ä½ å¥½ï¼Œæˆ‘æ˜¯æœ±ç»´å¨å€‚æ¬¢è¿Žä½ è·Ÿæˆ‘ä¸€èµ·é‡å¦ç º¿æ€§ä»£æ•°ï¼

åœ¨ä¸Šä¸€èŠ‚è¯¾ä¸ï¼Œæˆ‘è®²è§£äº†ç º¿æ€§æ–¹ç¨‹ç»„çš„å¦ä¸€ç§è¡¨è¾¾â€”â€”çŸ©é˜µã€‚é‚£ä¹ˆä»Šå¤©ï¼Œæˆ‘ä»¬å°±æ¥è®²è§£ä¸€ä¸‹å¦‚ä½•ä½¿ç"¨çŸ©é˜µæ¥è§£ç º¿æ€§æ–¹ç¨‹ç»„ï¼Œä¹Ÿå°±æ˜¯å¦‚ä½•æ±‚ç º¿æ€§æ–¹ç¨‹ç»„çš„ç‰¹æ®Šè§£å'Œé€šç"¨è§£ã€‚

ç®€å•çš„ç º¿æ€§æ–¹ç¨‹ç»„ï¼Œæˆ'ä»¬å½"ç„¶å¯ä»¥è¿ç"¨å'ä¸å¦è¯¾ç¨‹ä¸çš„çŸ¥è¯†æ¥æ±‚è§£ï¼Œé‚£å¤æ‚çš„å'¢ï¼Ÿç¬¬ä¸€ä¸ªæƒ³åˆ°çš„å°±æ˜¯ä¸Žè®¡ç®—æœºç»"åˆï¼Œå› ä¸ºè®¡ç®—æœºæ"¯æŒçš„è¿ç®—æ•°é‡çº§éžå¸¸å¤§ï¼Œé€šè¿‡å¾ªçŽ¯ï¼Œæ¯"æ¬¡è¿ç®—éƒ½ä¼šè¢«çŽ°å®žåŒ–ï¼Œä½ èƒ½çœ‹åˆ°ç»"æžœã€‚ä½ è‚¯å®šåœ¨æƒ³ï¼Œé€™ä¸ªå·¥ä½œé‡çœŸçš„å¾ˆå¤§ã€‚

è€ŒçŸ©é˜µå°±æ˜¯ä¸ºæˆ'ä»¬æä¾›äº†ä¸€ç§æ–°çš„æ€ç»´ï¼Œè®©æˆ'ä»¬å¯ä»¥ç"¨çŸ©é˜µçš„è¿ç®—æ¥è§£å†³å¤æ‚çš„é—®é¢˜ã€‚

ç º¿æ€§æ–¹ç¨‹ç»„çš„è§£çš„å¯»æ‰¾

çŽ°åœ¨ï¼Œå°±è®©æˆ'ä»¬å¼€å§‹å¯»æ‰¾ç º¿æ€§æ–¹ç¨‹ç»„çš„è§£ã€‚åœ¨ä¸Šå‰çš„è¯¾ç¨‹ä¸ï¼Œæˆ'ä»¬å·²ç»å¼•å…¥äº†ç º¿æ€§æ–¹ç¨‹ç»„çš„ä¸€èˆ¬è¡¨è¾¾ï¼Œä½ å¯ä»¥çœ‹çœ‹ä¸‹é¢çš„ä¾‹åã€‚

$$
\left\{\begin{array}{l}
a_{11} x_{1}+a_{12} x_{2}+\cdots+a_{1 n} x_{n}=b_{1} \\
a_{21} x_{1}+a_{22} x_{2}+\cdots+a_{2 n} x_{n}=b_{2} \\
\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \\
a_{m 1} x_{1}+a_{m 2} x_{2}+\cdots+a_{m n} x_{n}=b_{m}
\end{array}\right.
$$

å…¶ä¸ï¼Œa_{ij}å'Œ b_{i} å±žäºŽå®žæ•°ï¼Œè€Œä¸"æ˜¯å·²çŸ¥å¸¸æ•°ï¼Œè€Œx_{j}æ˜¯æœªçŸ¥å˜é‡ï¼Œiå'Œjçš„å–å€¼èŒƒå›´åˆ†åˆ«æ˜¯ï¼š$i=1,â€¦,m$ï¼›$j=1,â€¦,n$ã€‚å¦‚æžœæˆ'ä»¬ç"¨çŸ©é˜µçš„ç®€å•è¡¨è¾¾æ–¹å¼æ¥ç œçš„è¯ï¼Œå°±æ˜¯$Ax=B$ã€‚

è¦æžœ¸…æ¥šæ¦‚å¿µï¼Œæˆ'ä»¬è¿˜æ˜¯è¦å¤ç®€å•ä¸€ç‚¹ä¾‹åã€‚è®©æˆ'ä»¬å…ˆæ¥çœ‹ä¸€ä¸ªå®žä¾‹ï¼Œæ¥åŠ æ·±ä¸€ä¸‹ç†è§£ã€‚

$$
\left[\begin{array}{cccc}
1 & 0 & 8 & -4 \\
0 & 1 & 2 & 12
\end{array}\right]\left[\begin{array}{c}
x_{1} \\
x_{2} \\
x_{3} \\
x_{4}
\end{array}\right]=\left[\begin{array}{c}
42 \\
8
\end{array}\right]
$$

å¾ˆæ˜Žæ˜¾ï¼Œè¿™æ˜¯ä¸€ä¸ªçŸ©é˜µè¡¨è¾¾æ–¹å¼ã€‚å®ƒçš„ä¸€èˆ¬ç º¿æ€§æ–¹ç¨‹ç»„è¡¨è¾¾æ–¹å¼æ˜¯ä¸‹é¢çš„å½¢å¼ï¼Œä½ åº”è¯¥å¾ˆç†Ÿæ‚‰äº†ã€‚

$$
\left\{\begin{array}{l}
1 \times x_{1}+0 \times x_{2}+8 \times x_{3}+(-4) \times x_{4}=42 \\
0 \times x_{1}+1 \times x_{2}+2 \times x_{3}+12 \times x_{4}=8
\end{array}\right.
$$

