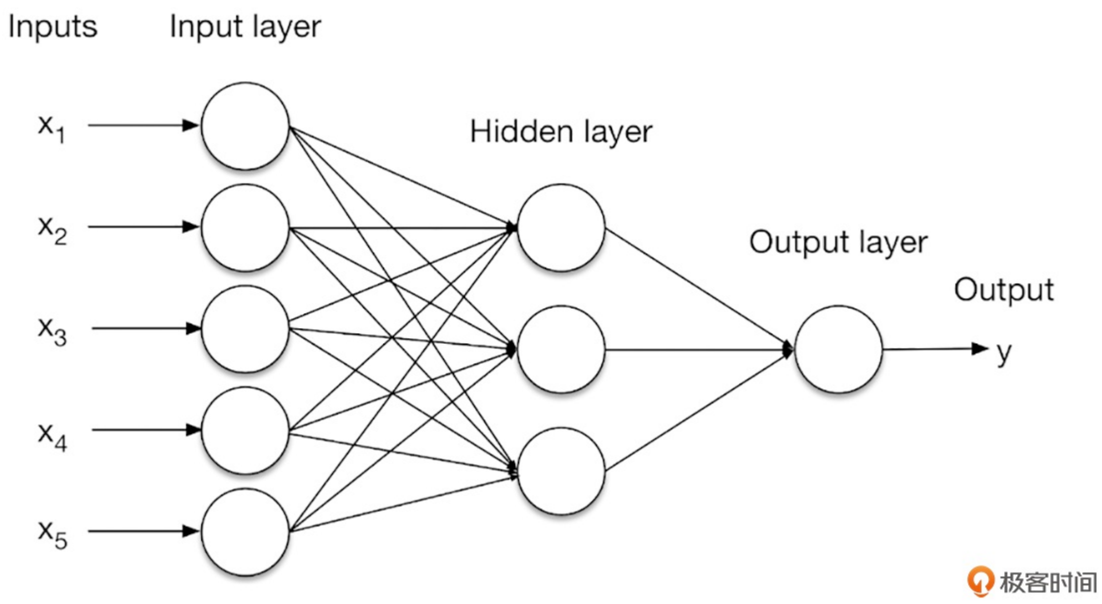
ä½ å¥½ï¼Œæˆ‘æ˜¯æœ±ç»´å˜ã€‚æ¬¢è¿Žä½ ç»§ç»è·Ÿæˆ‘å¦ä¹ çº¿æ€§ä»£æ•°ï¼Œä»Šå¤©æˆ‘ä»¬è¦è®²çš„å†…å®¹æ˜¯â€œçŸ©é˜µâ€ã€‚

åœ¨å¼€å§‹å¦ä¹ ä¹‹å‰ï¼Œæˆ‘æƒ³å…ˆé—®ä½ ä¸ªé—®é¢˜ï¼Œä½ è§‰å¾—ï¼Œå¦ä¹ çŸ©é˜µæœ‰ä»€ä¹ˆç”¨å‘¢ï¼Ÿä½ å¯ä»¥å…ˆè‡ªå·±æƒ³ä¸€æƒ³ã€‚ä¹‹åŽæˆ‘ä»¬è®²ä»»ä½•ä¸€ä¸ªçŸ¥è¯†ç‚¹çš„æ—¶å€™ï¼Œä½ éƒ½å¯ä»¥ä»Žè¿™ä¸ªè§’åº¦å‡ºå‘ï¼Œè‡ªå·±å…ˆæ€è€ƒä¸€ä¸‹ï¼Œè¿™æ ·æœ‰åŠ©äºŽä½ å¯¹æ‰€å¦å†…å®¹ç†è§£å¾—æ›´æ·±åˆ»ã€‚

å¯¹äºŽåˆšæ‰é‚£ä¸ªé—®é¢˜ï¼Œæˆ‘çš„ç”æ¡ˆå¾ˆç®€å•ï¼Œå°±ä¸€å¥è¯ï¼ŒäºŽæˆ‘ä»¬ç¨‹åºå‘˜çš„è§’åº¦å¤‡çŸ¥è¯†æ¥è¯´ï¼Œ**çŸ©é˜µå¯ä»¥æžå¤§åœ°æé«˜è®¡ç®—æœºçš„è¿ç®—æ•ˆçŽ‡**ã€‚æ€Žä¹ˆè¯´å‘¢ï¼Ÿæˆ‘ç»™ä½ ä¸¾ä¸ªä¾‹åã€‚åœ¨æœºå™¨å¦ä¹ ä¸ï¼ˆç‰¹åˆ«æ˜¯æ·±åº¦å¦ä¹ ï¼‰æˆ–è€…æ›´å…·ä½“ä¸€ç‚¹ï¼Œç¥žç»ç½‘ç»œï¼‰ï¼Œå¹¶è¡Œè®¡ç®—æ˜¯éžå¸¸æ˜‚è´µçš„ã€‚



ä¸Šå›¾æ˜¯ä¸€ä¸ªå…¸åž‹çš„ç¥žç»ç½‘ç»œæž¶æž„ï¼Œåœ¨è¿™æ—¶å€™ï¼ŒçŸ©é˜µå°±èƒ½å‘æŒ¥ç”¨æ¦äº†ï¼Œåœ¨ä¸ºï¼Œè®¡ç®—Héšè—å±‚è¾“å‡ºçš„å…¬å¼æ˜¯ï¼š$H = f(W.x + b)$ï¼Œå…¶ä¸Wæ˜¯æƒé‡çŸ©é˜µï¼Œfæ˜¯æ¿€æ´»å‡½æ•°ï¼Œbæ˜¯åç½®ï¼Œxæ˜¯è¾“å…¥å±‚çŸ©é˜µã€‚è€Œè¿™ä¸ªè®¡ç®—è¿‡ç¨‹å°±å«**向量化**ï¼ˆVectorizationï¼‰ï¼Œè¿™ä¹Ÿæ˜¯GPUåœ¨æ·±åº¦å¦ä¹ ä¸éžå¸¸é‡è¦çš„åŽŸå› ï¼Œå› ä¸ºGPUéžå¸¸æ“…é•¿åšç±»ä¼¼çŸ©é˜µä¹˜ç§¯çš„è¿ç®—ã€‚

$$
X=\left[\begin{array} {l}
x_{1} \\
x_{2}
\end{array}\right]
$$

$$
W=\left[\begin{array} {ll}
w_{1} & w_{2} \\
w_{4} & w_{5} \\
x_{3} & w_{6}
\end{array}\right]
$$

$$
H=f\left(\left[\begin{array} {ll}
w_{1} & w_{2} \\
w_{4} & w_{5} \\
x_{3} & w_{6}
\end{array}\right]\left[\begin{array} {l}
x_{1} \\
x_{2}
\end{array}\right]+b\right)
$$

ä¸è¿‡ï¼ŒçŸ©é˜µä¹Ÿä¸ä»…ä»…å±€é™äºŽç¥žç»ç½‘ç»œçš„åº”ç”¨ï¼ŒåŒæ—¶å®ƒä¹Ÿå¯ä»¥ç”¨åœ¨è®¡ç®—æœºå›¾å½¢å›¾åƒçš„åº”ç”¨ä¸ï¼Œæ¯”å¦‚ï¼Œä¸‰ç»´ç‰©ä½“ä»Žå–æ™¯åˆ°å±å¹•çš„æ˜¾ç¤ºï¼Œå°±éœ€è¦ç»åŽ†ä¸€ç³»åˆ—çš„ç©ºé—´å˜æ¢ï¼Œæ‰èƒ½ç”ŸæˆäºŒç»´å›¾åƒæ˜¾ç¤ºåœ¨æ˜¾ç¤ºå™¨ä¸Šã€‚åœ¨è¿™ä¸ªè®¡ç®—è¿‡ç¨‹ä¸ï¼Œæˆ‘ä»¬éƒ½éœ€è¦ç”¨åˆ°çŸ©é˜µã€‚

