

Magnetic Torque and Magnetic Force

Equipment- magnetic torque apparatus, calipers, scale

1 Introduction

In modern physics, the particles that make up matter, such as the electron, proton, and neutron, often have magnetic moments and angular momentum (spin). These vectors are parallel or anti-parallel. How these particles behave in uniform and inhomogeneous magnetic fields is of great interest. In a uniform magnetic field there is no net force on the particle, but there is a torque and the angular momentum will, in some sense, precess about the magnetic field. Quantum mechanics tells us that the projection of the angular momentum along the magnetic field can only be certain discrete values. By supplying a magnetic field of the right frequency it is possible to make the particle change its angular momentum projection, or "flip" the spin. In an inhomogeneous magnetic field, a particle with a magnetic moment experiences a net force. This enables a beam of particles to be deflected, such as in the Stern-Gerlach experiment.

This experiment does not deal with particles but with two macroscopic magnets. One magnet is just a bare magnet attached to a spring. When a uniform or inhomogeneous magnetic field is applied, the force on the magnet can be determined by measuring the deflection of the spring. The second magnet is buried at the center of a cue ball. The cue ball has a small handle, and for some of the experiments the handle is used to give the ball some rotation, and therefore angular momentum.

Many features of your experiments with these macroscopic magnets will be similar to features of experiments with elementary particles. Some features will be different. The quantization of the angular projection along the magnetic field will not be observable, but does play a major role in particle experiments.

2 Apparatus Overview

This section will give you an idea of what the apparatus looks like. Fig. 0 shows the apparatus with the cue ball. The cue ball sits in a spherical well that has a hole in the bottom through which air is pumped. The cue ball sits on a cushion of air that has very little friction. The black handle on the cue ball is clearly visible. The cue ball sits at the center of two coils that are connected in series. A power supply provides current to the coils, and the current can be read by a meter on the power supply. Depending on the coil connections, the field supplied by the coils can be very uniform or very non-uniform. A switch on the power supply makes the choice. There is a stroboscope mounted on the top coil that can be directed at a white spot on the handle of the cue ball. The stroboscope can be used to measure the rotation frequency of the cue ball. It is also possible to insert a rod that has a sliding weight into the cue ball. See Fig. 1. In this way a gravitational torque can be applied to the cue ball. If the cue ball is removed, a plastic tower can be inserted into the apparatus. See Fig. 5. This allows a magnet on a gimbal to be suspended from a spring in the magnetic field.

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Instruments Designed For Teaching

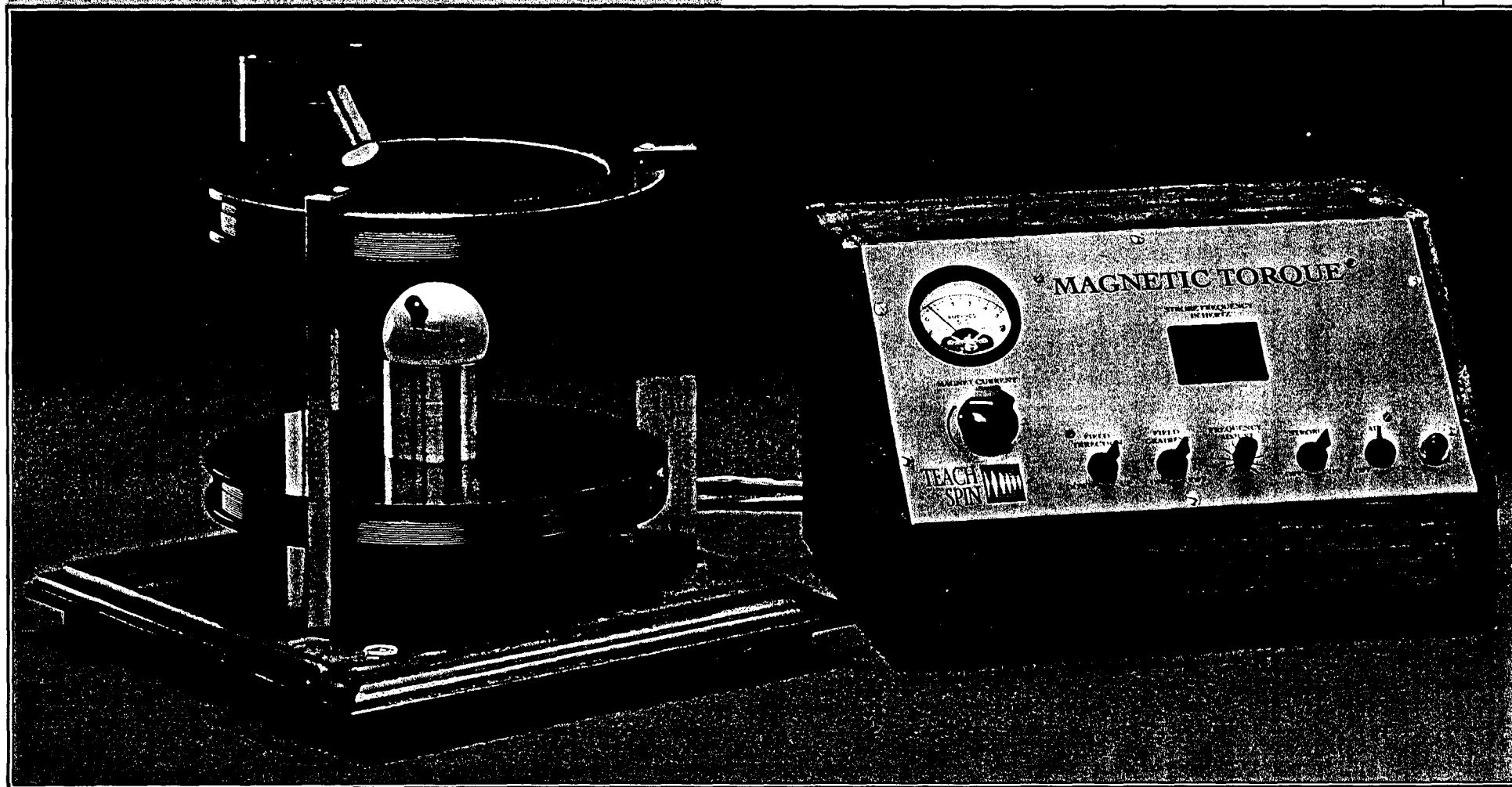


FIG. 0

MAGNETIC TORQUE - "A NEW CLASSIC"