åœ¨è¿™ä¸ªä¸€èˆ¬ç º¿æ€§æ–¹ç¨‹ç»„ä¸ï¼Œæœ‰å››ä¸ªæœªçŸ¥å˜é‡ï¼Œä½†åªæœ‰ä¸¤ä¸ªç‰å¼ï¼Œè¿™å°±æ„å'³ç€è¿™ä¸ªç º¿æ€§æ–¹ç¨‹ç»„æœ‰æ— ç©·å¤šä¸ªè§£ï¼ˆè¿™ä¸ªæˆ'ä»¬åŽé¢ä¼šè®²ï¼‰ã€‚é€šè¿‡ç»†å¿ƒè§‚å¯Ÿï¼Œæˆ'ä»¬å¯ä»¥å'çŽ°ç¬¬ä¸€å'Œç¬¬äºŒåˆ—éƒ½æ˜¯ç"±0å'Œ1ç»„æˆçš„ï¼Œå› æ¤ä½ å¾ˆå®¹æ˜"å°±èƒ½å'çŽ°å…¶ä¸ä¸€ä¸ªè§£ã€‚

$$
42\left[\begin{array}{l}
1 \\
0
\end{array}\right]+8\left[\begin{array}{l}
0 \\
1
\end{array}\right]=\left[\begin{array}{c}
42 \\
8
\end{array}\right]
$$

è¿™ä¸ªè§£å°±æ˜¯$\left[\begin{array}{llll}42 & 8 & 0 & 0\end{array}\right]^{T}$ï¼Œä¹Ÿå°±æ˜¯è¯´å››ä¸ªæœªçŸ¥å˜é‡åˆ†åˆ«ä¸º42ã€8ã€0ã€0ã€‚

$$
\left\{\begin{array}{l}
x_{1}=42 \\
x_{2}=8 \\
x_{3}=0 \\
x_{4}=0
\end{array}\right.
$$

è¿™ä¸ªè§£ä¹Ÿå«ç‰¹æ®Šè§£ã€‚æˆ'ä»¬åˆšæ‰å·²ç»è¯´è¿‡ï¼Œè¿™ä¸ªç º¿æ€§æ–¹ç¨‹ç»„æœ‰æ— ç©·å¤šä¸ªè§£ï¼Œé‚£æˆ'ä»¬ç¡®å®žéœ€è¦ä¸€ä¸ªè¡¨æ˜Žçš„æ–¹å¼æ¥è¡¨è¾¾æ‰€æœ‰çš„è§£ï¼Œæ˜¯å¦èƒ½ç»™è§£çš„æ–¹å¼å°±æ˜¯é€šè¿‡çŸ©é˜µçš„åˆ—æ¥ž,é€ 0ã€‚ä¾‹å¦‚ï¼Œå¯¹äºŽç¬¬ä¸‰åˆ—æ¥è¯´ï¼Œæˆ'ä»¬å¯ä»¥ç"¨ç¬¬ä¸€å'Œç¬¬äºŒåˆ—çš„ç»„åˆå½¢å¼æ¥è¡¨è¾¾ã€‚

$$
8\left[\begin{array}{l}
1 \\
0
\end{array}\right]+2\left[\begin{array}{l}
0 \\
1
\end{array}\right]=\left[\begin{array}{l}
8 \\
2
\end{array}\right]
$$

é€šè¿‡è®¡ç®—$Ax=0$ï¼Œæˆ'ä»¬å¾—å‡ºè§£$\left[\begin{array}{llll}8 & 2 & -1 & 0\end{array}\right]^{T}$ã€‚è€Œä¸"å®žä¸Šï¼Œè¿™ä¸ªè§£å¯ä»¥ä¹˜ä»¥ä»»ä½•å®žæ•°$λ_{1}$ï¼Œä½¿å¾—$Ax=0$æˆç«‹ã€‚

$$
\left[\begin{array}{cccc}
1 & 0 & 8 & -4 \\
0 & 1 & 2 & 12
\end{array}\right]
\left(\begin{array}{l}
\lambda_{1}\left[\begin{array}{l}
8 \\

2 \\\
-1 \\\
0
\end{array}\right]
\end{array}\right)=0
$$

åŒç†ï¼Œå¯¹äºŽç¬¬å››åˆ—æ¥è¯´ï¼Œæˆ‘ä»¬å¯ä»¥ä½¿ç”¨ç¬¬ä¸€åˆ—ç¬¬äºŒåˆ—çš„ç»„åˆå½¢å¼æ¥è¡¨ç¤ºï¼Œå¾—å‡ºå¦ä¸€å¥—è§£ï¼Œä½¿å¾—$Ax=0$ã€‚

$$
\left[\begin{array}{cccc}
1 & 0 & 8 & -4 \\\
0 & 1 & 2 & 12
\end{array}\right]
\left(\begin{array}{l}
\lambda_{2}\left[\begin{array}{l}
-4 \\\
12 \\\
0 \\\
-1
\end{array}\right]
\end{array}\right)=0
$$

çŽ°åœ¨ï¼Œæˆ‘ä»¬å¯ä»¥æŠŠä¹‹å‰çš„ç‰¹æ®Šè§£ä¸Žåˆšåˆšå¾—å‡ºçš„ä¸¤å¥—è§£ç»„åˆèµ·æ¥ï¼Œå¾—å‡ºæœ€ç»ˆè§£ï¼Œè¿™ä¸ªè§£ä¹Ÿå°±æ˜¯æˆ‘ä»¬æ‰€è¯´çš„é€šç”¨è§£äº†ã€‚

$$
x \in R^{4}: x=\left[\begin{array}{c}
42 \\\
8 \\\
0 \\\
0
\end{array}\right]+\lambda_{1}\left[\begin{array}{c}
8 \\\
2 \\\
-1 \\\
0
\end{array}\right]+\lambda_{2}\left[\begin{array}{c}
-4 \\\
12 \\\
0 \\\
-1
\end{array}\right], \lambda_{1}, \lambda_{2} \in R
$$

