

Carrei da Reis Barbara Domingues

Sinta 6A

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1.

$$a) \begin{pmatrix} 1+0 & 0+10 \\ 2+6 & 7+(-4) \end{pmatrix} = \begin{pmatrix} 10 & 7 \\ 8 & 3 \end{pmatrix}$$

$$b) \begin{pmatrix} 1 \cdot 0 + 0 \cdot 3 & 1 \cdot 5 + 0 \cdot (-2) \\ 2 \cdot 0 + 7 \cdot 3 & 2 \cdot 5 + 7 \cdot (-2) \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 21 & -4 \end{pmatrix} + \begin{pmatrix} 0 \cdot 1 + 5 \cdot 2 & 0 \cdot 0 + 5 \cdot 7 \\ 3 \cdot 1 + (-2) \cdot 2 & 3 \cdot 0 + (-2) \cdot 7 \end{pmatrix} = \begin{pmatrix} 10 & 35 \\ -1 & -14 \end{pmatrix}$$

$$\begin{pmatrix} 0 + (-10) & 5 + (-35) \\ 2 + 7 & -4 + 14 \end{pmatrix} = \begin{pmatrix} -10 & -30 \\ 9 & 10 \end{pmatrix}$$

c) não é possível não há matriz com a mesma dimensão

$$d) \begin{pmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{pmatrix} + \begin{pmatrix} -6 & 3 & 18 \\ -12 & 0 & 0 \\ 9 & 12 & 3 \end{pmatrix} = \begin{pmatrix} -6 + (-6) & 2+3 & -4+18 \\ 4+(-12) & 2+0 & 0+0 \\ 0+9 & 8+12 & 4+3 \end{pmatrix} = \begin{pmatrix} -12 & 5 & 14 \\ -8 & 2 & 0 \\ 9 & 20 & 7 \end{pmatrix}$$

$D:2 \quad 3:E^*$

e)

$$\begin{pmatrix} -3 \cdot (-3) + 2 \cdot 1 + 0 \cdot (-2) & -3 \cdot 2 + 2 \cdot 1 + 0 \cdot 0 & -3 \cdot 0 + 2 \cdot 4 + 0 \cdot 2 \\ 1 \cdot (-3) + 1 \cdot 1 + 4 \cdot (-2) & 1 \cdot 2 + 1 \cdot 1 + 4 \cdot 0 & 1 \cdot 0 + 1 \cdot 4 + 4 \cdot 2 \\ -2 \cdot (-3) + 0 \cdot 1 + 2 \cdot (-2) & -2 \cdot 2 + 0 \cdot 1 + 2 \cdot 0 & -2 \cdot 0 + 0 \cdot 4 + 2 \cdot 2 \end{pmatrix} = \begin{pmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} -3 \cdot 2 + 2 \cdot (-1) + 0 \cdot (-6) & -3 \cdot 4 + 2 \cdot 0 + 0 \cdot 0 & -3 \cdot (-3) + 2 \cdot (-4) + 0 \cdot (-1) \\ 1 \cdot 2 + 1 \cdot (-1) + 4 \cdot (-6) & 1 \cdot 4 + 1 \cdot 0 + 4 \cdot 0 & 1 \cdot (-3) + 1 \cdot (-4) + 4 \cdot (-1) \\ -2 \cdot 2 + 0 \cdot (-1) + 2 \cdot (-6) & -2 \cdot 4 + 0 \cdot 0 + 2 \cdot 0 & -2 \cdot (-3) + 0 \cdot (-4) + 2 \cdot (-1) \end{pmatrix} = \begin{pmatrix} -8 & -12 & 17 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 11 + (-8) & -4 + (-12) & 8 + 17 \\ -10 + (-23) & 3 + 4 & 12 + (-11) \\ 2 + (-16) & -4 + (-8) & 4 + 4 \end{pmatrix} = \begin{pmatrix} 3 & -16 & 25 \\ -33 & 7 & 1 \\ -14 & -12 & 8 \end{pmatrix}$$

$$f) \begin{pmatrix} -2 & 7 & 1 & 0 \\ 3 & -3 & 2 & 7 \\ -7 & -2 & & \end{pmatrix} = \begin{pmatrix} -2 \cdot 1 + 7 \cdot 2 & -2 \cdot 0 + 7 \cdot 7 \\ 3 \cdot 4 + (-3) \cdot 2 & 3 \cdot 0 + (-3) \cdot 7 \\ -7 \cdot 1 + (-2) \cdot 2 & -7 \cdot 0 + (-2) \cdot 7 \end{pmatrix} = \begin{pmatrix} 12 & 49 \\ -3 & -21 \\ -11 & -14 \end{pmatrix}$$