çŸ©é˜µæ˜¯éžå¸¸å®žç”¨çš„ï¼Œä½†å®ƒæ£å¼ä½œä¸ºæ•°å¦ä¸çš„ç ”ç©¶å¯¹è±¡å‡ºçŽ°ï¼Œå…¶å®žæ˜¯åœ¨è¡Œåˆ—å¼çš„ç ”ç©¶å‘å±•èµ·æ¥ä¹‹åŽã€‚è‹±å›½æ•°å¦å®¶ Arthur Cayley è¢«å…¬è®¤ä¸ºçŸ©é˜µè®ºçš„åˆ›ç«‹äººï¼Œä»–æå‡ºçš„çŸ©é˜µæ¦‚å¿µå¯èƒ½æ¥è‡ªäºŽè¡Œåˆ—å¼ã€‚ä½†æˆ‘ç›¸ä¿¡å¦å¤–ä¸€ä¸ªèµ·å› ，ä¹Ÿå°±æ˜¯ç¥žç»ç½‘ç»œçš„åº”ç”¨ã€‚

çŸ©é˜µçš„åŸºæœ¬æ¦‚å¿µ

çº¿æ€§æ–¹ç¨‹ç»„çš„æ¦‚å¿µå¾ˆæ—©å°±æœ‰äº†ï¼Œè€Œæˆ‘ä»¬ä»Žå°å¦ä¹ è¿‡äºŒå…ƒä¸€æ¬¡æ–¹ç¨‹ç»„ï¼Œä¸‰å…ƒä¸€æ¬¡æ–¹ç¨‹ç»„ã€‚

$$ax+by=c$$

$$
\left\{\begin{array} {l}
a_{1} x+b_{1} y+C_{1}=0 \\
a_{2} x+b_{2} y+C_{2}=0
\end{array}\right.
$$

åœ¨è¿™æ ·ä¸€ä¸ªæ–¹ç¨‹ç»„ä¸ï¼Œ$a1$ã€$a2$ã€$b1$ã€$b2$ä¸èƒ½åŒæ—¶ä¸º0ã€‚å½“æˆ‘ä»¬æŠŠäºŒå…ƒä¸€æ¬¡æ–¹ç¨‹ç»„å†æ‰©å±•ä¸€ä¸‹ï¼Œå˜æˆå¤šå…ƒä¸€æ¬¡æ–¹ç¨‹ç»„æ—¶ï¼Œæˆ‘ä»¬å°±èƒ½å¾—åˆ°çº¿æ€§æ–¹ç¨‹ç»„çš„ä¸€èˆ¬è¡¨è¾¾å¼ï¼Œå³$AX=B$ã€‚

$$
\left\{\begin{array} {l}
a_{11} x_{1}+a_{12} x_{2}+\cdots+a_{1 n} x_{n}=b_{1} \\
a_{21} x_{1}+a_{22} x_{2}+\cdots+a_{2 n} x_{n}=b_{2} \\
\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \\
a_{m 1} x_{1}+a_{m 2} x_{2}+\cdots+a_{m n} x_{n}=b_{m}
\end{array}\right.
$$

äºŽæ˜¯ï¼Œè¿™ä¸ªçº¿æ€§æ–¹ç¨‹ç»„çš„æ‰€æœ‰ç³»æ•°å°±æž„æˆäº†ä¸€ä¸ª$mÃ—n$çš„mè¡Œnåˆ—çŸ©é˜µï¼š

$$
A=\left[\begin{array} {cccc}

a_{11} & a_{12} & \ldots & a_{1 n} \\
a_{21} & a_{22} & \ldots & a_{2 n} \\
\ldots & \ldots & \ldots & \ldots \\
a_{m 1} & a_{m 2} & \ldots & a_{m n}
\end{array}\right]
$$

æˆ'ä»¬æŠŠAç§°ä¸ºè¯¥æ–¹ç¨‹ç»„çš„ç³»æ•°çŸ©é˜µï¼Œè€Œå½"æˆ'ä»¬æŠŠç‰å¼å³è¾¹çš„å¸¸æ•°bæ'¾å…¥çŸ©é˜µåŽï¼Œå°±æˆ¯ä¸‹é¢è¿™æ ·ï¼š

$$
\widetilde{A}=\left[\begin{array}{ccccc}
a_{11} & a_{12} & \ldots & a_{1 n} & b_{1} \\
a_{21} & a_{22} & \ldots & a_{2 n} & b_{2} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
a_{m 1} & a_{m 2} & \ldots & a_{m n} & b_{m}
\end{array}\right]
$$

è¿™æ ·æˆ'ä»¬å°±å¾—åˆ°äº†AçŸ©é˜µçš„å¢žå¹¿çŸ©é˜µ$\widetilde{A}$ ï¼Œå¯ä»¥è¡¨ç¤ºä¸º(A, B)ï¼Œè¿™é‡Œçš„Bè¡¨ç¤ºçš„æ˜¯æ–¹ç¨‹ç»„å¸¸æ•°é¡¹æž„æˆçš„åˆ—å'é‡ï¼Œä¹Ÿå°±æ˜¯$mÃ—1$çš„mè¡Œ1åˆ—çŸ©é˜µï¼š

$$
B=\left[\begin{array}{l}
b_{1} \\
b_{2} \\
\cdots \\
b_{m}
\end{array}\right]
$$

å¦‚æžœè®¾Xä¸º$nÃ—1$çš„nè¡Œ1åˆ—çŸ©é˜µï¼š

$$
X=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\cdots \\
x_{n}
\end{array}\right]
$$

é‚£ä¹ˆçº¿æ€§æ–¹ç¨‹ç»„Aï¼Œå°±å¯ä»¥è¡¨ç¤ºä¸º$AX=B$çš„çŸ©é˜µå½¢å¼ã€‚å¦‚æžœæˆ'ä»¬å†æ¢ä¸€ç§è¡¨ç¤ºå½¢å¼ï¼Œè®¾ï¼š$a_{1} ,a_{2},\ldots, a_{n},\beta$è¡¨ç¤ºå¢žå¹¿çŸ©é˜µ$\widetilde{A}$ çš„åˆ—å'é‡ï¼Œå'Œçº¿æ€§æ–¹ç¨‹ç»„Aå°±å¯è¡¨ç¤ºä¸º$a_{1} x_{1}+a_{2} x_{2}+\cdots+a_{n} x_{n}=Î²$ã€‚

çº¿æ€§æ–¹ç¨‹ç»„çš„çŸ©é˜µå'Œå'é‡å½¢å¼ƒ½æ˜¯çº¿æ€§æ–¹ç¨‹ç»„çš„å…¶ä»–è¡¨ç¤ºå½¢å¼ï¼Œå®ƒå·²ç»è¿œè¿œè¶…å‡ºäº†æ–¹ç¨‹çš„æ¦‚å¿µï¼Œè¿™ä¸€ç‚¹å¯¹æˆ'ä»¬ç†è§£çŸ©é˜µå¾ˆé‡è¦ã€‚

é€šè¿‡å‰é¢çš„è®²è§£ï¼Œæˆ'ç›¸ä¿¡ä½ å¯¹çŸ©é˜µæœ‰äº†ä¸€å®šçš„äº†è§£ï¼ŒçŽ°åœ¨æˆ'ä»¬æ›´æ·±å…¥çš„æŽ¢ç´¢çŸ©é˜µçš„å®šä¹‰ã€‚