æˆ‘æ¥æ€»ç»“ä¸€ä¸‹å¯»æ‰¾é€šç”¨è§£çš„è¿‡ç¨‹ï¼Œè¿™ä¸ªè¿‡ç¨‹åˆ†ä¸ºä¸‰æ¥ï¼š

1. æˆ‘ä»¬è¦å¯»æ‰¾ä¸€ä¸ªç‰¹æ®Šè§£ï¼Œä½¿å¾—$Ax=b$ï¼›
2. æ‰¾å‡º$Ax=0$çš„æ‰€æœ‰è§£ï¼›
3. ç»„åˆç¬¬ä¸€å’Œç¬¬äºŒæ¥çš„è§£å½¢æˆé€šç”¨è§£ã€‚

çœ‹åˆ°äº†è¿™é‡Œï¼Œä½ æœ‰æ²¡æœ‰å‘çŽ°æœ‰äº›å¥‡æ€ªå‘¢ï¼Ÿæˆ–è€…è¯´ï¼Œæœ‰æ²¡æœ‰è§‰å¾—å“ªé‡Œæœ‰ç‚¹å„¿ä¸å¯¹ï¼Ÿæˆ‘ç›¸ä¿¡ï¼Œå¥½å¥‡æœ‰ç‚¹å„¿é¢å¤–çš„ä½ ä¸€å®šåœ¨æƒ³ï¼Œè¿™ä¸ªçº¿æ€§æ–¹ç¨‹ç»„æ˜¯ç‰¹æ®Šçš„ï¼Œç¬¬ä¸€åˆ—å’Œç¬¬äºŒåˆ—æ˜¯ç”±1å’Œ0ç»„æˆçš„ã€‚æ‰€ä»¥ï¼Œæˆ‘ä»¬åªé€šè¿‡è§‚å¯Ÿå°±èƒ½å¾—å‡ºç‰¹æ®Šè§£å’Œé€šç”¨è§£ã€‚

ç¡®å®žï¼Œä½ è¯´å¾—æ²¡é”™ï¼Œå› ä¸ºè¿™ä¸ªæ–¹ç¨‹ç»„æ˜¯æˆ‘ç‰¹åœ°æŒ‘é€‰çš„ï¼Œç›®çš„å°±æ˜¯ä¸ºäº†è®©ä½ æ›´å®¹æ˜“ç†è§£ã€‚ä½†çŽ°å®žä¸ç³»ç»Ÿä¸çš„çº¿æ€§æ–¹ç¨‹ç»„ï¼Œå¾ˆå¤šéƒ½ä¸æ˜¯è¿™æ ·çš„ã€‚é‚£æ€Žä¹ˆåŠžå‘¢ï¼Ÿæˆ‘ä»¬å°±éœ€è¦ç”¨é«˜æ–¯æ¶ˆå…ƒæ³•ã€‚

é«˜æ–¯æ¶ˆå…ƒæ³•çš„æ ¸å¿ƒå°±æ˜¯**ç°¿æ€§æ–¹ç¨‹ç»„çš„åˆç‰å˜æ¢**ï¼Œä¹Ÿå°±æ˜¯ï¼Œæˆ‘ä»¬å¯ä»¥通过高斯消元法，将线性方程组转化为最简形式，这种形式就是我们要介绍的简化阶梯形矩阵。

åˆç‰å˜æ¢çš„ä¸‰ç§å½¢å¼

æ—¢ç„¶é«˜æ–¯æ¶ˆå…ƒæ³•çš„æ ¸å¿ƒå°±æ˜¯çº¿æ€§æ–¹ç¨‹ç»„çš„åˆç‰å˜æ¢ï¼Œé‚£ä¸ºäº†æ–¹ä¾¿ä½ ç†è§£é«˜æ–¯æ¶ˆå…ƒæ³•ï¼Œæˆ‘å…ˆæ¥è®²ä¸€ä¸‹åˆç‰å˜æ¢çš„ä¸‰ç§å½¢å¼æœ‰å“ªäº›ï¼š

1. ä¸¤ä¸ªç‰å¼çš„äº¤æ¢ï¼Œä¹Ÿå°±æ˜¯çŸ©é˜µè¡Œäº¤æ¢ï¼›
2. ä¸€ä¸ªç‰å¼ï¼Œæˆ–è€…è¯´çŸ©é˜µè¡Œä¹˜ä»¥ä¸€ä¸ªå¸¸æ•°ï¼›
3. ä¸¤ä¸ªç‰å¼ç›¸åŠ ï¼Œæˆ–è€…è¯´çŸ©é˜µçš„ä¸¤è¡Œç›¸åŠ ã€‚

é“ç†æ˜¯è¿™æ ·çš„ï¼Œé‚£æˆ‘ä»¬é€šè¿‡ä¸€ä¸ªä¾‹å来看看，如何使用高斯消元法求解线性方程组的特殊解和通用解。

$$
\left\{\begin{array}{c}
-2 x_{1}+4 x_{2}-2 x_{3}-x_{4}+4 x_{5}=-3 \\\
4 x_{1}-8 x_{2}+3 x_{3}-3 x_{4}+x_{5}=2 \\\
x_{1}-2 x_{2}+x_{3}-x_{4}+x_{5}=0 \\\
x_{1}-2 x_{2}-3 x_{4}+4 x_{5}=a
\end{array}\right.
$$

1.æˆ‘ä»¬è¦æŠŠè¿™ä¸ªçº¿æ€§æ–¹ç¨‹ç»„ï¼Œè½¬æ¢æˆçŸ©é˜µçš„è¡¨ç¤ºå½¢å¼ï¼Œ$Ax=b$ã€‚

$$
\left[\begin{array}{cccccc}
-2 & 4 & -2 & -1 & 4 & \mid & -3 \\\
4 & -8 & 3 & -3 & 1 & \mid & 2 \\\
1 & -2 & 1 & -1 & 1 & \mid & 0 \\\
1 & -2 & 0 & -3 & 4 & \mid & a
\end{array}\right]
$$

2.æŽ¥ç€æˆ‘ä»¬æ¥äº¤æ¢ç¬¬ä¸€å’Œç¬¬ä¸‰è¡Œã€‚

$$
\left[\begin{array}{cccccc}
1 & -2 & 1 & -1 & 1 & \mid & 0 \\\
4 & -8 & 3 & -3 & 1 & \mid & 2 \\\
-2 & 4 & -2 & -1 & 4 & \mid & -3 \\\
1 & -2 & 0 & -3 & 4 & \mid & a
\end{array}\right]
$$

