ä½ å¥½ï¼Œæˆ‘æ˜¯æœ±ç»´å˜šã€‚æ¬¢è¿Žä½ ç»§ç»è·Ÿæˆ‘å¦ä¹ ç º¿æ€§ä»£æ•°ï¼Œä»Šå¤©æˆ‘ä»¬è¦è®²çš„å†…å®¹æ˜¯â€œçº¿æ€§æ˜ å°„â€ã€‚

å‰é¢æˆ‘ä»¬å¦çš„å†…å®¹éƒ½æ˜¯å±€é™åœ¨ä¸€ä¸ªçº¿æ€§ç©ºé—´ï¼Œæˆ–è€…è¯´ä¸€ä¸ªå‘é‡ç©ºé—´ä¸Šï¼Œä½†ä»Šå¤©ä¸€æ ·å“¦ï¼Œæˆ‘ä»¬è¦æ¥çœ‹çœ‹ä¸¤ä¸ªå‘é‡ç©ºé—´ä¹‹é—´çš„å…³ç³»ï¼Œä¹Ÿå°±æ˜¯çº¿æ€§æ˜ å°„ã€‚

ä¹‹å‰æˆ‘è¯´è¿‡ï¼Œå‘é‡ä¹Ÿæ˜¯å¯¹è±¡ï¼Œæ˜¯èƒ½å¤Ÿç¦»å¼€åæ ‡ç³»ç‹¬ç«‹å˜åœ¨çš„ã€‚ä»Žåæ ‡è¿ç®—åˆ°æŠ½è±¡è¿ç®—ï¼Œæ˜¯å‘é‡ã€è€Œå**ä»Žæ•°å€¼è¿ç®—åˆ°çœŸæ£ç†è§£ç©ºé—´å…³ç³»ï¼Œæ˜¯çº¿æ€§æ˜ å°„**ã€‚æ˜¯çš„ï¼šå‡è®¾ä¸¤ä¸ªå‘é‡ç©ºé—´Vå’ŒWï¼Œæœ‰ä¸€ä¸ªå‡½æ•°ϕæ¥å®Œæˆå‘é‡ç©ºé—´Våˆ°Wçš„æ˜ å°„ï¼Œå¦‚æžœæˆ‘ä»¬æƒ³è¦å®Œæ—¶ä¿æŒå‘é‡ç©ºé—´ç»“æž„ä¸å˜ï¼Œé‚£ä¹ˆϕåº”è¯¥æ»¡è¶³ï¼š

$$\begin{array} {l}
\phi(x+y)=\phi(x)+\phi(y)\\
\phi(\lambda x)=\lambda \phi(x)
\end{array}$$

å…¶ä¸ï¼Œæ‰€æœ‰xå’Œyå±žäºŽå‘é‡ç©ºé—´Vï¼Œ$Î»$å±žäºŽå®žæ•°ã€‚äºŽæ˜¯ï¼Œæˆ‘ä»¬å¾—åˆ°äº†çº¿æ€§æ˜ å°„çš„å®šä¹‰ã€‚

çº¿æ€§æ˜ å°„å®šä¹‰

å‡è®¾æœ‰ä¸¤ä¸ªå‘é‡ç©ºé—´Vå’ŒWï¼Œϕæ˜¯ä¸€ä¸ªå‡½æ•°ï¼Œå®ƒå®Œæˆäº†å‘é‡ç©ºé—´Våˆ°Wçš„çº¿æ€§æ˜ å°„ï¼Œé‚£ä¹ˆçº¿æ€§æ˜ å°„å¿…é¡»æ»¡è¶³ä¸‹é¢çš„æ¡ä»¶ï¼š

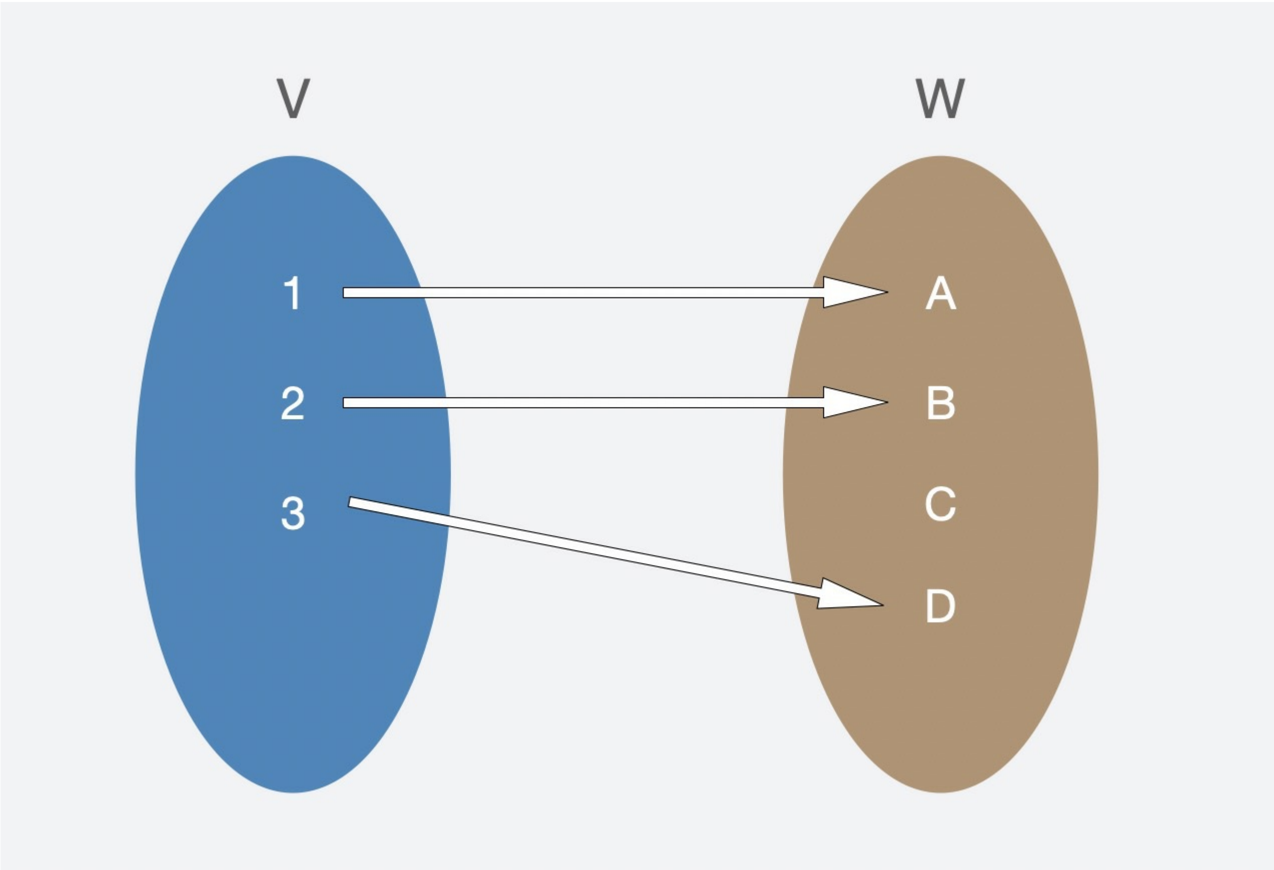
$$\phi(\lambda x+\varphi y)=\lambda \phi(x)+\varphi \phi(y)$$

