$27$
$27$
$21$
$49 \quad 1$

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1.

$r: X = (-5, \frac{2}{3}, 0) + \lambda(\frac{1}{2}, 1, 1)$.

a)

$$\text{sen}\,\alpha = \frac{|\vec{v} \cdot \vec{n}|}{\|\vec{v}\| \cdot \|\vec{n}\|}$$

$\theta = \text{ang}(s, r)$

$\vec{v}$: vetor diretor de $r$: $\vec{v} = (\frac{1}{2}, 1, 1)$

$s: z = 3x = 2y - 16$

$z = 3x \rightarrow z - 3x = 0 \rightarrow \alpha_1: (-3, 0, 1)$

$z = 2y - 16 \rightarrow z - 2y + 16 = 0 \rightarrow \alpha_2: (0, -2, 1)$

$\vec{n} = (-3, 0, 1) \times (0, -2, 1) = (0 - (-2), -(-3 - 0), -6 - 0) = (2, 3, -6)$

$|\vec{v} \cdot \vec{n}| = |(\frac{1}{2}) \cdot 2 + 1 \cdot 3 + 1 \cdot (-6)| = |1 + 3 - 6| = |-2| = 2$

$\|\vec{v}\| = \sqrt{(\frac{1}{2})^2 + 1^2 + 1^2} = \sqrt{\frac{1}{4} + 1 + 1} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$\|\vec{n}\| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

$$\text{sen}\,\alpha = \frac{2}{\frac{3}{2} \cdot 7} = \frac{2}{\frac{21}{2}} = \frac{4}{21}$$

b)

$r: \vec{v} = (0, -1, 1)$

$\alpha: x - y + 3 = z = 4$

$x - y + 3 - z = 0 \rightarrow (1, -1, -1)$

$z - 4 = 0 \rightarrow (0, 0, 1)$

$\vec{n} = (1, -1, -1) \times (0, 0, 1) = (-1, -1, 0)$

$|\vec{v} \cdot \vec{n}| = |0 \cdot (-1) + (-1) \cdot (-1) + 1 \cdot 0| = 1$

$\|\vec{v}\| = \sqrt{0^2 + (-1)^2 + (1)^2} = \sqrt{2}$

$\|\vec{n}\| = \sqrt{(-1)^2 + (-1)^2 + 0^2} = \sqrt{2}$

$$\text{sen}\,\alpha = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$$

Credeal

c)

Reta r:

$$\begin{cases} x+3z=7 \\ y=0 \end{cases}$$

Seja $z=t$

$x=7-3t$

$y=0$

$z=t$ vetor diretor $\vec{v}=(-3,0,1)$

Plano 2:

$$\begin{cases} x-4y-2z=5 \\ y=0 \end{cases} \rightarrow n_1=(1,-4,-2)$$

$n_2=(0,1,0)$

$\rightarrow$ produto vetorial de um e com outro

$\vec{n}=(1,-4,-2)\times(0,1,0)=(4,0,1)$

$\vec{n}=\begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

$|\vec{v}\cdot\vec{n}|=|(-3\cdot 4)+(0+0)+(1\cdot 1)|=|-12+1|=11$

$\|\vec{v}\|=\sqrt{(-3)^2+0^2+1^2}=\sqrt{10}$

$\|\vec{n}\|=\sqrt{4^2+0^2+1^2}=\sqrt{17}$ $\rightarrow \cos\alpha=\dfrac{11}{\sqrt{10}\cdot\sqrt{17}}=\dfrac{11}{\sqrt{170}}$

d)

Reta r:

Paramétrica:

$x=t$

$y=1-2t$

$z=3t$

Então vetor diretor $\vec{v}=(1,-2,3)$

Plano 2:

Sistema:

$$\begin{cases} 3x+y-5z=0 \\ x-2y+3z+1=0 \end{cases}$$

$\vec{n_1}=(3,1,-5)$

$\vec{n_2}=(1,-2,3)$

$\vec{n}=(3\cdot 1)$

Produto vetorial:

$(y_1\cdot z_2-z_1\cdot y_2), -(x_1\cdot z_2-z_1\cdot x_2), (x_1\cdot y_2-y_1\cdot x_2)$

$\vec{n}$ $1\cdot 3-((-5)\cdot(-2)), -((3\cdot 3)-((-5)\cdot 1)), (3\cdot(-2)-1\cdot 1)=(-7,-14,-7)$

$(1,-2,3)=\vec{v}$ $\quad \vec{n}=(-7,14,-7)$

$|\vec{v}\cdot\vec{n}| = |(1\cdot(-7)) + ((-2)\cdot(-14)) + (3\cdot(-7))| = |-7+28-21| = 0$

Independente do $(\|\vec{v}\|\cdot\|\vec{n}\|)$ o resultado de $\boxed{\cos\alpha = 0}$

2.