æ³¨æ„ï¼Œä½ çŸ¥é“æˆ‘ä»¬ä¸ºä»€ä¹ˆé€‰æ‹©第一行和第三行交换吗？其实，这是为了便于计算。而其中的一行是有且只有一个1的，所以我们就把这一行放到第一行。

3.æˆ‘ä»¬ä»¥ç¬¬ä¸€è¡Œä¸ºåŸºç¡€ï¼Œå¼€å§‹æ‰§è¡Œä¹˜å'ŒåŠ å˜æ¢ï¼Œå°†ç¬¬ä¸€è¡Œä¹˜ä»¥-4çš„ç»“æžœå'Œç¬¬äºŒè¡Œç›¸åŠ ï¼Œä»Žè€ŒèŽ·å¾—äº†ä¸‹é¢è¿™æ ·çš„ç»“æžœã€‚

$$

\left[\begin{array} {ccccccc}
1 & -2 & 1 & -1 & 1 & \mid & 0 \\
0 & 0 & -1 & 1 & -3 & \mid & 2 \\
-2 & 4 & -2 & -1 & 4 & \mid & -3 \\
1 & -2 & 0 & -3 & 4 & \mid & a
\end{array}\right]
$$

4.ç„¶åŽï¼Œæˆ'ä»¬ç"¨åŒæ ·çš„æ–¹æ³•ï¼Œå°†ç¬¬ä¸€è¡Œä¹˜ä»¥2çš„ç»"æžœï¼Œå†åŠ ç¬¬ä¸‰è¡Œç›¸åŠ ï¼Œå¾—åˆ°äº†ä¸‹é¢è¿™æ ·çš„ç»"æžœã€‚

$$
\left[\begin{array} {ccccccc}
1 & -2 & 1 & -1 & 1 & \mid & 0 \\
0 & 0 & -1 & 1 & -3 & \mid & 2 \\
0 & 0 & 0 & -3 & 6 & \mid & -3 \\
1 & -2 & 0 & -3 & 4 & \mid & a
\end{array}\right]
$$

5.ä»¥æ¤ç±»æŽ¨ï¼Œæˆ'ä»¬å°†ç¬¬ä¸€è¡Œä¹˜ä»¥-1çš„ç»"æžœï¼ŒåŠ ç¬¬å››è¡Œç›¸åŠ ï¼Œç»§ç»èŽ·å¾—æ–°çŸ©é˜µã€‚

$$
\left[\begin{array} {ccccccc}
1 & -2 & 1 & -1 & 1 & \mid & 0 \\
0 & 0 & -1 & 1 & -3 & \mid & 2 \\
0 & 0 & 0 & -3 & 6 & \mid & -3 \\
0 & 0 & -1 & -2 & 3 & \mid & a
\end{array}\right]
$$

6.å°†ç¬¬äºŒè¡Œä¹˜ä»¥-1çš„ç»"æžœï¼ŒåŠ ç¬¬å››è¡Œç›¸åŠ ï¼Œå¾—åˆ°ä¸‹é¢è¿™æ ·çš„ç»"æžœã€‚

$$
\left[\begin{array} {ccccccc}
1 & -2 & 1 & -1 & 1 & \mid & 0 \\
0 & 0 & -1 & 1 & -3 & \mid & 2 \\
0 & 0 & 0 & -3 & 6 & \mid & -3 \\
0 & 0 & 0 & -3 & 6 & \mid & a-2
\end{array}\right]
$$

7.å°†ç¬¬ä¸‰è¡Œä¹˜ä»¥-1çš„ç»"æžœï¼ŒåŠ ç¬¬å››è¡Œç›¸åŠ ã€‚
$$
\left[\begin{array} {ccccccc}
1 & -2 & 1 & -1 & 1 & \mid & 0 \\
0 & 0 & -1 & 1 & -3 & \mid & 2 \\
0 & 0 & 0 & -3 & 6 & \mid & -3 \\
0 & 0 & 0 & 0 & 0 & \mid & a+1
\end{array}\right]
$$

8.ç¬¬äºŒè¡Œä¹˜ä»¥-1ï¼Œç¬¬ä¸‰è¡Œä¹˜ä»¥$-\frac{1} {3}$ã€‚
$$
\left[\begin{array} {ccccccc}
1 & -2 & 1 & -1 & 1 & \mid & 0 \\
0 & 0 & 1 & -1 & 3 & \mid & -2 \\
0 & 0 & 0 & 1 & -2 & \mid & 1 \\
0 & 0 & 0 & 0 & 0 & \mid & a+1
\end{array}\right]
$$

9.çŽ°åœ¨ï¼Œè¿™ä¸ªçŸ©é˜µå°±æ˜¯ä¸€ä¸ªç®€å•å½¢å¼çš„çŸ©é˜µï¼Œä¹Ÿå«åš**è¡Œé˜¶æ¢¯å½¢çŸ©é˜µ**ï¼ˆRow-Echelon Formï¼ŒREFï¼‰ã€‚

$$
\left\{\begin{array} {r}
x_{1}-2 x_{2}+x_{3}-x_{4}+x_{5}=0 \\
x_{3}-x_{4}+3 x_{5}=-2 \\
x_{4}-2 x_{5}=1 \\
0=a+1
\end{array}\right.
$$

ä¸€ä¸ªçŸ©é˜µæˆä¸ºè¡Œé˜¶æ¢¯å½¢çŸ©é˜µéœ€æ»¡è¶³ä¸¤ä¸ªæ¡ä»¶ï¼š

- å¦‚æžœå®ƒæ—¢æœ‰é›¶è¡Œï¼Œåˆæœ‰éžé›¶è¡Œï¼Œåˆ™é›¶è¡Œåœ¨ä¸‹ï¼Œéžé›¶è¡Œåœ¨ä¸Šï¼›
- å¦‚æžœå®ƒæœ‰éžé›¶è¡Œï¼Œåˆ™æ¯ä¸ªéžé›¶è¡Œçš„ç¬¬ä¸€ä¸ªéžé›¶å…ƒç´ æ‰€åœ¨åˆ—å·è‡ªä¸Šè€Œä¸‹ä¸¥æ ¼å•è°ƒé€'å¢žï¼Œæ£å¦‚ä¸Šå›¾çš„è¿™ä¸ªçŸ©é˜µï¼Œåˆ—å·è‡ªä¸Šè€Œä¸‹æ˜¯1ã€3ã€4ï¼Œæ˜¯ä¸¥æ ¼å•è°ƒé€'å¢žçš„ã€‚

