10.0847
UserKNN CFCBF asymmetric	0.9924	0.9985	0.4571	0.1736	9.8420
UserKNN CFCBF tversky	0.9936	0.9987	0.5378	0.2049	10.0801
DeepCF	0.9950	0.9990	0.5882	0.2548	10.3870
CoupledCF	0.9952	0.9990	0.6124	0.2655	10.4473

Table 112. Experimental results for the CoupledCF method for the Tafeng dataset on beyond accuracy metrics.

	@ 5				
	Div. MIL	Div. HHI	Cov. Item	Div. Gini	Div. Shannon
Random	**0.9998**	**1.0000**	**0.9987**	**0.7777**	**14.4213**
TopPopular	0.9967	0.9993	0.1376	0.0540	10.7100
UserKNN CF cosine	**0.9981**	**0.9996**	**0.6084**	**0.1538**	**12.1032**
UserKNN CF dice	**0.9982**	**0.9996**	**0.6383**	**0.1636**	**12.1700**
UserKNN CF jaccard	**0.9982**	**0.9996**	**0.6395**	**0.1643**	**12.1758**
UserKNN CF asymmetric	**0.9981**	**0.9996**	**0.6076**	**0.1535**	**12.1022**
UserKNN CF tversky	**0.9981**	**0.9996**	**0.6218**	**0.1566**	**12.1154**
ItemKNN CF cosine	**0.9984**	**0.9996**	**0.7495**	**0.2179**	**12.5110**
ItemKNN CF dice	**0.9982**	**0.9996**	**0.7409**	**0.1989**	**12.3097**
ItemKNN CF jaccard	**0.9980**	**0.9996**	**0.6757**	**0.1693**	**12.1120**
ItemKNN CF asymmetric	**0.9984**	**0.9997**	**0.7461**	**0.2213**	**12.5418**
ItemKNN CF tversky	**0.9980**	**0.9996**	**0.6644**	**0.1597**	**12.0657**
$P^3\alpha$	**0.9982**	**0.9996**	**0.6720**	**0.1749**	**12.2433**
$RP^3\beta$	**0.9981**	**0.9996**	**0.6726**	**0.1655**	**12.1182**
$EASE^R$	**0.9982**	**0.9996**	**0.5924**	**0.1516**	**12.1238**
SLIM BPR	**0.9980**	**0.9996**	**0.6421**	**0.1535**	**12.0256**
SLIM ElasticNet	**0.9981**	**0.9996**	**0.5985**	**0.1474**	**12.0482**
MF BPR	0.9970	0.9993	0.2135	0.0629	10.9588
MF FunkSVD	0.9967	0.9993	0.1404	0.0541	10.7171
PureSVD	0.9970	0.9993	**0.3393**	0.0696	11.0290
NMF	**0.9989**	**0.9998**	**0.6044**	**0.2052**	**12.6505**
iALS	**0.9982**	**0.9996**	**0.3436**	**0.1121**	**11.8099**
ItemKNN CBF cosine	**0.9995**	**0.9999**	**0.8286**	**0.4142**	**13.6428**
ItemKNN CBF dice	**0.9997**	**0.9999**	**0.8081**	**0.4715**	**13.8256**
ItemKNN CBF jaccard	**0.9997**	**0.9999**	**0.8073**	**0.4713**	**13.8249**
ItemKNN CBF asymmetric	**0.9994**	**0.9999**	**0.7866**	**0.3683**	**13.4382**
ItemKNN CBF tversky	**0.9997**	**0.9999**	**0.8078**	**0.4715**	**13.8255**
UserKNN CBF cosine	**0.9978**	**0.9995**	**0.5440**	**0.1277**	**11.8532**
UserKNN CBF dice	**0.9980**	**0.9996**	**0.6041**	**0.1473**	**12.0191**
UserKNN CBF jaccard	**0.9980**	**0.9996**	**0.6087**	**0.1490**	**12.0334**
UserKNN CBF asymmetric	**0.9978**	**0.9995**	**0.5434**	**0.1275**	**11.8515**
UserKNN CBF tversky	**0.9980**	**0.9996**	**0.5986**	**0.1448**	**11.9960**
ItemKNN CFCBF cosine	**0.9984**	**0.9996**	**0.7519**	**0.2184**	**12.5130**
ItemKNN CFCBF dice	**0.9981**	**0.9996**	**0.7055**	**0.1788**	**12.1742**
ItemKNN CFCBF jaccard	**0.9980**	**0.9996**	**0.6946**	**0.1753**	**12.1441**
ItemKNN CFCBF asymmetric	**0.9985**	**0.9997**	**0.7485**	**0.2298**	**12.6041**
ItemKNN CFCBF tversky	**0.9981**	**0.9996**	**0.7127**	**0.1910**	**12.2602**
UserKNN CFCBF cosine	**0.9977**	**0.9995**	**0.4118**	**0.1041**	**11.6422**
UserKNN CFCBF dice	**0.9977**	**0.9995**	**0.4257**	**0.1098**	**11.7135**
UserKNN CFCBF jaccard	**0.9977**	**0.9995**	**0.4249**	**0.1092**	**11.7057**
UserKNN CFCBF asymmetric	**0.9977**	**0.9995**	**0.4034**	**0.1025**	**11.6247**
UserKNN CFCBF tversky	**0.9978**	**0.9995**	**0.4590**	**0.1181**	**11.8036**
DeepCF	0.9972	0.9994	0.2122	0.0708	11.1506
CoupledCF	0.9971	0.9994	0.2190	0.0670	11.0656