$r$: $\leftarrow P$

$X=(0,2,0)+\lambda(0,1,0)$ $\leftarrow$ vetor diretor

$s: X=(1,2,0)+\mu(0,0,1)$

$Q$

coordenada do ponto

$$P\begin{cases} x = 0+\lambda\cdot 0 \\ y = 2+\lambda\cdot 1 = (0,2+\lambda,0) \\ z = 0+\lambda\cdot 0 \end{cases}$$

$$Q\begin{cases} x = 1+\mu\cdot 0 \\ y = 2+\mu\cdot 0 = (1,2,\mu) \\ z = 0+\mu\cdot 1 \end{cases}$$

reta $\overline{PQ}$ $45^\circ$ com $r$

$60^\circ$ com $s$

$\vec{PQ} = Q-P = (1-0, 2-(2+\lambda), \mu-0)$

$PQ = (1,-\lambda,\mu)$

Ângulo entre $\vec{PQ}$ e $r$ é $45^\circ$:

$\vec{v}_r=(0,1,0)$ $\quad \cos 45^\circ = \dfrac{|\vec{PQ}\cdot\vec{v}_r|}{\|\vec{PQ}\|\cdot\|\vec{v}_r\|} = \dfrac{-\lambda \text{ (óbvio)} (0+\lambda+0)}{\sqrt{1^2+\lambda^2+\mu^2}\cdot 1}$ $\leftarrow$ módulo de $(\vec{v}_r)$

$\dfrac{-\lambda}{\sqrt{1+\lambda^2+\mu^2}} = \dfrac{\sqrt{2}}{2}$

Ângulo entre $\vec{PQ}$ e $\vec{v}_s = 60^\circ$

$\mu$ é óbvio também $(0+0+\mu)$ $\|u\|=1$

$\dfrac{\mu}{\sqrt{1+\lambda^2+\mu^2}\cdot 1} = \dfrac{1}{2} \Rightarrow \mu = \dfrac{1}{2}\cdot\sqrt{1+\lambda^2+\mu^2}$

$-\lambda = \dfrac{\sqrt{2}}{2}\cdot\sqrt{1+\lambda^2+\mu^2}$

Segunda equação:

$$\sqrt{1+\lambda^2+\mu^2} = 2\mu \xrightarrow{x^2} 1+\lambda^2+\mu^2 = 4\mu^2$$

$$1+\lambda^2 = 3\mu^2 \quad (1)$$

Agora da primeira equação:

$$-\lambda = \frac{\sqrt{2}}{2} \cdot \sqrt{1+\lambda^2+\mu^2} \xrightarrow{\text{x multiplicando}} -2\lambda = \sqrt{2} \cdot \sqrt{1+\lambda^2+\mu^2}$$

elevando ao quadrado $x^2$

$$4\lambda^2 = 2(1+\lambda^2+\mu^2)$$

$$4\lambda^2 = 2+2\lambda^2+2\mu^2$$

$$2\lambda^2-2\mu^2 = 2 \rightarrow \lambda^2-\mu^2 = 1 \quad (2)$$

Substituindo (1) em (2):

$$(3\mu^2-1)-\mu^2 = 1$$

$$2\mu^2-1 = 1$$

$$2\mu^2 = 2$$

$$\mu^2 = 1$$

$$\mu = \pm 1 \xrightarrow{Q} \mu = 1$$

$$Q = (1,2,\mu)$$

$$Q = (1,2,1)$$

P

$$\lambda^2 = 3\mu^2-1 = 2$$

$$\lambda = \pm\sqrt{2} \text{ usa } (-)$$

$$\lambda = -\sqrt{2}$$

$$P = (0, 2+\lambda, 0)$$

$$\rightarrow P = (0, 2-\sqrt{2}, 0)$$

Resposta: $P = (0, 2-\sqrt{2}, 0)$
$Q = (1,2,1)$

3.

ang r e 2

$$\operatorname{sen}\theta = \frac{|\vec{r_0} \cdot \vec{n}|}{\|\vec{r_0}\| \cdot \|\vec{n}\|} \rightarrow \theta = \operatorname{arcsen}\left(\frac{|\vec{r_0} \cdot \vec{n}|}{\|\vec{r_0}\| \cdot \|\vec{n}\|}\right)$$

a) $x = y - z = 0 \quad \pi: z = 0$ / /

$\vec{v} = (1, -1, -1)$

$z = 0 \rightarrow \vec{n} = (0, 0, 1)$

$\|\vec{v} \cdot \vec{n}\| = -1$

módulo:

$\|\vec{v}\| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3} \quad \|\vec{n}\| = 1$

$\operatorname{sen} \theta = \frac{|-1|}{\sqrt{3} \cdot 1} = \frac{1}{\sqrt{3}} \quad \theta \approx \operatorname{arcsen}\left(\frac{1}{\sqrt{3}}\right) \approx 0{,}615 \text{ rad}$

b)