å…¶ä¸ï¼Œä»»æ„xå’Œyéƒ½å±žäºŽå‘é‡ç©ºé—´Vï¼Œè€Œä»»æ„$Î»$å’Œ$Ï†$éƒ½å±žäºŽå®žæ•°ã€‚

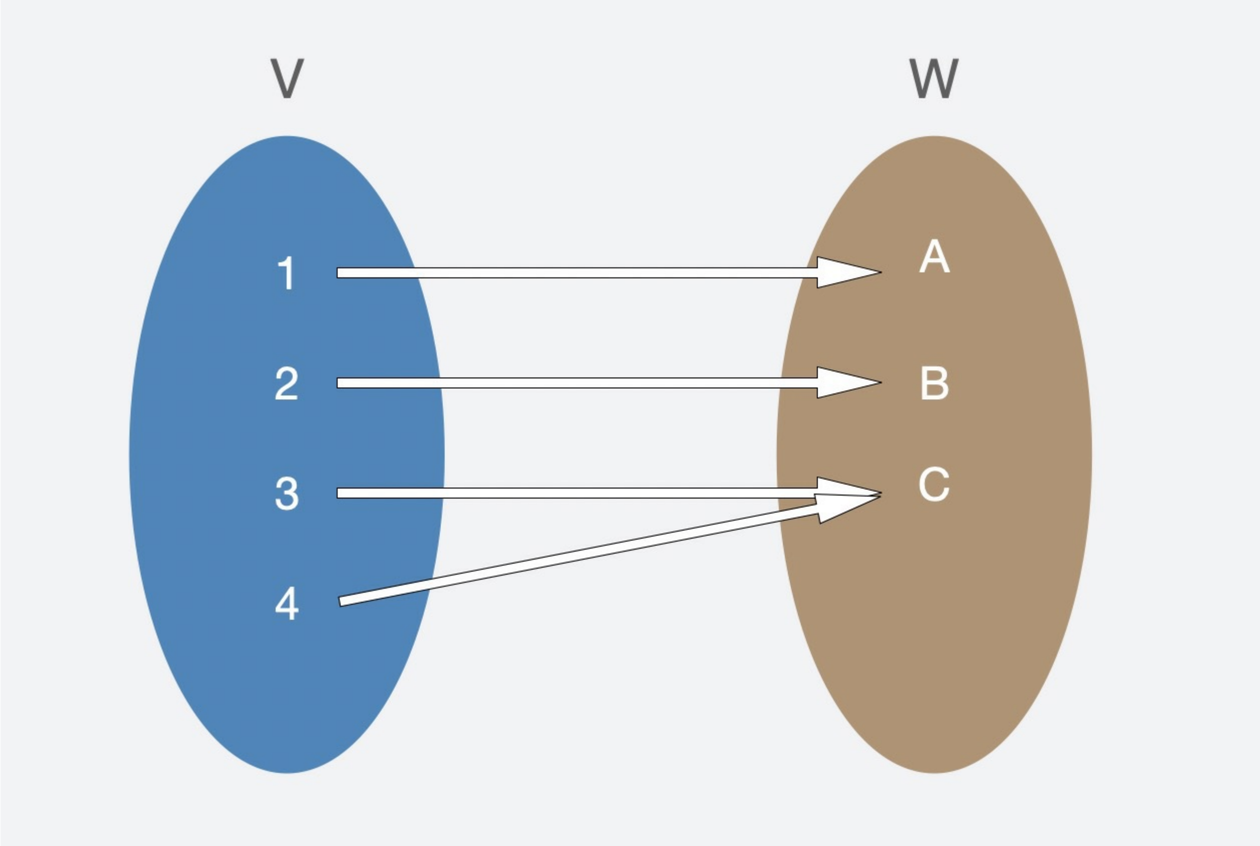
å½“ç„¶ï¼Œæˆ‘ä»¬ä¸èƒ½æŠŠçº¿æ€§æ˜ å°„å’ŒçŸ©é˜µå‰²è£‚å¼€ä¾†çœ‹ï¼Œå…¶å®žçº¿æ€§æ˜ å°„å°±æ˜¯çŸ©é˜µï¼Œæˆ–者说另一种形式的矩阵。但在理解线性映射时，把矩阵当作函数来看也许更容易理解。

æŽ¥ä¸‹æ¥æˆ‘ä»¬æ¥çœ‹çœ‹ä¸¤ä¸ªä»»æ„é›†åˆVåˆ°Wçš„ä¸‰ç§ç‰¹æ®Šæ˜ å°„ï¼Œä½†é¦–å…ˆæˆ‘éœ€è¦ä»‹ç»å‡ ä¸ªæ¦‚å¿µã€‚