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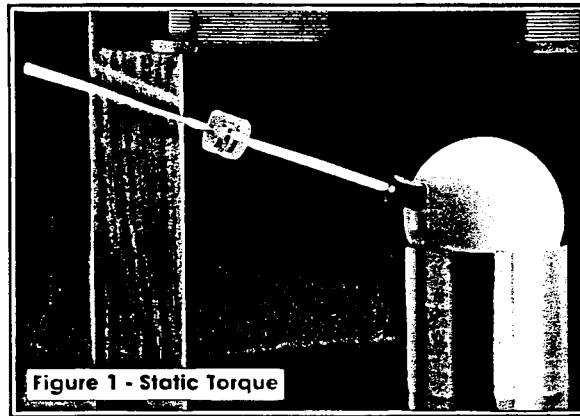


Figure 1 - Static Torque

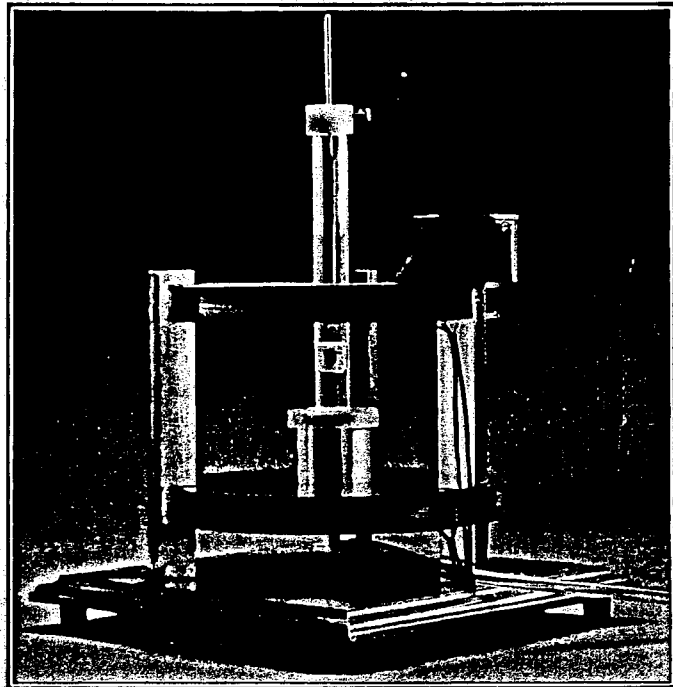
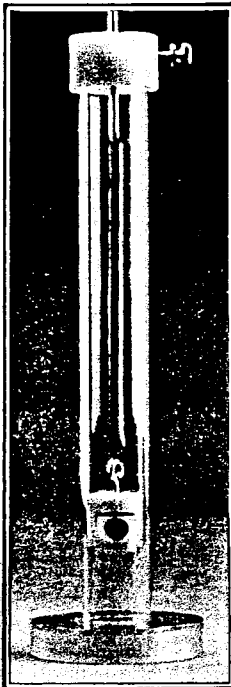


Figure 5 - Magnetic Force Apparatus with Detail of Gimbal and Calibrated Spring

The Instrument

This section of the manual provides a description of the various component parts.

A. The Magnet

What we refer to as "the magnet" is the component of the instrument that houses the two co-axial coils, the air bearing, and the strobe light.

1. -- Coils

The coils are composed of #18 copper wire that is wound on phenolic bobbins. **Each coil has 195 turns.** The two coils are always connected in series so that the same current flows through each turn. This current is displayed on the analog ammeter. It is important to note that the coils have some resistance, and that resistance is temperature-dependent. If significant current (~3-4 amps), is allowed to flow through the coils for a long time, the coils' temperature begins to rise. You can feel the increase in temperature. As the copper heats up, its resistance increases, and the current subsequently decreases, since the power supply is **not** current-regulated. It is therefore a good idea to have the

students turn the current down to zero when the magnet is not being used. Try to avoid using high currents for any appreciable length of time. The instrument is designed to sustain the full output power of the supply without any danger. However, since the maximum current will be decreased as the temperature of the coils increases, students may not be able to obtain the highest fields if they allow the coils to get too hot.

In order to calculate the magnetic field at the center of the apparatus, one needs to perform an integral, since each of the turns has a different radius and a different distance from the center of the pair. Such a calculation may be a bit tedious for students, so we have calculated an equivalent radius and equivalent distance between the two coils, such that each coil can be represented by a single turn whose radius and separation are:

$$\text{equivalent radius } \langle r \rangle = 0.109 \text{ m}$$

$$\text{equivalent separation between the coils } \langle d \rangle = 0.138 \text{ m}$$

For those dimensions, and using the Biot-Savart Law, if one ampere of current flows through the pair of coils, then the magnetic field at the center of the two coils is:

$$B = 1.36 \pm .03 \times 10^{-3} \text{ Tesla (one Ampere)}$$

So,

$$B = 1.36 \pm .03 \times 10^{-3} \text{ Tesla/Ampere}$$

Using the equivalent radius and separation along with the Biot-Savart Law, the students should be able to calculate for themselves this magnetic field at the center, which is where the magnetic dipole will reside. *It is only the magnetic field at the center of the instrument that is important in all of these experiments.*

The value for the magnetic field *gradient* at the center of the apparatus will be needed for one of the experiments. The field gradient can be calculated by differentiating the expression of the magnetic field with respect to z , if the center axis of the coils is the z -axis. The expression for the on-axis magnetic field gradient due to one current loop is:

$$\frac{dB}{dz} = -\frac{3}{2} \mu_0 I r^2 \frac{z}{(r^2 + z^2)^{5/2}} \quad (\text{Field Gradient/loop of wire})$$

Using the equivalent radius, and separation, the value of $\frac{dB}{dz}$ at the center of Mtl-A for a single turn is:

$$\frac{dB}{dz} = 4.33 \times 10^{-5} \text{ I Tesla/meter (per loop of wire)}$$

For both coils the z-axis field gradient of the entire magnet is:

$$\frac{dB}{dz} = 1.69 \times 10^{-2} \text{ Tesla/Meter (for magnet)}$$

2. -- Air bearing

The air bearing is a spherical hollow in the cylindrical brass rod that is supported on the bottom coil form. The bearing has a narrow opening that allows air to be pumped into the spherical hollow. The ball sits in this hollow and floats on a cushion of air providing support with only a small amount of friction. The air pump is housed inside of the power supply. A vinyl hose connects the air supply to the under-side of the air bearing. Make sure the students are careful not to restrict air flow by accidentally "kinking" the hose.

3. -- Strobe light

The strobe light is housed in its own insulating housing on top of the upper coil. The student can vary the frequency of the strobe flashes by a control located on the power supply. The frequency of these flashes is automatically measured and read out to two significant figures on the power supply's front panel. This data is updated every 10 seconds.

