9a Lista 3  / /

1.

$$A = \begin{pmatrix} 1 & 3 & 4 \\ -1 & 2 & 5 \\ -2 & -1 & 3 \end{pmatrix}_{3\times3} = \begin{pmatrix} 2\times(1)-1 & -1 & 1+2 & 1+3 \\ 1-2 & 2(2)-2 & 2+3 & \\ -1+3 & 2-3 & 2(3)-3=3 & \end{pmatrix}_{3\times3}$$

$\nabla^{+}$ $\Delta^{-}$

$$\det A = \begin{vmatrix} 1 & 3 & 4 & 1 & 3 \\ -1 & 2 & 5 & -1 & 2 \\ -2 & -1 & 3 & -2 & -1 \end{vmatrix} = 10$$

$(+16+5+9)\ (6-30\ 4)$

$30 + -20 = 10$

$$\det(A_{x1}) = \begin{vmatrix} 1 & 3 & 4 & 1 & 3 \\ 2 & 2 & 5 & 2 & 2 \\ -3 & -1 & 3 & -3 & -1 \end{vmatrix} = \quad 6(-45)(-8) + (-18)(5)(24) = -36$$

subt. B  $\Delta A$

$$\det(A_{x2}) = \begin{vmatrix} 1 & 1 & 4 & 1 & 1 \\ -1 & 2 & 5 & -1 & 2 \\ -2 & -3 & 3 & -2 & -3 \end{vmatrix} \quad \begin{aligned} &6+(-10)+12+3+15+16 \\ &-4 + 15 + 31 \\ &11 + 31 = 42 \end{aligned}$$

$$\det(A_{x3}) = \begin{vmatrix} 1 & 3 & 1 & 1 & 3 \\ -1 & 2 & 2 & -1 & 2 \\ -2 & -1 & -3 & -2 & -1 \end{vmatrix} \quad \begin{aligned} &-6+(-12)+1+(-9)+2+4 \\ &(-18 + (-8)\ 6 \\ &-20 \end{aligned}$$

$$X_1 = \frac{-36}{10} = -3{,}6 \qquad X_2 = \frac{42}{10} = 4{,}2 \qquad X_3 = \frac{-20}{10} = -2{,}0$$

$R: X = (X_1, X_2, X_3) = (-3{,}6\ ;\ 4{,}2\ ;\ -2{,}0)$

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2) Achar a inversa precisa do determinante

2 (a) $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} = \det A = -5$ $\quad \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix} \quad \frac{1}{-5} \cdot \underbrace{\begin{bmatrix} \frac{3}{-5} & \frac{-4}{-5} \\ \frac{-2}{-5} & \frac{1}{-5} \end{bmatrix}}_{\text{Inversa da A}}$ $\quad X = \begin{bmatrix} \frac{3}{-5} \cdot 2 & \frac{-4}{-5} \cdot -1 \\ \frac{-2}{-5} \cdot 2 & \frac{1}{-5} \cdot -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Fórmula: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad$ Fórmula: $\frac{1}{\det}$

$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Ao invés de dividir inverte a matriz

(b) $\begin{pmatrix} 2 & 3 \\ 5 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} Y = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} Y = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ 5 & 5 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} Y = \begin{pmatrix} -1 & 0 \\ -3 & 2 \end{pmatrix}}$

$\det \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = 5 - 6 = -1 \quad \left(\frac{1}{-1}\right) \cdot \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ -3 & 2 \end{pmatrix}$

(c) $\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 2 & -1 & 0 & 2 & -1 \\ 2 & 3 & 1 & 2 & 3 \end{array}\right] = -1 \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$

$c_{1\times1} = \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} = -1 \quad c_{2\times1} = \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix} = 0 \quad c_{3\times1} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = 0$

$c_{1\times2} = \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix} = 2 \quad c_{2\times2} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = 1 \quad c_{3\times2} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} = 0$

$c_{1\times3} = \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix} = 8 \quad c_{2\times3} = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = 3 \quad c_{3\times3} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = -1$

$= \begin{bmatrix} -1 & 0 & 0 \\ 2 & 1 & 0 \\ 8 & 3 & -1 \end{bmatrix} \Rightarrow A^t \begin{bmatrix} -1 & 2 & 8 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{bmatrix}$