$r: -x = y = \frac{z-1}{2}, \quad \pi: 2x - y = 0$

$\vec{v}: x = -t \qquad \pi: 2x - y = 0 \rightarrow \vec{n} = (2, -1, 0)$

$y = t$

$z = 1 + 2t \rightarrow \vec{v}(-1, 1, 2)$

$\|\vec{v} \cdot \vec{n}\| = |-2 - 1 + 0| = |-3| = 3$

$\|\vec{v}\| = \sqrt{(-1)^2 + 1^2 + 2^2} = \sqrt{6}$

$\|\vec{n}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5} \quad \operatorname{sen} \theta = \frac{|-3|}{\sqrt{6} \cdot \sqrt{5}} = \frac{3}{\sqrt{30}}$

$\theta \approx \operatorname{arcsen}\left(\frac{3}{\sqrt{30}}\right) \approx 0{,}720 \text{ rad}$

/ /

c)

$r: X=(1,0,0)+\lambda\cdot(1,1,-2)$ & $\vec{v}=(1,1,-2)$

$\pi: x+y-z-1=0$ & $\vec{n}=(1,1,-1)$

$\|\vec{v}\cdot\vec{n}\|=1+1+2=4$

$\|\vec{v}\|=\sqrt{1^2+1^2+(-2)^2}=\sqrt{6}$

$\|\vec{n}\|=\sqrt{1^2+1^2+(-1)^2}=\sqrt{3}$

$$\sin\theta=\frac{|4|}{\sqrt{6}\cdot\sqrt{3}}=\frac{4}{\sqrt{18}}=\frac{4}{3\sqrt{2}}=\frac{2\sqrt{2}}{3}$$

$$\theta\approx\arcsin\left(\frac{2\sqrt{2}}{3}\right)\approx 1{,}003 \text{ rad}$$

4.

$\pi_1: x+y+z=0 \to \vec{n_1}(1,1,1)$

$\pi_2: x-y=0 \to n_2(1,-1,0)$

Ângulo de 45° com o plano $\pi_2$:

$$\cos(45°)=\frac{\|\vec{v}\cdot\vec{n_2}\|}{\|\vec{v}\|\cdot\|\vec{n_2}\|} \to |\vec{v}|=1$$

Resolvendo o sistema:

$\vec{v}=(x,y,z)$ $\quad x+y+z=0$ $\quad \dfrac{|x-y|}{\sqrt{(1^2+(-1)^2)}}=\cos(45°)=\dfrac{\sqrt{2}}{2}$

$$\frac{|x-y|}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$|x-y|=1$$

$\vec{v}$ unitário:

$$x^2+y^2+z^2=1$$

Sistema:

$$\begin{cases} x+y+z=0 \ (1) \\ |x-y|=1 \\ x^2+y^2+z^2=1 \end{cases}$$

$x=y+1$ (4)

$(y+1)+y+z=0$

$2y+1+z=0$

$z=-2y-1$ (5)

$x^2+y^2+z^2=1$ → substituindo (4) e (5): $(y+1)^2+y^2+(-2y-1)^2=1$

$(y^2+2y+1)+y^2+(4y^2+4y+1)=1$ Somando os termos:

$6y^2+6y+2=1$ → Equação correta

$6y^2+6y+1=0$

$$\Delta=6^2-4(6)(1)=36-24=12$$

$$y=\frac{-6\pm\sqrt{12}}{12}=\frac{-6\pm2\sqrt{3}}{12}=\frac{-3\pm\sqrt{3}}{6}$$

Calculando $x$ e $z$:

$y=\left(\frac{-3+\sqrt{3}}{6}\right)$

$x=y+1=\frac{-3+\sqrt{3}}{6}+1=\frac{3+\sqrt{3}}{6}$

$$z=-2y-1=-2\left(\frac{-3+\sqrt{3}}{6}\right)-1=\frac{6-2\sqrt{3}}{6}-1=\frac{6-2\sqrt{3}-6}{6}=\frac{-2\sqrt{3}}{6}=\frac{-\sqrt{3}}{3}$$

$$\vec{v}=\left(\frac{3+\sqrt{3}}{6},\frac{-3+\sqrt{3}}{6},\frac{-\sqrt{3}}{3}\right)$$

5.

a)

$$\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}|\cdot|\vec{n_2}|}$$

b)

Planos:

$\pi_1: 2x + y - z - 1 = 0 \rightarrow \vec{n_1} = (2, 1, -1)$

$\pi_2: x - y + 3z - 10 = 0 \rightarrow \vec{n_2} = (1, -1, 3)$

$$\vec{n_1} \cdot \vec{n_2} = 2(1) + 1(-1) + (-1)(3) = 2 - 1 - 3 = |-2| \rightarrow 2$$

módulos:

$|\vec{n_1}| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$

$|\vec{n_2}| = \sqrt{1^2 + (-1)^2 + 3^2} = \sqrt{11}$

$$\cos\theta = \frac{2}{\sqrt{6}\cdot\sqrt{11}} = \frac{2}{\sqrt{66}} \rightarrow \theta \approx \arccos\left(\frac{2}{\sqrt{66}}\right) \approx 1{,}30 \text{ rad}$$

c)

$\pi_1: X = (1, 0, 0) + \lambda(1, 0, 1) + \mu(-1, 0, 0)$

$\pi_2: x + y + z = 0$

$\pi_1: (1, 0, 1)$ e $(-1, 0, 0)$

$$\vec{n_1} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix} = (0, -1, 0) \quad \vec{n_1} = (0, -1, 0) \quad \pi_2 = (1, 1, 1) \rightarrow \vec{n_2} = (1, 1, 1)$$

$$|\vec{n_1} \cdot \vec{n_2}| = 0 - 1 + 0 = |-1| \rightarrow 1$$

$\|\vec{n_1}\| \rightarrow 1$

$\|\vec{n_2}\| \rightarrow \sqrt{3}$

$$\cos\theta = \frac{1}{\sqrt{3}} \quad \theta \approx \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 0{,}955 \text{ rad}$$

Credeal

5)

$\pi_1: X = (0,0,0) + \lambda(1,0,0) + \mu(1,1,1)$,

$\pi_2: X = (1,0,0) + \lambda(-1,2,0) + \mu(0,1,0)$

$\pi_1$:

$(1,0,0)$ e $(1,1,1)$

$$\vec{n_1} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (0,-1,1)$$

$\pi_2$:

$(-1,2,0)$ e $(0,1,0)$

$$\vec{n_2} = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,-1)$$

$\|\vec{n_1} \cdot \vec{n_2}\| = 0 + 0 + (-1) = +1 = 1$

$\|\vec{n_1}\| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ $\quad \|\vec{n_2}\| = 1$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \theta \approx \arccos\left(\frac{1}{\sqrt{2}}\right) \approx 0{,}785\,rad$

6.

$\pi_1: 2x - y + z = 0 \rightarrow \vec{n_1}(2,-1,1)$ $\pi$ 

$P = (1,2,3)$ $\quad \vec{v_1} = (1,-2,1)$ contido

$\vec{u} = (0,0,1)$ R

mult. multiplicando $\vec{v}$ e paralela para Z

$$\vec{n_2} = \vec{v} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & -2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = (-2,-1,0)$$

$\|\vec{n_1} \cdot \vec{n_2}\| = -4 + 1 + 0 = |-3| = 3$

$\|\vec{n_1}\| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$ $\rightarrow \frac{3}{\sqrt{30}}$ $\cos\theta = \frac{3}{\sqrt{30}} \approx 0{,}5477 \approx 56{,}8°$

$\|\vec{n_2}\| = \sqrt{(-2)^2 + (-1)^2 + 0^2} = \sqrt{4+1} = \sqrt{5}$

7. $dist(P, A) = dist(P, B) \Rightarrow \|\vec{PA}\| = \|\vec{PB}\|$

isto é

$$\|P - A\| = \|P - B\| \Rightarrow \|P - A\|^2 = \|P - B\|^2$$

a)

$$r: x - 1 = 2y = z \; ; \; A = (1,1,0) \text{ e } B = (0,1,1)$$

$$x = 1 + t, \quad y = \frac{t}{2}, \quad z = t$$

$$P(t) = \left(1 + t, \frac{t}{2}, t\right)$$

$$\|P - A\|^2 = \|P - B\|^2$$

Ponto $A = (1,1,0)$:

$$\|P - A\|^2 = (1 + t - 1)^2 + \left(\frac{t}{2} - 1\right)^2 + (t - 0)^2 = t^2 + \left(\frac{t - 2}{2}\right)^2 + t^2$$

$$= t^2 + \frac{(t-2)^2}{4} + t^2 = 2t^2 + \frac{t^2 - 4t + 4}{4} = 2t^2 + \frac{t^2}{4} - t + 1$$

$$= \frac{8t^2 + t^2}{4} - t + 1 = \frac{9t^2}{4} - t + 1$$

$B = (0,1,1)$:

$$\|P - B\|^2 = (1 + t - 0)^2 = (1 + 2t + t^2) + \frac{(t-2)^2}{4} + (t^2 - 2t + 1)$$

$$= 1 + 2t + t^2 + \frac{t^2 - 4t + 4}{4} + t^2 - 2t + 1 = 2t^2 + \frac{t^2}{4} - t + 2 = \frac{8t^2 + t^2}{4} - t + 2 = \frac{9t^2}{4} - t + 2$$

$$\frac{9t^2}{4} - t + 1 = \frac{9t^2}{4} - t + 2 \Rightarrow 1 = 2 \text{ (falsa!)}$$

Não existe valor de t que satisfaça a igualdade

$\therefore$ não há ponto na reta r que seja equidistante de A e B

/ /

b)