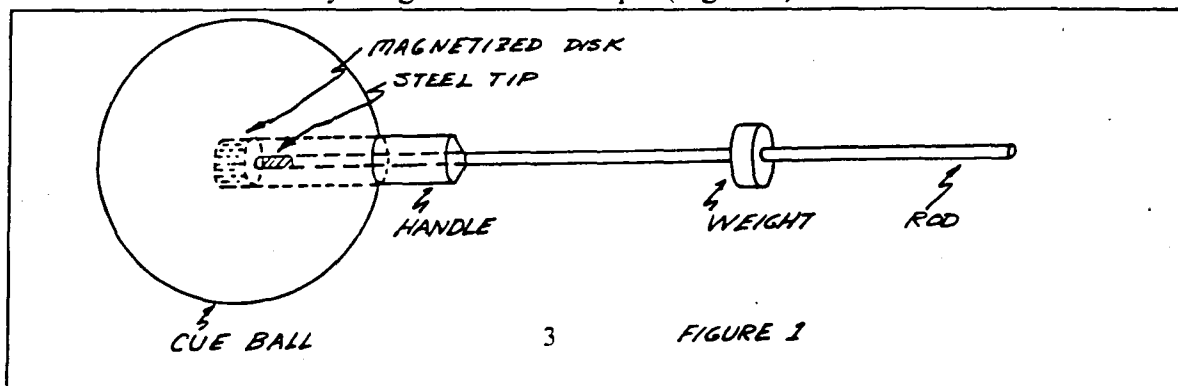
B. The Accessories

The accessories are those components of the Magnetic Torque instrument that are not permanently attached to the two main parts of the instrument. They're the "toys" that the student has to manipulate in order to perform the various experiments. Let's examine them.

1. -- Cue balls

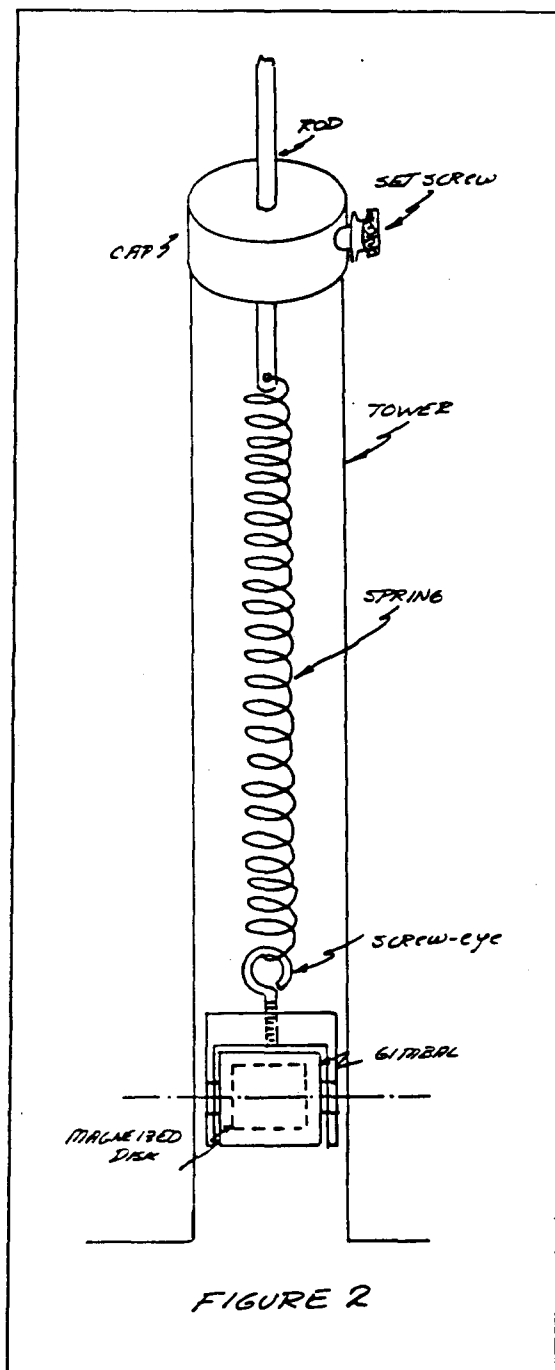
These cue balls are simply aramith snooker balls with a small cylindrical permanent magnet at their centers. The magnet acts as if it were a magnetic dipole whose magnetic dipole moment points in the direction of the ball's handle. A handle embedded in the ball allows the student to spin the ball, measure its rotation frequency, and determine the direction of the magnetic moment.

The handles on the balls have a small axial hole drilled in them. A thin aluminum rod with an attached weight is placed in this hole as part of a static magnetic torque experiment. The rod has a steel-tipped end that holds fast to the magnetic inside of the ball. The movable weight is a small clear plastic cylinder with an O-ring inside that keeps the weight from involuntarily slipping on the rod. The weight is meant to be moved up and down the rod to vary the gravitational torque (Figure 1).



2. -- Plastic tower

The clear plastic tube attached to a cylindrical base can be placed on top of the air bearing.



This apparatus is used for the magnetic force experiments. A nylon cap placed on the top of the tube holds the rod that supports the spring. The other end of the spring is connected to the magnet. The position of the suspended magnet inside of the tube can be adjusted by moving the rod inside the cap. There is also a small screw-eye attached above the magnetic dipole that can be used to prevent the magnet from rotating on its gimbals (Figure 2).

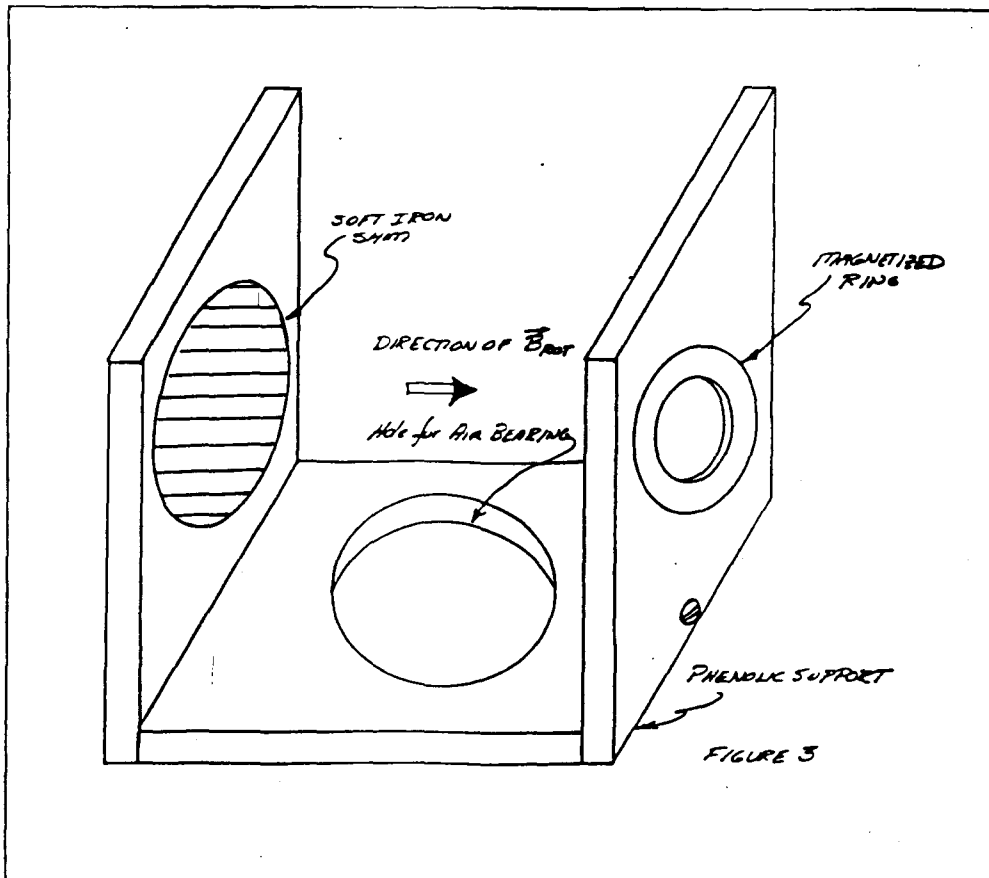
Ball bearings that weigh one gram each are provided for calibration of the spring.