$\begin{matrix} (-1)^{1+1} \cdot (-1) & (-1)^{2+1} \cdot 2 & (-1)^{3+1} \cdot 8 \\ (-1)^{1+2} \cdot 0 & (-1)^{2+2} \cdot 1 & (-1)^{3+2} \cdot 3 \\ (-1)^{1+3} \cdot 0 & (-1)^{2+3} \cdot 0 & (-1)^{3+3} \cdot (-1) \end{matrix} = \begin{pmatrix} -1 & -2 & 8 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{pmatrix} = \operatorname{adj}(C) = \begin{pmatrix} -1 & 0 & 0 \\ -2 & 1 & 0 \\ 8 & -3 & -1 \end{pmatrix}$

MATRIZ dos cofatores

$$(C)^{-1} = \frac{1}{-1} \cdot \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & 0 \\ 8 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -8 & 3 & 1 \end{bmatrix} \quad W = \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -8 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix} = W = \begin{bmatrix} 5 & 0 & 0 \\ 10 & -7 & 0 \\ -40 & 21 & 2 \end{bmatrix}$$

3.

a) $AXB = C$

$A^{-1} AXB = A^{-1} C \Rightarrow XB = A^{-1} C$

$XB \cdot B^{-1} = A^{-1} CB^{-1} \Rightarrow X = A^{-1} CB^{-1}$

R: $X = A^{-1} CB^{-1}$

f) $2AX - X = 3B$

$2AX - X = 3B \Rightarrow (2A - I)X = 3B$

$X = (2A - I)^{-1} 3B$

R: $X = 3(2A - I)^{-1} B$

b) $A(B + X) = A$

$AB + AX = A \Rightarrow AX = A - AB = A(I - B)$

$X = A^{-1} A(I - B) = I - B$

R: $X = I - B$

c) $ACXB = C$

$CXB = A^{-1} C$

$X = C^{-1} A^{-1} CB^{-1}$

R: $X = C^{-1} A^{-1} CB^{-1}$

d) $(AB)^{-1}(AX) = CC^{-1}$

$CC^{-1} = I$

$(AB)^{-1}(AX) = I \Rightarrow (AB)^{-1} AX = I$

$B^{-1} A^{-1} AX = I \Rightarrow B^{-1} X = I \Rightarrow X = B$

R: $X = B$

e) $AB^{t} X B^{-1} = A^{t}$

$XB^{-1} = (AB^{t})^{-1} A^{t} \Rightarrow X = (AB^{t})^{-1} A^{t} B$

R: $X = (AB^{t})^{-1} A^{t} B$

/ /

Fórm: $x = \frac{\det(A_x)}{\det A}$

4. a) $3x - 4y = 1$, $2x + 6y = 18$

$a = \begin{matrix} 3 & -4 \\ 2 & 6 \end{matrix}$ $18 + 8 = 26$ $x = \frac{78}{26} = 3$ $y = \frac{52}{26} = 2$

$a_x = \begin{matrix} 1 & -4 \\ 18 & 6 \end{matrix}$ $6 + 72 = 78$ R: $x = 3$ $y = 2$

$a_y = \begin{matrix} 3 & 1 \\ 2 & 18 \end{matrix}$ $54 - 2 = 52$

$x + 2y + 3z = 10$
$3x + 4y + 6z = 23$
$3x + 2y + 3z = 10$

$y$ $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 3 & 2 & 3 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 4 \\ 3 & 2 \end{matrix}$

$-36 - 12 - 18 / 12\ 36\ 18 = 0$

$\det y = 0$ ?!

b) $b = \begin{matrix} 5 & 8 \\ 10 & 16 \end{matrix} = 0$ $\quad -80 \quad 80$

$x = \frac{144}{0} = 0$ $y = \frac{-90}{0} = 0$

$x = \begin{vmatrix} 10 & 2 & 3 \\ 23 & 4 & 6 \\ 10 & 2 & 3 \end{vmatrix} \begin{matrix} 10 & 2 \\ 23 & 4 \\ 10 & 2 \end{matrix}$

$b_x = \begin{matrix} 34 & 8 \\ 50 & 16 \end{matrix}$ $544 - 400 = 144$ $\quad -400 \quad 544$

R: $x = 0$ $y = 0$

$120 + 120 + 138 - 138 - 120 - 120$

$\det(0)$

$b_y = \begin{matrix} 5 & 34 \\ 10 & 50 \end{matrix}$ $250 - 340 = -90$ $\quad -340 \quad 250$

$\begin{vmatrix} 1 & 10 & 3 \\ 3 & 23 & 6 \\ 3 & 10 & 3 \end{vmatrix} \begin{matrix} 1 & 10 \\ 3 & 23 \\ 3 & 10 \end{matrix} = 69 + 180 + 90 - 90 - 60 - 207$