Reta: $\vec{X}(\lambda) = (0,0,4) + \lambda(4,2,-3)$

$A = (2,2,5)$, $B = (0,0,1)$

$$P(\lambda) = (4\lambda, 2\lambda, 4-3\lambda)$$

$* \; P-A = (4\lambda-2, 2\lambda-2, 4-3\lambda-5) = (4\lambda-2, 2\lambda-2, -3\lambda-1)$

$\|P-A\|^2 = (4\lambda-2)^2 + (2\lambda-2)^2 + (3\lambda-1)^2$

$= 16\lambda^2 - 16\lambda + 4 + 4\lambda^2 - 8\lambda + 4 + 9\lambda^2 + 6\lambda + 1 = 29\lambda^2 - 18\lambda + 9$

$*$ distância $P-B = (4\lambda, 2\lambda, 4-3\lambda-1) = (4\lambda, 2\lambda, 3-3\lambda)$

$\|P-B\|^2 = (4\lambda)^2 + (2\lambda)^2 + (3-3\lambda)^2 = 16\lambda^2 + 4\lambda^2 + 9 - 18\lambda + 9\lambda^2 = 29\lambda^2 - 18\lambda + 9$

$\|P-A\|^2 = \|P-B\|^2 \Rightarrow$ Sempre verdade!

c)

Reta: $X(\lambda) = (2,3,-3) + \lambda(1,1,1)$

$A = (1,1,0)$, $B = (2,2,4)$

$$P(\lambda) = (2+\lambda, 3+\lambda, -3+\lambda)$$

$P-A = (1+\lambda, 2+\lambda, -3+\lambda)$

$\|P-A\|^2 = (1+\lambda)^2 + (2+\lambda)^2 + (-3+\lambda)^2 = \lambda^2 + 2\lambda + 1 + \lambda^2 + 4\lambda + 4 + \lambda^2 - 6\lambda + 9 =$

$3\lambda^2 + 14$

8. $d = \frac{|\vec{P_0 P} \times \vec{v}|}{|\vec{v}|}$ | Onde: $P_0$ = ponto da reta $P$ = ponto dado
$\vec{v}$ = vetor diretor da reta

a)
$P = (-2,0,1)$, $M: X = (1,-2,0) + \lambda(3,2,1)$

$|(1,-2,0) \cdot (-2,0,1) \times (3,2,1)| \Rightarrow$

$\vec{P_0 P} = (-3,2,1)$

$\vec{v} \times \vec{P_0 P} = (0,-6,12) = \sqrt{0^2 + (-6)^2 + 12^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}$

$\|\vec{v}\| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

$d = \frac{6\sqrt{5}}{\sqrt{14}} = \frac{6\sqrt{5} \cdot \sqrt{14}}{14} = \frac{6\sqrt{70}}{14} = \boxed{\frac{3\sqrt{70}}{7}}$

b)
$P = (1,-1,4)$

$\frac{x-2}{4} = \frac{y}{-3} = \frac{1-z}{2}$

$\vec{v} = (4,-3,-2)$

$P_0 = (2,0,1)$ $\vec{P_0 P} = (1-2, -1-0, 4-1) = (-1,-1,3)$

Produto vetorial:

$|\vec{v} \times \vec{P_0 P}| = (-9,10,-7)$

$\sqrt{81+100+49} = \sqrt{230}$

$|\vec{v}| = \sqrt{16+9+4} = \sqrt{29}$

$d = \frac{\sqrt{230}}{\sqrt{29}} = \boxed{\sqrt{\frac{230}{29}}}$

c)
$P = 0,-1,0)$

Reta: Equações:

$x = 2y - 3$

$x = 2z - 1$

Parametrizando (t):

$x = t, y = \frac{t+3}{2}, z = \frac{t+1}{2}$

Vetor diretor: $\vec{v} = (1, (0,5), (0,5))$

$\vec{P_0 P} = (0-0, -1-(1,5), 0-(-0,5)) = (0, (-2,5), (-0,5))$

Produto vetorial: $\vec{v} \times \vec{P_0 P} \approx ((-1,25), (0,5), (-2,5))$

Módulo:

$\approx \sqrt{(-1,25)^2 + (0,5)^2 + (-2,5)^2} \approx \sqrt{(1,56) + (0,25) + (6,25)} = \sqrt{8.06} \approx 2,84$

Credeal

$|\vec{v}| \simeq \sqrt{1^2+(0,25)+(0,25)} = \sqrt{(1,5)} \simeq 1,22$

$d \simeq \frac{2,89}{1,22} \simeq 2,83$

9.

$\pi_1$ e $\pi_2$ distam $\frac{\sqrt{14}}{3}$ da reta $r$.