3. -- Rotating magnetic field

This is an optional accessory, which may not be included. The rotating magnetic field is simply a special configuration of permanent magnets and soft iron shims that provide a horizontal magnetic field. This uniform horizontal magnetic field (~ 1.0 mT) can be manually rotated around the air bearing. It has a hole in its base that allows the air bearing to act as its rotation axis (Figure 3). This magnetic field is used to demonstrate nuclear magnetic resonance. (The lowest frequency NMR ever!)

4. -- Bulls-eye level

If the air bearing is not level, an additional torque due to unequal air flow can result. Such a torque can produce erroneous data. The bulls-eye level is simply a fluid-filled region that has a bubble in it. When the bubble is within the circle, the apparatus is reasonably level. The leveling can easily be accomplished by placing shims underneath the rubber feet below the magnet.



C. Power Supply

ON the front panel, going from left to right:

1. -- Analog ammeter display

Reads the current passing through the coils (since the coils are connected in series). The knob below the display is used to adjust the current. Turning the knob clockwise increases the current through the coils.

2. -- Field direction switch

Controls whether the magnetic field at the center is directed up or down.

3. -- Field gradient switch

Controls whether there is a magnetic field **gradient** at the center of the instrument (on), or a **uniform** magnetic field (gradient off).

4. -- Strobe light frequency display

Displays the frequency of the strobe light in Hertz. Below the display is the frequency-adjust. Turning the knob clockwise will increase the frequency. After adjusting the frequency, one needs to wait for the instrument to count up to the actual frequency.

Next to the frequency-adjust knob is the on-off switch for the strobe light.

5. -- Air switch

Allows the student to turn off the air pump when they are performing the magnetic *force* experiments.

6. -- Pilot light

Indicates when the ac power is on for the entire system.

On the back of the power supply are the following:

1. -- on-off ac switch for all of the components inside.
2. -- cord that plugs into the ac electrical socket.
3. -- Cinch Jones connector that connects the power supply to the magnet.
4. – male air hose connection (the air hose has a female connection that mates to it).

EXPERIMENT 1: Magnetic torque equals gravitational torque

Objectives

The main objective of this experiment is to measure the magnetic moment (μ) of the magnetic dipole (which is the magnet inside the cue balls). You will also verify the functional relationship: $\mu \times B = r \times mg$.

Equipment

Magnet, power supply, air bearing, cue ball, aluminum rod with a steel end, weight, ruler, balance, and calipers.

Theory

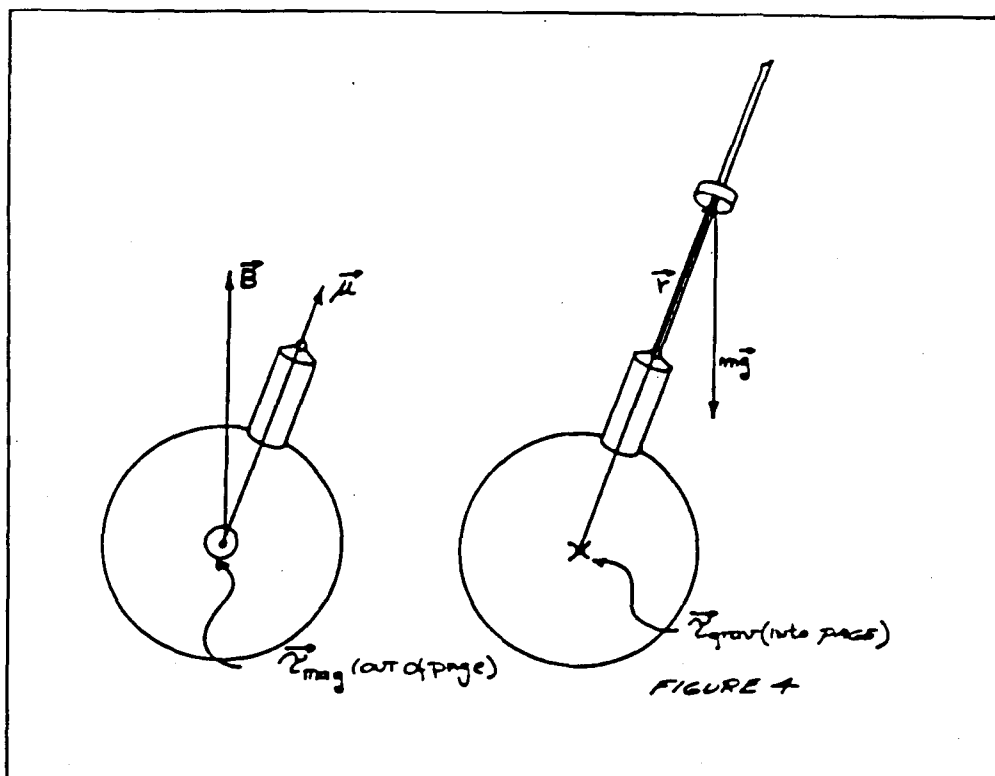
From your electricity and magnetism course, you should be aware that a loop of continuous current is referred to as a magnetic dipole. The neodymium iron boron magnetized disk inside the ball is not a loop of current. In fact it is a .375 inch diameter, .25 inch thick disk magnetized along the axis of the disk. But its magnetic field is such that it acts *as if it were* a magnetic dipole. In a uniform magnetic field (which is the case at the center of the two-coil configuration), a magnetic dipole experiences a magnetic torque that is given by the expression:

$$\tau = \mu \times B$$

Your magnet's dipole moment is aligned parallel to the handle on the ball; the magnetic field produced by the coils can be either up or down along the coils' axis. If the magnetic field points up, and the magnetic moment is aligned at some angle θ away from the direction of the magnetic field, the ball will experience a torque that will tend to rotate it so that the handle of the ball points upward. But if the aluminum rod is placed in the handle of the ball, there is now another torque due to the earth's gravitational field. The expression for this torque is:

$$\tau = r \times mg$$

The gravitational torque tends to cause the ball to rotate so that the ball's handle points downward (Figure 4). Since a net torque causes a change in angular momentum, the ball



will rotate if the gravitational torque is larger than the magnetic torque, or vice versa. But when the magnetic torque is equal to the gravitational torque, the ball will not rotate, since the net torque on the ball is zero. This configuration is mathematically represented by:

$$\mu B \sin \theta = r m g \sin \theta$$

Or:

$$\mu B = r m g$$

$$r = \frac{\mu}{m g} B$$

So if we measure r for various magnetic fields, the functional dependence of r to B should be a straight line with the slope being an expression that contains μ . But what's r , m , and B ? B is the magnetic field at the center of the coils, and can be calculated from the known current. But there are two m 's and two r 's that we need to consider. The first m is the mass of the weight, with its corresponding r being the distance from the center of the ball to the center of mass of the weight. The second m is the mass of the rod, with its corresponding r being the distance from the center of the ball to the center-of-mass of the rod. But remember that we're trying to measure μ using the slope of a line. *It turns out that the mass of the rod and its center-of-mass distance, r , are combined in a constant in the graph of r vs. B , and do not affect the slope of the line.* So the only r that one needs to measure is the r of the weight, and the only m that must be measured is the mass of the

weight. This is the advantage of a slope measurement of μ rather than a single point determination where the mass and center-of-mass of the rod would be essential.

So μ is the unknown in this experiment. The independent variable is the B-field at the center of the instrument, and the dependent variable is r , the displacement of the center of mass of the weight from the center of the ball.

Procedure

a. First measure all of the constants that are involved in the experiment. Use a balance to determine the mass of the weight, calipers to measure the diameter of the ball, and a ruler to measure the length of the ball's handle. From these measurements, r of the weight can be determined. *Make sure that you remember to keep all of your measurements in SI units, specifically keeping the magnetic field in Teslas, the length measurements in meters, and the mass measurements in kilograms.*

b. Next measure r of the weight for various magnetic fields. Turn on the power supply and the air. Keep both the field gradient and the strobe light off. Set the direction of the magnetic field on "up" so that the handle of the ball points upward when the ball rests on the air bearing. For small currents the gravitational torque is greater than the magnetic torque, so the current to start at is about 2.5 amps. With that current, adjust the position of the weight until the ball and the tip remain stationary at about 90 degrees with respect to the vertical. You might have to steady the rod, weight, and ball with your hand, because the system tends to oscillate and drift due in part to the earth's magnetic field. When the gravitational torque equals the magnetic torque (remember to continuously check the current to make sure that it remains at the set value), take the ball off of the air bearing and turn the current down to zero. Measure the length from the end of the handle to the center of mass of the weight. Add this value to the length from the center of the ball to the end of the handle, and the resulting value is the r of the weight.

Repeat these steps for at least six different currents up to 4 amps. The magnetic field can be calculated from the currents. A data table with column headings such as the ones below is a good way to organize the measurements:

I (amps)	B (teslas)	r (meters)
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Report

1. Compose a data table.
2. Include a sample calculation of your determination of the magnetic field from the measured current.
3. Graph r vs. B .
4. Calculate the magnitude of your magnetic moment, μ .
5. Extrapolate your graph in order to determine its y-intercept. From this value, determine the location of the rod's center of mass. Compare this calculated value with a measured location of the rod's center of mass.

EXPERIMENT 2: Harmonic oscillation of a spherical pendulum

Objectives

The primary objectives of this experiment are to determine the dipole moment of the magnet inside of the ball, and to study the behavior of a physical pendulum's small amplitude oscillation.

Equipment

Magnet, power supply, air bearing, cue ball, stopwatch, calipers, and balance.

Theory

This experiment involves *dynamics* principles. From classical mechanics, you are familiar that a net torque on an object causes a change in that object's angular momentum, given by the expression:

$$\sum \tau = \frac{d\mathbf{L}}{dt}$$

For our particular system, if the cue ball is placed in the air bearing with a uniform magnetic field present, and if the "intrinsic" dipole moment of the ball is displaced some angle away from the direction of the magnetic field, the ball will experience a net torque and will change its angular momentum. However, it's important to note the *direction* of the magnetic moment *relative* to the magnetic field. If the magnetic moment in the ball is displaced an angle θ from the axis of the coils (the direction of the field), it experiences a restoring torque that acts *against* the angular displacement of μ . Thus, the differential equation that describes the motion of the ball having moment of inertia I is:

$$-\mu \times \mathbf{B} = I \frac{d^2\theta}{dt^2}$$

Where θ is the angular displacement from the direction of \mathbf{B} . The minus sign indicates that the torque is restoring in nature. In scalar form we have:

$$-\mu B \sin \theta = I \frac{d^2\theta}{dt^2}$$

But for small angle displacements, $\sin \theta \approx \theta$, so

$$-\mu B \theta = I \frac{d^2\theta}{dt^2}$$

We'll guess the solution of this equation to be $\theta(t) = A \cos \omega t$, where ω and A are constants. Substituting into the differential equation we have:

$$-\mu B A \cos \omega t = -I A \omega^2 \cos \omega t$$

For this equation to be true for all times t ,

$$\omega^2 = \frac{\mu}{I} B$$

where ω is the angular frequency of oscillation. If T is the period of oscillation,

$$T = \frac{2\pi}{\omega}$$

Thus, the final expression is:

$$T^2 = \frac{4\pi^2 I}{\mu B}$$

Remember that this expression is only applicable for a small angle displacement. I can be well approximated as the moment of inertia of a uniform solid sphere, namely:

$$I = \frac{2}{5} m r^2$$

where m is the mass of the ball and r is the ball's radius. B is again the independent variable, and T can be measured using a stopwatch. A graph of T^2 vs. $\frac{1}{B}$ should yield a straight line whose slope includes the magnetic moment.

Procedure

- a. First, determine the moment of inertia of the cue ball using its mass and its radius. The mass can be determined using a balance, and the radius can be determined using the calipers.
- b. For this experiment, the field gradient should be off; the strobe light, off; the air on; and the field "up". Because the magnetic torque is the only torque involved in this experiment, the experiment can be performed at low currents (small B). Place the cue ball on the air bearing and set the current at or near one amp. Give the handle of the ball a small angular displacement from the vertical. Release the ball from rest, and it will oscillate. With a stopwatch, measure the amount of time it takes the ball to complete twenty full cycles of motion. **Make sure you include the counting of the first full cycle.** This measured time divided by twenty will be the period of oscillation for the ball at that particular applied magnetic field.