Table 113. Computation time for the algorithms in the selected results for the CoupledCF method on the Movielens 1M dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.02 [sec]	11.42 [sec]	529
TopPopular	0.03 [sec]	11.95 [sec]	505
UserKNN CF cosine	3.37 ± 0.17 [sec]	16.89 ± 0.52 [sec]	361
UserKNN CF dice	3.24 ± 0.15 [sec]	16.86 ± 0.58 [sec]	376
UserKNN CF jaccard	3.26 ± 0.16 [sec]	16.79 ± 0.67 [sec]	374
UserKNN CF asymmetric	3.36 ± 0.20 [sec]	16.82 ± 0.86 [sec]	369
UserKNN CF tversky	3.36 ± 0.17 [sec]	17.06 ± 0.39 [sec]	359
ItemKNN CF cosine	1.58 ± 0.11 [sec]	16.85 ± 2.03 [sec]	382
ItemKNN CF dice	1.57 ± 0.09 [sec]	15.62 ± 0.20 [sec]	390
ItemKNN CF jaccard	1.57 ± 0.09 [sec]	15.99 ± 1.03 [sec]	393
ItemKNN CF asymmetric	1.63 ± 0.07 [sec]	18.02 ± 0.99 [sec]	344
ItemKNN CF tversky	1.57 ± 0.10 [sec]	15.92 ± 0.67 [sec]	387
$P^3\alpha$	3.24 ± 0.93 [sec]	15.72 ± 0.26 [sec]	388
$RP^3\beta$	4.10 ± 0.95 [sec]	16.50 ± 0.74 [sec]	350
$EASE^R$	5.67 ± 0.36 [sec]	44.87 ± 0.28 [sec]	134
SLIM BPR	467.50 [sec] / 7.79 ± 7.86 [min]	17.19 ± 0.17 [sec]	351
SLIM ElasticNet	155.93 [sec] / 2.60 ± 1.12 [min]	15.97 ± 0.16 [sec]	378
MF BPR	504.64 [sec] / 8.41 ± 7.70 [min]	21.44 ± 5.17 [sec]	247
MF FunkSVD	2275.27 [sec] / 37.92 ± 42.06 [min]	17.40 ± 5.10 [sec]	442
PureSVD	1.32 ± 0.84 [sec]	15.63 ± 4.40 [sec]	443
NMF	671.50 [sec] / 11.19 ± 7.70 [min]	20.11 ± 5.89 [sec]	246
iALS	1155.74 [sec] / 19.26 ± 25.07 [min]	17.47 ± 5.04 [sec]	435
ItemKNN CBF cosine	0.29 ± 0.05 [sec]	15.73 ± 0.67 [sec]	365
ItemKNN CBF dice	0.27 ± 0.05 [sec]	15.17 ± 0.51 [sec]	383
ItemKNN CBF jaccard	0.27 ± 0.04 [sec]	15.92 ± 0.22 [sec]	383
ItemKNN CBF asymmetric	0.31 ± 0.05 [sec]	16.10 ± 0.15 [sec]	378
ItemKNN CBF tversky	0.26 ± 0.05 [sec]	15.82 ± 0.35 [sec]	387
UserKNN CBF cosine	0.74 ± 0.15 [sec]	17.47 ± 0.95 [sec]	333
UserKNN CBF dice	0.77 ± 0.10 [sec]	17.65 ± 0.68 [sec]	333
UserKNN CBF jaccard	0.74 ± 0.10 [sec]	17.85 ± 0.76 [sec]	326
UserKNN CBF asymmetric	0.81 ± 0.12 [sec]	17.50 ± 1.06 [sec]	335
UserKNN CBF tversky	0.77 ± 0.10 [sec]	18.21 ± 0.25 [sec]	328
ItemKNN CFCBF cosine	1.60 ± 0.10 [sec]	16.59 ± 1.09 [sec]	378
ItemKNN CFCBF dice	1.62 ± 0.09 [sec]	16.11 ± 0.89 [sec]	389
ItemKNN CFCBF jaccard	1.60 ± 0.09 [sec]	15.54 ± 0.24 [sec]	390
ItemKNN CFCBF asymmetric	1.62 ± 0.10 [sec]	15.95 ± 0.60 [sec]	391
ItemKNN CFCBF tversky	1.61 ± 0.08 [sec]	15.63 ± 0.18 [sec]	391
UserKNN CFCBF cosine	3.30 ± 0.16 [sec]	16.91 ± 0.67 [sec]	352
UserKNN CFCBF dice	3.24 ± 0.14 [sec]	16.72 ± 0.66 [sec]	371
UserKNN CFCBF jaccard	3.26 ± 0.16 [sec]	17.08 ± 0.80 [sec]	373
UserKNN CFCBF asymmetric	3.31 ± 0.15 [sec]	17.45 ± 0.19 [sec]	344
UserKNN CFCBF tversky	3.33 ± 0.16 [sec]	17.45 ± 0.93 [sec]	353
DeepCF	5220.13 [sec] / 1.45 [hour]	31.44 [sec]	192
CoupledCF	18009.05 [sec] / 5.00 [hour]	40.42 [sec]	149

