Week 2 Hand-in

NumIntro 2019

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1. Exercise

The decimal number $\frac{4}{5}$ cannot be displayed in binary with a finite amount of numbers. It looks as follows:

$$
\begin{aligned}
\frac{4}{5} &= 0.8_{10} \\
0.8 * 2 &= 1.6, \text{Integral part} = 1 \\
0.6 * 2 &= 1.2, \text{Integral part} = 1 \\
0.2 * 2 &= 0.4, \text{Integral part} = 0 \\
0.4 * 2 &= 0.8, \text{Integral part} = 0 \\
0.8 * 2 &= 1.6, \text{Integral part} = 1
\end{aligned}
$$

We can see that it begins repeating here, so the binary representation will look as follows:

$$0.\overline{1100}$$

Meaning that the sequence "1100" repeats forever.

2. Exercise

(a)

10^{40} is not a machine number in Marc-32, since the number is larger than what can be represented. The max value is roughly $3.4 * 10^{38}$.

(b)

$2^{-1} + 2^{-29}$ is not a machine number in Marc-32, due to the fact that numbers with more than 6 decimal digits are approximated, because there are no more than 23 bits in the mantissa part of Marc-32 representation.

(c)

$\frac{1}{3}$ is not a machine number in Marc-32, because it is impossible for it to represent infinitely repeating numbers ($0.\overline{0101}_2$) accurately.

(d)

$\frac{1}{5}$ is not a machine number in Marc-32, because it is impossible for it to represent infinitely repeating numbers ($0.\overline{0011}_2$) accurately.

3. Exercise

(a)

$$11001_2 = (1 * 2^4) + (1 * 2^3) + (0 * 2^2) + (0 * 2^1) + (1 * 2^0) = 25_{10}$$

(b)

$$\begin{aligned} 1101.001_2 &= \\ &= (1 * 2^3) + (1 * 2^2) + (0 * 2^1) + (1 * 2^0) + (0 * 2^{-1}) + (0 * 2^{-2}) + (1 * 2^{-3}) \\ &= 13.125_{10} \end{aligned}$$

4. Exercise

This exercise has been submitted on Absalon under Q0008

5. Exercise

To show this approximation, we first use a truncated Taylor series for $cos(x)$. We can do this because $x \to 0$:

$$cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$$

Then we can say:

$$1 \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$$

This means that our error is:

$$\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$$

We can then solve:

$$\frac{1}{2} * 10^{-8} = \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$$

The solution for this approximates to $x \approx 0.0001$. This means that for the approximation $cos(x) \approx 1$, to be accurate, x would have to be less than or equal to roughly 0.0001.

6. Exercise

This exercise has been submitted on Absalon under Q0006