ä½ å¥½ï¼Œæˆ‘æ˜¯æœ±ç»´åˆšã€‚æ¬¢è¿Žä½ ç»§ç»è·Ÿæˆ‘å¦ä¹ çº¿æ€§ä»£æ•°ã€‚

ä»Šå¤©è¿™ä¸€èŠ‚è¯¾ç›¸å†…å®¹æ˜¯åŸºç¡€ç¯‡å…³ã€‚è¿™é‡Œä¼šç”¨5é“å…¸åž‹ä¾‹é¢˜ï¼Œè®©ä½ å·©å›ºä¸€ä¸‹çº¿æ€§ä»£æ•°ç›¸åŸºç¡€çŸ¥è¯†ï¼Œè¿™ä¹Ÿæ˜¯è¿›å…¥åº”ç”¨ç¯‡å¦ä¹ ä¹‹å‰çš„ä¸€æ¬¡åŠ¨æ‰‹æœºä¼šã€‚ä»Žè¯¾ç¨‹ä¸Šçº¿åˆ°çŽ°åœ¨å¿«æœ‰ä¸€ä¸ªæœˆäº†ï¼Œè¿™æœŸé—´æˆ‘ä¹Ÿä¸€ç›´åœ¨ç•™è¨€åŒºè·Ÿä½ è®¨è®ºä¹Ÿæ˜¯æˆ‘æ²¡æœ‰æƒ³åˆ°çš„ï¼Œé‚£ä¹ˆæ·±å…¥ï¼Œè¿˜è¯è¿™è®©æˆ‘æ„Ÿè§‰æŒºæ„¤çš„ï¼Œå¸Œæœ›ä½ å†æŽ¥å†åŽ‰ã€‚

çŽ°åœ¨ï¼Œä½ å°±ä»¥ç»ƒä¹ æ¥ä½œä¸ºåŸºç¡€ç¯‡å…³çš„5é“ä¾‹é¢˜ä¸ºï¼Œé¢˜ç›®å’Œè§£æžéƒ½æ”¾åœ¨äº†æ–‡ç« ä¸ï¼Œä½ å¯ä»¥è‡ªå·±å…ˆè¯•ç€è§£ç”ä¸€ä¸‹ï¼ŒåŸºç¡€çŸ¥è¯†ï¼Œæˆ‘ä»¬ä¸€èµ·çœ‹çœ‹ã€‚

ä¾‹é¢˜ä¸€

æ‰¾åˆ°çº¿æ€§æ–¹ç¨‹ç»„$Ax=b$çš„æ‰€æœ‰è§£ï¼Œå…¶ä¸ï¼š

$$
A=\left[\begin{array}{cc}
1 & 2 \\
3 & 0 \\
-1 & 2
\end{array}\right], b=\left[\begin{array}{c}
1 \\
0 \\
1
\end{array}\right]
$$

è§£æžï¼š

è¿™é‡Œè€ƒå¯Ÿäº†è§£çº¿æ€§æ–¹ç¨‹ç»„çš„æ–¹æ³•ï¼Œç‰¹åˆ«æ˜¯é«˜æ–¯æ¶ˆå…ƒæ³•ï¼Œä½ å¯ä»¥å‚è€ƒç¬¬4èŠ‚çš„å†…å®¹ã€‚

é¦–å…ˆï¼Œå½¢æˆå¢žå¹¿çŸ©é˜µï¼š

$$
\left[\begin{array}{cccc}
1 & 2 & 1 \\
3 & 0 & 0 \\
-1 & 2 & 1
\end{array}\right]
$$

æŽ¥ç€ï¼Œåˆ†æ¥è®¡ç®—å¢žå¹¿çŸ©é˜µçš„è¡Œé˜¶æ¢¯å½¢çŸ©é˜µï¼š

1. ç¬¬ä¸€è¡Œä¹˜-3åŠ ç¬¬äºŒè¡Œç›¸åŠ ã€‚
2. ç¬¬ä¸€è¡ŒåŠ ç¬¬ä¸‰è¡Œç›¸åŠ ã€‚

$$
\left[\begin{array}{cccc}
1 & 2 & 1 \\
0 & -6 & -3 \\
0 & 4 & 2
\end{array}\right]
$$