$\pi_1: x+y=2$

$\pi: x=y+2$

reta $r: x=y=z+1$

Plano 1: $x+y=2$ (1)

Plano 2: $x-y-z=0$ (2)

Parametrize escolhendo $z=t$

de (2):

$x-y-t=0 \Rightarrow x-y=t$

de (1): $x+y=2$

Somando:

$(x-y)+(x+y)=t+2 \Rightarrow 2x=t+2 \Rightarrow x=\frac{t+2}{2}$

Substituindo em (1):

$\frac{t+2}{2}+y=2 \Rightarrow y=2-\frac{t+2}{2}=\frac{4-t-2}{2}=\frac{2-t}{2}$

Então a reta interseção $r$ é:

$X(t)=\left(\frac{t+2}{2}, \frac{2-t}{2}, t\right)$

Reta $r: x=y=z+1$

Parametrizando: seja $k$ o parâmetro

$z=k$

$y=k+1$

$x=+1$ Então o ponto genérico de $r$ é: $S(k)=(k+1, k+1, k)$

Vetor diretor de $r$: $(1,1,1)$

E ponto $P_0$ sobre $r_2$: por exemplo, quando $\lambda = 0$: $P_0 = (0,0,-1)$

Agora, seja $P(t)$ o ponto genérico da reta $r$:

$$P(t) = (2-t, t, 2-2t$$

A fórmula será:

$$d = \frac{|(P(t) - P_0) \times \vec{v_2}|}{|\vec{v_2}|}$$

Calculando:

1. vetor $(P(t) - P_0)$:

$$(2-t, t, 2-2t) - (0,0,-1) = (2-t, t, 3-2t)$$

Então:

$$\vec{u} = (2-t, t, 3-2t)$$

2. Produto vetorial $\vec{u} \times \vec{v_2}$:

$$\vec{v_2} = (1,1,1)$$

Determinante:

$$\vec{u} \times \vec{v_2} = \begin{vmatrix} i & j & k \\ 2-t & t & 3-2t \\ 1 & 1 & 1 \end{vmatrix}$$

Calculando:

• Componente $i$:

$$t \cdot 1 - (3-2t) \cdot 1 = t - 3 + 2t = 3t - 3$$

• Componente $j$: (sinal trocado):

$$-((2-t) \cdot 1 - (3-2t) \cdot 1) = -[2 - t - 3 + 2t] = -(-1+t) = 1-t$$

• Componente $k$:

$$(2-t) \cdot 1 - t \cdot 1 = 2 - t - t = 2 - 2t$$

Então o vetor resultante:

$$\vec{w} = (3t-3, 1-t, 2-2t)$$

/ /

3. módulo de $\vec{w}$

$$|\vec{w}|^2 = (3t-3)^2 + (1-t)^2 + (2-2t)^2$$

Expandindo:

$$(3t-3)^2 = 9t^2 - 18t + 9$$
$$(1-t)^2 = t^2 - 2t + 1$$
$$(2-2t)^2 = 4 - 8t + 4t^2$$

Somando:

$$(9t^2 + t^2 + 4t^2) = 14t^2$$
$$(-18t - 2t - 8t) = -28t$$
$$(9 + 1 + 4) = 14$$
$$|\vec{w}|^2 = 14t^2 - 28t + 14$$

4. $|\vec{v_2}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$$d^2 = \frac{14t^2 - 28t + 14}{3}$$

$d = \frac{\sqrt{14}}{3}$, então:

$$d^2 = \frac{14}{9}$$

Igualando:

$$\frac{14t^2 - 28t + 14}{3} = \frac{14}{9}$$

Multiplicando por 9:

$$3(14t^2 - 28t + 14) = 14$$
$$42t^2 - 84 + 42 = 14$$
$$42t^2 - 84t + 28 = 0$$

Dividindo por 14:

$$3t^2 - 6t + 2 = 0$$

Equação

$$\Delta = (-6)^2 - 4 \cdot 3 \cdot 2 = 36 - 24 = 12$$

$$t = \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm 2\sqrt{3}}{6} = 1 \pm \frac{\sqrt{3}}{3}$$

$\rightarrow t_1 = 1 + \frac{\sqrt{3}}{3}$

$\rightarrow t_2 = 1 - \frac{\sqrt{3}}{3}$

10.

distância

$d$ : $P=(x_0,y_0,z_0)$ a um plano $\pi$: $\dfrac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$

a)

Ponto $P=(1,3,4)$

Plano: dado na forma vetorial

$$X=(1,0,0)+\lambda(1,0,0)+\mu(-1,0,3)$$

Vetores diretores do plano:

$$\vec{v}_1=(1,0,0),\ \vec{v}_2=(-1,0,3)$$

Calculando o vetor normal ao plano ($\vec{n}$):

$$\vec{n}=\vec{v}_1\times\vec{v}_2=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{vmatrix}=\vec{i}(0\cdot3-0\cdot0)-\vec{j}(1\cdot3-0\cdot(-1))+\vec{k}(1\cdot0-0\cdot(-1))$$

$\vec{n}=(0,-3,0)$, simplificando: $(0,1,0)$

$$d=\frac{|0\cdot1+1\cdot3+0\cdot4+0|}{\sqrt{0^2+1^2+0^2}}=\frac{3}{1}=3$$

b)