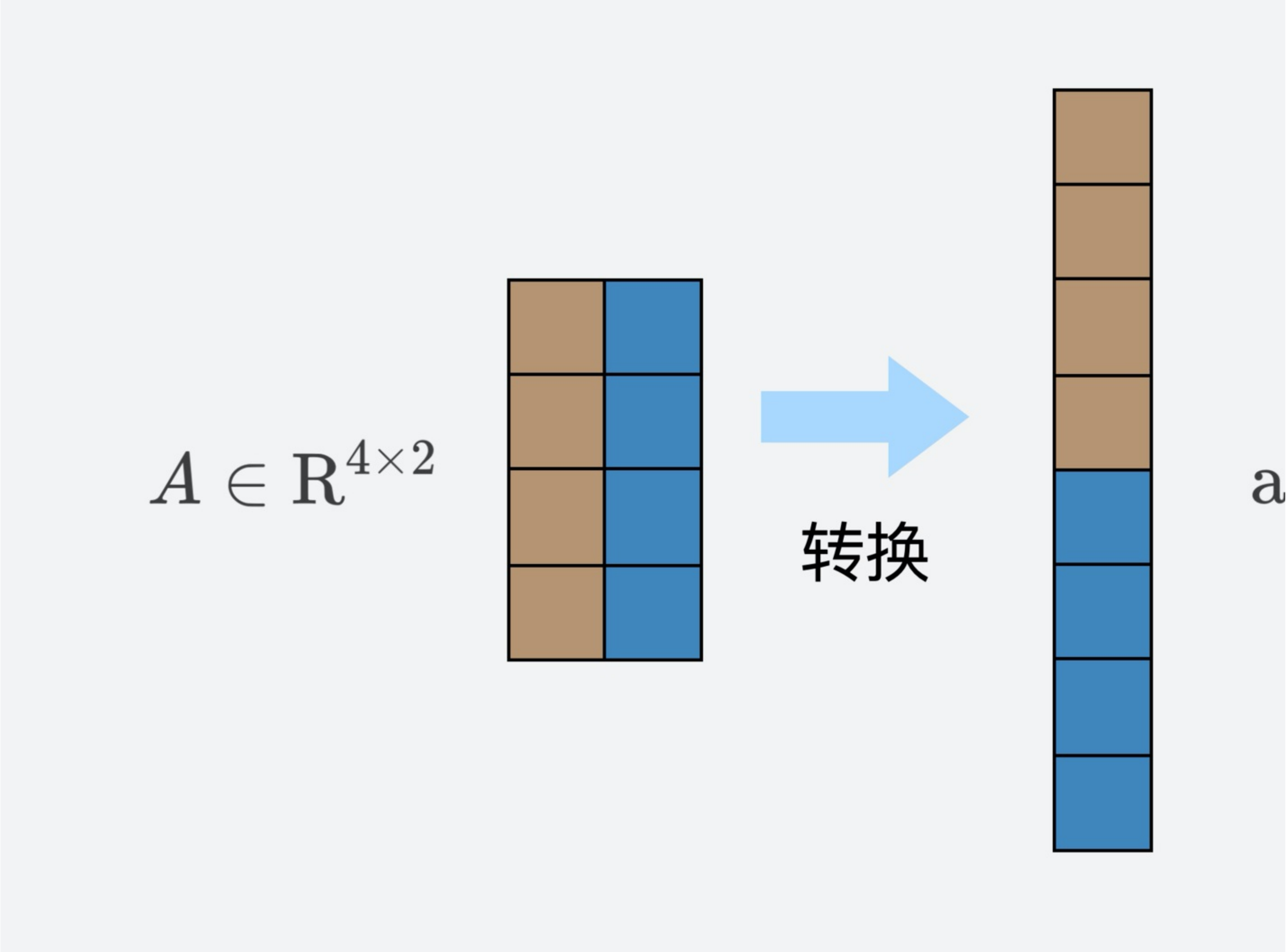
çŸ©é˜µçš„å®šä¹‰æ˜¯ï¼šä¸€ä¸ª(m, n)çŸ©é˜µAï¼Œæ˜¯ç"±$mÃ—n$ä¸ªå…ƒç´ ç»„æˆï¼Œmå'Œnæ˜¯å®žæ•°ï¼Œå…¶ä¸å…ƒç´ $a_{i j}, \mathrm{i}=1, \ldots, \mathrm{m}, \mathrm{j}=1, \ldots, \mathrm{n}$æŒ‰$m$è¡Œ$n$åˆ—çš„çŸ©å½¢æŽ'åˆ—ï¼Œå°±å¯ä»¥å½¢æˆçŸ©é˜µAï¼š

$$
A=\left[\begin{array}{cccc}
a_{11} & a_{12} & \ldots & a_{1 n} \\
a_{21} & a_{22} & \ldots & a_{2 n} \\
\ldots & \ldots & \ldots & \ldots \\
a_{m 1} & a_{m 2} & \ldots & a_{m n}
\end{array}\right]
$$

å…¶ä¸$a_{i j}$å±žäºŽå®žæ•°æˆ–å¤æ•°ï¼Œå¹¶ä¸"æˆ'ä»¬ç"¨çš„æ˜¯å®žæ•°Rï¼Œæ£å¦‚ä¸Šå›¾ä¸€æ ·ã€‚

å®šä¹‰å®ŒçŸ©é˜µåŽï¼Œæˆ'ä»¬ä»‹ç»ä¸€ä¸‹çŸ©é˜µè½¬æ¢ï¼ˆMatrix transformationï¼‰ã€‚çŸ©é˜µè½¬æ¢æ˜¯ä¸€ä¸ªå¾ˆé‡è¦çš„æ¦‚å¿µã€‚

æˆ'ä»¬ä¸€ä¸ªæ'¨ä¾‹ä¸€ä¸‹çŸ©é˜µçš„è½¬æ¢ã€‚è®¾$\mathrm{R}^{m \times n}$æ˜¯å®žæ•°çŸ©é˜µ(m, n)çš„é›†åˆï¼Œ$A \in \mathrm{R}^{m \times n}$å¯ä»¥è¡¨ç¤ºä¸ºå'é‡$a \in \mathrm{R}^{mn}$ ã€‚æˆ'ä»¬æŠŠçŸ©é˜µçš„nåˆ—å †å æˆä¸€ä¸ªé•¿å'é‡ï¼Œè¿™å°±æ˜¯reshapeã€‚



çŸ©é˜µçš„è¿ç®—

äº†è§£äº†çŸ©é˜µçš„åŸºæœ¬å®šä¹‰åŽï¼Œæˆ'ä»¬æ‰è¿›å…¥çŸ©é˜µçš„è¿ç®—çŽ¯èŠ‚ï¼Œå°±æ˜¯çŸ©é˜µçš„åŠ å'Œä¹˜ã€‚

åŠ è¿ç®—å¾ˆç®€å•ï¼Œä¸¤ä¸ªçŸ©é˜µ$A \in \mathrm{R}^{m \times n}$ï¼Œ$B \in \mathrm{R}^{m \times n}$çš„åŠ è¿ç®—å…¶å®žå°±æ˜¯çŸ©é˜µå„è‡ªå…ƒç´ çš„åŠ ã€‚

$$
A+B=\left[\begin{array}{ccc}
a_{11}+b_{11} & \ldots & a_{1 n}+b_{1 n} \\
\cdot & & \cdot \\
\cdot & & \cdot \\
\cdot & & \cdot \\
a_{m 1}+b_{m 1} & \ldots & a_{m n}+b_{m n}
\end{array}\right] \in R^{m \times n}
$$

æˆ'æŽ¨è使ç"¨NumPyçš„einsumæ¥é«˜æ•ˆåœ°åšè¿™ç±»è¿ç®—ï¼Œå›ä¸ºå®ƒåœ¨é€Ÿåº¦å'Œå†…å˜æ•ˆçŽ‡æ–¹é¢é€šå¸¸å¯ä»¥è¶…è¶Šæˆ'ä»¬å†™çš„arrayå‡½æ•°ã€‚

```
C= np.einsum('il, lj', A, B)
```

æŽ¥ä¸‹æ¥ï¼Œæˆ'ä»¬ä¸€èµ·æ¥çœ‹çœ‹çŸ©é˜µçš„ä¹˜ã€‚è¿™é‡Œä½ éœ€è¦æ³¨æ„ï¼ŒçŸ©é˜µçš„ä¹˜å'Œé€šå¸¸æˆ'ä»¬ç†ŸçŸ¥çš„ä¹˜æ³•ä¸ä¸€æ ·ï¼ŒçŸ©é˜µçš„ä¹˜æœ‰å¤šç§ç±»åž‹ï¼Œè¿™é‡Œæˆ'è®²ä¸‰ç§æœ€æ™®éï¼Œä¹Ÿæ˜¯åœ¨ä¸åŒé¢†åŸŸé‡Œç"¨å¾—æœ€å¤šçš„çŸ©é˜µä¹˜ã€‚