Table 114. Computation time for the algorithms in the selected results for the CoupledCF method on the Tafeng dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.03 [sec]	110.93 [sec] / 1.85 [min]	291
TopPopular	0.05 [sec]	116.85 [sec] / 1.95 [min]	276
UserKNN CF cosine	13.95 ± 1.42 [sec]	127.76 [sec] / 2.13 ± 0.06 [min]	246
UserKNN CF dice	13.72 ± 1.12 [sec]	127.08 [sec] / 2.12 ± 0.04 [min]	247
UserKNN CF jaccard	14.38 ± 1.08 [sec]	128.50 [sec] / 2.14 ± 0.03 [min]	247
UserKNN CF asymmetric	13.97 ± 1.28 [sec]	129.85 [sec] / 2.16 ± 0.05 [min]	244
UserKNN CF tversky	15.27 ± 1.02 [sec]	129.48 [sec] / 2.16 ± 0.05 [min]	245
ItemKNN CF cosine	5.64 ± 0.69 [sec]	134.13 [sec] / 2.24 ± 0.01 [min]	241
ItemKNN CF dice	5.64 ± 0.45 [sec]	132.98 [sec] / 2.22 ± 0.04 [min]	246
ItemKNN CF jaccard	5.84 ± 0.47 [sec]	132.44 [sec] / 2.21 ± 0.02 [min]	242
ItemKNN CF asymmetric	6.00 ± 0.46 [sec]	133.71 [sec] / 2.23 ± 0.02 [min]	242
ItemKNN CF tversky	6.27 ± 0.48 [sec]	130.25 [sec] / 2.17 ± 0.04 [min]	244
$P^3\alpha$	23.99 ± 4.64 [sec]	124.16 [sec] / 2.07 ± 0.01 [min]	261
$RP^3\beta$	25.14 ± 4.22 [sec]	123.33 [sec] / 2.06 ± 0.03 [min]	258
$EASE^R$	261.07 [sec] / 4.35 ± 0.07 [min]	229.71 [sec] / 3.83 ± 0.06 [min]	138
SLIM BPR	2965.11 [sec] / 49.42 ± 26.56 [min]	128.97 [sec] / 2.15 ± 0.03 [min]	249
SLIM ElasticNet	724.79 [sec] / 12.08 ± 5.12 [min]	132.95 [sec] / 2.22 ± 0.08 [min]	235
MF BPR	2089.76 [sec] / 34.83 ± 34.52 [min]	102.09 [sec] / 1.70 ± 0.02 [min]	319
MF FunkSVD	2277.99 [sec] / 37.97 ± 51.14 [min]	107.85 [sec] / 1.80 ± 0.07 [min]	290
PureSVD	1.00 ± 1.65 [sec]	101.64 [sec] / 1.69 ± 0.04 [min]	321
NMF	553.76 [sec] / 9.23 ± 7.94 [min]	101.13 [sec] / 1.69 ± 0.02 [min]	323
iALS	1522.30 [sec] / 25.37 ± 17.32 [min]	101.54 [sec] / 1.69 ± 0.01 [min]	317
ItemKNN CBF cosine	19.28 ± 10.55 [sec]	119.53 [sec] / 1.99 ± 0.02 [min]	265
ItemKNN CBF dice	17.57 ± 9.87 [sec]	119.03 [sec] / 1.98 ± 0.01 [min]	270
ItemKNN CBF jaccard	19.30 ± 9.99 [sec]	118.94 [sec] / 1.98 ± 0.01 [min]	270
ItemKNN CBF asymmetric	20.54 ± 10.30 [sec]	120.04 [sec] / 2.00 ± 0.02 [min]	266
ItemKNN CBF tversky	23.76 ± 9.79 [sec]	119.73 [sec] / 2.00 ± 0.00 [min]	269
UserKNN CBF cosine	17.52 ± 1.78 [sec]	135.30 [sec] / 2.26 ± 0.03 [min]	237
UserKNN CBF dice	16.55 ± 0.95 [sec]	135.20 [sec] / 2.25 ± 0.01 [min]	238
UserKNN CBF jaccard	16.99 ± 0.87 [sec]	134.23 [sec] / 2.24 ± 0.06 [min]	239
UserKNN CBF asymmetric	18.15 ± 1.36 [sec]	134.75 [sec] / 2.25 ± 0.04 [min]	236
UserKNN CBF tversky	17.47 ± 1.02 [sec]	130.40 [sec] / 2.17 ± 0.10 [min]	237
ItemKNN CFCBF cosine	18.52 ± 1.48 [sec]	137.35 [sec] / 2.29 ± 0.03 [min]	234
ItemKNN CFCBF dice	18.03 ± 0.88 [sec]	129.98 [sec] / 2.17 ± 0.02 [min]	248
ItemKNN CFCBF jaccard	18.50 ± 0.90 [sec]	128.86 [sec] / 2.15 ± 0.03 [min]	247
ItemKNN CFCBF asymmetric	19.09 ± 1.35 [sec]	135.91 [sec] / 2.27 ± 0.06 [min]	233
ItemKNN CFCBF tversky	18.60 ± 1.63 [sec]	130.70 [sec] / 2.18 ± 0.04 [min]	249
UserKNN CFCBF cosine	21.36 ± 1.94 [sec]	133.41 [sec] / 2.22 ± 0.10 [min]	235
UserKNN CFCBF dice	22.14 ± 1.30 [sec]	134.25 [sec] / 2.24 ± 0.04 [min]	237
UserKNN CFCBF jaccard	22.37 ± 1.17 [sec]	133.75 [sec] / 2.23 ± 0.05 [min]	237
UserKNN CFCBF asymmetric	21.85 ± 1.59 [sec]	135.13 [sec] / 2.25 ± 0.04 [min]	236
UserKNN CFCBF tversky	23.01 ± 1.34 [sec]	133.47 [sec] / 2.22 ± 0.06 [min]	240
DeepCF	4948.23 [sec] / 1.37 [hour]	197.08 [sec] / 3.28 [min]	164
CoupledCF	7785.86 [sec] / 2.16 [hour]	272.01 [sec] / 4.53 [min]	119

Table 115. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
UserKNN CF cosine	topK	376	1000
	shrink	0	0
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	TF-IDF	TF-IDF
UserKNN CF dice	topK	167	1000
	shrink	0	0
	similarity	dice	dice
	normalize	False	False
UserKNN CF jaccard	topK	198	1000
	shrink	0	0
	similarity	jaccard	jaccard
	normalize	True	True
UserKNN CF asymmetric	topK	270	1000
	shrink	0	0
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.1795	0.4536
	feature weighting	TF-IDF	TF-IDF
UserKNN CF tversky	topK	353	1000
	shrink	0	0
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	1.6894	1.1418
	tversky beta	1.1405	2.0000
ItemKNN CF cosine	topK	68	1000
	shrink	422	1000
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	TF-IDF	TF-IDF
ItemKNN CF dice	topK	48	615
	shrink	1	37
	similarity	dice	dice
	normalize	True	False
ItemKNN CF jaccard	topK	46	998
	shrink	0	38
	similarity	jaccard	jaccard
	normalize	False	True
ItemKNN CF asymmetric	topK	475	1000
	shrink	183	1000
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.3828	0.4300
	feature weighting	BM25	TF-IDF
ItemKNN CF tversky	topK	66	655
	shrink	449	48
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	0.0000	0.6733
	tversky beta	1.1856	0.5679