Repeat this for currents up to 4 amps. Because of the $1/B$ independent variable that will be graphed, it's a good idea to obtain data for quite a few different lower currents in order to obtain an even distribution of data points on the graph of T^2 vs. $1/B$. Plot your data in a table with column headings such as the ones below:

I (A)	B (T)	t (20) (s)	$T = t/20$ (s)	T^2 (s ²)	$1/B$ (T ⁻¹)
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Report

1. Compose a data table. Show sample calculations.
2. Graph the functional relationship of T^2 vs. $1/B$.
3. Calculate the magnitude of your magnetic moment from the slope of this graph.

EXPERIMENT 3: Precessional motion of a spinning sphere

Objectives

The main objective is to measure the dipole moment of a permanent magnet inside the cue ball. A secondary objective is to observe and quantify the motion of a spinning sphere subject to an external torque.

Equipment

Magnet, power supply, air bearing, cue ball, strobe light, stopwatch, calipers, and balance.

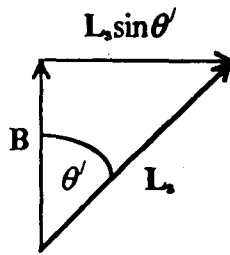
Theory

When the magnetic moment is displaced some angle from the direction of the magnetic field, the magnetic dipole (and subsequently the ball) experiences a torque that causes a change in the ball's angular momentum in the direction of the torque. This is the central principle of this experiment. The ball is displaced from the vertical position and spun, with its spin-axis the axis that runs through the handle of the ball. This creates a large spin angular momentum. The spin axis will remain in a fixed position until the uniform magnetic field is turned on. When the magnetic field is turned on, the magnetic dipole will experience a torque. This torque will cause a change of angular momentum *in the direction of the torque*, but because the ball already has a large spin angular momentum, it will precess. This motion is similar to that of the spinning gyroscope in the earth's gravitational field. You may be familiar with gyroscopic motion from your mechanics course.

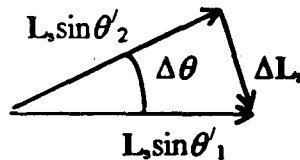
The differential equation for the motion of the ball is:

$$\boldsymbol{\mu} \times \mathbf{B} = \frac{d\mathbf{L}}{dt}$$

A side view of the angular momentum vectors is shown below:



If we look from above the spinning ball at the change of the angular momentum for a short time Δt , the picture looks like the one below:



Since $s = r\theta$, we can show that:

$$\Delta L_s = \Delta \theta L \sin \theta'$$

$$\frac{\Delta L}{\Delta t} = \frac{\Delta \theta}{\Delta t} L \sin \theta'$$

as $\Delta t \rightarrow 0$,

$$\frac{dL}{dt} = \Omega_p L \sin \theta'$$

where Ω_p is the precessional angular velocity. Since μ is in the same direction as L ,

$$\frac{dL}{dt} = \mu B \sin \theta'$$

So $\mu B \sin \theta' = \Omega_p L \sin \theta'$

and $\mu B = \Omega_p L$

or

$$\boxed{\Omega_p = \frac{\mu}{L} B}$$

The precessional frequency (in radians/second) is the dependent variable. It can be determined by measuring the time needed for the handle of the ball to precess through 2π radians, and then dividing that time by 2π . The magnetic field is the independent variable.

The magnitude of the angular momentum L is a constant that can be measured using the strobe light. The handle of the ball has a white dot on its top. As the ball spins the strobe light reflects off of this white dot. When the strobe light is flashing at the same frequency at which the white dot is spinning, the dot will appear stationary. Thus, from the displayed strobe frequency and the measurement of the moment of inertia, the spin angular momentum of the ball at that time can be calculated. The graph of Ω_p vs. B will yield a straight line if L is held constant throughout the experiment. From the slope of this line, μ can be determined.

Procedure

a. First measure the constants. The moment of inertia of the ball is assumed to be that of a solid sphere. The mass of the ball needs to be measured with the balance; its radius, with the calipers. The other constant is the spin angular momentum of the ball. A constant value of the spin angular momentum of the ball can be accomplished by fixing the frequency of the strobe light. We recommend choosing a frequency somewhere between 4.5 and 6 Hertz. Set the strobe light at a frequency in that range and check it throughout the experiment to make sure that it remains constant. In order for the strobe light to illuminate the white dot, the room should be darkened. It is not necessary to make the room completely dark in order to use the strobe light effectively. You can maintain ample light for writing and working with the instrument.

b. Once the strobe light has been set to some particular frequency, record that number. Turn the air on, leave the field gradient off, and set the field direction "up". Have a stopwatch ready, but don't turn on the current quite yet. Before you begin making measurements, practice spinning the ball. A good technique is to spin the ball (give it a good, hard spin!) and then use the tip of your fingernail to direct it to spin about the handle's axis with the handle in the strobe light. Notice that the frequency of the ball's spin *does change with time*. If you graph this change, it closely approximates an exponential decay. It's because of this decay that the range of frequency between 4.5 and 6 hertz is advised. In this range, the rotational frequency does not change significantly during the time it takes the ball to precess through one period.

When you master the spin technique, begin the measurements. Leave the current off, spin the ball up, adjust it so that it's bathed in the strobe light, and then watch the white dot. You'll see it all around the handle at first, but as the ball slows down, the white dot will begin to have a regular rotation of its own. This rotation will slow down until the white dot stops. As soon as it stops, turn the current up to 1 amp, and time a period of the ball's precession. Record this time for that current, turn the current off, and spin the ball up again. Do the same procedure, but this time use 1.5 amps, and continue on in 0.5 amp intervals until you reach 4 amps. This should give you enough data. Record your data in a table with columns such as the ones below:

I (A)	B (T)	T_p (s)	$\Omega_p = (2\pi)/T$ (s ⁻¹)
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Report

1. Compose a data table. Show sample calculations.
2. Graph the relationship Ω vs. B .
3. Calculate the magnitude of the your magnetic moment from the slope of this graph.

EXPERIMENT 4: Net force in a magnetic field gradient

Objectives

There are three main objectives to this experiment. The first is to demonstrate that in a uniform magnetic field, there is *no net force on a magnetic dipole*, only a net torque. Secondly, this experiment demonstrates that there *is a net force on a magnetic dipole when it is in the presence of a magnetic field gradient*. The third objective is to measure the dipole moment, using the fact that the net force on a magnetic dipole in a magnetic field gradient is proportional to the magnetic moment.