g) não é possível as matrizes não possuem mesma dimensão

$$h) \begin{pmatrix} 1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & -3 \\ -10 & -4 \\ 60 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 5 \end{pmatrix}$$

$$i) \begin{pmatrix} 0 & (-2) + 5 \cdot 4 & 0 \cdot 3 + 5 \cdot (-3) & 0 \cdot (-7) + 5 \cdot (-2) \\ 3 \cdot (-2) + (-2) \cdot 7 & 3 \cdot 3 + (-2) \cdot (-3) & 3 \cdot (-7) + (-2) \cdot (-2) \\ 3 \cdot 5 \cdot 1 + (-1) \cdot 5 \cdot (-2) + (-10) \cdot 0 & -20 \cdot 1 + 15 \cdot (-2) + (-17) \cdot 0 \end{pmatrix} = \begin{pmatrix} 35 & -13 & -10 \\ -20 & 15 & -17 \\ 65 & -50 \end{pmatrix}$$

2.

~~a) não é definida por nã~~

a) $C_m = 2$ e $n = 4$, ou seja, BA não está definido $B_m = 4$ e $A_m = 2$, $B_m \neq A_m$

b) $C_m = 4$ e $n = 3$, ou seja, BA não está definido $B_m = 2$, $A_m = 4$, $B_m \neq A_m$

c) $C_m = 1$ e $n = 1$, ou seja, BA está definido $B_m = 1$ e $A_m = 1$, $B_m = A_m$

d) $C_m = 5$ e $n = 3$, ou seja, BA não está definido $B_m = 3$, $A_m = 5$, $B_m \neq A_m$

e) $C_m = 4$ e $n = 3$, ou seja, BA não está definido $B_m = 3$ e $A_m = 4$, $B_m \neq A_m$

f) $C_m = 4$ e $n = 4$, ou seja, BA está definido $B_m = 4$ e $A_m = 4$, $B_m = A_m$

3.

$$a) \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} 3i-2j \\ 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$$

$$b) \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 4 & -1 & -2 \\ 3 & 8 & 1 \\ 8 & 7 & 12 \end{pmatrix}$$

$$c) \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$$

$$d) \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{pmatrix}$$

4.

a) $\Rightarrow (2 \cdot 1) + (-1 \cdot 2) + (4 \cdot 5) = \boxed{BA}_{23} = 20$

b) $(-2 \cdot 3) + (-3 \cdot 4) + (2 \cdot (-7)) = 0$

$\Rightarrow BA_{23} = -32$

c) $(-3 \cdot 1) + (-1 \cdot 2) + (-7 \cdot (-3)) = 0 \quad \begin{matrix} p^2 = 16 \\ p_1 = 4 \end{matrix}$

d) $a_{11} + a_{22} + a_{33} = 1 + (-3) + 5 = \text{tr}(A) = 3$

e) $B^T = \begin{bmatrix} 15 & 2 & -3 \\ 0 & -1 & -1 \\ 3 & 4 & -7 \end{bmatrix} \Rightarrow 1 \cdot 1 - 7 = 0 \Rightarrow \text{tr } B^T = -7$

f) $(-1) + (-3 + 7) + (5 + 7) = \boxed{\text{tr}(A-B) = 10}$

g) $[(1 \cdot 1) + (2 \cdot 2) + (1 \cdot (-5))] + [(-3 \cdot (-1)) + (2 \cdot (-1)) + (4 \cdot 3)] + [(1 \cdot 3) + (4 \cdot 4) + (5 \cdot (-7))]$
 $-2 + 1 - 16 = \text{tr}(AB) = -17$

5.

a) $3B = \begin{pmatrix} 6 & 3 \\ 12 & 9 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 10 & 13 \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 22 & 22 \end{pmatrix} \quad 2x = \begin{pmatrix} 6 & 5 \\ 22 & 22 \end{pmatrix} + \begin{pmatrix} 11 & -7 \\ -2 & -6 \end{pmatrix}$