Table 116. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
$P^3\alpha$	topK	317	1000
	alpha	1.7378	0.5688
	normalize similarity	True	False
$RP^3\beta$	topK	881	931
	alpha	0.8272	0.4851
	beta	0.7356	0.0842
	normalize similarity	True	False
$EASE^R$	l2 norm	9.73E+02	2.79E+03
SLIM BPR	topK	872	942
	epochs	145	290
	symmetric	True	False
	sgd mode	adagrad	adagrad
	lambda i	1.00E-05	1.00E-05
	lambda j	1.70E-03	1.00E-02
	learning rate	4.41E-02	1.00E-04
SLIM ElasticNet	topK	541	999
	l1 ratio	1.83E-03	1.00E-05
	alpha	0.0637	0.1242
MF BPR	sgd mode	adam	sgd
	epochs	890	200
	num factors	192	1
	batch size	16	1
	positive reg	7.70E-04	1.00E-05
	negative reg	3.06E-05	1.00E-05
	learning rate	1.31E-03	4.15E-02
MF FunkSVD	sgd mode	adagrad	sgd
	epochs	300	55
	use bias	False	True
	batch size	16	16
	num factors	41	1
	item reg	5.36E-04	1.96E-03
	user reg	6.59E-03	1.00E-02
	learning rate	4.01E-02	1.00E-02
	negative quota	0.1226	0.1016
PureSVD	num factors	63	1
NMF	num factors	141	20
	solver	mult. update	mult. update
	init type	random	nndsvda
	beta loss	kullback-leibler	kullback-leibler
iALS	num factors	84	14
	confidence scaling	linear	linear
	alpha	0.9005	1.8783
	epsilon	3.6269	0.0124
	reg	1.00E-02	1.69E-05
	epochs	55	15

Table 117. Hyperparameter values for our content based KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
ItemKNN CBF ICM all cosine	topK	895	532
	shrink	113	606
	similarity	cosine	cosine
	normalize	False	False
	feature weighting	TF-IDF	BM25
ItemKNN CBF ICM all dice	topK	427	113
	shrink	603	0
	similarity	dice	dice
	normalize	False	False
ItemKNN CBF ICM all jaccard	topK	366	161
	shrink	1000	1000
	similarity	jaccard	jaccard
	normalize	True	True
ItemKNN CBF ICM all asymmetric	topK	502	436
	shrink	20	1000
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.0099	0.0000
	feature weighting	TF-IDF	BM25
ItemKNN CBF ICM all tversky	topK	323	153
	shrink	578	180
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	2.0000	1.2762
	tversky beta	1.4391	1.2042
UserKNN CBF cosine	topK	905	999
	shrink	59	75
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	none	BM25
UserKNN CBF dice	topK	889	1000
	shrink	0	991
	similarity	dice	dice
	normalize	True	True
UserKNN CBF jaccard	topK	1000	1000
	shrink	0	971
	similarity	jaccard	jaccard
	normalize	True	False
UserKNN CBF asymmetric	topK	882	1000
	shrink	1000	182
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	2.0000	0.0000
	feature weighting	none	TF-IDF
UserKNN CBF tversky	topK	1000	1000
	shrink	0	0
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	2.0000	0.0000
	tversky beta	0.0000	0.0000

Table 118. Hyperparameter values for our hybrid KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
ItemKNN CFCBF ICM all cosine	topK	111	1000
	shrink	0	1000
	similarity	cosine	cosine
	normalize	False	False
	feature weighting	BM25	BM25
	ICM weight	0.0100	100.0000
ItemKNN CFCBF ICM all dice	topK	54	574
	shrink	5	248
	similarity	dice	dice
	normalize	False	False
	ICM weight	37.4751	0.0100
ItemKNN CFCBF ICM all jaccard	topK	54	676
	shrink	0	227
	similarity	jaccard	jaccard
	normalize	False	True
	ICM weight	0.0100	0.0235
ItemKNN CFCBF ICM all asymmetric	topK	82	1000
	shrink	44	31
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.2269	0.3969
	feature weighting	TF-IDF	none
	ICM weight	0.9996	1.8780
ItemKNN CFCBF ICM all tversky	topK	47	740
	shrink	610	291
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	0.0000	0.5378
	tversky beta	2.0000	1.6396
	ICM weight	0.1974	0.1392

Table 119. Hyperparameter values for our hybrid KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
UserKNN CFCBF cosine	topK	393	1000
	shrink	52	1000
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	BM25	BM25
	UCM weight	0.0147	1.9173
UserKNN CFCBF dice	topK	152	950
	shrink	1	22
	similarity	dice	dice
	normalize	False	False
	UCM weight	10.5354	0.0194
UserKNN CFCBF jaccard	topK	182	970
	shrink	0	14
	similarity	jaccard	jaccard
	normalize	True	False
	UCM weight	0.0100	97.7807
UserKNN CFCBF asymmetric	topK	547	1000
	shrink	249	1000
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.0000	0.5012
	feature weighting	BM25	BM25
	UCM weight	0.0100	2.5128
UserKNN CFCBF tversky	topK	383	828
	shrink	155	25
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	0.0000	1.6936
	tversky beta	1.2914	2.0000
	UCM weight	100.0000	0.0100

Table 120. Hyperparameter values for our content based KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
ItemKNN CBF ICM original cosine	topK	-	995
	shrink	-	13
	similarity	-	cosine
	normalize	-	True
	feature weighting	-	none
ItemKNN CBF ICM original dice	topK	-	538
	shrink	-	126
	similarity	-	dice
	normalize	-	True
ItemKNN CBF ICM original jaccard	topK	-	594
	shrink	-	695
	similarity	-	jaccard
	normalize	-	True
ItemKNN CBF ICM original asymmetric	topK	-	955
	shrink	-	961
	similarity	-	asymmetric
	normalize	-	True
	asymmetric alpha	-	1.9332
	feature weighting	-	none
ItemKNN CBF ICM original tversky	topK	-	597
	shrink	-	336
	similarity	-	tversky
	normalize	-	True
	tversky alpha	-	0.9637
	tversky beta	-	1.0719