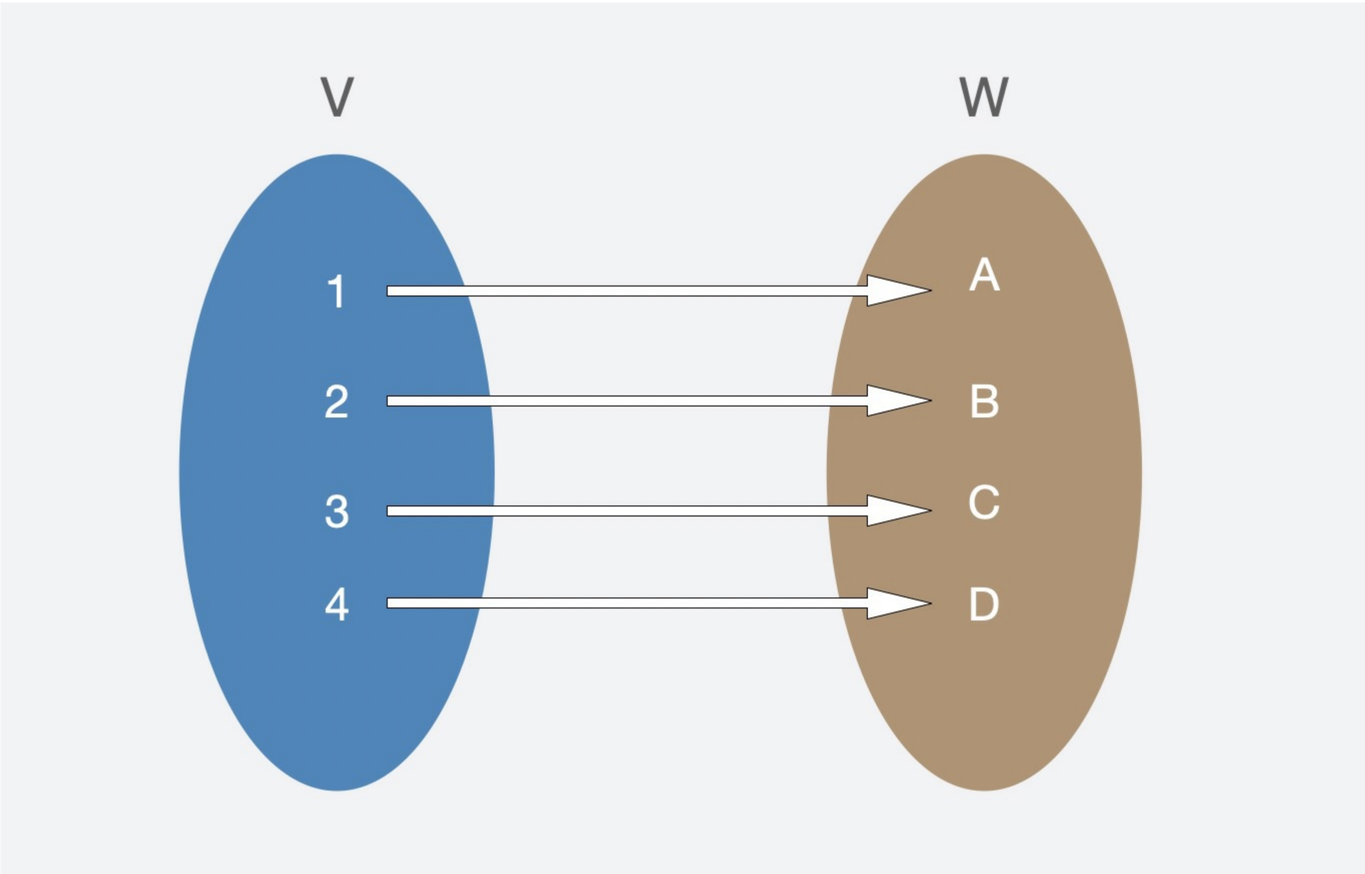
1. å‡½æ•°ϕæ˜¯å•å°„ï¼ˆInjectiveï¼‰æ—¶ï¼šå¦‚æžœ$\phi(x)= \phi(y)$ ï¼Œé‚£ä¹ˆ$x=y$ ï¼Œå…¶ä¸ä»»æ„xå’Œyéƒ½å±žäºŽé›†åˆVï¼Œä»Žå›¾ä¸Šåˆä¸Šçœ‹ï¼Œå¯ä»¥ç”¨å·¦è¾¹çš„å…ƒç´ æŒ‡å‘å³è¾¹çš„å…ƒç´ æ¥ç†è§£ã€‚



2. å‡½æ•°ϕæ˜¯æ»¡å°„ï¼ˆSurjectiveï¼‰ï¼šä¹Ÿå°±æ˜¯æ»¡è¶³ç‰ˆå¼ $\phi(V)= W$ï¼Œä»Žå›¾ä¸Šçœ‹ï¼Œå°±æ˜¯å³è¾¹çš„æ‰€æœ‰å…ƒç´ éƒ½è¢«å·¦è¾¹çš„å…ƒç´ æŒ‡å‘äº†ã€‚



3. å‡½æ•°ϕæ˜¯åŒå°„ï¼ˆBijectiveï¼‰ï¼Œå°±æ˜¯å‘³ç€å®ƒæ—¢æ˜¯å•å°„åˆæ˜¯æ»¡å°„ï¼Œä»Žå›¾ä¸Šæ¥çœ‹å°±æ˜¯ä¸€ä¸€å¯¹åº”ï¼Œå®ƒè¡¨è¾¾çš„æ˜¯ï¼Œæ‰€æœ‰Vé›†åˆçš„å…ƒç´ é½å’ŒWé›†åˆçš„å…ƒç´ ä¸€ä¸€å¯¹åº”ï¼Œä¸€å¯¹ä¸€ã€‚