1.æ™®é€šçŸ©é˜µä¹˜

æ™®é€šçŸ©é˜µä¹˜æ˜¯å®šä¹‰åœ¨ä¸¤ä¸ªçŸ©é˜µä¹‹é—´çš„ä¹˜ï¼Œä¸¤ä¸ªçŸ©é˜µ$A \in \mathrm{R}^{m \times n}$ï¼Œ$B \in \mathrm{R}^{n \times k}$ï¼Œæ™®é€šçŸ©é˜µå'Œä¹˜å¯ä»¥è¡¨ç¤ºä¸º$C=A B \in R^{m \times k}$ï¼ŒCä¸å…ƒç´ çš„è®¡ç®—è§„å'Œæ˜¯çŸ©é˜µAã€Bå¯¹åº"ä¸¤ä¸¤å…ƒç´ ä¹˜ç§¯ä¹‹å'Œã€‚

$$
c_{i j}=\sum_{k=1}^{n} a_{i k} b_{k j}, i=1, \ldots, m, j=1, \ldots, l
$$

æˆ'ä»¬ä¸¾ä¸ªä¾‹åæ¥è¯´æ˜Žã€‚Cçš„ç¬¬ä¸€ä¸ªå…ƒç´ $c_{11}=a_{11} \times b_{11}+a_{12} \times b_{21}+a_{13} \times b_{31}=1 \times 1+2 \times 2+3 \times 3$ã€‚

$$
C=A B=\left[\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6
\end{array}\right]\left[\begin{array}{ll}
1 & 4 \\
2 & 5 \\
3 & 6
\end{array}\right]=\left[\begin{array}{lll}
1 \times 1+2 \times 2+3 \times 3 & 1 \times 4+2 \times 5+3 \times 6 \\
4 \times 1+5 \times 2+6 \times 3 & 4 \times 4+5 \times 5+6 \times 6
\end{array}\right]=\left[\begin{array}{cc}
14 & 32 \\
32 & 77
\end{array}\right]
$$

è¿™é‡Œéœ€è¦ç‰¹åˆ«æ³¨æ„çš„æ˜¯ï¼Œåªæœ‰ç›¸é‚»é˜¶æ•°åŒ¹é…çš„çŸ©é˜µæ‰èƒ½ç›¸ä¹˜ï¼Œä¾‹å¦‚ï¼Œä¸€ä¸ª$nÃ—k$çŸ©é˜µAå'Œä¸€ä¸ª$kÃ—m$çŸ©é˜µBç›¸ä¹˜ï¼Œæœ€åŽå¾—å‡º$nÃ—

m$çŸ©é˜µ$C$ï¼Œè€Œè¿™é‡Œçš„$k$å°±æ˜¯ç›¸é‚»é˜¶æ•°ã€‚

$$AB=C$$

ä½†åè¿‡æ¥BA’ŒAç›¸ä¹˜å°±ä¸è¡Œäº†ï¼Œå› ä¸ºç›¸é‚»é˜¶æ•°mä¸ç‰äºŽnã€‚

2.å“ˆè¾¾çŽ›ç§¯

å“ˆè¾¾çŽ›ç§¯ç†è§£èµ·æ¥å°±å¾ˆç®€å•äº†ï¼Œå°±æ˜¯çŸ©é˜µå„å¯¹åº”å…ƒç´ çš„ä¹˜ç§¯ï¼Œ$c_{ij}=a_{ij} Ã— b_{ij}$ ã€‚ä¸¾ä¸ªä¾‹åï¼š

$$
C=A^{*} B=\left[\begin{array} {ll}
1 & 2 \\
4 & 5
\end{array}\right]\left[\begin{array} {ll}
1 & 4 \\
2 & 5
\end{array}\right]=\left[\begin{array} {cc}
1 * 1 & 2 * 4 \\
4 * 2 & 5 * 5
\end{array}\right]=\left[\begin{array} {cc}
1 & 8 \\
8 & 25
\end{array}\right]
$$

å“ˆè¾¾çŽ›ç§¯å…¶å®žåœ¨æ°å¦ä¸€ä¸ªçœå°±ä¼šè¢«ç§°ä¸ºå“ˆè¾¾çŽ›ç§¯é‚£ä¸ªæœ‰ç”¨ï¼Œå› ä¸ºå®ƒå¯ä»¥ç”¨æ¥åšæ—¶é—´åºåˆ—çš„é¢„æµ‹ä¹Ÿå¯ä»¥ç”¨æ¥è®¡ç®—æ¯”çŽ‡å¼‚é«˜ã€‚

3.å…‹ç½—å†…å…‹ç§¯

å…‹ç½—å†…å…‹ç§¯æ˜¯ä»¥å¾·å›½æ•°å¦å®¶åˆ©å¥¥æ³¢å¾··å…‹ç½—å†…å…‹ï¼ˆLeopold Kroneckerï¼‰çš„åå—å‘½åçš„ã€‚å®ƒå¯ä»¥åº”ç”¨åœ¨è§£çº¿æ€§çŸ©é˜µæ–¹ç¨‹å’Œå›¾åƒå¤„ç†æ–¹é¢ï¼Œå½“ç„¶ä»Žæ›´æ·±å±‚çš„è§’åº¦è®²ï¼Œå®ƒè¿˜ä¼šåœ¨é‡åä¿¡æ¯é¢†åŸŸï¼Œæˆ‘ä»¬æš‚ä¸”ä¸è®¨è®ºã€‚

å’Œæ™®é€šçŸ©é˜µä¹˜ä¸Œå“ˆè¾¾çŽ›ç§¯ä¸åŒçš„æ˜¯ï¼Œå…‹ç½—å†…å…‹ç§¯æ˜¯ä¸¤ä¸ªä»»æ„å¤§å°çŸ©é˜µé—´çš„è¿ç®—ï¼Œè¡¨ç¤ºä¸º$AÃ—B$ï¼Œå¦‚æžœAæ˜¯ä¸€ä¸ª$m Ã— n$çš„çŸ©é˜µï¼Œè€ŒBæ˜¯ä¸€ä¸ª$pÃ—q$çš„çŸ©é˜µï¼Œå…‹ç½—å†…å…‹ç§¯åˆ™æ˜¯ä¸€ä¸ª$mpÃ—nq$çš„çŸ©é˜µã€‚

æŽ¥ä¸‹æ¥ä»‹ç»è¿ç®—ä¸å¾ˆé‡è¦çš„ä¸€ç±»**åœ¨çŸ©é˜µçš„ä¹˜æ³•ä¸èµ·ç‰¹æ®Šä½œç”¨**çš„çŸ©é˜µï¼Œå®ƒå°±æ˜¯**å•ä½çŸ©é˜µ**ã€‚é«˜ç‰ä»£æ•°ä¸ï¼Œåœ¨æ±‚è§£ç›¸åº”çš„çŸ©é˜µæ—¶ï¼Œè‹¥æ·»åŠ å•ä½çŸ©é˜µï¼Œé€šè¿‡åˆç‰å˜æ¢æ±‚è§£ä¼šæ›´ä¸ºæ–¹ä¾¿ï¼Œå®ƒæ˜¯ä¸ªä¸»å¯¹è§’çº¿ä¸Šçš„å…ƒç´ éƒ½ä¸º1ï¼Œè€Œå…¶ä½™å…ƒç´ å…¨ä¸º0ã€‚

åœ¨çº¿æ€§ä»£æ•°ä¸ï¼Œå¤§å°ä¸ºnçš„å•ä½çŸ©é˜µå°±æ˜¯åœ¨ä¸»å¯¹è§’çº¿ä¸Šå‡ä¸º1ï¼Œè€Œå…¶ä»–ä½ç½®å…¨éƒ½ä¸º0çš„$nÃ—n$çš„æ–¹é˜µï¼Œå®ƒç”¨I_{n}è¡¨ç¤ºï¼Œè¡¨è¾¾æ–¹å¼å¦‚ä¸‹ï¼Œå¯ä»¥ç®€å†™ä¸ºIæ¥è¡¨ç¤ºï¼š

$$
I_{1}=[1], I_{2}=\left[\begin{array} {ll}
1 & 0 \\
0 & 1
\end{array}\right], I_{3}=\left[\begin{array} {lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right], â€¦, I_{n}=\left[\begin{array} {cccc}
1 & 0 & â€¦ & 0 \\
0 & 1 & â€¦ & 0 \\
. & . & â€¦ & . \\
. & . & . & . \\
0 & 0 & â€¦ & 1
\end{array}\right]
$$