Table 121. Hyperparameter values for our hybrid KNN baselines on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
ItemKNN CFCBF ICM original cosine	topK	-	1000
	shrink	-	1000
	similarity	-	cosine
	normalize	-	True
	feature weighting	-	TF-IDF
	ICM weight	-	100.0000
ItemKNN CFCBF ICM original dice	topK	-	626
	shrink	-	334
	similarity	-	dice
	normalize	-	False
	ICM weight	-	0.2297
ItemKNN CFCBF ICM original jaccard	topK	-	796
	shrink	-	308
	similarity	-	jaccard
	normalize	-	True
	ICM weight	-	0.4195
ItemKNN CFCBF ICM original asymmetric	topK	-	1000
	shrink	-	1000
	similarity	-	asymmetric
	normalize	-	True
	asymmetric alpha	-	0.3680
	feature weighting	-	TF-IDF
	ICM weight	-	100.0000
ItemKNN CFCBF ICM original tversky	topK	-	1000
	shrink	-	405
	similarity	-	tversky
	normalize	-	True
	tversky alpha	-	1.0435
	tversky beta	-	1.8406
	ICM weight	-	0.9671

Table 122. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Movielens 1M ours	Tafeng ours
DeepCF	learning rate	1.00E-03	5.00E-03
	epochs	15	10
	n negative sample	4	4
	number model	3	3
CoupledCF	learning rate	1.00E-03	5.00E-03
	epochs	45	5
	n negative sample	4	4
	number model	2	2

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Relevant statistics on the dataset, which we mentioned in the paper, are reported in Table 123. The results of our evaluation can be seen in Table 124 (Amazon Music), Table 125 (Amazon Music, cold items removed) and Table 126 (Movielens 1M). The corresponding optimal hyperparameters are reported in Table 129 (collaborative KNNs), Table 130 (non-neural machine learning and graph based) and Table 131 (DELF).

Lastly, the time required to train and evaluate the models is reported in Table 128 (Amazon Music) and Table 127 (Movielens 1M).

Table 123. Dataset characteristics.

Dataset		Interactions	Items	Users	Density
Amazon Music	original	836K	266414	478235	$6.56 \cdot 10^{-4}$
Amazon Music	preprocessed	76K	41488	1835	0.100
Movielens 1M	-	1M	3706	6040	4.468

Table 124. Experimental results for the DELF method for the Amazon Music dataset.

	@ 5		@ 10		@ 20	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0490	0.0293	0.1014	0.0459	0.1973	0.0699
TopPopular	0.2452	0.1726	0.3057	0.1921	0.3744	0.2094
UserKNN CF cosine	0.3248	0.2544	0.3760	0.2708	0.4376	0.2864
UserKNN CF dice	0.3210	0.2522	0.3760	0.2700	0.4371	0.2854
UserKNN CF jaccard	0.3210	0.2526	0.3760	0.2704	0.4371	0.2858
UserKNN CF asymmetric	0.3188	0.2516	0.3749	0.2698	0.4365	0.2853
UserKNN CF tversky	0.3221	0.2527	0.3760	0.2701	0.4371	0.2855
ItemKNN CF cosine	0.3204	0.2528	0.3733	0.2698	0.4371	0.2858
ItemKNN CF dice	0.3117	0.2441	0.3717	0.2632	0.4338	0.2789
ItemKNN CF jaccard	0.3090	0.2439	0.3602	0.2604	0.4256	0.2767
ItemKNN CF asymmetric	0.3204	0.2566	0.3711	0.2731	0.4327	0.2886
ItemKNN CF tversky	0.3046	0.2431	0.3619	0.2615	0.4278	0.2780
$P^3\alpha$	0.3188	0.2524	0.3684	0.2684	0.4300	0.2839
$RP^3\beta$	0.3155	0.2494	0.3684	0.2663	0.4272	0.2811
$EASE^R$	-	-	-	-	-	-
SLIM BPR	0.3139	0.2446	0.3717	0.2632	0.4392	0.2801
SLIM ElasticNet	0.3199	0.2577	0.3678	0.2730	0.4354	0.2900
MF BPR	0.2376	0.1896	0.2768	0.2023	0.3520	0.2213
MF FunkSVD	0.2545	0.2035	0.2916	0.2155	0.3417	0.2280
PureSVD	0.2627	0.2141	0.3084	0.2290	0.3537	0.2405
NMF	0.2921	0.2306	0.3510	0.2498	0.4087	0.2644
iALS	0.3319	0.2604	0.3717	0.2732	0.4229	0.2860
DELF MLP	0.2986	0.2339	0.3619	0.2542	**0.4561**	0.2778
DELF EF	**0.5422**	**0.3632**	**0.7439**	**0.4290**	**0.8578**	**0.4583**

Table 125. Experimental results for the DELF method for the Amazon Music (cold items removed) dataset.

	@ 5		@ 10		@ 20	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0567	0.0353	0.1079	0.0516	0.2174	0.0789
TopPopular	0.2474	0.1730	0.3041	0.1913	0.3738	0.2090
UserKNN CF cosine	**0.3150**	**0.2495**	**0.3471**	**0.2600**	0.3738	**0.2668**
UserKNN CF dice	**0.3106**	**0.2470**	**0.3471**	**0.2590**	0.3744	**0.2659**
UserKNN CF jaccard	**0.3106**	**0.2474**	**0.3471**	**0.2594**	0.3738	**0.2663**
UserKNN CF asymmetric	**0.3084**	**0.2464**	**0.3460**	**0.2587**	0.3744	**0.2660**
UserKNN CF tversky	**0.3117**	**0.2475**	**0.3471**	**0.2591**	0.3744	**0.2661**
ItemKNN CF cosine	**0.3084**	**0.2464**	**0.3428**	**0.2576**	0.3744	**0.2655**
ItemKNN CF dice	**0.3025**	**0.2389**	**0.3422**	**0.2517**	0.3744	**0.2598**
ItemKNN CF jaccard	**0.2992**	**0.2382**	**0.3324**	**0.2489**	0.3575	**0.2552**
ItemKNN CF asymmetric	**0.3090**	**0.2506**	**0.3401**	**0.2609**	0.3717	**0.2689**
ItemKNN CF tversky	**0.2965**	**0.2380**	**0.3319**	**0.2495**	0.3591	**0.2564**
$P^3\alpha$	**0.3074**	**0.2465**	**0.3373**	**0.2564**	0.3689	**0.2644**
$RP^3\beta$	**0.3046**	**0.2434**	**0.3379**	**0.2543**	0.3651	**0.2611**
$EASE^R$	-	-	-	-	-	-
SLIM BPR	**0.3008**	**0.2380**	**0.3390**	**0.2504**	0.3673	**0.2576**
SLIM ElasticNet	**0.3101**	**0.2526**	**0.3411**	**0.2625**	0.3711	**0.2701**
MF BPR	0.2360	0.1888	0.2687	0.1995	0.3095	0.2099
MF FunkSVD	0.2545	0.2035	0.2899	0.2150	0.3292	0.2248
PureSVD	0.2627	0.2141	0.3084	0.2290	0.3542	0.2406
NMF	**0.2910**	**0.2294**	**0.3482**	**0.2480**	**0.4038**	**0.2621**
iALS	**0.3319**	**0.2604**	**0.3706**	**0.2729**	**0.4109**	**0.2831**
DELF MLP	0.2905	0.2239	0.3275	0.2361	0.3787	0.2489
DELF EF	0.2883	0.2224	0.3313	0.2364	0.3831	0.2496

Table 126. Experimental results for the DELF method for the Movielens 1M dataset.

