Canuê dos Reis Borges e Maniciano Lista 2 / /

1.

a) $\begin{vmatrix} 1 & 2 \\ -4 & 3 \end{vmatrix}$ $\quad -8 \quad 3$

$a_{11}\cdot(-1)^{1+1} M_{11} + a_{21}\cdot(-1)^{2+1} M_{21}$

$1(-1)^2 M_{11} + (-4(-1)^3 M_{21})$

$1\cdot 1 M_{11} + (-4\cdot(-1) M_{21})$

$1 M_{11} + 4 M_{21}$

$1\cdot 3 + 4\cdot 2 = 11$

b) $\begin{vmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{vmatrix}$

$\sqrt{2}(-1)^2\sqrt{3} + 2(-1)^3 3\sqrt{6}$

$\sqrt{2}\cdot 1\cdot\sqrt{3} + 2\cdot(-1)\cdot 3\sqrt{6}$

$1\sqrt{6} - 6\sqrt{6} = -5\sqrt{6}$

c) $\begin{vmatrix} \pi & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$

$\pi(-1)^2 \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 5(-1)^3 \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} + 1(-1)^4 \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix}$

$\cancel{\pi\cdot 1\cdot 0 + 5\cdot(-1)\cdot 0} + 1\cdot 1\cdot 2 \Rightarrow 2$

d) $\begin{vmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{vmatrix}$

$-2\cdot(-1)^{1+1}\cdot M_1 + 1\cdot(-1)^{2+1}\cdot M_2 + -3\cdot(-1)^{3+1}\cdot M_3$

$-2\cdot(-1)^2\cdot \begin{vmatrix} 5 & 4 \\ 4 & 2 \end{vmatrix} + 1\cdot(-1)^3 \begin{vmatrix} 1 & -1 \\ 4 & 2 \end{vmatrix} - 3\cdot(-1)^4\cdot \begin{vmatrix} 1 & -1 \\ 4 & 2 \end{vmatrix}$

$-2\cdot(-6) + (-1)\cdot(+6) + (-3)\cdot(9) \Rightarrow 12 + (-6) + (-27) = -21$

e) $\begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & -1 & 2 \end{vmatrix}$

$2(-1)^3 \begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix} \Rightarrow -2\cdot(-8) = 16$

$10 - \quad 2 - 8$

f) $\begin{vmatrix} 3 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix}$

$3(-1)^2 \begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} \Rightarrow 3(-1)^2 \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + -1\cdot(-1)^3 \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$

$3\cdot(-1-1)$

$3\cdot(-2) = -6$

/ /

g) $\begin{vmatrix} -1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 7 & 2 & \sqrt{5} & 0 & 0 \\ 1 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{vmatrix}$ $3(-1)^5 \cdot \begin{vmatrix} 2 & \sqrt{5} & 0 \\ -3 & 6 & 1 \\ -3 & 0 & 0 \end{vmatrix} = 3(-1)^5 (-3\sqrt{5})$

$-3 \cdot (-3\sqrt{5}) = 9\sqrt{5}$

h) $\begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix}$ $3(-1)^{1+1=2} \cdot (-2(-1)^5) \cdot (-2(-1)^4 \cdot 2 \cdot (-1)^2$

$3 \cdot 1 \cdot (-2 \cdot (-1)) \cdot (-2 \cdot 1) \cdot 2 \cdot 1$

$3 \cdot 2 \cdot (-2) \cdot 2 = -24$

2) a) $\begin{vmatrix} 7 & -2 & 14 \\ -3 & 2 & 10 \\ 4 & -8 & 2 \end{vmatrix}$ $7 \cdot \begin{vmatrix} 2 & 10 \\ -8 & 2 \end{vmatrix} + (-3) \begin{vmatrix} -2 & 14 \\ -8 & 2 \end{vmatrix} + 4 \cdot \begin{vmatrix} -2 & 14 \\ 2 & 10 \end{vmatrix}$

$7 \cdot 84 + (-3 \cdot 108) + 4 \cdot (-48)$

$588 + 224 + 192 = 920$

b) relação linha eliminar

$A = \begin{vmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{vmatrix}$ $3 \cdot \begin{vmatrix} 2 & 8 \\ -9 & 6 \end{vmatrix} + 4 \begin{vmatrix} -5 & 7 \\ -9 & 6 \end{vmatrix} + 1 \begin{vmatrix} -5 & 7 \\ 2 & 8 \end{vmatrix}$