2. ç¬¬äºŒè¡Œä¹˜$\frac{1}{3}$åŠ ç¬¬ä¸€è¡Œç›¸åŠ ã€‚
3. ç¬¬äºŒè¡Œä¹˜$\frac{2}{3}$åŠ ç¬¬ä¸‰è¡Œç›¸åŠ ã€‚
4. ç¬¬ä¸‰è¡Œä¹˜$-\frac{1}{6}$ã€‚

$$
\left[\begin{array}{llll}
1 & 0 & 0 \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0
\end{array}\right]
$$

æœ€åŽå¾—å‡ºè¯¥çº¿æ€§æ–¹ç¨‹ç»„çš„å”¯ä¸€è§£ï¼š

$$
x=\left[\begin{array}{l}
0 \\
\frac{1}{2}
\end{array}\right]
$$

ä¾‹é¢˜äºŒ

æ‰¾åˆ°çº¿æ€§æ–¹ç¨‹ç»„$Ax=b$çš„æ‰€æœ‰è§£ï¼Œå…¶ä¸ï¼š

$$
A=\left[\begin{array}{lll}
1 & 2 & 3 \\
0 & 2 & 2
\end{array}\right], b=\left[\begin{array}{l}
1 \\
1
\end{array}\right]
$$

è§£æžï¼š

è¿™é‡Œè€ƒå¯Ÿäº†è§£çº¿æ€§æ–¹ç¨‹ç»„çš„æ–¹æ³•ï¼Œç‰¹åˆ«æ˜¯é«˜æ–¯æ¶ˆå…ƒæ³•ã€‚ä½ å¯ä»¥å‚è€ƒç¬¬4èŠ‚çš„å†…å®¹ï¼Œå’Œä¾‹é¢˜ä¸€ä¸åŒçš„æ˜¯ï¼Œä¾‹é¢˜äºŒè¿™é‡Œå¾—åˆ°çš„ä¼šæ˜¯æ— ç©·è§£ã€‚æ‰€ä»¥ï¼Œè¿™ä¸€é¢˜é‡Œæ‰¾ç‰¹æ®Šè§£å'Œé€šç"¨è§£çš„æ–¹æ³•æ˜¯å…³é"®ã€‚

é¦–å…ˆï¼Œå½¢æˆå¢žå¹¿çŸ©é˜µï¼š

$$
\left[\begin{array}{lllll}
1 & 2 & 3 & 1 & 1 \\
0 & 2 & 2 & 1 & 1
\end{array}\right]
$$

æŽ¥ç€ï¼Œå½¢æˆå¢žå¹¿çŸ©é˜µï¼šåˆ†æ¥è®¡ç®—å¢žå¹¿çŸ©é˜µçš„è¡Œé˜¶æ¢¯å½¢çŸ©é˜µï¼š

1. ç¬¬ä¸€è¡Œä¹˜-1å'Œç¬¬äºŒè¡Œç›¸åŠ ï¼›
2. ç¬¬äºŒè¡Œä¹˜1/2ã€‚

$$
\left[\begin{array}{lllll}
1 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & \frac{1}{2}
\end{array}\right]
$$

ä½¿ç"¨ä¸»å…ƒåˆ—ï¼Œå¾—åˆ°ç‰¹æ®Šè§£ï¼š

$$
x=\left[\begin{array}{l}
0 \\
\frac{1}{2} \\
0
\end{array}\right]
$$

ä¸‹ä¸€æ¥ï¼ŒèŽ·å–çº¿æ€§æ–¹ç¨‹ç»„$Ax=0$çš„é€šç"¨è§£ï¼Œä»Žå¢žå¹¿çŸ©é˜µçš„å·¦è¾¹ï¼Œèƒ½å¤Ÿç«‹å³å¾—å‡ºï¼š

$$
\lambda\left[\begin{array}{c}
1 \\
1 \\
-1
\end{array}\right]
$$

æœ€åŽï¼ŒæŠŠç‰¹æ®Šè§£å'Œé€šç"¨è§£ç»"åˆèµ·æ¥å°±æ˜¯ï¼š

$$
x=\left[\begin{array}{l}
0 \\
\frac{1}{2} \\
0
\end{array}\right]+\lambda\left[\begin{array}{c}
1 \\
1 \\
-1
\end{array}\right]
$$