$= \det(-18$

c) $x + 2y = 5$
$2x - 3y = -4$

$x = \frac{-7}{-7} = 1$

$y = \frac{-14}{-7} = 2$

SI: sistema sem solução

$c = \begin{matrix} 1 & 2 \\ 2 & -3 \end{matrix}$ $-3 - 4 = -7$

$c_x = \begin{matrix} 5 & 2 \\ -4 & -3 \end{matrix}$ $-15 + 8 = -7$ $\quad +8 \quad -15$

$x = 1$ $y = 2$

$c_y = \begin{matrix} 1 & 5 \\ 2 & -4 \end{matrix}$ $= -10 - 4 = -14$ $\quad -10 \quad -4$

d) $\begin{cases} 3x+2y-5z=8 \\ 2x-4y-2z=-4 \\ x-2y-3z=-4 \end{cases}$

$$d=\left|\begin{array}{ccc|cc} 3 & 2 & -5 & 3 & 2 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -2 & -3 & 1 & -2 \end{array}\right| = 40$$

$36+4+20+\cancel{12}-\cancel{12}-20$

$d=40$

$-20 \quad -12 \quad 12 \quad 36 \quad 4 \quad 20$

$$dx=\left|\begin{array}{ccc|cc} 8 & 2 & -5 & 8 & 2 \\ -4 & -4 & -2 & -4 & -4 \\ -4 & -2 & -3 & -4 & -2 \end{array}\right| = 96+16-40-24-32+80$$

$dx=96$

$x=\frac{96}{40}=2,4$

$80 \quad -32 \quad -24 \quad 96 \quad 16 \quad -40$

$$dy=\left|\begin{array}{ccc|cc} 3 & 8 & -5 & 3 & 8 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -4 & -3 & 1 & -4 \end{array}\right| = 36-16+40+48-24-20=64$$

$dy=64 \qquad y=\frac{64}{40}=1,6$

$-20 \quad -24 \quad 48 \quad 36 \quad -16 \quad 40$

$$dz=\left|\begin{array}{ccc|cc} 3 & 2 & 8 & 3 & 2 \\ 2 & -4 & -4 & 2 & -4 \\ 1 & -2 & -4 & 1 & -2 \end{array}\right| = 48-8-32+16-24+32$$

$dz=32 \qquad z=\frac{32}{40}=0,8$

$32 \quad -24 \quad 16 \quad 48 \quad -8 \quad -32$

R: $x=2,4$, $y=1,6$, $z=0,8$

e) $1x+2y-1z=2$

$2x-1y+3z=9$

$3x+3y-2z=3$

$$e\left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = 2+18-6+8-9-3=10$$

$-3 \quad -9 \quad 8 \quad 2 \quad 18 \quad -6$

$$dx = \begin{vmatrix} 2 & 2 & -1 & 2 & 2 \\ 9 & -1 & 3 & 9 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix} = 4 + 18 - 27 + 36 - 18 - 3$$

$dx = 10 \quad \frac{10}{10} = 1$

$-3 \quad -18 \quad 36 \quad 4 \quad 18 \quad -27$

$$dy = \begin{vmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 9 & 3 & 2 & 9 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix} = \cancel{-18 + 18}\ 0 - 6 + 8 - 9 + 27 = 20$$

$dy = 20 \quad \frac{20}{10} = 2$

$-9 \quad 8 \quad -18 \quad 18 \quad -6$

$$dz = \begin{vmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & -1 & 9 & 2 & -1 \\ 3 & 3 & 3 & 3 & 3 \end{vmatrix} = -3 + 54 \cancel{+12} \cancel{-12}$$

$-3 + 54 + 0 - 27 + 6 = 30$

$\frac{30}{10} = 3$

$0 \quad -27 \quad -12 \quad -3 \quad 54 \quad 12$

R: $x = 1, y = 2, z = 3$

b) $1x + 3z + 0 = -2$

$2x - 4y = -4 \quad z = 0$

$3x - 2y - 5z = 26$

$$D = \begin{vmatrix} 1 & 0 & 3 & 1 & 0 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & -2 & -5 & 3 & -2 \end{vmatrix} = 20 + 0 - 12 + 0 + 36$$

$D = 44$

$-36 \quad 0 \quad 0 \quad 20 \quad 0 \quad -12$

$$dx = \begin{vmatrix} -2 & 0 & 3 & -2 & 0 \\ -4 & -4 & 0 & -4 & -4 \\ 26 & -2 & -5 & 26 & -2 \end{vmatrix} = -40 + 24 + 312 = 296 \quad \frac{296}{44} = 6{,}72$$