çŸ©é˜µçš„æ€§è´¨

åœ¨äº†è§£äº†çŸ©é˜µå¦‚ä½•å’Œå…¶ä»–å•ä½çŸ©é˜µåšäº†ä¹˜ç§¯ä¹‹åŽï¼Œæˆ‘ä»¬æ¥è®¨è®ºçŸ©é˜µçš„ä¸€äº›æ€§è´¨ã€‚äº†è§£çŸ©é˜µçš„æ€§è´¨å¯¹äºŽçŸ©é˜µè®¡ç®—éžå¸¸é‡è¦ï¼Œå°±åƒæˆ‘ä»¬å°æ—¶å€™å¦ä¹ ä¹˜æ³•å£è¯€å’Œä¹˜æ³•åˆ†é…å¾‹ç‰è¿ç®—æ³•åˆ™ä¸€æ ·ï¼Œäº†è§£ä»Žå¤´åˆ°å°¾å¾ˆæœ‰å¸®åŠ©ã€‚

1.ç»“åˆå¾‹

ä»»æ„å®žæ•°$mÃ—n$çŸ©é˜µAï¼Œ$nÃ—p$çŸ©é˜µBï¼Œ$pÃ—q$çŸ©é˜µCä¹‹é—´ç›¸ä¹˜ï¼Œæ»¡è¶³ç»“åˆå¾‹$(AB)C=A(BC)$ã€‚è¿™ä¸ªå¾ˆå¥½ç†è§£ï¼Œæˆ‘å°±ä¸å¤šè¯´äº†ã€‚

$$\forall A \in R^{m \times n}, B \in R^{n \times p}, C \in R^{p \times q}:(A B) C=A(B C)$$

2.åˆ†é…å¾‹

ä»»æ„å®žæ•°$mÃ—n$çŸ©é˜µAå’ŒBï¼Œ$nÃ—p$çŸ©é˜µCå’ŒDä¹‹é—´ç›¸ä¹˜ï¼Œæ»¡è¶³åˆ†é…å¾‹$(A+B)C=AC+BC$ï¼Œ$A(C+D)=AC+AD$ã€‚

$$
\forall \mathrm{A}, B \in \mathrm{R}^{m \times n}, C, D \in \mathrm{R}^{n \times p}:(A+B) C=A C+B C, A(C+D)=A C+A D
$$

3.å•ä½çŸ©é˜µä¹˜

ä»»æ„å®žæ•°$mÃ—n$çŸ©é˜µAå’Œå•ä½çŸ©é˜µä¹‹é—´çš„ä¹˜ï¼Œç‰äºŽå®ƒæœ¬èº«Aã€‚

$$
\forall A \in R^{m \times n}: I_{m} A=A I_{n}=A
$$

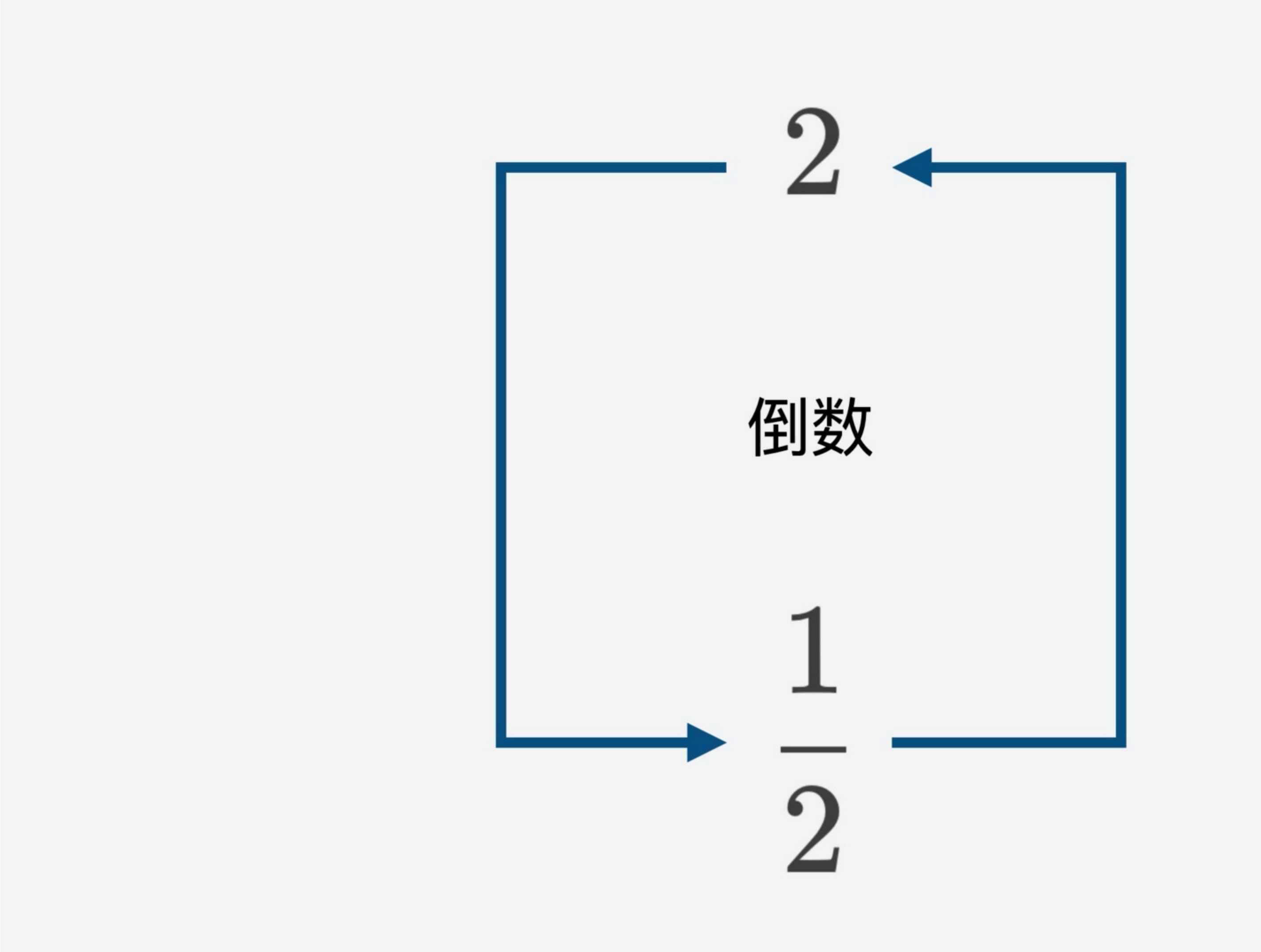
æ³¨æ„ï¼Œè¿™é‡Œçš„ï¼Œå’Œå¹¶ä¸ä¸€å®šç›¸ç‰ï¼Œ$m \neq n$æ—¶ï¼Œæ ¹æ®çŸ©é˜µä¹˜ç§¯ï¼Œå·¦ä¹˜å’Œå³ä¹˜å•ä½çŸ©é˜µä¸åŒï¼Œå³ä¹Ÿå°±æ˜¯$I_{m} \neq I_{n}$ã€‚