	@ 5		@ 10		@ 20	
	HR	NDCG	HR	NDCG	HR	NDCG
Random	0.0525	0.0307	0.1002	0.0460	0.1972	0.0703
TopPopular	0.3302	0.2229	0.4696	0.2674	0.6577	0.3148
UserKNN CF cosine	**0.5186**	**0.3633**	0.6796	**0.4156**	0.8246	**0.4524**
UserKNN CF dice	0.5150	**0.3611**	0.6796	**0.4145**	0.8218	0.4506
UserKNN CF jaccard	0.5166	**0.3622**	0.6788	**0.4148**	0.8227	**0.4513**
UserKNN CF asymmetric	**0.5205**	**0.3635**	**0.6852**	**0.4168**	0.8329	**0.4542**
UserKNN CF tversky	0.5161	**0.3620**	0.6748	**0.4136**	0.8256	**0.4518**
ItemKNN CF cosine	0.4936	0.3426	0.6677	0.3989	0.8243	0.4387
ItemKNN CF dice	0.4895	0.3370	0.6667	0.3943	0.8276	0.4352
ItemKNN CF jaccard	0.4958	0.3408	0.6725	0.3979	0.8197	0.4354
ItemKNN CF asymmetric	0.4946	0.3437	0.6718	0.4009	0.8266	0.4401
ItemKNN CF tversky	0.4936	0.3418	0.6620	0.3964	0.8038	0.4324
$P^3\alpha$	0.4945	0.3438	0.6574	0.3965	0.7952	0.4313
$RP^3\beta$	0.5138	0.3559	0.6809	0.4102	0.8276	0.4475
$EASE^R$	**0.5716**	**0.4064**	**0.7258**	**0.4566**	**0.8516**	**0.4887**
SLIM BPR	**0.5380**	**0.3742**	**0.7077**	**0.4292**	**0.8452**	**0.4640**
SLIM ElasticNet	**0.5706**	**0.4038**	**0.7306**	**0.4557**	**0.8586**	**0.4882**
MF BPR	0.4844	0.3310	0.6595	0.3877	0.8152	0.4275
MF FunkSVD	**0.5312**	**0.3708**	**0.6948**	**0.4239**	0.8245	**0.4569**
PureSVD	**0.5513**	**0.3891**	**0.7021**	**0.4382**	0.8303	**0.4708**
NMF	**0.5339**	**0.3746**	**0.6965**	**0.4272**	**0.8385**	**0.4635**
iALS	**0.5643**	**0.3975**	**0.7228**	**0.4489**	**0.8354**	**0.4776**
DELF MLP	0.5168	0.3587	0.6809	0.4119	0.8342	0.4508
DELF EF	0.4805	0.3305	0.6504	0.3852	0.8043	0.4243

Table 127. Computation time for the algorithms in the selected results for the DELF method on the Movielens 1M dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.03 [sec]	8.92 [sec]	677
TopPopular	0.05 [sec]	9.73 [sec]	621
UserKNN CF cosine	4.63 ± 0.23 [sec]	16.11 ± 0.45 [sec]	383
UserKNN CF dice	4.44 ± 0.19 [sec]	15.72 ± 0.68 [sec]	389
UserKNN CF jaccard	4.45 ± 0.20 [sec]	15.18 ± 0.35 [sec]	393
UserKNN CF asymmetric	4.50 ± 0.19 [sec]	15.84 ± 0.66 [sec]	387
UserKNN CF tversky	4.56 ± 0.22 [sec]	15.86 ± 0.87 [sec]	386
ItemKNN CF cosine	2.13 ± 0.12 [sec]	16.08 ± 0.26 [sec]	375
ItemKNN CF dice	2.09 ± 0.12 [sec]	15.20 ± 0.88 [sec]	414
ItemKNN CF jaccard	2.10 ± 0.13 [sec]	15.73 ± 1.94 [sec]	417
ItemKNN CF asymmetric	2.19 ± 0.15 [sec]	16.66 ± 0.98 [sec]	375
ItemKNN CF tversky	2.13 ± 0.10 [sec]	15.14 ± 0.65 [sec]	420
$P^3\alpha$	4.64 ± 1.45 [sec]	14.66 ± 0.36 [sec]	415
$RP^3\beta$	5.03 ± 1.47 [sec]	14.87 ± 0.49 [sec]	407
$EASE^R$	5.45 ± 0.30 [sec]	17.93 ± 0.07 [sec]	335
SLIM BPR	781.38 [sec] / 13.02 ± 10.84 [min]	15.74 ± 1.06 [sec]	354
SLIM ElasticNet	207.72 [sec] / 3.46 ± 2.28 [min]	15.33 ± 0.52 [sec]	401
MF BPR	537.89 [sec] / 8.96 ± 6.05 [min]	10.06 ± 0.24 [sec]	598
MF FunkSVD	2005.64 [sec] / 33.43 ± 36.79 [min]	10.10 ± 0.08 [sec]	600
PureSVD	0.91 ± 0.66 [sec]	10.15 ± 0.10 [sec]	597
NMF	277.51 [sec] / 4.63 ± 14.49 [min]	10.33 ± 0.09 [sec]	587
iALS	315.26 [sec] / 5.25 ± 4.21 [min]	10.13 ± 0.04 [sec]	594
DELF MLP	12436.79 [sec] / 3.45 [hour]	553.49 [sec] / 9.22 [min]	11
DELF EF	24083.34 [sec] / 6.69 [hour]	550.98 [sec] / 9.18 [min]	11

Table 128. Computation time for the algorithms in the selected results for the DELF method on the Amazon Music dataset.