10.ä½ å¯ä»¥çœ‹å‡ºï¼Œåªæœ‰åœ¨$a=-1$çš„æƒ…况下ï¼Œè¿™ä¸ªçº¿æ€§æ–¹ç¨‹ç»„æ‰æœ‰è§£ï¼Œç‰¹æ®Šè§£æ˜¯$\left[\begin{array} {lllll}2 & 0 & -1 & 1 & 0\end{array}\right]^{\mathrm{T}} $ã€‚

11.æœ€åŽï¼Œæˆ'ä»¬å¾—å‡ºè¿™ä¸ªçº¿æ€§æ–¹ç¨‹ç»„çš„é€šç"¨è§£ï¼Œå¦‚ä¸‹å›¾æ‰€ç¤ºã€‚

$$
x \in R^{5}: x=\left[\begin{array} {c}
2 \\
0 \\
-1 \\
1 \\
0
\end{array}\right]+\lambda_{1}\left[\begin{array} {l}
2 \\
1 \\
0 \\
0 \\
0
\end{array}\right]+\lambda_{2}\left[\begin{array} {c}
2 \\
0 \\
-1 \\
2 \\
1
\end{array}\right], \lambda_{1}, \lambda_{2} \in R
$$

æ³¨æ„ï¼Œè¿™é‡Œæœ‰ä¸€ä¸ªæ¦‚å¿µéœ€è¦ä»‹ç»ï¼Œé‚£å°±æ˜¯**ä¸»å…ƒ**ã€‚ä¸»å…ƒå°±æ˜¯åœ¨çŸ©é˜µæ¶ˆå…ƒè¿‡ç¨‹ä¸ï¼Œæ¯åˆ—è¦ä¿ç•™çš„éžé›¶å…ƒç´ ï¼Œæˆ'ä»¬å¯ä»¥ç"¨å®ƒæŠŠè¯¥åˆ—å…¶ä»–å…ƒç´ æ¶ˆåŽ»ã€‚åœ¨é˜¶æ¢¯åž‹çŸ©é˜µä¸ï¼Œæ¯ä¸ªéžé›¶è¡Œç¬¬ä¸€ä¸ªéžé›¶å…ƒç´ å°±æ˜¯ä¸»å…ƒã€‚

æ‹¿ä¹‹å‰çš„ç¬¬8æ¥è®¡ç®—åŽçš„ç»"æžœæ¥ä¸¾ä¾‹ï¼Œç¬¬ä¸€è¡Œçš„ç¬¬ä¸€ä¸ªå…ƒç´ 1å°±æ˜¯ä¸»å…ƒï¼Œç¬¬äºŒè¡Œç¬¬ä¸‰ä¸ªå…ƒç´ 1æ˜¯ä¸»å…ƒï¼Œç¬¬ä¸‰è¡Œçš„ç¬¬å››ä¸ªå…ƒç´ 1ä¹Ÿæ˜¯ä¸»å…ƒã€‚

$$
\begin{bmatrix}
1 & -2 & 1 & -1 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

å¯¹åº”è¡Œé˜¶æ¢¯å½¢çŸ©é˜µä¸»å…ƒçš„åˆ—å«åšåŸºæœ¬åˆ—ï¼Œè€Œå…¶ä»–çš„åˆ—å«åšè‡ªç”±åˆ—ï¼Œè¿™ä¸ªä¾‹åä¸ï¼Œx_{1}ã€x_{3}ã€x_{4}å°±æ˜¯åŸºæœ¬åˆ—ï¼Œx_{2}ã€x_{5}å°±æ˜¯è‡ªç”±åˆ—ã€‚ä½¿ç”¨è¡Œé˜¶æ¢¯å½¢çŸ©é˜µè¿ƒæ›´ç®€å•åœ°å¾—åˆ°æ–¹ç¨‹çš„è§£ï¼Œæ‰€ä»¥æˆ‘ä»¬å¯ä»¥ä½¿ç”¨ä¸»å…ƒåˆ—æ¥è¡¨è¾¾æ€§æ–¹ç¨‹ç»„ï¼š

$$
b=\sum_{i=1}^{P} \lambda_{i} \mathrm{p}_{i}, i=1, \ldots, P
$$

åœ¨ä¹‹å‰çš„ä¾‹åï¼Œæˆ‘ä»¬ä½¿ç”¨ä¸»å…ƒåˆ—æ¥è¡¨è¾¾æˆä¸‹é¢è¿™æ ·çš„çŸ©é˜µå½¢å¼ï¼š

$$
\lambda_{1}\left[\begin{array}{l}
1 \\
0 \\
0 \\
0
\end{array}\right]+\lambda_{2}\left[\begin{array}{l}
1 \\
1 \\
0 \\
0
\end{array}\right]+\lambda_{3}\left[\begin{array}{c}
-1 \\
-1 \\
1 \\
0
\end{array}\right]=\left[\begin{array}{c}
0 \\
-2 \\
1 \\
0
\end{array}\right]
$$

äºŽæ˜¯ï¼Œæˆ‘ä»¬æœ€ç»ˆå¾—å‡º $Î»_{3}=1$ï¼Œ$Î»_{2}=-1$ï¼Œ$Î»_{1}=2$ ï¼Œåˆ†åˆ«å¯¹åº”äºŽx_{4}ã€x_{3}ã€x_{1}ã€‚è€Œå¯¹äºŽè‡ªç”±éžä¸»å…ƒåˆ—ï¼Œæˆ‘ä»¬å·²ç»éšå¼åœ°æŠŠç³»æ•°è®¾ç½®æˆäº†0ï¼Œæ‰€ä»¥è¿™ä¸ªçº¿æ€§æ–¹ç¨‹ç»„çš„ç‰¹æ®Šè§£æ˜¯$x=\left[\begin{array}{lllll}2 & 0 & -1 & 1 & 0\end{array}\right]^{\mathrm{T}}$ã€‚