é€†çŸ©é˜µä¸Žè½¬ç½®çŸ©é˜µ

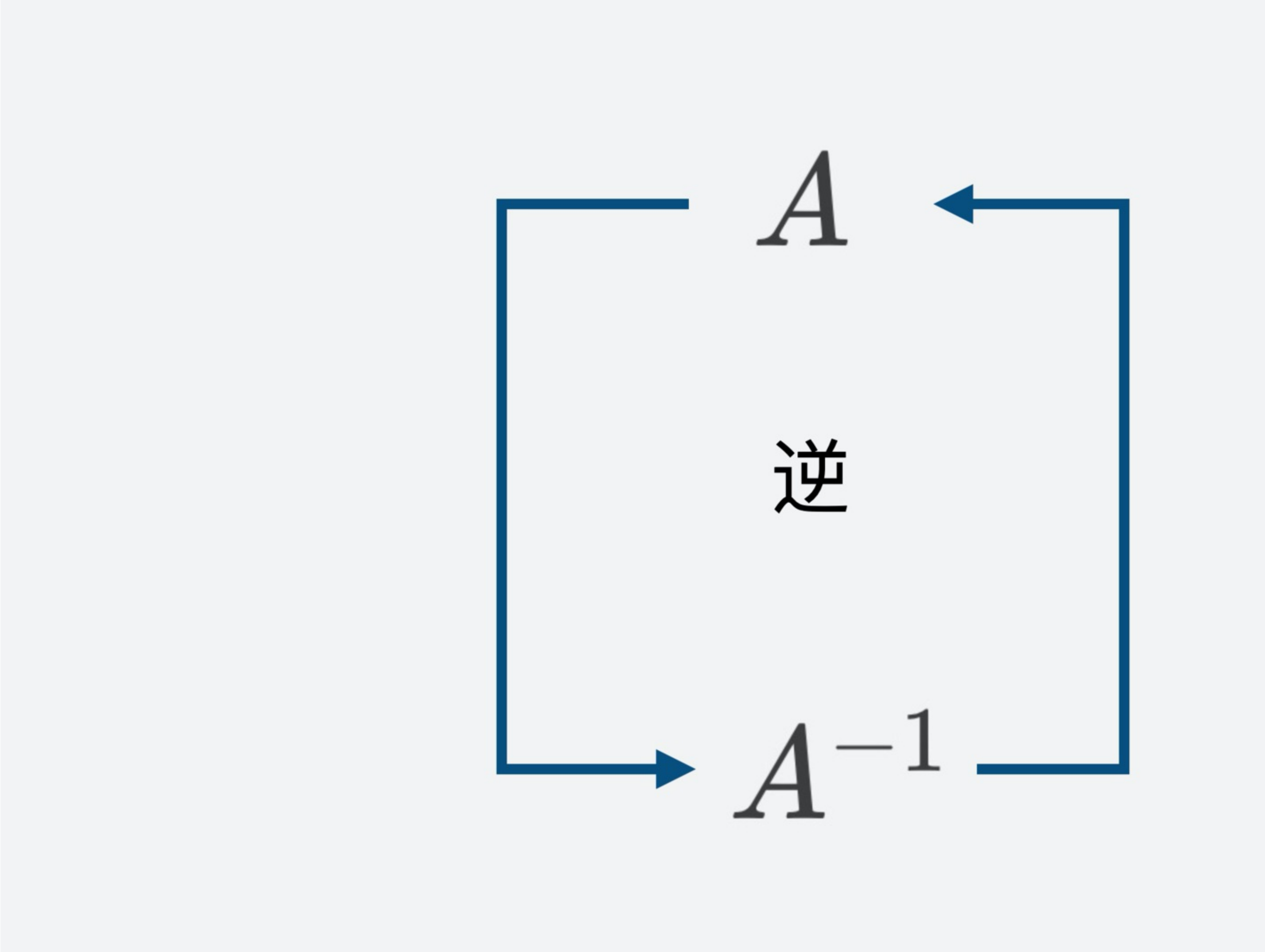
äº†è§£çŸ©é˜µçš„æ€§è´¨ä¹‹åŽï¼Œæˆ‘ä»¬å†æ¥è¯´è¯´é€†çŸ©é˜µå’Œè½¬ç½®çŸ©é˜µã€‚é€†çŸ©é˜µå’Œè½¬ç½®çŸ©é˜µåœ¨å®žé™…åº”ç”¨ä¸ä¹Ÿå¾ˆé‡è¦ï¼Œæ¯”å¦‚ï¼Œæˆ‘ä»¬ä¼šåœ¨ä¹‹åŽçš„ç« èŠ‚é‡Œè®²è§£å…¶ç”¨æ³•ã€‚

é€†çŸ©é˜µ

ä¸‹é¢è¿™ä¸ªä¾‹ååº”è¯¥é½å®¶éƒ½å¾ˆç†Ÿæ‚‰ï¼Œå³ä¸€ä¸ªæ•°2çš„å€’æ•°æ˜¯$\frac{1}{2}$ï¼Œ$\frac{1}{2}$çš„å€’æ•°æ˜¯2ã€‚



å…¶å®žé€†çŸ©é˜µä¹Ÿæœ‰ç€ç±»ä¼¼çš„æ¦‚å¿µï¼Œåªä¸è¿‡æ˜¯å†™æ³•ä¸ä¸€æ ·ï¼Œæˆ‘ä»¬æŠŠé€†çŸ©é˜µå†™æˆA^{-1}ã€‚é‚£ä¸ºä»€ä¹ˆä¸æ˜¯$\frac{1}{A}$å‘¢ï¼Ÿé‚£æ˜¯å› ä¸ºæ•°å¦æ— æ³•è¢«çŸ©é˜µé™¤ã€‚



æˆ'ä»¬çŸ¥é"ï¼Œ2ä¹˜ä»¥å®ƒçš„å€'æ•°$\frac{1}{2}$ç‰äºŽ1ã€‚åŒæ ·çš„é"ç†ï¼ŒAä¹˜ä»¥å®ƒçš„é€†çŸ©é˜µA^{-1}å°±ç‰äºŽå•ä½çŸ©é˜µï¼Œå³$\mathrm{A} \times A^{-1}=\mathrm{I}$ï¼ˆ$I$å³å•ä½çŸ©é˜µï¼‰ï¼Œåè¿‡æ¥ä¹Ÿä¸€æ ·ï¼Œ$\mathrm{A}^{-1} \times A=\mathrm{I}$ã€‚

ä¸ºæ–¹ä¾¿ç†è§£ï¼Œæˆ'ç"¨ä¸€ä¸ª2×2çŸ©é˜µAæ¥è§£é‡Šä¸€ä¸‹é€†çŸ©é˜µçš„ç®—æ³•ã€‚é¦–å…ˆï¼Œæˆ'ä»¬äº¤æ¢a_{11}å'Œa_{22}çš„ä½ç½®ï¼Œç„¶åŽåœ¨a_{12}å'Œa_{21}å‰åŠ ä¸Šè´Ÿå·ï¼Œæœ€åŽé™¤ä»¥è¡Œåˆ—å¼$a_{11} a_{22}-a_{12} a_{21}$ã€‚

$$
A^{-1}=\left[\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right]^{-1}=\frac{1}{a_{11} a_{22}-a_{12} a_{21}}\left[\begin{array}{cc}
a_{22} & -a_{12} \\
-a_{21} & a_{11}
\end{array}\right]
$$

é‚£æˆ'ä»¬è¯¥å¦‚ä½•éªŒè¯è¿™æ˜¯ä¸æ˜¯æ£è§£å'¢ï¼Ÿ

æ–¹æ³•å…¶å®žå¾ˆç®€å•ï¼Œè®°å¾—åˆšæ‰çš„å…¬å¼å°±è¡Œï¼Œ$\mathrm{A} \times A^{-1}=\mathrm{I}$ã€‚çŽ°åœ¨æˆ'ä»¬å°±ä»£å…¥å…¬å¼éªŒè¯ä¸€ä¸‹ï¼ŒAå'Œå®ƒçš„é€†çŸ©é˜µç›¸ä¹˜ï¼Œé€šè¿‡åˆšæ‰çš„ç®—æ³•æœ€ç»ˆå¾—å‡ºçš„ç»“æžœæ˜¯å•ä½çŸ©é˜µã€‚

$$
A \times A^{-1}=\left[\begin{array}{llll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right]\left[\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right]^{-1}=\left[\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right]\left[\begin{array}{lll}
\frac{a_{22}}{a_{11} a_{22}-a_{12} a_{21}} & \frac{-a_{12}}{a_{11} a_{22}-a_{12} a_{21}} \\
\frac{-a_{21}}{a_{11} a_{22}-a_{12} a_{21}} & \frac{a_{11}}{a_{11} a_{22}-a_{12} a_{21}}
\end{array}\right]=\left[\begin{array}{ll}
\frac{a_{11} \times a_{22}}{a_{11} a_{22}-a_{12} a_{21}}+\frac{a_{12} Ã—(-a_{21})}{a_{11} a_{22}-a_{12} a_{21}} & \frac{a_{11} Ã—(-a_{12})}{a_{11} a_{22}-a_{12} a_{21}}+\frac{a_{12} \times a_{11}}{a_{11} a_{22}-a_{12} a_{21}} \\
\frac{a_{21} \times a_{22}}{a_{11} a_{22}-a_{12} a_{21}}+\frac{a_{22} Ã—(-a_{21})}{a_{11} a_{22}-a_{12} a_{21}} & \frac{a_{21} Ã—(-a_{12})}{a_{11} a_{22}-a_{12} a_{21}}+\frac{a_{22} Ã— a_{11}}{a_{11} a_{22}-a_{12} a_{21}}
\end{array}\right]=\left[\begin{array}{lll}
1 & 0 \\
0 & 1
\end{array}\right]
$$