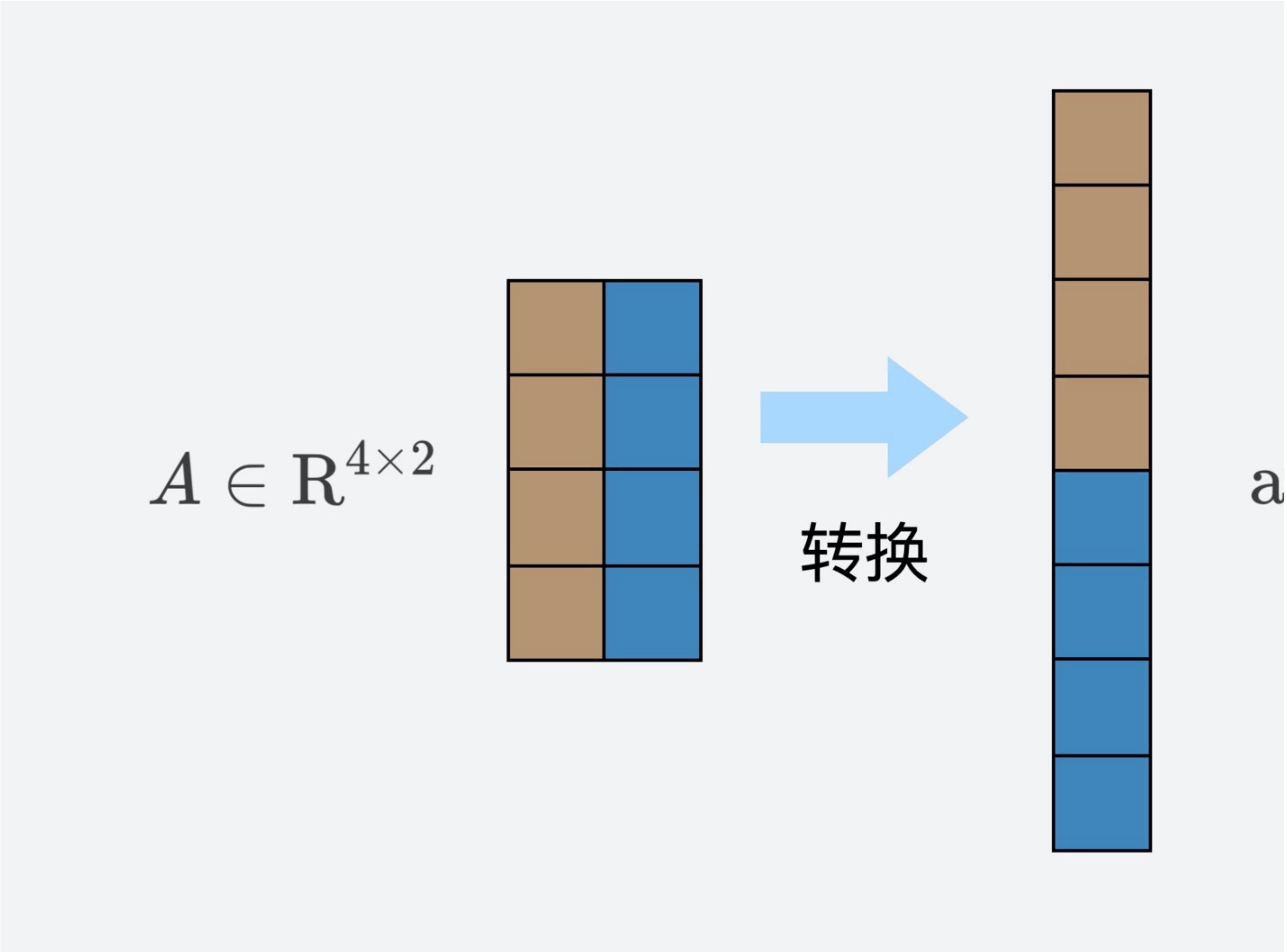


é€šè¿‡è¿™äº›å®šä¹‰ï¼Œæˆ‘ä»¬å°±å¯ä»¥å¼•å…¥å‡ ä¸ªçº¿æ€§æ˜ å°„çš„ç‰¹æ®Šæ¦‚å¿µäº†ã€‚

1. åŒæž„ï¼ˆ Isomorphismï¼‰ï¼šå³å‡½æ•° ϕä½¿Våˆ°Wæ˜¯çº¿æ€§ä¸”åŒå°„çš„ï¼›
2. è‡ªåŒæ€ï¼ˆEndomorphismï¼‰ï¼šå³å‡½æ•° ϕä½¿Våˆ°Væ˜¯çº¿æ€§çš„ï¼›
3. è‡ªåŒæž„ï¼ˆAutomorphismï¼‰ï¼šå³å‡½æ•°ϕä½¿Våˆ°Væ˜¯çº¿æ€§ä¸”åŒå°„çš„ï¼›
4. æŠŠVåˆ°Vï¼Œå…ƒç´ xåˆ°xçš„æ˜ å°„å®šä¹‰ä¸ºæ’ç‰æ˜ å°„ã€‚

é‚£ä¹ˆï¼Œä¸ºä»€ä¹ˆä½ éœ€è¦äº†è§£è¿™å‡ ä¸ªç‰¹æ®Šçš„æ¦‚å¿µå‘¢ï¼Ÿé‚£æ˜¯å› ä¸ºæˆ‘ä»¬éœ€è¦é€šè¿‡è¿™äº›æ¦‚å¿µå¼•å‡ºä¸€ä¸ªå®šç†ï¼šæœ‰é™ç»´åº¦çš„å‘é‡ç©ºé—´Vå’ŒWï¼Œå¦‚æžœå®ƒä»¬çš„ç»´åº¦ç›¸ç‰ï¼Œé‚£ä¹ˆå®ƒä»¬å°±æ˜¯åŒæž„çš„ã€‚é‚£å°±æ˜¯è¯´ï¼ŒåŒä¸€ç»´åº¦çš„å‘é‡ç©ºé—´æŸç§ç¨‹åº¦ä¸Šæ˜¯ä¸€æ ·çš„ï¼Œå› ä¸ºå®ƒä»¬èƒ½å¤Ÿåœ¨æ²¡æœ‰å‘ç”ŸæŸå¤±çš„æƒ…å†µä¸‹ç›¸äº’è½¬æ¢ã€‚

æ¯”å¦‚ï¼Œ$\mathrm{R}^{m \times n}$çŸ©é˜µå‘é‡ç©ºé—´ï¼Œå’ŒR^{mn}é•¿åº¦æ˜¯mnçš„å‘é‡ç©ºé—´ï¼Œæˆ‘ä»¬å¯ä»¥è®¤ä¸ºå®ƒä»¬æ˜¯ç›¸åŒçš„ï¼Œå› ä¸ºå®ƒä»¬ç»´åº¦é½æ˜¯mnï¼Œè€Œä¸”å˜åœ¨ä¸€ä¸ªçº¿æ€§æ˜ å°„åŒå°„ï¼Œä½¿å¾—å®ƒä»¬èƒ½ç›¸äº’è½¬æ¢ã€‚è¿™è®©å¾—åœ¨çŸ©é˜µé‚£è¾¹ä¸ï¼Œæˆ‘æ˜ å°„çš„çŸ©é˜µè½¬æ¢å—ï¼Ÿå¾ˆå¤šæ—¶å€™è¿™ç±»è½¬æ¢å°±æ˜¯ä¸ºäº†è®¡ç®—æ–¹ä¾¿ã€‚



çº¿æ€§æ˜ å°„çš„çŸ©é˜µè¡¨ç¤º

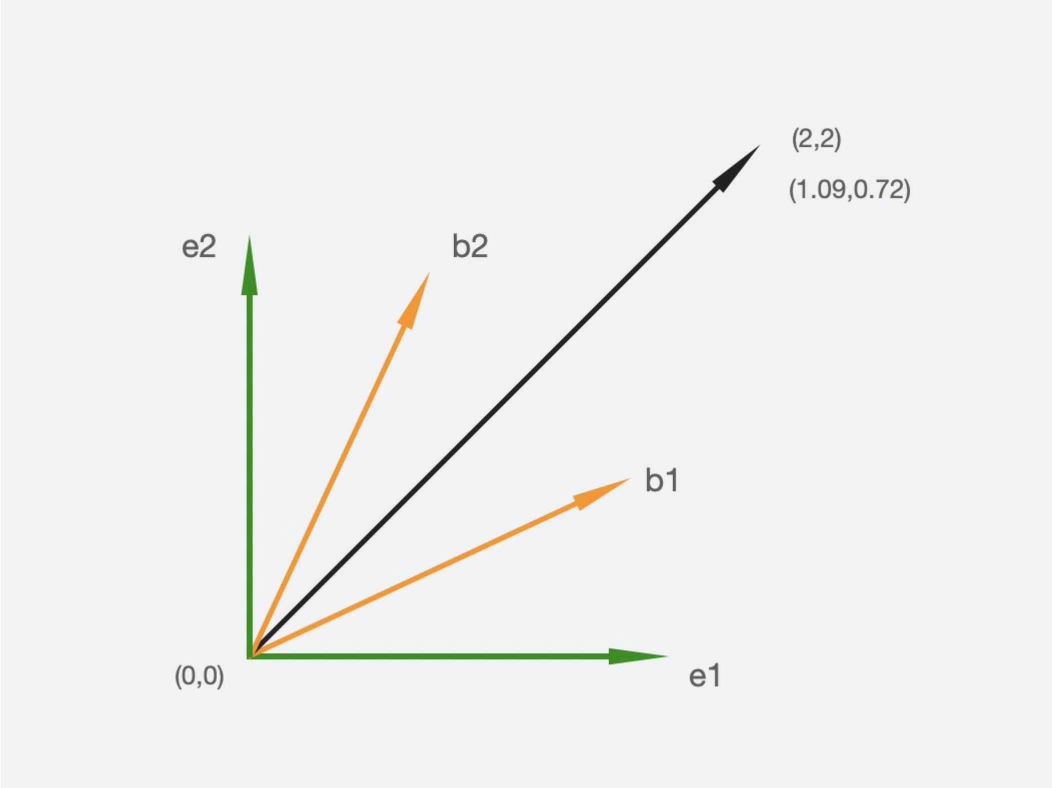
å‰æ‰æˆ‘ä»¬æŠŠçŸ©é˜µè§†ä¸ºçº¿æ€§æ˜ å°„çš„è¡¨è¾¾ï¼Œé‚£ä¹ˆç»´åº¦ä¸åŒçš„å‘é‡ç©ºé—´ä¹‹é—´çš„çº¿æ€§æ˜ å°„æ˜¯å¦ä¹Ÿå¯ä»¥ç”¨çŸ©é˜µè¡¨ç¤ºå‘¢ï¼Ÿç”æ¡ˆæ˜¯è‚¯å®šçš„ã€‚ä¸‹é¢æˆ'ä»¬å°±æ¥çœ‹çœ‹å¦‚ä½•ç”¨çŸ©é˜µè¡¨ç¤ºçº¿æ€§æ˜ å°„çš„è¿‡ç¨‹ã€‚