ä¾‹é¢˜ä¸‰

è®¡ç®—çŸ©é˜µä¹˜ABã€‚

$$
A=\left[\begin{array}{ccc}
1 & 2 & 3 \\
0 & -1 & 2
\end{array}\right], B=\left[\begin{array}{ccc}
4 & -1 & 2 \\
0 & 2 & 1
\end{array}\right]
$$

è§£æžï¼š

è¿™é‡Œè€ƒå¯Ÿäº†åŸºæœ¬çš„çŸ©é˜µä¹˜è¿ç®—ï¼Œç‰¹åˆ«æ˜¯æ™®é€šçŸ©é˜µä¹˜ï¼Œåªæœ‰ç¬¦åˆè§„åˆ™æ•°åŒ¹é…çš„çŸ©é˜µæ‰èƒ½ç›¸ä¹˜ï¼Œä½†å¯ä»¥è€ƒç‰3èŠ‚çš„å†…å®¹ã€‚

çŸ©é˜µä¹˜æ— æ³•å®Œæˆï¼Œå› ä¸ºAæ˜¯2è¡Œ3åˆ—çŸ©é˜µï¼ŒBä¹Ÿæ˜¯2è¡Œ3åˆ—çŸ©é˜µï¼ŒAå'Œé‚»å±…ç»´åº¦ä¸åŒã€‚

ä¾‹é¢˜å››

è®¡ç®—çŸ©é˜µä¹˜ABã€‚

$$
A=\left[\begin{array}{ccc}
1 & 2 & 3 \\
0 & -1 & 2
\end{array}\right], B=\left[\begin{array}{cc}
4 & -1 \\
2 & 0 \\
2 & 1
\end{array}\right]
$$

è§£æžï¼š

è¿™é‡Œè€ƒå¯Ÿäº†åŸºæœ¬çš„çŸ©é˜µä¹˜è¿ç®—ï¼Œç‰¹åˆ«æ˜¯æ™®é€šçŸ©é˜µä¹˜ï¼Œä½†å¯ä»¥è€ƒç‰3èŠ‚çš„å†…å®¹ã€‚

çŸ©é˜µä¹˜å¯ä»¥å®Œæˆï¼Œå› ä¸ºä¸¤ä¸ªçŸ©é˜µçš„ç»´åº¦ç›¸åŒï¼Œæ¿a_{11}ä¸ºä¾‹ï¼š$a_{11}=1 \times 4+2 \times 2+3 \times 2=14$ï¼Œç»"æžœï¼š

$$
A B=\left[\begin{array}{cc}
14 & 2 \\
2 & 2
\end{array}\right]
$$

ä¾‹é¢˜äº”

å‡è®¾R^{3}å’Œå®ƒçš„è¿ç®—$\langle\ Â·,Â· \rangle$ï¼Œ$x, y \in R^{3}$ï¼Œæˆ‘ä»¬æœ‰ï¼š

$$
\langle x, y\rangle=x^{T} A y, A=\left[\begin{array}{ccc}
4 & 2 & 1 \\
0 & 4 & -1 \\
1 & -1 & 5
\end{array}\right]
$$

é‚£ä¹ˆï¼Œ$\langle\ Â·,Â· \rangle$æ˜¯å†…ç§¯å—ï¼Ÿ

è§£æžï¼š

è¿™é‡Œè€ƒå¯Ÿçš„æ˜¯å†…ç§¯ï¼Œä»¥åŠå†…ç§¯çš„æ€§è´¨ä¹‹ä¸€ï¼šå¯¹ç§°æ€§ï¼Œä½ å¯ä»¥è€ƒç¬¬10èŠ‚çš„å†…å®¹ã€‚

é€‰æ‹©$x=\left[\begin{array}{lll}1 & 1 & 0\end{array}\right]^{T}$ï¼Œ$y=\left[\begin{array}{lll}1 & 2 & 0\end{array}\right]^{T}$ï¼Œé€šè¿‡è®¡ç®—ï¼Œèƒ½å¤Ÿå¾—åˆ°ï¼š

$$
\begin{array}{l}
\langle x, y\rangle=16 \\
\langle y, x\rangle=14 \\
\langle x, y\rangle \neq\langle y, x\rangle
\end{array}
$$

äºŽæ˜¯ï¼Œ$\langle\ Â·,Â· \rangle$æ˜¯ä¸å¯¹ç§°çš„ã€‚