	Train Time	Recommendation Time	Recommendation Throughput
Random	0.00 [sec]	2.84 [sec]	646
TopPopular	0.00 [sec]	3.44 [sec]	533
UserKNN CF cosine	0.10 ± 0.04 [sec]	4.99 ± 0.04 [sec]	366
UserKNN CF dice	0.10 ± 0.00 [sec]	4.98 ± 0.06 [sec]	366
UserKNN CF jaccard	0.10 ± 0.01 [sec]	5.00 ± 0.01 [sec]	367
UserKNN CF asymmetric	0.10 ± 0.01 [sec]	4.97 ± 0.08 [sec]	365
UserKNN CF tversky	0.10 ± 0.00 [sec]	5.01 ± 0.03 [sec]	365
ItemKNN CF cosine	12.41 ± 1.25 [sec]	5.14 ± 0.05 [sec]	354
ItemKNN CF dice	12.74 ± 0.14 [sec]	5.11 ± 0.03 [sec]	357
ItemKNN CF jaccard	12.72 ± 0.12 [sec]	5.01 [sec]	366
ItemKNN CF asymmetric	12.74 ± 0.18 [sec]	5.14 ± 0.02 [sec]	356
ItemKNN CF tversky	13.10 ± 0.17 [sec]	5.13 ± 0.02 [sec]	360
$P^3\alpha$	39.70 ± 1.80 [sec]	5.08 ± 0.01 [sec]	361
$RP^3\beta$	41.92 ± 1.71 [sec]	5.07 ± 0.00 [sec]	362
$EASE^R$	-	-	-
SLIM BPR	1145.90 [sec] / 19.10 ± 9.02 [min]	5.13 ± 0.09 [sec]	357
SLIM ElasticNet	1153.77 [sec] / 19.23 ± 6.67 [min]	5.19 ± 0.05 [sec]	354
MF BPR	112.91 [sec] / 1.88 ± 2.68 [min]	3.60 ± 0.02 [sec]	509
MF FunkSVD	285.07 [sec] / 4.75 ± 4.56 [min]	3.88 ± 0.14 [sec]	462
PureSVD	0.74 ± 0.65 [sec]	3.74 ± 0.12 [sec]	498
NMF	568.27 [sec] / 9.47 ± 7.02 [min]	4.14 ± 0.39 [sec]	489
iALS	888.30 [sec] / 14.80 ± 12.32 [min]	3.63 ± 0.02 [sec]	507
DELF MLP	14180.01 [sec] / 3.94 [hour]	604.42 [sec] / 10.07 [min]	3
DELF EF	26691.54 [sec] / 7.41 [hour]	596.87 [sec] / 9.95 [min]	3

Table 129. Hyperparameter values for our collaborative KNN baselines on all datasets.

Algorithm	Hyperparameter	Amazon Music	Movielens 1M
UserKNN CF cosine	topK	1000	461
	shrink	0	0
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	none	TF-IDF
UserKNN CF dice	topK	916	339
	shrink	9	0
	similarity	dice	dice
	normalize	False	True
UserKNN CF jaccard	topK	1000	329
	shrink	0	0
	similarity	jaccard	jaccard
	normalize	False	True
UserKNN CF asymmetric	topK	1000	374
	shrink	1000	0
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	2.0000	0.1258
	feature weighting	none	TF-IDF
UserKNN CF tversky	topK	997	414
	shrink	9	71
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	0.2340	1.1580
	tversky beta	0.1063	1.9364
ItemKNN CF cosine	topK	998	283
	shrink	978	765
	similarity	cosine	cosine
	normalize	True	True
	feature weighting	TF-IDF	BM25
ItemKNN CF dice	topK	1000	78
	shrink	29	2
	similarity	dice	dice
	normalize	True	True
ItemKNN CF jaccard	topK	335	42
	shrink	39	0
	similarity	jaccard	jaccard
	normalize	True	False
ItemKNN CF asymmetric	topK	1000	277
	shrink	544	644
	similarity	asymmetric	asymmetric
	normalize	True	True
	asymmetric alpha	0.0000	0.6317
	feature weighting	TF-IDF	BM25
ItemKNN CF tversky	topK	409	66
	shrink	51	0
	similarity	tversky	tversky
	normalize	True	True
	tversky alpha	0.0216	0.5465
	tversky beta	1.9479	2.0000

Table 130. Hyperparameter values for our non-neural machine learning and graph based baselines on all datasets.

Algorithm	Hyperparameter	Amazon Music	Movielens 1M
$P^3\alpha$	topK	914	406
	alpha	0.5072	1.3317
	normalize similarity	True	True
$RP^3\beta$	topK	833	265
	alpha	0.6294	1.2847
	beta	0.0343	0.5993
	normalize similarity	True	True
SLIM BPR	topK	1000	1000
	epochs	140	595
	symmetric	True	True
	sgd mode	adam	adagrad
	lambda i	9.31E-04	1.00E-02
	lambda j	1.00E-02	9.42E-03
	learning rate	1.00E-01	9.93E-03
SLIM ElasticNet	topK	1000	502
	l1 ratio	1.30E-04	1.86E-05
	alpha	0.2789	0.0689
MF BPR	sgd mode	adam	adam
	epochs	845	800
	num factors	88	171
	batch size	128	1024
	positive reg	9.10E-04	1.00E-05
	negative reg	4.69E-03	1.00E-05
	learning rate	2.16E-03	2.16E-03
MF FunkSVD	sgd mode	adam	adagrad
	epochs	225	320
	use bias	True	False
	batch size	2	128
	num factors	132	47
	item reg	1.00E-02	2.14E-05
	user reg	1.00E-02	1.28E-03
	learning rate	1.11E-03	3.39E-02
	negative quota	0.3648	0.0941
PureSVD	num factors	58	49
NMF	num factors	64	77
	solver	coord. descent	coord. descent
	init type	nndsvda	random
	beta loss	frobenius	frobenius
iALS	num factors	13	60
	confidence scaling	log	log
	alpha	50.0000	0.5425
	epsilon	0.5407	0.0010
	reg	1.00E-05	1.00E-05
	epochs	90	10
$EASE^R$	l2 norm	-	9.36E+02

Table 131. Hyperparameter values for the neural algorithm on all datasets.