å¦‚æžœå¨ä¸€ä¸ªnç»´å'é‡ç©ºé—´Vå'Œå®ƒçš„ä¸€ä¸ªæœ‰åºåŸº$B=(b_{1},\cdots,b_{n})$ï¼Œé‚£ä¹ˆå¯¹äºŽä»»æ„ä¸€ä¸ªå±žäºŽVçš„xï¼Œæˆ'ä»¬èƒ½å¾—åˆ°ä¸€ä¸ªè¿™æ ·çš„çº¿æ€§ç»„åˆï¼š$x=\alpha_{1} b_{1}+\cdots+\alpha_{n} b_{n}$ï¼Œæˆ'ä»¬å°±ä»¥è¯´

$$\alpha=\left[\begin{array}{c}
\alpha_{1} \\
\cdot \\
\cdot \\
\cdot \\
\alpha_{n}
\end{array}\right]$$

æ˜¯xçš„åæ ‡å'é‡æˆ–åæ ‡è¡¨ç¤ºã€‚

æˆ'ä»¬éƒ½çŸ¥é"ç»´åº¦ä¸åŒçš„å'é‡ç©ºé—´ä¹‹é—´çš„çº¿æ€§æ˜ å°„çš„è¡¨ç¤ºæ–¹æ³•ï¼Œè¿™é‡Œæˆ'ä»¬ä¸»è¦å…³æ³¨çº¿æ€§æ˜ å°„çš„çŸ©é˜µè¡¨ç¤ºã€‚



å‡è®¾å›¾ä¸ç»¿è‰²æ˜¯åæ ‡ç³»Vï¼Œé»„è‰²æ˜¯åæ ‡ç³»Wã€‚

åœ¨Vä¸ï¼Œe_{1}å’Œe_{2}æ˜¯Vçš„æ ‡å‡†åŸºï¼Œå‘é‡xç”±çº¿æ€§ç»„åˆe_{1}å’Œe_{2}è¡¨ç¤ºï¼Œxçš„åæ ‡æ˜¯(2,2)ï¼Œä»Žæ˜¯ï¼Œxåœ¨Vä¸å¯ä»¥è¢«è¡¨ç¤ºæˆï¼š$x=2 e_{1}+2 e_{2}$ã€‚

åœ¨Wä¸ï¼Œb_{1}å’Œb_{2}æ˜¯Wçš„æ ‡å‡†åŸºï¼Œå‘é‡xç”±çº¿æ€§ç»„åˆb_{1}å’Œb_{2}è¡¨ç¤ºï¼Œè€Œè¿™é‡Œçš„$x$åæ ‡å°±ä¸åŒäº†ï¼Œå˜æˆäº†(1.09,0.72)ï¼Œä»Žæ˜¯ï¼Œxåœ¨Wä¸å¯ä»¥è¢«è¡¨ç¤ºæˆï¼š$x=1.09 b_{1}+0.72 b_{2}$ã€‚

è¯´åˆ°è¿™ï¼Œä½ æ˜¯ä¸æ˜¯å¯¹çº¿æ€§æ˜ å°„æœ‰äº†ä¸€ä¸ªæ›´ç›´è§‚çš„æ„Ÿå—ï¼ŸçŽ°åœ¨æˆ‘ä»¬å°±æŠŠçŸ©é˜µå¼•å…¥åˆ°çº¿æ€§æ˜ å°„ä¸ï¼Œä»Žæ˜¯ï¼Œæˆ‘ä»¬å°±æœ‰äº†**å˜æ¢çŸ©é˜µ**ï¼Œå®ƒå°±æ˜¯ç”¨çŸ©é˜µæ¥è¡¨ç¤ºçº¿æ€§å˜æ¢çš„è¿‡ç¨‹ã€‚

å˜æ¢çŸ©é˜µçš„å®šä¹‰æ˜¯ï¼šæˆ‘ä»¬æœ‰å‘é‡ç©ºé—´Vå'ŒWï¼Œå®ƒä»¬å„è‡ªæœ‰æœ‰åºåŸº $B=(b_{1},\cdots,b_{n})$å'Œ $C=(c_{1},\cdots,c_{m})$ ï¼Œè€Œϕå°±æ˜¯Våˆ°Wçš„çº¿æ€§æ˜ å°„ï¼š$\phi\left(b_{j}\right)=\alpha_{1 j} c_{1}+\cdots+\alpha_{m j} c_{m}$ã€‚

çº¿æ€§æ˜ å°„ϕä¸çš„jæ˜¯ä»Ž1åˆ°nï¼Œä»Žæ˜¯ï¼Œæˆ‘ä»¬å°±èƒ½å¾—åˆ°ä¸€ä¸ªϕçš„$m \times n$çš„å˜æ¢çŸ©é˜µ A_{ϕ}ï¼Œè¿™ä¸ªå˜æ¢çŸ©é˜µä¸çš„å…ƒç´ æ˜¯ $A_{\phi}(i, j)=\alpha_{i j}$ï¼Œä¹Ÿå°±æ˜¯è¯´ï¼Œ$\phi\left(b_{j}\right)$çš„åæ ‡æ˜¯ A_{ϕ}çš„ç¬¬jåˆ—ã€‚

æˆ‘ä»¬å¯ä»¥æŠŠå®ƒå†™ä¸€ä¸‹ï¼Œè¿™æ ·å®šä¹‰çœ‹èµ·æ¥ä¼šæ›´æ¸…æ¥šä¸€ç‚¹ï¼š $y=A_{\phi}(x)$ã€‚å…¶ä¸ï¼Œxæ˜¯VåŸºäºŽBåŸºçš„åæ ‡å'é‡ï¼Œyæ˜¯WåŸºäºŽCåŸºçš„åæ ‡å'é‡ã€‚æ‰€ä»¥ï¼Œå˜æ¢çŸ©é˜µåˆ™å¯ä»¥è¢«ç”¨æ¥æ˜ å°„åŸºäºŽæœ‰åºåŸºçš„åæ ‡ã€‚