ç®€åŒ–è¡Œé˜¶æ¢¯å½¢çŸ©é˜µ

è¿™é‡Œæˆ‘ä»¬å†å¼•å…¥ä¸€ä¸ªæ¦‚å¿µï¼Œç®€åŒ–è¡Œé˜¶æ¢¯å½¢çŸ©é˜µï¼Œå› ä¸ºå¼•å…¥ç®€åŒ–è¡Œé˜¶æ¢¯å½¢çŸ©é˜µå¯¹äºŽçº¿æ€§æ–¹ç¨‹ç»„çš„æ±‚è§£æœ‰å¾ˆå¤§çš„ç®€åŒ–ã€‚å…¶å®žï¼Œé«˜æ–¯æ¶ˆå…ƒæ³•çš„æ ¸å¿ƒå°±æ˜¯é€šè¿‡åˆç‰å˜æ¢ï¼ŒæŠŠçº¿æ€§æ–¹ç¨‹ç»„è½¬æ¢æˆç®€åŒ–è¡Œé˜¶æ¢¯å½¢çŸ©é˜µã€‚é‚£ä¹ˆä¸€ä¸ªæ–¹ç¨‹ç»„æ˜¯ç®€åŒ–è¡Œé˜¶æ¢¯å½¢çŸ©é˜µï¼Œåˆ™å¿…é¡»æ»¡è¶³å“ªäº›æ¡ä»¶å‘¢ï¼Ÿ

1. è¿™ä¸ªæ–¹ç¨‹ç»„å¿…é¡»æ˜¯è¡Œé˜¶æ¢¯å½¢çŸ©é˜µï¼›
2. æ–¹ç¨‹ç»„çš„æ¯ä¸€ä¸ªä¸»å…ƒéƒ½æ˜¯1ï¼›
3. ä¸»å…ƒåœ¨å®ƒçš„åˆ—ä¸æ˜¯å”¯ä¸€çš„éž0å…ƒç´ ã€‚

çŽ°åœ¨ï¼Œæˆ‘ä»¬å†é€šè¿‡ä¸€ä¸ªå®žä¾‹ï¼Œçœ‹çœ‹è¯¥å¦‚ä½•é€šè¿‡é«˜æ–¯æ¶ˆå…ƒæ³•è®¡ç®—ä¸€ä¸ªçŸ©é˜µçš„é€†çŸ©é˜µã€‚è®¾çŸ©é˜µAå¦‚ä¸‹æ‰€ç¤ºï¼š

$$
A=\left[\begin{array}{llll}
1 & 0 & 2 & 0 \\
1 & 1 & 0 & 0 \\
1 & 2 & 0 & 1 \\
1 & 1 & 1 & 1
\end{array}\right]
$$

é¦–å…ˆï¼Œæˆ‘ä»¬å½¢æˆAçš„å¢žå¹¿çŸ©é˜µï¼ˆå…·ä½“æ–¹æ³•å¯ä»¥è§ä¸Šé¢ï¼‰ã€‚

$$
\left[\begin{array}{llllllll}
1 & 0 & 2 & 0 & \mid & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & \mid & 0 & 1 & 0 & 0 \\
1 & 2 & 0 & 1 & \mid & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & \mid & 0 & 0 & 0 & 1
\end{array}\right]
$$

å…¶æ¬¡ï¼Œä½¿ç”¨æˆ‘ä»¬åˆšåˆšè®¨è®ºè¿‡çš„é«˜æ–¯æ¶ˆå…ƒæ³•è®¡ç®—å‡ºç®€åŒ–è¡Œé˜¶æ¢¯å½¢çŸ©é˜µã€‚

$$
\left[\begin{array}{ccccccccc}
1 & 0 & 0 & 0 & \mid & -1 & 2 & -2 & 2 \\
0 & 1 & 0 & 0 & \mid & 1 & -1 & 2 & -2 \\
0 & 0 & 1 & 0 & \mid & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 1 & \mid & -1 & 0 & -1 & 2
\end{array}\right]
$$

æœ€åŽï¼Œæˆ‘ä»¬å°±å¾—åˆ°Açš„é€†çŸ©é˜µï¼Œå¦‚ä¸‹æ‰€ç¤ºã€‚

$$
A^{-1}=\left[\begin{array}{cccc}
-1 & 2 & -2 & 2 \\
1 & -1 & 2 & -2 \\
1 & -1 & 1 & -1 \\
-1 & 0 & -1 & 2
\end{array}\right]
$$

æŽ¥ä¸‹æ¥ï¼Œæˆ‘ä»¬åªè¦ä½¿ç”¨å…¬å¼$A A^{-1}=I$ å°±å¯ä»¥å¯¹ç»“æžœè¿›è¡Œé ªŒè¯äº†ã€‚

æ›´å¤šè§£çº¿æ€§æ–¹ç¨‹ç»„çš„æ–¹æ³•

å°ç®—æ³•ä¸ªæ•°ï¼Œç›¸ä¼¼å¯ä»¥ç†è§£ä¸ºè¿™æ ·ä½¿è§£çº¿æ€§æ–¹ç¨‹ç»„ï¼Œå¹¶ä¸”æœ‰è®¸å¤šæ–¹æ³•ç”¨äºŽè§£å†³ç”¨è§£ï¼Œä¸ºä»€ä¹ˆæˆ‘ä»¬ä¸è€ƒè™‘é«˜æ–¯æ¶ˆå…ƒæ³•æ¥è§£çº¿æ€§æ–¹ç¨‹ç»„ã€‚æœ€åŽï¼Œæˆ‘ä»¬å°†æ€»ç»“ä¸€äº›è§£æ–¹æ³•æ¥ä½œä¸ºç»“æŸè¯ã€‚