$3 \cdot (-84) + 4 \cdot 33 + 1 \cdot (-54) = 252 - 132 - 54 = 66$

$B = \begin{vmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{vmatrix}$ $-1 \cdot \begin{vmatrix} 3 & 7 \\ 1 & -4 \end{vmatrix} + 0 \begin{vmatrix} 4 & 7 \\ 3 & -4 \end{vmatrix} + 2 \begin{vmatrix} 4 & 3 \\ 3 & 1 \end{vmatrix}$ $\det A \times \det B = 594$

$-1 \cdot (-19) + 2 \cdot (-5) = 9$

c)

$\det B = 9$ $\det A = 66$ $\det(B^t \times A^t) = 594$

d)

d) $\begin{pmatrix} 9 & -15 & 21 \\ 12 & 6 & 24 \\ 3 & -27 & 18 \end{pmatrix} - \begin{pmatrix} 4 & 6 & -2 \\ 12 & 18 & -4 \\ 16 & 24 & -6 \end{pmatrix} + \begin{vmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{vmatrix} \Rightarrow \begin{vmatrix} 9 & -18 & 30 \\ -1 & -12 & 30 \\ -10 & -50 & 20 \end{vmatrix}$

$9 \cdot \begin{vmatrix} -12 & 30 \\ -50 & 20 \end{vmatrix} + (-1) \begin{vmatrix} -18 & 30 \\ -50 & 20 \end{vmatrix} + (-10) \begin{vmatrix} -18 & 30 \\ -12 & 30 \end{vmatrix}$

$9 \cdot (-240 + 1500) + 1(-360 + 1500) + -10(-540 + 360)$

$9 \cdot 1260 + 1140 + 1800 \Rightarrow 14.280$

e)

$C^t = \begin{pmatrix} 2 & 6 & 8 \\ 3 & 9 & 12 \\ -1 & -2 & -3 \end{pmatrix}$ $(-1) \cdot \begin{vmatrix} 6 & 8 \\ 9 & 12 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & 8 \\ 3 & 12 \end{vmatrix} + (-3) \begin{vmatrix} 2 & 6 \\ 3 & 9 \end{vmatrix}$

$-1(\cancel{72 - 72}) + 2(\cancel{24 - 24}) + -3(\cancel{18 - 18}) = 0$

$0 \cdot 0 \cdot 0 = 0$

3.

a) $\det(A^4) = -2$

b) $\det 6A = 6^4 \cdot (-2) = 1296 \cdot (-2) = -2592$

c) $\det(A^T) = -128$

d) $\det(A^{-1}) = \frac{1}{\det A}$

4.

$\begin{vmatrix} a & b & -2c \\ 3d & 3e & -6f \\ g & h & -2i \end{vmatrix} = b$

a) $4 \cdot (\det(\text{original}))$

$4 \cdot (-3) = \det(A) = -12$

b) $3 \cdot (-2) = -6 \Rightarrow -6 \cdot \det(\text{org}) \Rightarrow -6 \cdot (-3$

$\det(B) = 18$

c)

$\begin{vmatrix} -a & -b & -c \\ g & h & i \\ -d & -e & -f \end{vmatrix} = c$ $-1 \cdot (-1) = 2 = 1 \cdot (-3) = -3 \Rightarrow \det(C) = -3$

d) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} = D$ troca sinal $\Rightarrow \det(D) = 3$

e) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix}$ $L_2 \Leftarrow 2 \cdot L_2 + L1 \Rightarrow |\det A = -3$ $= e$

b) $\begin{vmatrix} ka+a & kb+b & kc+c \\ d & e & f \\ g & h & i \end{vmatrix}$ $k+1$ $k \cdot l1 + l1 = -3 \cdot (k+1$ $= 6$

5 $\begin{vmatrix} 10 & 8 & 40 & -2 \\ 4 & 6 & 20 & -2 \\ -5 & -7 & -30 & 1 \\ -2 & -6 & -30 & 1 \end{vmatrix}$

$-4 \cdot \begin{vmatrix} -7 & -30 \\ -6 & -30 \end{vmatrix} + 1 \cdot \begin{vmatrix} 6 & 20 \\ -6 & -30 \end{vmatrix} + 12 \cdot \begin{vmatrix} 6 & 20 \\ -4 & 30 \end{vmatrix}$

$-4(210-180) + -1(-180+120) + 12(-180+140)$
$-4 \cdot 30 + 60 - 48 = -540 \cdot -5400$

$-2 \begin{vmatrix} -7 & -30 \\ -6 & -30 \end{vmatrix} + 1 \begin{vmatrix} 8 & 40 \\ -6 & -30 \end{vmatrix} + 12 \begin{vmatrix} 8 & 40 \\ -7 & -30 \end{vmatrix}$ (10)

