MASD Final Exam

MASD 2018
Department of Computer Science
University of Copenhagen

Eksamensnummer: 35

Version 1
Due: November 4th, 2018, 16:00

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(a)

The expression *pmf* stands for the "probability mass function".
The *pmf* of $X \sim Poiss(\lambda)$ can be written as follows, as per table 1.1 in the "MASD_essentials_ch1-updated.pdf" on Absalon:

$$p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

This expression is used to calculate the probability of x events happening, when there is an average of λ events happening.

(b)

The expectation of Y, $\mathbb{E}Y$ can be written as follows, as per definition 1.8 in the "MASD_essentials_ch1-updated.pdf" on Absalon:

$$\mathbb{E}Y = \sum_y yp(y)$$

The variance of Y, $var(Y)$, can be written as follows, as per definition 1.10 in the "MASD_essentials_ch1-updated.pdf" on Absalon:

$$var(Y) = \mathbb{E}(Y - \mathbb{E}Y)^2 = \sum_y y^2 p(y) - (\sum_y yp(y))^2$$

To find $\mathbb{P}(a < V \leq b)$ we first have to make the observation that the cdf $F_V(x)$ is defined as being the probability that the variable V will be *less than or equal to* x, thus we can say that:

$$F_V(x) = \mathbb{P}(V \leq x)$$

This means that to calculate the probability of it being inside the interval $a < V \leq b$, we calculate $\mathbb{P}(V \leq b)$, and subtract $\mathbb{P}(V \leq a)$.

$$\mathbb{P}(a < V \leq b) = F_V(a) - F_V(b)$$

4. Exercise

(a)

For this, I assume that the given values for x and y are all the values possible.
If that assumption is true, then the final joint pmf will look as follows:

$p(x,y)$	y=-1	y=1	y=3
x=-1	0.1	0.1	0.4
x = 1	0.3	0.05	0.05

The reason for this is because the joint pmf must be equal to 1. So we simply calculate:

$$1 - (0.1 + 0.1 + 0.3 + 2(0.05)) = 0.4$$

(b)

To compute $\mathbb{E}(X+Y)$ we have to make the following observation:

$$\mathbb{E}(X+Y) = \mathbb{E}X + \mathbb{E}Y$$

This means that we simply have to compute:

$$\mathbb{E}X + \mathbb{E}Y = \sum_x x p_X(x) + \sum_y y p_Y(y)$$

To find the marginal pmfs $p_X(x)$ and $p_Y(y)$ we use the following calculations:

$$p_X(x) = \sum_y p_{X,Y}(x,y)$$

$$p_Y(y) = \sum_x p_{X,Y}(x,y)$$

First we will find $\mathbb{E}X$, then $\mathbb{E}Y$:

$$\mathbb{E}X = -1(0.1 + 0.1 + 0.4) + 1(0.3 + 0.05 + 0.05) = -0.2$$

$$\mathbb{E}Y = -1(0.1 + 0.3) + 1(0.1 + 0.05) + 3(0.4 + 0.05) = 1.1$$

Then we simply write:

$$\mathbb{E}(X+Y) = -0.2 + 1.1 = 0.9$$

This should be interpreted as expecting to make back 90% of the money put in.

(c)

First we will compute the pmf p_Z of $Z = X * Y$.
To do this, first we will define what p_Z looks like:

$$p_Z(z) = \mathbb{P}(Z = z)$$

We can see from the table from Exercise 4a that there are six possible values for Z, since there are 2 values for X and 3 for Y. These values are the following:

$X * Y$	$y=-1$	$y=1$	$y=3$
$x=-1$	1	-1	-3
$x=1$	-1	1	3

Now we simply compute $\mathbb{P}(X * Y)$ for each value of x and y, and get the following table:

$p_Z(z)$	$\mathbb{P}(X * Y = z)$
$z = -3$	0.4
$z = -1$	0.4
$z = 1$	0.15
$z = 3$	0.05

To figure out if X and Y are independent, we have to look at their covariance, which is defined as thus:

$$cov(X, Y) = \mathbb{E}XY - \mathbb{E}X\mathbb{E}Y$$

We start off by finding $\mathbb{E}XY$:

$$\mathbb{E}XY = \sum_{x,y} xyp(x, y)$$

I have done this calculation in the attached Jupyter Notebook named: "Calculations" since it would be tedious to do by hand. It has given me the following:

$$\mathbb{E}XY = -1.3$$

We have already calculated $\mathbb{E}X$ and $\mathbb{E}Y$ further up, in exercise 4b. So we can write the following:

$$cov(X, Y) = (-1.3) - (-0.2) * 1.1 = -1.08$$

Since we now know that the covariance $cov(X, Y) \neq 0$, we also know that X and Y are dependant on each other.