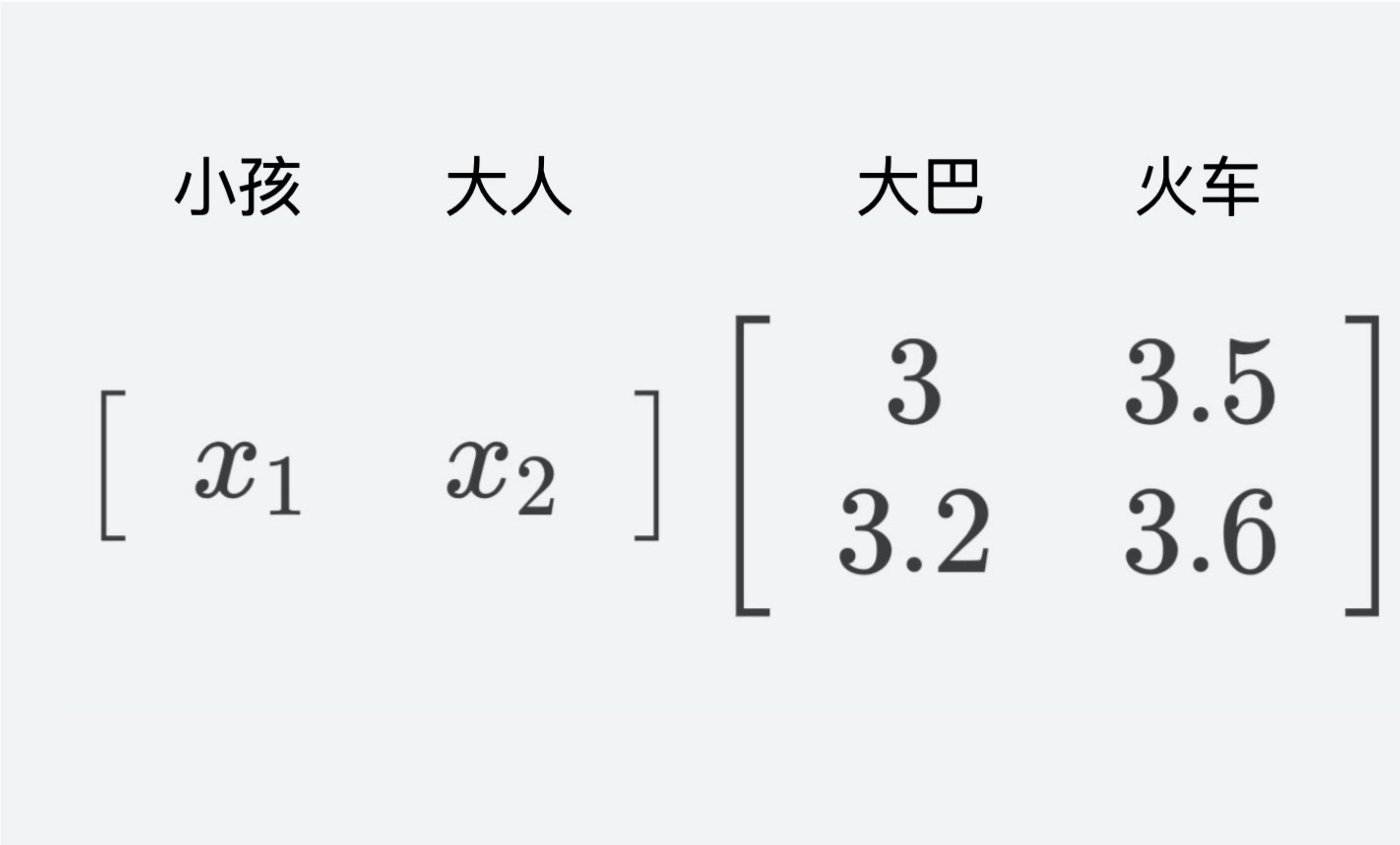
è¿™é‡Œæœ‰ä¸€ç‚¹éœ€è¦ç‰¹åˆ«è¯´æ˜Žï¼Œä¸æ˜¯æ‰€æœ‰çŸ©é˜µéƒ½æ˜¯å¯é€†çš„ã€‚å¦‚æžœä¸€ä¸ªçŸ©é˜µæ˜¯å¯é€†çš„ï¼Œé‚£è¿™ä¸ªçŸ©é˜µæˆ'ä»¬å«å**éžå¥‡å¼‚çŸ©é˜µ**ï¼Œå¦‚æžœä¸€ä¸ªçŸ©é˜µæ˜¯ä¸å¯é€†çš„ï¼Œé‚£è¿™ä¸ªçŸ©é˜µæˆ'ä»¬å°±å«å**å¥‡å¼‚çŸ©**
é˜µï¼Œæ˜¯ä¸€ä¸ªç‰¹æ®Šçš„ä¾‹åã€‚

è¿˜è®°å¾—è¡Œåˆ—å¼$a_{11} a_{22}-a_{12} a_{21}$å—ï¼Ÿå¦‚æžœæˆ'ä»¬è¦è¯æ˜ŽçŸ©é˜µæ˜¯å¯é€†çš„ï¼Œåªè¦è¯æ˜Žè¡Œåˆ—å¼ä¸ç‰äºŽé›¶å°±è¡Œã€‚æ›´é«˜é˜¶çš„é€†çŸ©é˜µçš„ç®—æ³•ä¹Ÿæ˜¯ä¸€æ ·çš„åŽŸç†ã€‚

æœ€åŽï¼Œæˆ‘æƒ³é€šè¿‡ä¸€ä¸ªçŽ°å®žç”Ÿæ´»ä¸çš„æ¡ˆä¾‹æ¥è®©ä½ æ›´å¥½åœ°ç†è§£é€†çŸ©é˜µã€‚

ä¸€ä¸ªæ—…æ¸¸å›¢ç”±å©åå'Œå¤§äººç»„æˆï¼ŒåŽ»ç¨‹ä»–ä»¬ä¸€èµ·åå¤§å·´ï¼Œæ¯ä¸ªå°å©ç¥¨ä»·3å…ƒï¼Œå¤§äººç¥¨ä»·3.2å…ƒï¼Œæ€»å…±èŠ±è´¹$118.4$å…ƒã€‚å›žç¨‹ä¸€èµ·åç«è½¦ï¼Œæ¯ä¸ªå°å©ç¥¨ä»·3.5å…ƒï¼Œå¤§äººç¥¨ä»·3.6å…ƒï¼Œæ€»å…±èŠ±è´¹$135.2$å…ƒã€‚è¯·é—®æ—…æ¸¸å›¢ä¸æœ‰å¤šå°‘å°å©å'Œå¤§äººï¼Ÿ

é¦–å…ˆï¼Œæˆ‘ä»¬è®¾ç½®ä¸€äº›çŸ©é˜µï¼Œç»„æˆæ–¹ç¨‹$XA=B$ã€‚



è¦è§£Xï¼Œæˆ‘ä»¬éœ€è¦å…ˆè®¡ç®—Açš„é€†çŸ©é˜µA^{-1}ï¼š

$$
A^{-1}=\left[\begin{array}{cc}
3 & 3.5 \\
3.2 & 3.6
\end{array}\right]^{-1}=\frac{1}{3 \times 3.6-3.5 \times 3.2}\left[\begin{array}{cc}
3.6 & -3.5 \\
-3.2 & 3
\end{array}\right]=\left[\begin{array}{cc}
-9 & 8.75 \\
8 & -7.5
\end{array}\right]
$$