$-2(210-180) - 1(\cancel{-240+240}) + 12 \cdot (-240+280)$
$-60 + 480 = 420 \cdot \triangleright 1680$

$-2 \begin{vmatrix} 6 & 20 \\ -6 & -30 \end{vmatrix} + -4 \begin{vmatrix} 8 & 40 \\ -6 & 30 \end{vmatrix} + 12 \begin{vmatrix} 8 & 40 \\ 6 & 20 \end{vmatrix}$

$-2(-180+120) + (-4)(\cancel{-240+240}) + 12 \cdot (160-240$
$+120 - 960 = -840 \times (-5) = 4200$

somando!
$-5400 \quad 1680$
$+ 4200 \quad + 1200$
$-1200 + 0480$

$+480$
180
960

$2 \begin{vmatrix} 6 & 20 \\ -7 & 30 \end{vmatrix} + (-4) \begin{vmatrix} 8 & 40 \\ -7 & -30 \end{vmatrix} + 1 \begin{vmatrix} 8 & 40 \\ 6 & 20 \end{vmatrix}$

$\det |A| = 960$

$-2 \begin{vmatrix} 6 & 20 \\ -7 & -30 \end{vmatrix} + (-4) \begin{vmatrix} 8 & 40 \\ -7 & -30 \end{vmatrix} + 1 \begin{vmatrix} 8 & 40 \\ 6 & 20 \end{vmatrix}$

$-2(180 + 140) + 4(240 + 280) + 1(160 - 240)$
$\cancel{80} + 160 \cancel{80} = 160 \times 3 = 480$

6.

a)
$$-128 = \begin{vmatrix} 4 & 6 & x \\ 4 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix}\begin{matrix} 4 & 6 \\ 4 & 4 \\ 5 & 2 \end{matrix}$$

$(-16x + 60x + 14x) - 20x - 16x + 42x$

$-52x + 116x = 64$

$x = \frac{-128}{64} \Rightarrow x = -2$

b)
$$39 = \begin{vmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{vmatrix}\begin{matrix} 3 & 5 \\ 2x & x \\ 4 & 6 \end{matrix}$$

$21x + 60x + 84x - 70x - 54x - 28x$

$-152x + 165x = 13$

$x = \frac{39}{13} \Rightarrow x = 3$

c)
$$-D = \begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix}\begin{matrix} x+3 & x+1 \\ 4 & 5 \\ 9 & 10 \end{matrix}$$

$35x + 105 + 27 + 27 + 40x + 160$

$-28x - 28\ 30x - 90 - 45x - 180$

$-103x - 298 + 102 + 292 = 6$

$-x - 6 = 7 \Rightarrow -x = -7 + 6 \Rightarrow x = 1$

d) $\begin{pmatrix} x & x+2 \\ 1 & x \end{pmatrix}$ é singular

$\Delta = 1^2 - 4 \cdot 1 \cdot (-2)$
$= 1 + 8$
$= 9$

$x_1 = \frac{1+3}{2} = \frac{4}{2} = 2$

$x_2 = \frac{1-3}{2} = \frac{-2}{2} = -1$

$\{x = 2 \text{ ou } x = -1\}$

e) $\begin{pmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{pmatrix}$ é inversível

$$\begin{vmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{vmatrix}\begin{matrix} x-4 & 0 \\ 2 & 0 \\ 0 & 3 \end{matrix}$$

$x_1 = \frac{13+7}{2} \Rightarrow \frac{20}{2} = 10$

$x_2 = \frac{13-7}{2} \Rightarrow \frac{6}{2} = 3$

Só é inversível quando $x \neq 10$ e 3, ~~...~~ x só pode ser qualquer outro valor que ~~não~~ ~~...~~ não sejam 3 ou 10.

$\Delta = 13^2 - 4 \cdot 1 \cdot 30 = 169 - 120 = 49$

7)

a) Fórmula é: $A^{-1} = \frac{1}{\det(A)} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ pois A matriz $A_{2\times 2}$ é inversível se o determinante for diferente de zero;

b) $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}_{-5\ \ 6} \Rightarrow A^{-1} = \frac{1}{1} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} =$

$B = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix}_{7\ \ 8} \Rightarrow B^{-1} = \frac{1}{1} \begin{pmatrix} 2 & 7 \\ -2 & 4 \end{pmatrix}$

$(AB)^{-1} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} (3\cdot 4) + (1\cdot 1) & (3\cdot 7) + (1\cdot 2) \\ (5\cdot 4) + (2\cdot 1) & (5\cdot 7) + (2\cdot 2) \end{pmatrix} = \begin{pmatrix} 13 & 23 \\ 22 & 39 \end{pmatrix} \Rightarrow 507$