Ponto: $P=(0,0,-6)$

Plano: $x-2y-2z-6=0$ Coeficientes: $A=1, B=-2, C=-2, D=-6$

Calculando:

$$d=\frac{|1\cdot0+(-2)\cdot0+(-2)\cdot(-6)-6|}{\sqrt{1^2+(-2)^2+(-2)^2}}=\frac{|0+0+12-6|}{\sqrt{1+4+4}}=\frac{6}{3}=2$$

c) Ponto: $P=(1,1,1)$ Coeficientes: $A=2, B=-1$ $C=2$ $D=-3$

Plano: $2x-y+2z-3=0$

$$d=\frac{|2\cdot1+(-1)\cdot1+2\cdot1-3|}{\sqrt{2^2+(-1)^2+2^2}}=\frac{|2-1+2-3|}{\sqrt{4+1+4}}=\frac{0}{3}=0$$

/ /

11.

r: $x = 2 - y = y + z$

$\pi_1$: $x - 2y - z = 1$

$x = t$ (1)

$2 - y = t \Rightarrow y = 2 - t$ (2)

$y + z = t \Rightarrow z = t - y = t - (2 - t) = 2t - 2$ (3)

$P(t) = (t, 2-t, 2t-2)$

$$d = \frac{|x_0 - 2y_0 - z_0 - 1|}{\sqrt{1^2 + (-2)^2 + (-1)^2}} = \frac{|x_0 - 2y_0 - z_0 - 1|}{\sqrt{6}}$$

$d = \sqrt{6}$ $P = (x_0, y_0, z_0)$

$|x_0 - 2y_0 - z_0 - 1| = 6$

Substituindo pelos pontos da reta:

$x_0 = t$, $y_0 = 2 - t$, $z_0 = 2t - 2$

$x_0 = (t)$
$y_0 = (2 - t)$
$z_0 = (2t - 2)$

$\rightarrow |x_0 - 2y_0 - z_0 - 1| \rightarrow |t - 2(2 - t) - (2t - 2) - 1| = $ (1) (2)

$|t - 4 + 2t - 2t + 2 - 1|$

$t + 2t - 2t = t$

$-4 + 2 - 1 = -3 \Rightarrow \frac{|t - 3|}{\sqrt{6}} = \sqrt{6}$

$|t - 3| = 6$

$G_1 = t - 3 = 6 \Rightarrow t = 9$

$G_2 = t - 3 = -6 \Rightarrow t = -3$

$P_1$: $x_1 = 9$; $y_1 = 2 - 9 = -7$; $z_0 = 2 \cdot 9 - 2 = 16 \Rightarrow P_1(9, -7, 16)$

$P_2$: $x_0 = -3$; $y_0 = 2 + 3 = 5$; $z_0 = 2 \cdot (-3) - 2 = -6 - 2 = -8 \Rightarrow P_2(-3, 5, -8)$

12. $d = \frac{|(\vec{PQ}) \cdot (\vec{v_r} \times \vec{v_s})|}{|\vec{v_r} \times \vec{v_s}|}$

a) $r: X = (2,1,0) + \lambda(1,-1,1)$; $s: x+y+z = 2x-y-1 = 0$

$P = (2,1,0)$

$\vec{v_r} = (1,-1,1)$

$Q = (0,1,-1)$

$\begin{vmatrix} i & -j & k \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = (1), (-2), (-3) = \vec{v_s}$

$\vec{v_s} = (1,+2,-3)$

$|(\vec{PQ} = (-2,0,-1) \cdot \vec{v_r} \times \vec{v_s} = (5,4,-1))| = 9$

$|\vec{v_r} \times \vec{v_s}| = \sqrt{42}$

$d = \frac{9}{\sqrt{42}}$

b)

$r: \frac{x+4}{3} = \frac{y}{4} = \frac{z+5}{-2}$, $s: X = (21,-5,2) + \lambda(6,-4,-1)$

$P = (-4,0,-5)$, $\vec{v_r} = (3,4,-2)$ $= \frac{|(25,-5,7) \cdot (-12,9,-36)|}{|-12,9,-36|} = \frac{597}{39}$

$Q = (21,-5,2)$, $\vec{v_s} = (6,-4,-1)$

c)

$\vec{v_r} = \vec{v_s} = (-4,1,2)$ (paralelas) ¨

$P = (1,0,0)$ $Q = (0,0,2)$

$\vec{PQ} = (-1,0,2)$

$\vec{v} \times \vec{PQ} = (1,-2,-1)$

$|\vec{v} \times \vec{PQ}| = \sqrt{1+4+1} = \sqrt{6}$ $|\vec{v_r}| = \sqrt{21}$

c) : $\frac{\sqrt{6}}{\sqrt{21}}$

13.