æŽ¥ä¸‹æ¥è®¡ç®—$X=B A^{-1}$ï¼š

$$
\left[\begin{array}{ll}
x_{1} & x_{2}
\end{array}\right]=\left[\begin{array}{ll}
118.4 & 135.2
\end{array}\right]\left[\begin{array}{cc}
-9 & 8.75 \\
8 & -7.5
\end{array}\right]=\left[\begin{array}{ll}
16 & 22
\end{array}\right]
$$

æœ€ç»ˆï¼Œæˆ‘ä»¬å¾—å‡ºè¿™ä¸ªæ—…æ¸¸å›¢æœ‰16ä¸ªå°å©å'Œ22ä¸ªå¤§äººã€‚

è¿™ä¹Ÿæ˜¯è§£çº¿æ€§æ–¹ç¨‹ç»„çš„ä¸€ç§æ–¹æ³•ï¼Œç±»ä¼¼è¿™æ ·çš„è®¡ç®—è¢«å¹¿æ³›åº"ç"¨åœ¨å„ç§é¢†åŸŸä¸ï¼Œæ¯"å¦‚åºç"¨å·¥ç¨‹è®¾è®¡ã€æ¸¸æˆå¼€å'ç"Ÿç‰©ä¿¡æ¯ç‰ï¼Œä¾‹å¦‚æœºå™¨äººåŠ¨ä½œæŽ§åˆ¶ã€3Dæ¸¸æˆä¸çš„å›¾å½¢å¤„ç†ç‰ã€‚

Last but not leastï¼Œæ–¹ç¨‹æ±‚è§£æ—¶è¦ç‰¹åˆ«æ³¨æ„ï¼Œ$AX=B$å'Œ$XA=B$çš„ç»"æžœæ˜¯ä¸åŒçš„ï¼Œè¿™ä¸ªä¸€å®šè¦è®°ä½ï¼

è½¬ç½®çŸ©é˜µ

ä¸€è¯´ä½ éƒ½æ‡‚ï¼Œè½¬ç½®çŸ©é˜µå°±æ˜¯æŠŠä¸€ä¸ªçŸ©é˜µçš„è¡Œå'Œåˆ—äº¤æ¢ã€‚

å°†çŸ©é˜µçš„è¡Œåˆ—äº'æ¢ï¼Œå¾—åˆ°çš„æ–°çŸ©é˜µå°±å«åšè½¬ç½®çŸ©é˜µï¼ˆtransposeï¼‰ã€‚è½¬ç½®çŸ©é˜µçš„è¡Œåˆ—å¼ä¸å˜ã€‚æˆ'ä»¬æŠŠ$mÃ—n$çŸ©é˜µAçš„è¡Œåˆ—äº'æ¢ï¼Œå¾—åˆ°è½¬ç½®çŸ©é˜µA^{T}ã€‚

$$
A=\left[\begin{array}{cccc}
a_{11} & a_{12} & \ldots & a_{1 n} \\
a_{21} & a_{22} & \ldots & a_{2 n} \\
\ldots & \ldots & \ldots & \ldots \\
a_{m 1} & a_{m 2} & \ldots & a_{m n}
\end{array}\right]
$$

$$

A^{T}=\left[\begin{array}{cccc}
a_{11} & a_{21} & \ldots & a_{m1} \\
a_{12} & a_{22} & \ldots & a_{m2} \\
\ldots & \ldots & \ldots & \ldots \\
a_{1 n} & a_{2 n} & \ldots & a_{mn}
\end{array}\right]
$$

æœ€åŽï¼Œä¸ºäº†æ–¹ä¾¿ä½ ç†è§£ï¼Œæˆ‘ä»¬å†æ€»ç»“ä¸€ä¸‹é€†çŸ©é˜µå’Œè½¬ç½®çŸ©é˜µçš„æ€§è´¨ã€‚ä½ ä¸ç”¨æ»è®°ç¡¬èƒŒï¼Œéšåœ¨ç†è§£ã€‚

1. çŸ©é˜µå’Œè‡ªèº«é€†çŸ©é˜µç›¸ä¹˜å¾—å•ä½çŸ©é˜µï¼Œ$A A^{-1}=I=A^{-1} A$ï¼›
2. $A$$B$ä¸¤çŸ©é˜µç›¸ä¹˜çš„é€†ï¼Œç‰äºŽé€†çŸ©é˜µBå’Œé€†çŸ©é˜µAç›¸ä¹˜ï¼Œè¿™é‡Œå¼ºè°ƒä¸€ä¸‹ä¹˜çš„é¡ºåºå¾ˆé‡è¦ï¼Œ$(A B)^{-1}=B^{-1} A^{-1}$ï¼›
3. ABä¸¤çŸ©é˜µç›¸åŠ åŽçš„é€†çŸ©é˜µï¼Œä¸ç‰äºŽå„è‡ªé€†çŸ©é˜µçš„ç›¸åŠ ï¼Œ$(A+B)^{-1} \neq A^{-1}+B^{-1}$ï¼›
4. çŸ©é˜µè½¬ç½®çš„è½¬ç½®è¿˜æ˜¯å®ƒæœ¬èº«ï¼Œ$\left(A^{T}\right)^{\mathrm{T}}=A$ï¼›
5. ABä¸¤çŸ©é˜µç›¸åŠ åŽçš„è½¬ç½®çŸ©é˜µï¼Œç‰äºŽå„è‡ªè½¬ç½®çŸ©é˜µçš„ç›¸åŠ ï¼Œ$(A+B)^{T}=A^{T}+B^{T}$ï¼›
6. ABä¸¤çŸ©é˜µç›¸ä¹˜åŽçš„è½¬ç½®çŸ©é˜µï¼Œç‰äºŽè½¬ç½®çŸ©é˜µBå’Œè½¬ç½®çŸ©é˜µAçš„ç›¸ä¹˜ï¼Œè¿™é‡Œå†æ¬¡å¼ºè°ƒä¹˜çš„é¡ºåºå¾ˆé‡è¦ï¼Œ$(A B)^{T}=B^{T} A^{T}$ã€‚

æœ¬èŠ‚å°ç»“

å¥½äº†ï¼Œåˆ°è¿™é‡ŒçŸ©é˜µè¿™ä¸€è®²å°±ç»“æŸäº†ï¼Œæœ€åŽæˆ‘æ¥å¸¦ä½ æ€»ç»“ä¸€ä¸‹å‰é¢è®²è§£çš„å†…å®¹ã€‚

ä»Šå¤©çš„çŸ¥è¯†ï¼Œä½ å¯ä»¥ä»ŽçŸ©é˜µæ˜¯çº¿æ€§æ–¹ç¨‹ç»„çš„å¦ä¸€ç§è¡¨è¾¾ï¼Œä»¥è§£å†³æ–¹ç¨‹ç»„é—®é¢˜ä¸ºå¼€å§‹ï¼Œç„¶åŽè¿›ä¸€æ¥å¦ä¹ äº†çŸ©é˜µçš„å®šä¹‰å’Œæ€§è´¨ã€‚å…¶ä¸ï¼ŒçŸ©é˜µä¹˜æ³•æ˜¯é‡ç‚¹å†…å®¹ï¼Œå°½ç®¡æ‰‹ç®—ä¸æ˜¯è®²è§£çš„ä¸»è¦ç›®çš„ï¼Œä½†ä½ è¿˜æ˜¯éœ€è¦è¿žåŒä¸‰ç§çŸ©é˜µä¹˜çš„æ–¹å¼å›žåŽ»åŠ¨æ‰‹ç®—ä¸€ä¸‹ï¼Œè¿™æ ·æ‰èƒ½çœŸæ£ç†è§£çº¿æ€§ä»£æ•°çš„æ€è·¯ã€‚

çº¿æ€§ä»£æ•°ç»ƒä¹ åœº

å¯¹äºŽ10ç»´å‘é‡$x=\left(x_{1}, \ldots, x_{10}\right)^{T}$ï¼Œ$v=\left(v_{1}, \ldots, v_{10}\right)^{T}$ï¼Œå¦‚æžœè¦è®¡ç®—$y=x x^{T}\left(I+v v^{T}\right) x$ï¼Œå…¶ä¸$I$æ˜¯10é˜¶å•ä½çŸ©é˜µã€‚ä½ ä¼šæ€Žä¹ˆåšï¼Ÿ

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