$dx = 296$

$-312 \quad 0 \quad 0 \quad -40 \quad (0) \quad 24$

$$dy = \begin{vmatrix} 1 & -2 & 3 & 1 & -2 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & 26 & -5 & 3 & 26 \end{vmatrix} = 20 + 0 + 156 + 20 + 0 + 36$$

$y = 192 \quad \frac{192}{44} = 4{,}45$

$0 \quad -20 \quad 0 \quad 156$

$$dz = \begin{vmatrix} 1 & 0 & -2 & 1 & 0 \\ 2 & -4 & -4 & 2 & -4 \\ 3 & -2 & 26 & 3 & -2 \end{vmatrix} = Z = -104 + 0 + 8 + 0 + 8 - 24 = \frac{-128}{44} = -2{,}90$$

$-24 \quad -8 \quad 0 \quad -104 \quad 0 \quad -8$

R: $x = 6{,}72,\ y = 4{,}45,\ Z = -2{,}90$

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5.

a) $\begin{cases} 3x_1 - 4x_2 = 0 \\ -6x_1 + 8x_2 = 0 \end{cases}$ $\begin{bmatrix} 3 & -4 \\ -6 & 8 \end{bmatrix}$ det(A) = 0

$24 - 24$

Equações dependentes, uma é múltiplo da outra, sistema homogêneo com determinante 0 → infinitas soluções: sistema possível indeterminado (SPI)

b) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{vmatrix}\begin{matrix} 1 & 1 \\ 2 & 2 \\ 1 & 1 \end{matrix}$ = $6+4+2-6-4-2 = 0 = \det(b) = 0$

$-2 \ -4 \ -6 \ 6 \ 4 \ 2$   $x = 0 \quad y = 0 \quad z = 0$

Sistema possível indeterminado   0   Infinitas soluções

c) $\begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & +4 & 0 \end{vmatrix}\begin{matrix} 1 & 1 \\ 1 & -1 \\ 1 & +4 \end{matrix}$ =   $0 - 3 + 8 + 0 + 12 + 2 = 22$

Sistema possível ~~linear~~ indeterminado

6.

a) $\begin{cases} 3x + m \cdot y = 2 \\ x - y = 1 \end{cases}$ $A = \begin{bmatrix} 3 & m & 2 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow{L_1 - 3L_2} \begin{bmatrix} 0 & m+3 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$\Rightarrow m + 3 \neq 0 \Rightarrow m \neq -3$

Sistema possível e determinado

b) $\begin{cases} 3x + 2(m-1)y = 1 \\ mx - 4y = 0 \end{cases}$

$\begin{bmatrix} 3 & 2m-1 \\ m & -4 \end{bmatrix} \Rightarrow \text{Det} = -2m^2 + 2m - 12 \neq 0$

SPD para todo $m \in \mathbb{R}$ para todo $m \Rightarrow$ Todos m

c) $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 1 & m & 1 & 0 \\ -1 & 1 & -1 & 4 \end{array}\right] \underset{\substack{L_2-L_1 \\ L_3+L_1}}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & m+1 & 1 & -2 \\ 0 & 0 & -1 & 6 \end{array}\right]$ $m \neq -1$

SPD se $m \neq -1$

d) $\begin{bmatrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow \det = 2m + 2 \neq 0 \Rightarrow m \neq -1$

SPD se $m \neq -1$

7. Dado:

- Cada peça correta dá R$6,00
- Cada peça com defeito dá prejuízo de R$2,00
- Produziu 225 peças ao todo
- Ganhou R$750,00

$x$: número de peças corretas $y$: número de peças com defeito

Total:

$$x+y=225$$

Total arrecadado: $6x-2y=750$

$$\begin{vmatrix}1 & 1 \\ 6 & -2\end{vmatrix} \quad \det = -6-2=-8$$

$$\frac{-600}{-8}=y=75$$

$$y\begin{vmatrix}1 & 225 \\ 6 & 750\end{vmatrix} \quad y=-1350+750=-600$$

$$x\begin{vmatrix}225 & 1 \\ 750 & -2\end{vmatrix} \quad -450-750=-1200$$

$$\frac{-1200}{-8}=150 \quad x=150$$

Peças produzidas corretamente R: 150

8. Dado:

Tak percorre 540/mês

Custo por km

- Automóvel (Toyota AE86): R$0,60/km
- Motocicleta (Scooter Suzuki): R$0,20/km
- Custo total no mês R$300,00