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

a)

$r: (1,9,4) + \lambda(3,3,3)$ ← vetor diretor

Plano: $X = (5,7,9) + \lambda(1,0,0) + \mu(0,1,0)$ diretores

$$\vec{n}: \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,1) \rightarrow z = k$$ Equação do plano: $z - 9 = 0$

$$d = \frac{|4 - 9|}{1} = 5$$

b)

$\begin{cases} x - y + z = 0 \\ 2x + y - z - 3 = 0 \end{cases}$ ← reta   Plano: $y - z = 4 \rightarrow y - z - 4 = 0$

← somando os dois

$\begin{matrix} x & y & z \\ (0, & 1, & -1) \end{matrix}$

$(3x + 0y + 0z = 3) \rightarrow x = 1$

$\rightarrow 1 - y + z = 0 \rightarrow y = 1 + z$

$\rightarrow z = 0$

$\rightarrow y = 1, x = 1$

$\rightarrow P = (1,1,0)$

← vetor diretor da reta $\rightarrow z = t$

$y = 1$

$x = 1 \rightarrow (0,1,1) : (0,1,-1)$

Ponto da reta: $P = (1,1,0)$

Plano: $y - z - 4 = 0$

$$d = \frac{|1 - 0 - 4|}{\sqrt{1^2 + (-1)^2}} = \boxed{\frac{3}{\sqrt{2}}}$$

c)

$r: x = y-1 = z+3$; $\pi: 2x+y-3z-10=0$

$x = t$
$y = t+1$ } Ponto: $(0,1,-3)$ da reta
$z = t-3$

vetor diretor $(1,1,1)$ · vetor normal $(2,1,-3)$ $= 2+1-3 = 0$

$$d = \frac{|2\cdot 0+1\cdot 1-3\cdot(-3)-10|}{\sqrt{2^2+1^2+(-3)^2}} = \frac{|1+9-10|}{\sqrt{4+1+9}} = \frac{0}{\sqrt{14}}$$

14.

a) $\pi_1: 2x-y+2z=0 = \pi_2: 4x-2y+4z-21=0$

$\pi_1: 2x-y+2z-10.5=0$

$$d = \frac{|D_1-D_2|}{\sqrt{A^2+B^2+C^2}}$$ ou usa a distância entre ponto e plano ~~caso paralelos~~

$$\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

$(2,-1,2)$

$$d = \frac{|0-(-10,5)|}{\sqrt{2^2+(-1)^2+2^2}} = \frac{10.5}{\sqrt{9}} = \frac{10.5}{3} = 3,5$$

b)

$X = (2,1,2) + \lambda(-1,0,2) + \mu(1,1,0)$

$2x+2y+2z=5 \Rightarrow x+y+z = \frac{5}{2}$

$(2,1,2)$

$$d = \frac{|2+1+2-\frac{5}{2}|}{\sqrt{3}} = \frac{|5-2,5|}{\sqrt{3}} = \boxed{\frac{2.5}{\sqrt{3}}}$$

c)

$\cdot \pi_1: x+y+z=0$

$\cdot \pi_2: 2x+y+z+2=0$

$\vec{n}_1=(1,1,1)$; $\vec{n}_2=(2,1,1)$ $\rightarrow (1,-1,0)$

$$d=\frac{|2x+y+z+2|}{\sqrt{2^2+1^2+1^2}}=\frac{|2(1)+(-1)+0+2|}{\sqrt{6}}=\frac{|2-1+2|}{\sqrt{6}}=\frac{3}{\sqrt{6}}$$

15.

$r: x+z=5=y+4$

$s: X=(4,1,1)+\lambda(4,2,-3)$

$r_1: x+z=5 \Rightarrow x=t, z=5-t, y=1 \rightarrow \vec{v}_1=(1,0,-1)$

$\vec{v}_2=(4,2,-3)$

Normal $\rightarrow$

$$\vec{n}=\vec{v}_1 \times \vec{v}_2=\begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 4 & 2 & -3 \end{vmatrix}=(0\cdot(-3)-(1)\cdot 2,\ -(1\cdot(-3)-(-1)\cdot 4),\ 1\cdot 2-0\cdot 4)=(2,-(-3+4),2)=(2,-1,2)$$

$P_s=(4,1,1)$, $\vec{n}=(2,-1,2)$ $\rightarrow 2(x-4)-y-1+2(z-1)=0 \Rightarrow 2x-8-y+1+2z-2=0 \Rightarrow 2x-y+2z-9=0 \rightarrow 2x-y+2z-9=0=\pi$

$$\boxed{d=2=\frac{|D-9|}{\sqrt{2^2+(-1)^2+2^2}}=\frac{|D-9|}{3}}$$

$|D-9|=6 \Rightarrow D-9=\pm 6 \Rightarrow D \rightarrow D=15$ ou $D=3$

Planos:

$\boxed{2x-y+2z-15=0}$ e $\boxed{2x-y+2z-3=0}$