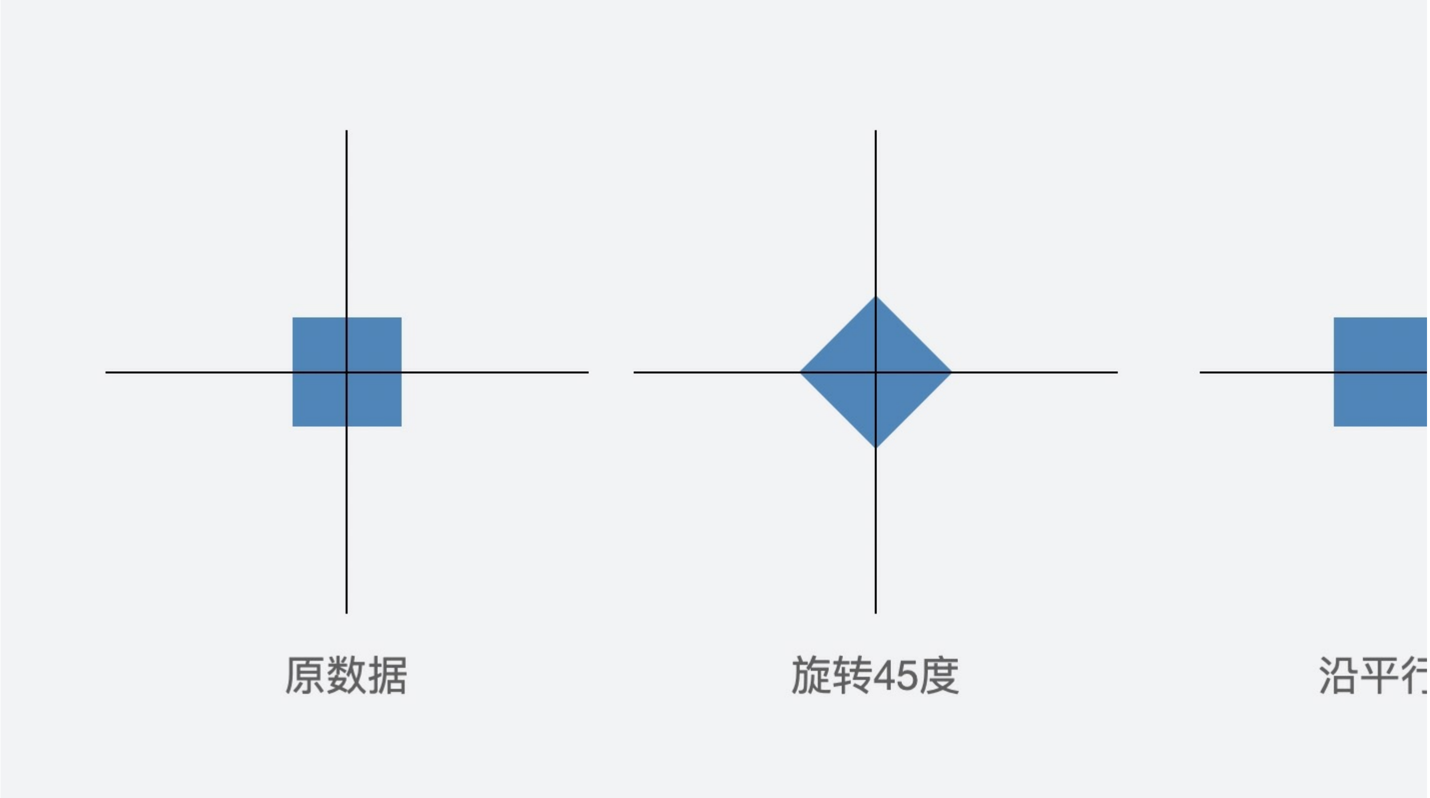
æˆ‘ä»¬æ¥çœ‹ä¸€ä¸ªå˜æ¢çŸ©é˜µçš„ä¾‹åï¼šå·²çŸ¥ä¸¤ä¸ªå'é‡ç©ºé—´Vå'ŒWï¼Œå®ƒä»¬å„è‡ªæœ‰æœ‰åºåŸºæ˜¯ $B=(b_{1},\cdots,b_{3})$å'Œ $C=(c_{1},\cdots,c_{4})$ ï¼Œçº¿æ€§æ˜ å°„ϕè¡¨ç¤ºå¦‚ä¸‹ï¼š

$$\begin{array}{l}
\phi\left(b_{1}\right)=c_{1}-c_{2}+3 c_{3}-c_{4} \\
\phi\left(b_{2}\right)=2 c_{1}+c_{2}+7 c_{3}+2 c_{4} \\
\phi\left(b_{3}\right)=3 c_{2}+c_{3}+4 c_{4}
\end{array}$$

äºŽæ˜¯ï¼Œæˆ‘ä»¬å°±å¯ä»¥è¿™æ ·æ±‚å¾—å˜æ¢çŸ©é˜µA_{ϕ}å¦‚ä¸‹ã€‚

$$ A_{\phi}=\left[\begin{array}{ccc}
1 & 2 & 0 \\
-1 & 1 & 3 \\
3 & 7 & 1 \\
-1 & 2 & 4
\end{array}\right]$$

çº¿è§£åˆ°è¿™é‡Œè¿˜ä¸ç®—å®Œï¼Œæˆ‘ä»¬è¦æ·±è¿›ä¸€æ¥ï¼Œçœ‹çœ‹åœ¨çŽ°å®žå›¾å½¢å›¾åƒå¤„ç†ä¸ï¼Œçº¿æ€§å˜æ¢æ˜¯æ€Žä¹ˆæ ·çš„ã€‚æŽ¥ä¸‹æ¥æˆ'ä»¬å†é€šè¿‡ä¸€ä¸ªä¾‹åï¼Œé€šè¿‡å›¾åƒå¤„ç†ä¸çš„ä¸€ä¸ªä¾‹åæ¥ç†è§£ä¸€ä¸‹ã€‚



ç¬¬ä¸€ä¸ªå›¾å½¢æ˜¯åŽŸå§‹æ•°æ®ï¼Œä½ å¯ä»¥æŠŠå®ƒçœ‹æˆæ˜¯ç”±å‡ ç™¾ä¸ªå‘é‡çš„å¤´éƒ¨ç‚¹ç»„æˆçš„å›¾ã€‚

ç¬¬äºŒä¸ªå›¾å½¢çš„æ•°æ®çŸ©é˜µä¹˜ä»¥ç®€å•çš„çŸ©é˜µç»è¿‡45åº¦è§’æ—‹è½¬åŽçš„ï¼Œå®ƒçš„å˜æ¢çŸ©é˜µæ˜¯ä¸‹é¢è¿™æ ·çš„ã€‚

$$A_{1}=\left[\begin{array}{cc}
\cos \left(\frac{\pi}{4}\right) & -\sin \left(\frac{\pi}{4}\right) \\
\sin \left(\frac{\pi}{4}\right) & \cos \left(\frac{\pi}{4}\right)
\end{array}\right]$$