$2x = \begin{pmatrix} 17 & -2 \\ 33 & 16 \end{pmatrix} \Rightarrow x = \begin{pmatrix} \frac{17}{2} & -1 \\ \frac{33}{2} & 8 \end{pmatrix}$

b) $B - C = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} + \begin{pmatrix} 0 & -2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & 3 \end{pmatrix} \Rightarrow (B-C)^T = \begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix} \Rightarrow \frac{(B-C)^T}{2} = \begin{pmatrix} 1 & \frac{3}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$

$y = \begin{pmatrix} 2 & -\frac{1}{2} \\ -\frac{5}{2} & \frac{9}{2} \end{pmatrix}$

c) $3x + A = B - x \quad (B-A) = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -7 \\ -2 & -6 \end{pmatrix} = \begin{pmatrix} 3 & -6 \\ 2 & -3 \end{pmatrix}$
 $4x = \begin{pmatrix} 3 & -6 \\ 2 & -3 \end{pmatrix} \Rightarrow x = \begin{pmatrix} \frac{3}{4} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$

$$\begin{aligned}
 & \begin{cases} x + y = 3A \\ x - y = 2B + C \end{cases} \Rightarrow \begin{cases} x = 3A \\ x = 2B + C \end{cases} \Rightarrow 2 \times 3A + 2B + C \\
 & 3A = \begin{pmatrix} -3 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot 2B = \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix} \quad x = \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{pmatrix}
 \end{aligned}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 1 & 0 & 15 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 5 \\ 15 & 2 & 4 \end{pmatrix}$$

$$x = \begin{bmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{bmatrix}; y = \begin{bmatrix} \frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{bmatrix}$$

$$6) A^2 = 2A$$

$$\begin{bmatrix} 1 & \frac{1}{x} \\ x & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{1}{x} \\ x & 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{bmatrix} \text{ ou seja } A^2 = 2A.$$

obtemos expressão para A^m para $m = 1, 2, 3, \dots$

$$A^3 = A^2 \cdot A$$

$$A^3 = A^2 \cdot A$$

$$A^3 = (2A)A$$

$$A^4 = (4A)A$$

$$A^3 = 2(AA) = 2A^2$$

$$A^4 = 4(AA) = 4A^2$$

$$A^3 = 2(2A) = 4A$$

$$A^4 = 4(2A) = 8A$$

$$\text{fórmula} = A^m = 2^{m-1}A$$

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$$\begin{aligned}
 a) A(B+C) &= AB+AC \\
 x+y &
 \end{aligned}$$

$$\begin{aligned}
 b) B^t A^t &= (AB)^t \\
 x^t &= A^t B^t
 \end{aligned}$$

$$\begin{aligned}
 c) C^t A^t &= (CA)^t \\
 y^t &= A^t C^t
 \end{aligned}$$

$$\begin{aligned}
 d) (AB+A)(C) &= (x+y)C \\
 (xy) &
 \end{aligned}$$

8)

$$a) 20A = A^6 \text{ en la 0.}$$

$$x+2=2x-6$$

$$x-2x=-8-2$$

$$-x=-5$$

$$x=5$$

$$x+2=7$$

$$x=5, A = \begin{bmatrix} 4 & 7 \\ 7 & 4 \end{bmatrix}$$

$$a) \begin{pmatrix} 6 & -4 & 2 \\ 2 & 0 & 1-z \\ 4 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1 & -1 & 0 \end{pmatrix}$$

$$-B = \begin{pmatrix} 0 & 4 & -2 \\ -x & 0 & -(1-z) \\ y & -2 & 0 \end{pmatrix}$$

$$x=4, y=-2, z=-1$$

$$b) \begin{pmatrix} x & 6 \\ 4 & 2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 4 & x & y \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & x & 3y \\ 2 & 2 & 3z \end{pmatrix}$$

$$\begin{aligned} -1+2+3 &= 3z \\ 2z &= 2 \Rightarrow z=1 \end{aligned}$$

$$\begin{aligned} x+4 &= 3x \\ 2x &= 4 \end{aligned}$$

$$x=2$$

$$6+2+y=3y$$

$$8=2y$$

$$y=4$$

$$2x+3z=3y$$

$$-4=3$$

$$x=2, y=4$$

$$z=1$$

10

a)

$$A^4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^2 A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$b) \begin{cases} x=0 \\ y=\sqrt{2} \\ z=\frac{1}{2} \end{cases}$$