$(AB) \Rightarrow \frac{1}{1} \begin{pmatrix} 39 & -23 \\ -22 & 13 \end{pmatrix} \Rightarrow (AB)^{-1} = \begin{pmatrix} 39 & -23 \\ -22 & 13 \end{pmatrix}$

8)

a) $A = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix}$

$C_{11} = (-1)^{1+1} \cdot \det(1) = 1 \cdot 1 = (1)$

$C_{12} = (-1)^{1+2} \cdot \det(3) = 1 \cdot 3 = (-3)$

$C_{21} = (-1)^{2+1} \cdot \det(-2) = -1 \cdot (-2) = (2)$

$C_{22} = (-1)^{2+2} \cdot \det(2) = 1 \cdot 2 = (2)$

$C(A) = \begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$

b) $B = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

$M_{11} = \det\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} \Rightarrow M_{11} = -3$ | $M_{21} = \det\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}$ | $M_{31} = \det\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = M_{31} = -2$

$M_{12} = \det\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} \Rightarrow M_{12} = -1$ | $M_{22} = \det\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix}$ | $M_{32} = \det\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = M_{32} = -2$

$M_{13} = \det\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \Rightarrow M_{13} = 1$ | $M_{23} = \det\begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix}$ | $M_{33} = \det\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = M_{33} = 6$

$\begin{pmatrix} (-1)^{1+1}(-3) & (-1)^{2+1}(-1) & (-1)^{1+3}(1) \\ (-1)^{2+1}(2) & (-1)^{2+2}(-2) & (-1)^{2+3}(2) \\ (-1)^{3+1}(-2) & (-1)^{3+2}(2) & (-1)^{3+3}(6) \end{pmatrix} \Rightarrow C(B) = \begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$

/ /

9)

a) $A = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix} \Rightarrow C_{11} = (-1)^{1+1} \cdot \det(1) = 1$ | $C_{21} = (-1)^{2+1} \cdot \det(-2) = 2$

$C_{12} = (-1)^{1+2} \cdot \det(3) = -3$ | $C_{22} = (-1)^{2+2} \cdot \det(2) = 2$

$C(A) = \begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix} \to adj(A) = [C]^t \Rightarrow adj(A) = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}$ $A^{-1} = \frac{1}{\det(A)} \cdot adj\, A = \frac{1}{8} \cdot \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}$

$= \boxed{\begin{pmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{-3}{8} & \frac{1}{4} \end{pmatrix} = A^{-1}}$

b) $B = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ $adj(B) = [C]^t \Rightarrow adj(B) = \begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ -1 & -2 & 6 \end{pmatrix}$

$B^{-1} = \frac{1}{\det(B)} \cdot adj\, B \to \frac{1}{8} \cdot \begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{pmatrix}$

$\Rightarrow B^{-1} \boxed{\begin{pmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ \frac{-1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{-3}{4} \end{pmatrix}}$

c) $\begin{pmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

$M_{11} = \det\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} \Rightarrow 1$ | $M_{21} = \det\begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} \Rightarrow -1$ | $M_{31} = \det\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} \Rightarrow 1$

$M_{12} = \det\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1$ | $M_{22} = \det\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \Rightarrow -1$ | $M_{32} \Rightarrow \det\begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} \Rightarrow -2$

$M_{13} = \det\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$ | $M_{23} = \det\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} \Rightarrow 1$ | $M_{33} \Rightarrow \det\begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} \Rightarrow 2$

$\begin{Bmatrix} (-1)^{1+1}(1) & (-1)^{1+2}(1) & (-1)^{1+3}(2) \\ (-1)^{2+1}(-1) & (-1)^{2+2}(-1) & (-1)^{2+3}(1) \\ (-1)^{3+1}(1) & (-1)^{3+2}(-2) & (-1)^{3+3}(2) \end{Bmatrix} \Rightarrow C(C) = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$ $adj(C) = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{bmatrix}$

$C^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{bmatrix} \Rightarrow C^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{-1}{3} & \frac{2}{3} \end{bmatrix}$

d) $\begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{vmatrix}$

$M_{11} = det \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 2 \\ 0 & -1 \end{matrix} = (3)$

$M_{12} = det \begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = (0)$

$M_{21} = det \begin{vmatrix} 0 & 0 & 0 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = (0)$

$M_{22} = det \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 2 \\ 0 & -1 \end{matrix} = (1)$