ç¬¬ä¸‰ä¸ªå›¾å½¢æ˜¯åŽŸå§‹æ•°æ®æ²¿å¹³è¡Œè½´æ‹‰ä¼¸ä¸¤å€çš„ï¼Œæ•°æ®ä¹˜ä»¥å˜æ¢çŸ©é˜µæ˜¯ä¸‹é¢è¿™æ ·çš„ã€‚

$$A_{2}=\left[\begin{array}{cc}
2 & 0 \\
0 & 1
\end{array}\right]$$

å½“ç„¶ï¼Œè¿™ä¸ªå›¾å½¢æ˜¯æ”¾å¤§çš„æ•°æ®ä¹‹åŽï¼Œè¿™æ˜¯å¤šä¸ªæ•°å—ç»„åˆçš„çŸ©é˜µï¼Œä¸ºäº†æ–¹ä¾¿ç›´è§‚çš„æ˜¾ç¤ºï¼Œç¼©å°ã€‚

åŸºçš„æ”¹å˜

ä¸‹é¢æˆ‘ä»¬è®¨è®ºçš„ç©ºé—´å°±æ˜¯å®ƒæ‰€åœ¨çš„åŸºæ˜¯ä¸€ä¸ªå®šç©ºé—´ï¼Œå¹¶ä¸”è¿™ä¸ªåŸºæ˜¯äºŒå¯¹è§’çš„æ—¶å€™ã€‚

$$B=\left\{\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]\right\}, C=\left\{\left[\begin{array}{l}
1 \\
0 \\
0 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
1 \\
0 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
0 \\
1 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
0 \\
0 \\

1
\end{array}\right]\right\}$$

åŸºäºŽå®ƒä»¬å„è‡ªçš„æ ‡å‡†åŸºBå’ŒCï¼Œå®ƒçš„å˜æ¢çŸ©é˜µæ˜¯ï¼š

$$A_{\phi}=\left[\begin{array}{ccc}
1 & 2 & 0 \\
-1 & 1 & 3 \\
3 & 7 & 1 \\
-1 & 2 & 4
\end{array}\right]$$

é‚£ä¹ˆçŽ°åœ¨ï¼Œæˆ‘ä»¬æ¥çœ‹ä¸€ä¸‹ï¼ŒåŸºBå’ŒCæ”¹å˜ä¸º$\widetilde{B}$å’Œ$\widetilde{C}$ä¹‹åŽï¼Œä¼šæœ‰æ€Žæ ·çš„å˜åŒ–ã€‚

$$\widetilde{B}=\left\{\left[\begin{array}{l}
1 \\
1 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
1 \\
1
\end{array}\right],\left[\begin{array}{l}
1 \\
0 \\
1
\end{array}\right]\right\}, \widetilde{C}=\left\{\left[\begin{array}{l}
1 \\
1 \\
0 \\
0
\end{array}\right],\left[\begin{array}{l}
1 \\
0 \\
1 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
1 \\
1 \\
0
\end{array}\right],\left[\begin{array}{l}
1 \\
0 \\
0 \\
1
\end{array}\right]\right\}$$

å¯¹äºŽæ–°åŸº$\widetilde{B}$å’Œ$\widetilde{C}$ï¼Œæˆ‘ä»¬å¾—åˆ°Så’ŒTï¼š

$$S=\left[\begin{array}{lll}
1 & 0 & 1 \\
1 & 1 & 0 \\
0 & 1 & 1
\end{array}\right], T=\left[\begin{array}{llll}
1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]$$

äºŽæ˜¯ï¼Œæˆ‘ä»¬å°±å¯ä»¥é€šè¿‡å…¬å¼å¾—åˆ°æƒ³è¦çš„$\tilde{A}_{\phi}$äº†ã€‚

$$\tilde{A}_{\phi}=T^{-1} A_{\phi} S=\left[\begin{array}{ccc}
-4 & -4 & -2 \\
6 & 0 & 0 \\
4 & 8 & 4 \\
1 & 6 & 3
\end{array}\right]$$

ä¸¤ä¸ªé‡è¦çš„åç©ºé—´

æœ€åŽï¼Œæˆ‘ä»¬æ¥è®²ä¸¤ä¸ªé‡è¦çš„åç©ºé—´â€”â€”æ ¸ç©ºé—´å’Œåƒç©ºé—´ï¼Œè¿™å¯¹äºŽé‡è¦æ˜¯æœ‰åŸºå› çš„ã€‚æ ¸ç©ºé—´å¯ä»¥å¸®åŠ©æˆ‘ä»¬ç†è§£çº¿æ€§æ–¹ç¨‹ç»„çš„æ€§è´¨ï¼Œåœ¨æ¶è¿˜å¯ä»¥å¸®åŠ©æˆ‘ä»¬æŠŠå¤æ‚é—®é¢˜ç®€åŒ–ï¼Œä¹Ÿå°±æ˜¯å°†å¤æ‚çš„é—®é¢˜åˆ†è§£æˆå°é—®é¢˜åˆ†åˆ«ç†è§£æ‰€ä»¥å¹¶ç”¨ç©¶ã€‚è€Œåƒç©ºé—´å¯ä»¥å¸®åŠ©æˆ‘ä»¬å¿«é€Ÿå¾—åˆ°ç©ºé—´çš„ç§©ï¼Œè¿™æ ·å°±èƒ½å¿«é€Ÿå¤æ–çº¿æ€§æ–¹ç¨‹çš„è§£çš„æƒ…å†µã€‚çŽ°åœ¨æˆ‘ä»¬æ¥å…·ä½“äº†è§£ä¸€ä¸‹ã€‚