Algorithm	Hyperparameter	Amazon Music	Movielens 1M
DELF MLP	epochs	35	30
	learning rate	1.00E-03	1.00E-03
	batch size	256	256
	num negatives	4	4
	layers	[256, 128, 64]	[256, 128, 64]
	regularization layers	[0, 0, 0]	[0, 0, 0]
	learner	adam	adam
	verbose	False	False
DELF EF	epochs	90	75
	learning rate	1.00E-03	1.00E-03
	batch size	256	256
	num negatives	4	4
	layers	[256, 128, 64]	[256, 128, 64]
	regularization layers	[0, 0, 0]	[0, 0, 0]
	learner	adam	adam
	verbose	False	False

P HYPERPARAMETER RANGE

Table 132. Hyperparameter values for our KNN and graph based baselines.

Algorithm	Hyperparameter	Range	Type	Distribution
UserKNN, ItemKNN cosine	topK	5 - 1000	Integer	uniform
	shrink	0 - 1000	Integer	uniform
	similarity	cosine	Categorical	
	normalize[a]	True, False	Categorical	
	feature weighting	none, TF-IDF, BM25	Categorical	
UserKNN, ItemKNN dice	topK	5 - 1000	Integer	uniform
	shrink	0 - 1000	Integer	uniform
	similarity	dice	Categorical	
	normalize[a]	True, False	Categorical	
UserKNN, ItemKNN jaccard	topK	5 - 1000	Integer	uniform
	shrink	0 - 1000	Integer	uniform
	similarity	jaccard	Categorical	
	normalize[a]	True, False	Categorical	
UserKNN, ItemKNN asymmetric	topK	5 - 1000	Integer	uniform
	shrink	0 - 1000	Integer	uniform
	similarity	asymmetric	Categorical	
	normalize[a]	True	Categorical	
	asymmetric alpha	0 - 2	Real	uniform
	feature weighting	none, TF-IDF, BM25	Categorical	
UserKNN, ItemKNN tversky	topK	5 - 1000	Integer	uniform
	shrink	0 - 1000	Integer	uniform
	similarity	tversky	Categorical	
	normalize[a]	True	Categorical	
	tversky alpha	0 - 2	Real	uniform
	tversky beta	0 - 2	Real	uniform
P3alpha	topK	5 - 1000	Integer	uniform
	alpha	0 - 2	Real	uniform
	normalize similarity[b]	True, False	Categorical	
RP3beta	topK	5 - 1000	Integer	uniform
	alpha	0 - 2	Real	uniform
	beta	0 - 2	Real	uniform
	normalize similarity[b]	True, False	Categorical	

[a]The *normalize* hyperparameter in KNNs refers to the use of the denominator when computing the similarity.
[b]The *normalize similarity* hyperparameter in P3alpha and RP3beta refers to applying L1 regularisation on the rows of the similarity matrix

Table 133. Hyperparameter values for our machine learning baselines.

Algorithm	Hyperparameter	Range	Type	Distribution
SLIM BPR	topK	5 - 1000	Integer	uniform
	epochs	1 - 1500	Integer	early-stopping
	symmetric	True, False	Categorical	
	sgd mode	sgd, adam, adagrad	Categorical	
	lambda i	10^{-5} - 10^{-2}	Real	log-uniform
	lambda j	10^{-5} - 10^{-2}	Real	log-uniform
	learning rate	10^{-4} - 10^{-1}	Real	log-uniform
SLIMElasticNet	topK	5 - 1000	Integer	uniform
	l1 ratio	10^{-5} - 10^{0}	Real	log-uniform
	alpha	10^{-3} - 10^{0}	Real	uniform
MF BPR	num factors	1 - 200[a]	Integer	uniform
	epochs	1 - 1500	Integer	early-stopping
	sgd mode	sgd, adam, adagrad	Categorical	
	batch size	2^{0} - 2^{10}	Integer	log-uniform
	positive reg	10^{-5} - 10^{-2}	Real	log-uniform
	negative reg	10^{-5} - 10^{-2}	Real	log-uniform
	learning rate	10^{-4} - 10^{-1}	Real	log-uniform
MF FunkSVD	num factors	1 - 200[a]	Integer	uniform
	epochs	1 - 500[b]	Integer	early-stopping
	use bias	True, False	Categorical	
	sgd mode	sgd, adam, adagrad	Categorical	
	batch size	2^{0} - 2^{10}	Integer	log-uniform
	item reg	10^{-5} - 10^{-2}	Real	log-uniform
	user reg	10^{-5} - 10^{-2}	Real	log-uniform
	learning rate	10^{-4} - 10^{-1}	Real	log-uniform
	negative quota[c]	0.00 - 0.50	Real	uniform
PureSVD	num factors	1 - 350	Integer	uniform
NMF	num factors	1 - 350	Integer	uniform
	solver	mult. update, coord. descent	Categorical	
	init type	nndsvda, random	Categorical	
	beta loss	kullback-leibler, frobenius	Categorical	
IALS	num factors	1 - 200[a]	Integer	uniform
	epochs	1 - 500[b]	Integer	early-stopping
	confidence scaling	linear, log	Categorical	
	alpha	10^{-3} - $5 \cdot 10^{+1}$ [d]	Real	log-uniform
	epsilon	10^{-3} - 10^{+1} [d]	Real	log-uniform
	reg	10^{-5} - 10^{-2}	Real	log-uniform
EASE R	l2 norm	10^{0} - 10^{+7}	Real	log-uniform

[a] The number of factors is lower than PureSVD or NFM due to the algorithm being slower.
[b] The number of epochs is lower than SLIM BPR or MF BPR due to the algorithm being slower.
[c] The *negative quota* is the percentage of samples chosen among items unobserved by the user, having a target rating of 0.
[d] The maximum value of this hyperparameter had been suggested in the article proposing the algorithm.

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