ç¬¬ä¸€ä¸ªæ–¹æ³•ï¼Œå‡è®¾ä¸€ä¸ªçŸ©é˜µAæ˜¯æ–¹é˜µï¼ˆè¡Œæ•°ä¸Žåˆ—æ•°ç›¸ç‰çš„çŸ©é˜µï¼‰ï¼Œå¹¶ä¸”æ˜¯å¯é€†çš„ï¼Œ$Ax=B$ ï¼Œé‚£ä¹ˆxè§£å°±å¯ä»¥å†™æˆ$x=A^{-1}B$ï¼Œä½†æ˜¯ï¼Œå¦‚æžœAçŸ©é˜µä¸å¯é€†ï¼Œå°±æ˜¯æ–¹é˜µï¼Œé‚£ä¹ˆæˆ‘ä»¬å°±å¯ä»¥ä½¿ç”¨ä¸‹é¢çš„æ–¹æ³•æ±‚xè§£äº†ã€‚

$$Ax=B\Rightarrow A^{T}Ax=A^{T}B\Rightarrow x=(A^{T}A)^{-1}A^{T}B$$

å…¶ä¸ï¼ŒçŸ©é˜µAçš„è½¬ç½®çŸ©é˜µå’ŒAç›¸ä¹˜çš„é€†çŸ©é˜µï¼Œå†å’ŒAçš„è½¬ç½®çŸ©é˜µç›¸ä¹˜ï¼Œæˆ‘ä»¬æŠŠå®ƒå«åšçŸ©é˜µçš„ä¼ªé€†çŸ©é˜µï¼ˆMoore-Penrose pseudo inverseï¼‰ï¼Œç®€ç§°ä¼ªé€†ã€‚

$$(A^{T}A)^{-1}A^{T}$$

è¿™ä¸ªæ–¹æ³•æœ‰ä¸¤ä¸ªå¼Šç«¯ï¼šç¬¬ä¸€ï¼ŒçŸ©é˜µå’Œé€†çŸ©é˜µçš„è®¡ç®—å¤æ‚ï¼›ç¬¬äºŒï¼Œæ•°å€¼ç¨³å®šæ€§ä¸è¶³ã€‚å› æ¤ï¼Œä»Žå®žè·µè§’åº¦æ¥çœ‹ï¼Œæˆ‘ä»¬ä¸€èˆ¬ä¸æŽ¨èä½¿ç”¨ã€‚

ç¬¬äºŒä¸ªæ–¹æ³•æ˜¯é«˜æ–¯æ¶ˆå…ƒæ³•ã€‚é«˜æ–¯æ¶ˆå…ƒæ³•æ˜¯éžå¸¸ç›´è§‚çš„ï¼Œå®ƒåœ¨å¾ˆå¤šè®¡ç®—ä¸Šéƒ½è¡¨çŽ°å¾—å……åˆ†ï¼Œä½†æ˜¯ä¹Ÿæœ‰ç¼ºç‚¹ï¼š

1. è®¡ç®—é‡å¤§ï¼›
2. æ•°å€¼ä¸ç¨³å®šæ€§ï¼›
3. è®¡ç®—çŸ©é˜µçš„é€†çŸ©é˜µï¼›
4. è®¡ç®—çŸ©é˜µçš„ç§©ï¼›
5. å†³å®šå‡ºçŸ©é˜µçš„åŸºã€‚

ä½†å½“é«˜æ–¯æ¶ˆå…ƒæ³•é¢å¯¹ç™¾ä¸‡çº§ç”šè‡³æ›´å¤§è§„æ¨¡çš„çº¿æ€§æ–¹ç¨‹ç»„æ—¶ï¼Œå®ƒçš„è®¡ç®—æ•ˆçŽ‡ä¼šä¸‹é™å¾ˆå¤šã€‚

æ‰€ä»¥è¿™ç§æƒ…å†µä¸‹ï¼Œæˆ‘ä»¬å°±éœ€è¦è€ƒè™‘è¿ä»£æ³•ã€‚

è¿ä»£æ³•æ˜¯ä¸€ç§é€šè¿‡è¿ä»£æ¥æ±‚è§£çº¿æ€§æ–¹ç¨‹ç»„çš„æ–¹æ³•ã€‚Stationary iterative methodï¼‰å’ŒKrylovåç©ºé—´æ–¹æ³•ã€‚

å®šå¸¸è¿ä»£æ³•ï¼šç†æŸ¥æ£®è¿ä»£æ³•ï¼ˆRichardson methodï¼‰ã€é›…å¯æ¯”æ–¹æ³•ï¼ˆJacobi methodï¼‰ã€GauÃŸ-Seidelæ–¹æ³•ã€é€æ¬¡è¶…æ¾å¼›æ–¹æ³•ï¼ˆSuccessive over-relaxation methodï¼Œç®€ç§°SORï¼‰ã€‚

Krylovåç©ºé—´æ–¹æ³•ï¼šå…±è½æ¢¯åº¦æ³•ï¼ˆConjugate gradientï¼‰ã€å¹¿ä¹‰æœ€å°æ®‹å·®æ³•ï¼ˆGeneralized minimal residualï¼‰ã€åŒå…±è½æ¢¯åº¦æ³•ï¼ˆBiconjugate gradientï¼‰ã€‚

è¿™é‡Œæˆ‘ä»¬åªåˆ—ä¸¾äº†ä¸€äº›å¸¸ç”¨çš„è¿ä»£æ³•ï¼Œå¦‚æžœä½ å¯¹è¿™äº›æ–¹æ³•æ„Ÿå…´è¶£ï¼Œå¯ä»¥é€šè¿‡ä¹¦ç±æ·±å…¥äº†è§£ã€‚

å¦‚æžœä½ æƒ³è¦æ·±å…¥ç ”ç©¶ï¼Œè¿™é‡ŒæŽ¨èä¸¤æœ¬ä¹¦ï¼šã€ŠIntroduction to Numerical Analysisã€‹ï¼Œä»¥åŠã€ŠLinear Algebraã€‹ã€‚