Equipment

Magnet, power supply, plastic tower apparatus, calibrated spring that is supported inside the plastic tower, permanent magnet disk mounted in a gimbal, ball bearings that serve as weights, ruler, and balance.

Theory

The principal player in this experiment is the magnetic dipole. If we model the magnetized disk inside the cue ball as a current loop, and place it in a uniform magnetic field that is directed along the axis of the loop, the force on any infinitesimal section $d\mathbf{l}$ of the loop is given by:

$$d\mathbf{F} = i d\mathbf{l} \times \mathbf{B}$$

where i is the loop current. The same amount of current passes through each infinitesimal section of the current loop. Using the right-hand rule one can see that every $d\mathbf{F}$ adds to another $d\mathbf{F}$ of equal magnitude that points in the opposite direction. Therefore, there is no net force on a current loop in a uniform magnetic field. If this model represents our magnetized disk then there should be no net force on the disk in a uniform magnetic field.

However, if the disk is placed in a non-uniform magnetic field, that is, a spatial magnetic field gradient, then there *is* a net force on it. To show this, let's assume that the direction of the magnetic field is a direction parallel to the z -axis. This magnetic field changes *only* in the z -direction, where it changes its magnitude with increasing z . But Maxwell's second equation,

$$\oint \mathbf{B} \cdot \mathbf{n} \, da = 0$$

says that the flux of the magnetic field through a closed surface S must equal zero. If a closed surface was constructed around the field along the z -axis, there would be a net flux of the magnetic field through the surface, which would violate Maxwell's second equation. We can thus conclude that if there is a magnetic field that is changing in space, *it must change in more than one direction*. It follows that in our particular situation, the field lines are no longer parallel to the axis of the magnetic dipole, but are bowed, and curve away from the z -axis. Using the expression for the force on an infinitesimal section of the current loop, it can be shown that there must be a net translational force on the magnetic dipole*. It turns out that the expression for the magnitude of this force is:

$$F_z = \mu \frac{dB_z}{dz}$$

We can measure this force. Using the plastic tower apparatus and applying a field gradient, the suspended magnet experiences a translational magnetic force. For our apparatus, this force is nearly constant over a fairly wide range of z -values, since the magnetic field gradient is reasonably constant. However, there is another force on the suspended magnet, the force that the spring applies. The force due to the spring is given by Hooke's Law,

$$F = kz$$

where k is the spring constant, and z is the displacement of the magnet from its equilibrium position (the position where the magnet resides in the absence of a magnetic field gradient). The fact that there are two forces acting on the magnet means that the magnet will displace either upward or downward until the spring force equals the magnetic force. At that point the magnet will cease its acceleration, and if one stops its motion, it will come to rest. At this displacement z , the following expression can be written:

$$F_{\text{spring}} = F_{\text{field gradient}}$$

or

$$kz = \mu \frac{dB_z}{dz}$$

The magnetic field gradient can be calculated. First calculate the on-axis magnetic field produced by two coils with currents in the opposite directions. Then differentiate that expression with respect to z , with the axis of the coils being the z -axis. The spring constant k can be measured using the ball bearings as weights to calibrate the spring. The displacement z is then the dependent variable, and $\frac{dB}{dz}$ is the independent variable,

* See "Electricity and Magnetism", Edward M. Purcell, McGraw-Hill, page 411, ISBN 0-07-004908-4.

proportional to the current. The graph of z vs. $\frac{dB}{dz}$ can be plotted, and from the slope of the expected line, the magnetic moment can be determined.

Procedure

a. For this experiment, you'll need both the power supply and the magnet coils, but the air bearing is not needed, so the air should be turned off. Place the clear plastic tower on top of the air bearing. Take the cap for the tower, and insert the rod with the spring and suspended magnet into the hole in the cap. There is a screw in the cap that can be used to hold the rod in place. Place the cap on the top of the tower. Now adjust the length of the rod so that the suspended magnet is in the center of the coils, that is, where the center of the ball was when it rested on the air bearing. For the first part of the experiment, loosen the screw on the suspended magnet so the magnet is free to rotate about a horizontal axis. Set the field gradient off, and turn up a current. Now keep your eye on the magnet, and change the field direction. Observe what happens and write it down.

b. For the next portion of the experiment, the same equipment will be used, except that now the suspended magnet should be screwed down to prevent it from rotating. Tighten the screw eye so that it is in a position where the flat sides of the magnet are facing up and down.

Determine the spring constant k . To do this the magnetic field must be turned off. Hang the magnet down as far as possible, so that only a small portion of the rod is showing above its support. Mark the position of the magnet on the side of the plastic tower with some masking tape. Measure the length of the rod above the plastic holder. Next, take the cap-rod-spring device out of the tower, and add one ball bearing to the magnet. Then place the cap-rod-spring device onto the tower and adjust the rod so that the magnet is again at the tape mark. Measure the length of the rod above the holder. Subtract from this length the length of the rod when the magnet was in equilibrium, and the resulting length will be the displacement of the magnet from equilibrium due to one ball/weight. (The balls have a mass very close to one gram. You can check that with a balance.) Repeat this for two, three, and four ball/weights hanging from the suspended magnet. Then graph the force on the magnet (mg) vs. displacement. The slope of that line will be k .

c. Now that the spring constant has been determined, you can proceed with the field gradient experiment. First, use the set screw to lock the dipole in place to prevent it from rotating. With all of the weights off of the magnetic dipole, adjust the position of the rod so that the magnetic dipole is at the center of the coils. Again, place a piece of masking tape on the side of the plastic tube so that you will know the location of this center. Measure the length of the rod when the magnet is at this center position. With the field gradient switch in the "on" position, slowly turn the current knob to 0.5 amps. The dipole will displace some amount up or down, depending on which direction the field is relative to μ . It is probably

best to switch the field direction so that the magnet displaces downward. When the magnet is at the center of the coils, there's more rod inside the tube than there is outside.

Once the magnetic dipole has displaced, return it to the center of the coils by adjusting the support rod. Then measure the length of the rod showing above the tower. The difference between this length and the length of rod showing when there is no magnetic field gradient is the displacement of the magnetic dipole due to the magnetic field gradient. Record this displacement z for the current I . Proceed from an initial current of 0.5 amps to 4 amps, in 0.5 amp increments. Again, continuously check the ammeter at high currents to make sure that it is staying at the value that you need for your particular measurement.