$x$ = km percorridos com automóvel

$y$ = km percorridos com a motocicleta

$$\begin{vmatrix}1 & 1 \\ 0,60 & 0,20\end{vmatrix} = -0,40$$

$$x\begin{vmatrix}540 & 1 \\ 300 & 0,20\end{vmatrix} = -192$$

$$y\begin{vmatrix}1 & 540 \\ 0,60 & 300\end{vmatrix} = -24$$

$$x+y=540$$

$$0,60+0,20=300$$

$$x=\frac{-192}{-0,40}=480$$ Automóvel (AE86): 480 km percorridos

$$y=\frac{-24}{-0,40}=60$$ Motocicleta (Suzuki): 60 km percorridos

9. Dados:
Total: 92 cédulas
Valor total: R$ 500,00
Quantidade de cédulas de R$ 2 e de R$ 10 é igual
Precisamos descobrir quantas cédulas de R$ 5 são usadas.
$x$ = número de cédulas de R$ 2,00
$y$ = número de cédulas de R$ 5,00
$z$ = número de cédulas de R$ 10,00
Total de cédulas: 92

$$x + y + z = 92$$
$$2x + 5y + 10z = 500$$
$$x - z = 0$$

$$\begin{matrix} 1 & 1 & 1 & = 92 \\ 2 & 5 & 10 & = 500 \\ 1 & 0 & -1 & = 0 \end{matrix}$$

$\det(N)$
$$\left|\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 5 & 10 & 2 & 5 \\ 1 & 0 & -1 & 1 & 0 \end{matrix}\right| \quad -5 + 10 + 0 + 2 + 0 - 5 = 2$$

$\det(N)_x$
$$\left|\begin{matrix} 92 & 1 & 1 & 92 & 1 \\ 500 & 5 & 10 & 500 & 5 \\ 0 & 0 & -1 & 0 & 0 \end{matrix}\right| = -460 + 0 + 0 + 500 + 0 + 0 = 960$$

$\det(N)_y =$
$$\left|\begin{matrix} 1 & 92 & 1 & 1 & 92 \\ 2 & 500 & 10 & 2 & 500 \\ 1 & 0 & -1 & 1 & 0 \end{matrix}\right| \quad -500 + 920 + 0 + 184 + 0 - 500 = 104$$

$\det(N)_z$
$$\left|\begin{matrix} 1 & 1 & 92 & 1 & 1 \\ 2 & 5 & 500 & 2 & 5 \\ 1 & 0 & 0 & 1 & 0 \end{matrix}\right| \quad 40 + 50 + 184 + 0 + 0 - 460 = -226$$

$$x = \frac{960}{2} = 480, \quad y = \frac{104}{2} = 52, \quad z = \frac{-226}{2} = -113$$

Ele precisará de 52 cédulas de R$ 5,00

10.

dada:

Kiba + Akamaru = 109 kg

Kiba + Tamaki = 142 kg

Tamaki + Akamaru = 97 kg

K = peso de Kiba

T = peso de Tamaki

A = peso de Akamaru

$$\begin{cases} K + 0T + A = 109 \\ K + T + 0A = 142 \\ 0K + T + A = 97 \end{cases} \quad \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det(D) \left| \begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{array} \right| = 1 + 0 + 1 + 0 + 0 + 0 = 2$$

$$\det(D)_K \left| \begin{array}{ccc|cc} 109 & 0 & 1 & 109 & 0 \\ 142 & 1 & 0 & 142 & 1 \\ 97 & 1 & 1 & 97 & 1 \end{array} \right| \quad 109 + 0 + 142 + 0 + 0 - 97 = 154$$

$$\det(D)_T \left| \begin{array}{ccc|cc} 1 & 109 & 1 & 1 & 109 \\ 1 & 142 & 0 & 1 & 142 \\ 0 & 97 & 1 & 0 & 97 \end{array} \right| \quad 142 + 0 + 97 - 109 + 0 + 0 = 130$$

$$\det(D)_A \left| \begin{array}{ccc|cc} 1 & 0 & 109 & 1 & 0 \\ 1 & 1 & 142 & 1 & 1 \\ 0 & 1 & 97 & 0 & 1 \end{array} \right| = 97 + 0 + 109 + 0 - 142 + 0 = 64$$

$$K = \frac{154}{2} = 77 \qquad T = \frac{130}{2} = 65 \qquad A = \frac{64}{2} = 32$$

Kiba pesa 77 kg

Tamaki pesa 65 kg

Akamaru: pesa 32kg