$M_{31} \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = (0)$

$M_{32} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$

$M_{13} = det \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 0 & 0 \\ 1 & 0 \\ 0 & -1 \end{matrix} = (-1)$

$M_{23} = det \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = (0)$

$M_{33} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$

$M_{14} = det \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 3 \end{vmatrix} \begin{matrix} 0 & 1 \\ 1 & 0 \\ 0 & 2 \end{matrix} = (2)$

$M_{24} = det \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 2 & 3 \end{vmatrix} = (0)$

$M_{34} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{matrix} = 1$

$M_{41} = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} \begin{matrix} 0 & 1 \\ 2 & 0 \\ 0 & 0 \end{matrix} = 2$

$M_{42} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix} = -1$

$M_{43} \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} = 0$

$M_{44} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{matrix} = 2$

$\begin{bmatrix} (-1)^{1+1}(3) & (-1)^{1+2}(0) \\ (-1)^{2+1}(0) & (-1)^{2+2}(1) \\ (-1)^{3+1}(0) & (-1)^{3+2}(0) \\ (-1)^{4+1}(2) & (-1)^{4+2}(0) \end{bmatrix} \quad \begin{bmatrix} (-1)^{1+3}(-1) & (-1)^{1+4}(2) \\ (-1)^{2+3}(0) & (-1)^{2+4}(0) \\ (-1)^{3+3}(0) & (-1)^{3+4}(1) \\ (-1)^{4+3}(-1) & (-1)^{4+4}(2) \end{bmatrix}$

$= C(D) = \begin{bmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{bmatrix}$

$adj(D) = \begin{bmatrix} 3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & 1 & 2 \end{bmatrix}$

$det\ D: 1 \cdot (-1)^2 \cdot 3 \quad 1 \cdot (-1)^3 \cdot 1$

$3^1 = (3) \quad -1 = -2 = 1$

$D^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{bmatrix} \Rightarrow D^{-1} \begin{bmatrix} 3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{bmatrix}$

10) $2A^t = C - XB$

Nesta equação, para ela ser resolvida, B precisa ser inversível, $= det(B) \neq 0$.

a)

$2A^t = C - XB$

$XB = C - 2A^t$

$X = (C - 2A^t)B^{-1}$

b) $2A = \begin{bmatrix} 2 & 0 & -2 \\ 4 & 6 & 0 \\ 0 & -2 & 4 \end{bmatrix} \Rightarrow 2A^t = \begin{bmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad C - 2A^t = \begin{bmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{bmatrix} = \begin{matrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{matrix}$

$B = \begin{bmatrix} 3 & -2 & 6 \\ 2 & -1 & 5 \\ 1 & 0 & 3 \end{bmatrix}$

$M_{11} = \det \begin{vmatrix} -1 & 5 \\ 0 & 3 \end{vmatrix} = 0-3$

$M_{21} = \det \begin{vmatrix} -2 & 6 \\ 0 & 3 \end{vmatrix} = 0-6$

$M_{31} = \det \begin{vmatrix} -2 & 6 \\ -1 & 5 \end{vmatrix} = -4$

$M_{12} = \det \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 1$

$M_{22} = \det \begin{vmatrix} 3 & 6 \\ 1 & 3 \end{vmatrix} = 3$

$M_{32} = \det \begin{vmatrix} 3 & 6 \\ 2 & 5 \end{vmatrix} = 3$

$M_{13} = \det \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$M_{23} = \det \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 0\,2$

$M_{33} = \det \begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix} = 0\,1$

$$\begin{pmatrix} (-1)^{1+1}(-3) & (-1)^{1+2}(1) & (-1)^{1+3}(1) \\ (-1)^{2+1}(-6) & (-1)^{2+2}(3) & (-1)^{2+3}(2) \\ (-1)^{3+1}(-4) & (-1)^{3+2}(3) & (-1)^{3+3}(1) \end{pmatrix} C(B) = \begin{bmatrix} -3 & -1 & 1 \\ 6 & 3 & -3 \\ -4 & -3 & 1 \end{bmatrix}$$

$$(C-2A^{-1})B^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -3 & -1 & 1 \\ 6 & 3 & -2 \\ -4 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 2\cdot(-3) & 2\cdot(-1) & 2\cdot 1 \\ 2\cdot 6 & 2\cdot 3 & 2\cdot(-2) \\ 2\cdot(-4) & 2\cdot(-3) & 2\cdot 1 \end{bmatrix}$$

$$\Downarrow$$

$$\boxed{x = \begin{bmatrix} -6 & -2 & 2 \\ 12 & 6 & -4 \\ -8 & -6 & 2 \end{bmatrix}}$$