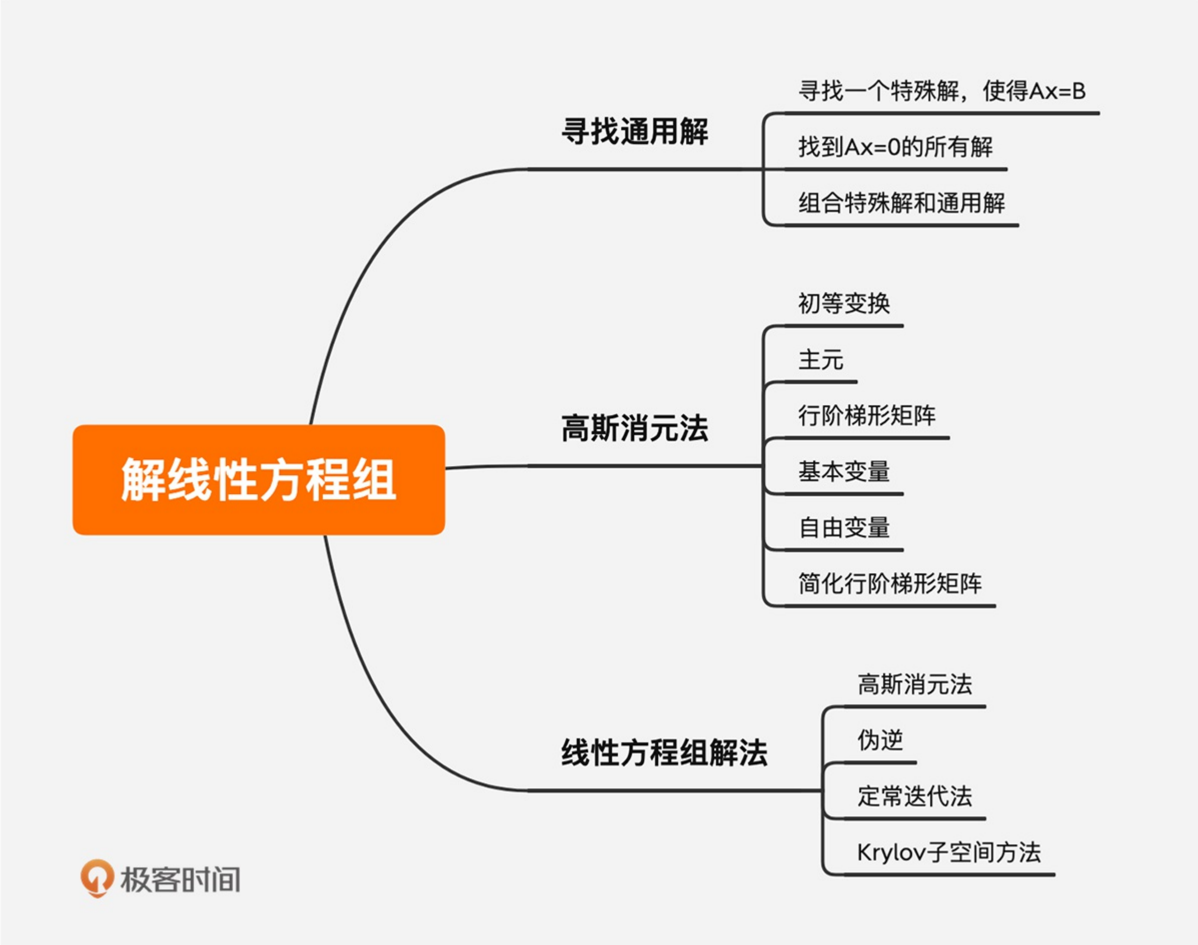
1.ã€ŠIntroduction to Numerical Analysisã€‹
ä½œè€…ï¼šStoer, Josef, Bulirsch, R.
2002å¹´å‡ºç‰ˆ
2.ã€ŠLinear Algebraã€‹
ä½œè€…ï¼šLiesen, JÃ¶rg, Mehrmann, Volker
2015å¹´å‡ºç‰ˆ

æœ¬èŠ‚å°ç»“

å¥½äº†ï¼Œåˆ°è¿™é‡Œè§£çº¿æ€§æ–¹ç¨‹ç»„ï¼Œè¿™ä¸€è®²å°±ç»“æŸäº†ï¼Œæœ€åŽæˆ‘æ¥æ€»ç»“ä¸€ä¸‹æœ¬èŠ‚è¯¾çš„æ ¸å¿ƒå†…å®¹ã€‚

é¦–å…ˆï¼Œæˆ‘ä»¬è®²è§£äº†è§£çº¿æ€§æ–¹ç¨‹ç»„çš„æ–¹æ³•ï¼Œè¿™äº›æ–¹æ³•æœ‰å…³é”®è¯ã€‚

çº¿æ€§æ–¹ç¨‹ç»„çš„æ±‚è§£è¿‡ç¨‹ä¹Ÿæ˜¯ä¸€ä¸ªè®¡ç®—è¿‡ç¨‹ã€‚



çº¿æ€§ä»£æ•°ç»ƒä¹ åœº

ç»ƒä¹ æ—¶å›»åˆ°äº†ï¼Œä»Šå¤©çš„ç»ƒä¹ é¢˜æ¯”è¾ƒç®€å•ï¼Œè¯·ä½ ç”¨é«˜æ–¯æ¶ˆå…ƒæ³•æ±‚ä¸‹é¢çš„çº¿æ€§æ–¹ç¨‹ç»„ã€‚

$$
\left\{\begin{array}{c}
x_{1}+x_{2}-2 x_{3}-x_{4}=-1 \\
x_{1}+5 x_{2}-3 x_{3}-2 x_{4}=0 \\
3 x_{1}-x_{2}+x_{3}+4 x_{4}=2 \\
-2 x_{1}+2 x_{2}+x_{3}-x_{4}=1
\end{array}\right.
$$

æ¬¢è¿Žåœ¨ç•™è¨€åŒºæ™’å‡ºä½ çš„è¿ç®—è¿‡ç¨‹å’Œç»“æžœï¼Œç•™ä¸‹ä½ çš„å¦ä¸€ç‰è¿¹ã€‚å¦‚æžœä½ æœ‰æ”¶èŽ·ï¼Œä¹Ÿæ¬¢è¿Žä½ æŠŠè¿™ç¯‡æ–‡ç« åˆ†äº«ç»™ä½ çš„æœ‹å‹ã€‚