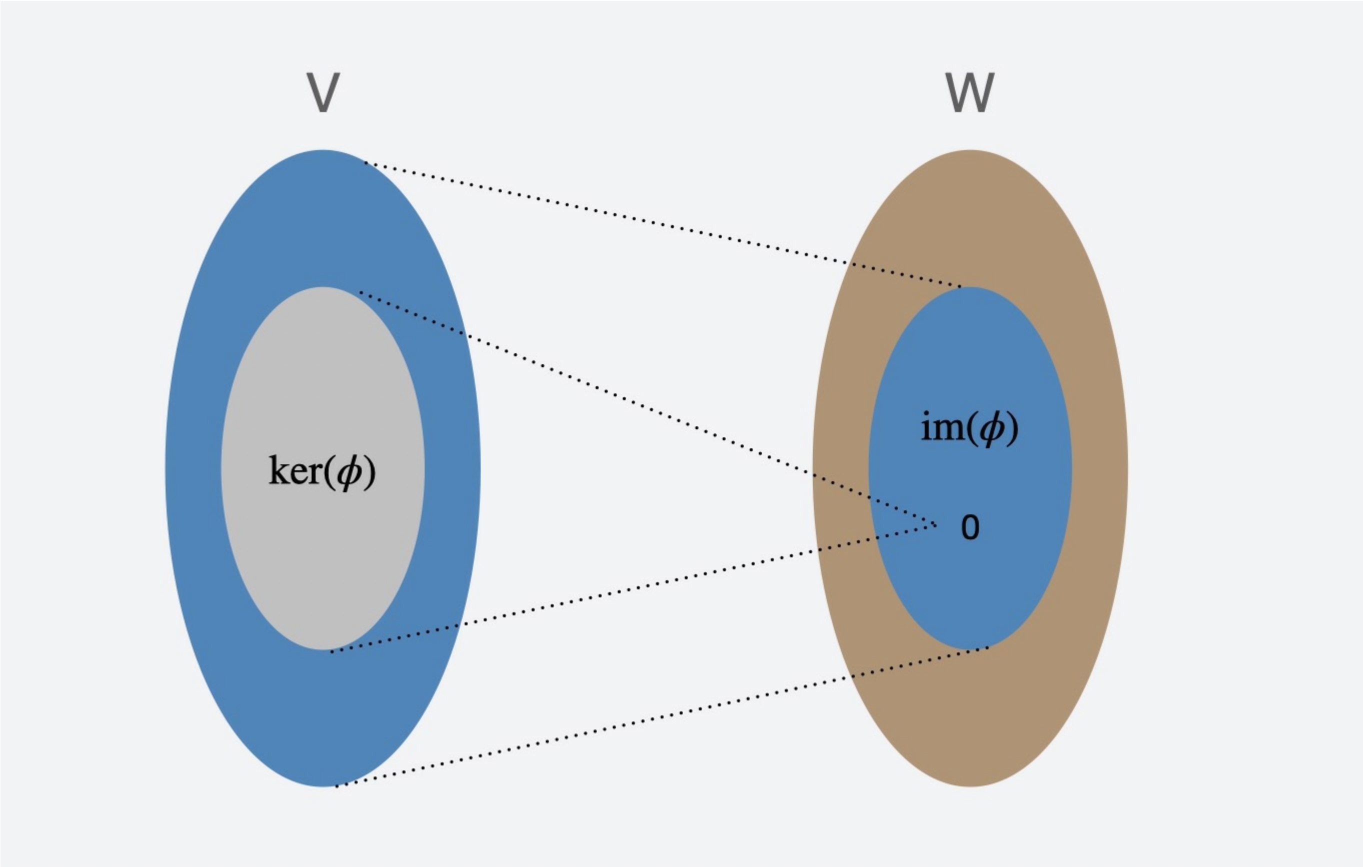
æ ¸ç©ºé—´

æ ¸ç©ºé—´ä¹Ÿå«åšé›¶ç©ºé—´ï¼Œä½ è¿˜è®°å¾—$Ax=b$å—ï¼Ÿæ ¸ç©ºé—´å…³æ³¨çš„å°±æ˜¯$Ax=0$ï¼Œä¹Ÿå°±æ˜¯å‘é‡ç©ºé—´Vä¸æ‰€æœ‰ç»è¿‡ϕæ˜ å°„ä¸ºé›¶çš„å‘é‡é›†åˆï¼Œç”¨ç¬¦å·è¡¨ç¤ºä¸ºæ˜¯ï¼š$\operatorname{ker}(\phi)$ ã€‚æ ¸çš„ç»´æ•°å«åšé›¶åŒ–åº¦ï¼ˆnullityï¼‰ï¼Œè¡¨ç¤ºä¸ºï¼š$\operatorname{dim}(\operatorname{ker}(\phi))$ã€‚

åƒç©ºé—´

å‘é‡ç©ºé—´Vä¸所有经过ϕæ˜ å°„åŽçš„å‘é‡é›†åˆï¼Œå«åšåƒç©ºé—´ï¼Œç”¨ç¬¦å·è¡¨ç¤ºä¸ºæ˜¯ï¼š$\operatorname{im}(\phi)$ï¼Œåƒç©ºé—´çš„ç»´æ•°å°±æ˜¯ç§©ï¼Œè¡¨ç¤ºä¸ºæ˜¯ï¼š$\operatorname{rk}(\phi)$ã€‚

é€šè¿‡å›¾å½¢è¡¨ç¤ºå‡ºæ¥ï¼Œä½ å°±èƒ½å¤Ÿæ›´å¥½åœ°ç†è§£ã€‚



æœ€åŽæˆ‘ä»¥ä¸€ä¸ªå®šç†æ¥ç»“æŸæœ¬èŠ‚çš„å†…å®¹ï¼Œç§©-é›¶åŒ–åº¦å®šç†ï¼šVçš„ç»´æ•°ç‰äºŽæ ¸çš„ç»´æ•°ä¸Žåƒçš„ç»´æ•°ä¹‹å’Œ$\operatorname{dim}(\mathrm{V})=\operatorname{dim}(\operatorname{ker}(\phi))+\operatorname{rk}(\phi)$ã€‚

æœ¬èŠ‚å°ç»“

å¥½äº†ï¼Œåˆ°è¿™é‡Œçº¿æ€§æ˜ å°„è¿™ä¸€è®²å°±ç»“æŸäº†ï¼Œæœ€åŽæˆ‘å†æ€»ç»“ä¸€ä¸‹å‰é¢è®²è§£çš„å†…å®¹ã€‚

çº¿æ€§æ˜ å°„è¿™ä¸ªæ¦‚å¿µæ˜¯åœ¨çº¿æ€§ä»£æ•°ä¸ï¼ŒåŸºäºŽè®©åƒƒåœ¨è®¡ç®—æœºç§‘å¦é‡Œæ˜¯å¤§çš„ä½œç”¨ã€‚çº¿æ€§æ˜ å°„çš„çŸ©é˜µæ˜¯è¿™ä¸€èŠ‚çš„é‡ç‚¹ï¼Œä½ è¦ç‰¢å›ºæŽŒæ¡ï¼Œå› ä¸ºå®ƒåœ¨æ—¥å¸¸è®¡ç®—å’Œæœºå™¨å¦ä¹ ä¸æ˜¯é«˜é¢‘ä½¿ç”¨çš„ï¼Œä¹Ÿæ˜¯ä¸€å¹¶æž„ä»¶ä¸ªå¼ºåŠ›åŸºç¡€çš„å·¥å…·ï¼Œå¿…é¡»ç‰¢ç‰¢è®°ä½ã€‚

çº¿æ€§ä»£æ•°ç»ƒä¹ åœº

å¥½ï¼Œä»Šå¤©ç»ƒä¹ æ—¶é—´åˆ°äº†ï¼Œåšåšæˆ‘ä»¬é€šè¿‡å›¾ç‰‡çš„å½¢å¼æ¥çœ‹çº¿æ€§æ˜ å°„çš„ä½œç”¨ï¼Œè®²è§£ç‰©ç†å›¾åƒã€‚çŽ°åœ¨æˆ‘ä»¬è¦æ±‚ä½ è¿™ä¸ªä¾‹åä¸çš„å„ä¸ªè¿™ä¸€æ¬¡è®¡ç®—ä½ æ¥è§£ç”é—®é¢˜ä¸Šã€‚å¦‚æžœæˆ‘ä»¬ææ˜Žå¼•æ•°æ®æ›¿ä½œæ²¿åˆ°è¯è½´æ¶ç©ä¸€åˆï¼Œé‚£ä¹ˆç»“æžœæ˜¯ä»€ä¹ˆå‘¢ï¼Ÿ

æ¬¢è¿Žåœ¨ç•™è¨€åŒºæ™’å‡ºä½ çš„ç”æ¡ˆï¼Œä¹Ÿæ¬¢è¿Žä½ æŠŠè¿™ç¯‡æ–‡ç« åˆ†äº«ç»™ä½ çš„æœ‹å‹ã€‚