The displacement z is the dependent variable for the experiment, while $\frac{dB}{dz}$ is the independent variable that varies with different currents. The spring constant k is a constant, so μ can be obtained from the slope of the graph of z vs. $\frac{dB}{dz}$. Record the data in a table with columns such as the ones below:

I (A)	dB/dz (T/m)	z (m)
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Report

1. Write down your observations from part a. of the procedure. Offer an explanation for these observations.
2. Compose a data table for the second part of the experiment, showing sample calculations.
3. Graph z vs. $\frac{dB}{dz}$.
4. Calculate the magnitude of the magnetic dipole moment from the slope of this graph.

Another unique feature of M τ 1-A is its ability to visually demonstrate magnetic resonance. By “demonstrate” we mean that M τ 1-A can be used to show the classical behavior of a particle with both a magnetic moment and a collinear angular momentum when it is simultaneously subjected to a static magnetic field and a magnetic field rotating at the Larmor precession frequency perpendicular to the static field. That is, it can be used to show students a classical “spin flip”.

Magnetic resonance can *only* be observed on particles that have both an intrinsic angular momentum and a magnetic moment. These include electrons, protons, neutrons, and many nuclei. Particles having only a magnetic moment do not exhibit this resonance phenomenon. M τ 1-A's billiard balls have an “intrinsic” magnetic moment supplied by their imbedded magnetized disk, but the demonstrator must supply the collinear angular momentum. This is done by manually by setting the sphere spinning with a quick twist of the handle. Thus, the operator chooses the value of angular momentum to use for the demonstration.

The ratio of the magnetic moment μ to the angular momentum L to is called the “gyromagnetic ratio γ ” or

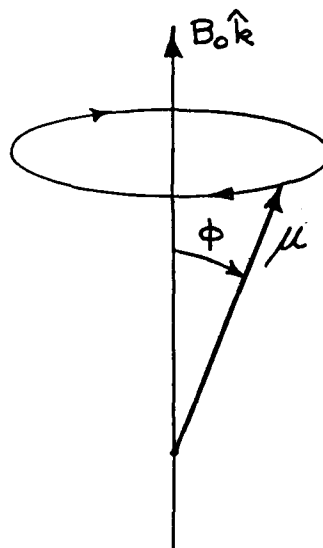
$$\gamma = \frac{\mu}{L}$$

If such an object is subjected to a magnetic field B (which may be time dependent), its classical motion is described by the expression:

$$\mu \times B = \frac{dL}{dt} = \frac{1}{\gamma} \frac{d\mu}{dt}$$

If B_0 is a constant field in the z direction, the object will execute precessional motion with the direction of the magnetic moment sweeping out a circle in the x - y plane as shown in the diagram. This motion has already been explained in the earlier sections of the manual.

Suppose we now add the “rotating magnetic field” shown in Figure 3 on page 5. This unit will produce a magnetic field of about 1.0 mT in the x - y plane perpendicular to the field from the coils B_0 . It is called “rotating magnetic field” because it can be rotated by the demonstrator at speeds appropriate to this apparatus. That's right, you rotate it *by hand*. There is no fancy electronics here! However, seeing you rotate the magnets will leave no doubts in



the students' mind that a rotating magnetic field has been applied. *

When this horizontal field is rotated **at the Larmor precession frequency** ω_0 , the angle of the magnetic moment with respect to the static field B_0 , ϕ will become time dependent. If the horizontal field is rotated for a sufficient time, ϕ can be made to change by 180° . This is the classical analog of a **spin-flip**.

The explanation of this spin-flip process can best be understood by examining the system in a noninertial rotating reference frame. Suppose we ask the student to visualize the motion of the sphere in a frame of reference rotating at the Larmor precession frequency, where the rotation axis is along the z-axis. In this frame, the ball is still spinning with angular momentum L but it appears to be at rest. How is it possible for an object which has both μ and L to be in a magnetic field B_0 and to remain at rest, not precessing? The answer, of course, is that in this noninertial frame the "effective magnetic field" is $B_{\text{effective}} = B_0 - \frac{\omega}{\gamma} = 0$. The effective magnetic field in the frame rotating at the Larmor precession frequency is zero.

But now we add B_{rot} in the x-y plane. In the rotating frame of reference this field is constant. The effective magnetic field becomes:

$$B_{\text{eff}} = B_{\text{rot}}$$

Observed from the rotating frame the sphere precesses about B_{rot} at the precession frequency $\Omega = \gamma B_{\text{rot}}$. It is this precession about the rotating field that causes the spin-flip, that changes the angle ϕ .

This simple analysis is only applicable if the rotating magnetic field is rotated at the Larmor precession frequency. The demonstrator can easily show that spin-flips do not occur if the field is not rotated, rotated with the opposite angular velocity (the wrong direction), or rotated in the right direction at the wrong frequency. In all those cases the angle ϕ between μ and B will remain constant, on average. It is even possible to calculate the time it will take for a 180° angle change (a 180° "pulse", in pulsed NMR

* In essentially all magnetic resonance apparatus the rotating magnetic field is created with a small coil and an oscillating current. This produces a linear oscillating magnetic field. However, it is easy to show that a linear oscillating field is exactly equivalent to two rotating fields; rotating in opposite directions. Only *one* of the two rotating fields causes the magnetic resonance. It is the one rotating in the same direction as the particle's precession. This can be verified for the student by showing that turning the rotating field in the opposite direction to the precession produces no effect.

jargon) using the magnitude of the magnetic field stamped on the “rotating magnetic field” and the value of γ calculated from the Larmor precession frequency.

There are several important techniques to be mastered before attempting to demonstrate this classical analog of magnetic resonance. First, it is important to spin the ball with as little nutation (wobbling of the μ -axis) as possible. To do this, place the rotating magnetic field over the air bearing, turn the coil current to its maximum value, and give the ball a spin with your thumb and first finger, using the black handle. Typically the handle will wobble rather profusely. If you touch the handle of the ball with your finger nail (or some other low friction smooth surface) you can get it to spin with a minimal wobble. Because it's not possible to perfectly balance each sphere, some wobble will be present, but it will not significantly effect the overall demonstration. The finger nail technique works quite well but it does require some practice.

With both fields present, the ball will precess about the resultant field which is the vector sum of $B_o \hat{k}$ and $B_{rot} \hat{j}$. Now rotate B_{rot} at the observed precession frequency always keeping μ transverse to B_{rot} . That is, keep the sphere handle pointing in the direction of the “gap” between the two uprights that supports the permanent magnets. This is important since if μ is parallel to B_{rot} there is *no torque* on μ and thus no spin-flip! This may take some practice, but you will soon get the hang of the procedure. You can achieve a 90° “flip” easily but a 180° nutation is a little more difficult because the process stops when the ball handle hits the air bearing. However, if you start the experiment with the handle already horizontal you can bring it up vertical and then down horizontal creating a complete 180° pulse or a complete spin flip, whichever you wish to call it.

Good luck watching “*the worlds lowest frequency magnetic resonance experiment*” Your students will flip, like the ball!