Cariel dos Reis Barbosa Domiciano

2025.1.08.035

1.

a) $\vec{BF} = \vec{BA} + \vec{AF} \to \vec{BA} = -\vec{AB} = -\vec{b}$ / $\vec{AF} = \vec{BF}$: $-b = -b + \vec{f} = \vec{f} - b$

b) $\vec{AG} = \vec{AG} = \vec{AB} + \vec{BG}$ / $\vec{BG} = \vec{BC} + \vec{CG} = \vec{BC} - \vec{BG} = \vec{a} + \vec{f}$ /
$\vec{AG} = \vec{b} + \vec{a} + \vec{f}$ / $\vec{a} = c - \vec{b}$ / $\vec{AG} = b + (c - b) + f = \vec{AG} = c + f$

c) $\vec{AE} = \vec{AB} + \vec{BE}$ / $\vec{BE} = \vec{AD} = \vec{a}$ / $\vec{AE} = \vec{b} + \vec{a} = \vec{b} + (\vec{c} - \vec{b}) = \vec{c}$

d) $\vec{BG} = \vec{BC} + \vec{CG} = \vec{a} + f = (\vec{c} - \vec{b}) + f = \vec{c} - \vec{b} + \vec{f}$

e) $\vec{HB} = \vec{HG} + \vec{GB}$ / $\vec{HG} = \vec{b}$ / $\vec{GB} = -\vec{BG} = -(\vec{c} - \vec{f})$ /
$\vec{HB} = b - (\vec{c} - \vec{b} + \vec{f}) = 2\vec{b} - c - f$

f) $\vec{AB} + \vec{FG} = \vec{AB} = \vec{b}$ / $\vec{FG} = \vec{AC} - \vec{AB} = \vec{c} - \vec{b}$
$\vec{AB} + \vec{FG} = \vec{b} + (\vec{c} - \vec{b}) = \vec{c}$

g) $\vec{AB} + \vec{HG} = \vec{AD} = \vec{a} = \vec{c} - \vec{b}$ / $\vec{HG} = \vec{b}$ / $(\vec{c} - \vec{b}) + \vec{b} = \vec{c}$

h) $\vec{HF} + \vec{AG} - \vec{EF} = \vec{HF} = \vec{f}$ / $\vec{AG} = \vec{c} + \vec{f}$ / $\vec{EF} = \vec{b}$ / $\vec{HF} + \vec{AG} + \vec{EF} = \vec{f}$
$\vec{HF} + \vec{AG} - \vec{EF} = \vec{f} + (\vec{c} + \vec{f}) - \vec{b} = 2\vec{f} + \vec{c} - \vec{b}$

i) $2\vec{AD} - \vec{FG} - \vec{BH} + \vec{GH} =$ / $2\vec{AD} = 2(\vec{c} - \vec{b})$ / $-\vec{FG} = -\vec{a}$ / $-\vec{BH} = -\vec{f}$
$\vec{GH} = -\vec{b}$

$i = 2(\vec{c} - \vec{b}) - (\vec{c} - \vec{b}) - \vec{f} - \vec{b} = 2\vec{c} - 2\vec{b} - \vec{b} - \vec{c} + \vec{b} - \vec{f} - \vec{b}$
$i = \vec{c} - 2\vec{b} - \vec{f}$

/ /

2.

a) $2\overline{DE} + \overline{DC}$

b) $2(\overline{DE} + \overline{DC})$

c) $\overline{2DC} + \overline{DE}$

d) $DC + DE$

e) $-DE +$

f) $-DE + 2DC$

g) $DO = (DE + DC)/ -DO + DC$

h) $DA = (2DE + 2DC)/ -DA + (-DC$

3.

a) $\vec{e} + \bar{d} + \bar{e} + \bar{d} + \bar{e} + \bar{d} = 3\bar{e} + 3\bar{d}$

b) $-\bar{e} + \bar{d} - \bar{d} + \bar{e} - \bar{e} + \bar{d} - \bar{d} + \bar{e} - \bar{e} + \bar{d} - \bar{d} + \bar{e} = 0$

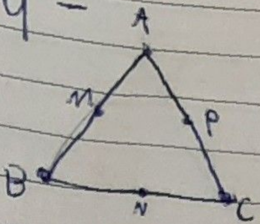
c) $-\bar{e} + \bar{d} - \bar{d} + \bar{e} - \bar{e} + \bar{d} - \bar{d} + \bar{e} - \bar{e} + \bar{d} = \bar{d} - \bar{e}$

d) $\bar{e} + \bar{d} + \bar{d} + \bar{e} = 2\bar{e} + 2\bar{d}$

e) $\bar{e} - \bar{e} + \bar{d} - \bar{e} + \bar{d} = 2\bar{d} - \bar{e}$

f) $e + d - d + e = 0$

4 -

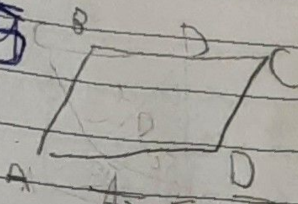


1. $\overline{BP} = \overline{BA} + \overline{AP}$; $\overline{BA} = -\overline{AB}$;
$\overline{AP} = 1/2\ \overline{AC}$
$BP = 1/2\ \overline{AC} - \overline{AB}$

2. $AN = 1/2\ (\overline{AB} + \overline{BC})$

3. $\overline{CM} = \overline{CA} + \overline{AM}$; $\overline{CA} = -\overline{AC}$; $\overline{AM} = 1/2\ \overline{AB}$
$CM = 1/2\ AB - \overline{AC}$

5



$\overline{AD} = 5u$, $\overline{BC} = 3u$, $AB = 2\overline{v}$

a) $\overline{CD} = \overline{CB} + \overline{BA} + \overline{AD}$
$= -\overline{BC} - \overline{AB} + AD$
$\Rightarrow CD = -3u - 2v + 5u$

a1) $\overline{BD} = \overline{BA} + \overline{AD}$
$= -\overline{AB} + AD = -2\overline{v} + 5\overline{u}$

• $\overline{CA} = \overline{CB} + \overline{BA} = -\overline{BC} - AB = 3\overline{u} - 2\overline{v}$

b) lados: $\overline{AD} = 5u$
$\overline{BC} = 3u$
$\overline{AB} = 2\overline{v}$
$\overline{CD} = 2\overline{u} - 2\overline{v}$

Note que $\{\overline{AD}, \overline{BC}\}$ é L.D. (os dois são paralelos) e por outro lado $\{\overline{AB}, \overline{CD}\}$ é L.I., ou seja, $\nexists\ \lambda \in \mathbb{R}$ tal que $\overline{AB} = \lambda \overline{CD}$
$2\overline{v} = \lambda(2\overline{u} - 2\overline{v})$
$2v = 2\lambda u - 2\lambda v$

Credeal

6) $\bar{a} = \overline{OA}$; $\bar{b} = \overline{OB}$; $\bar{c} = \overline{OC}$; $\overline{AD} = \frac{1}{4}\vec{c}$; $\overline{BE} = \frac{5}{6}\vec{a}$

$\overline{DE} = \overline{DO} + \overline{OE}$; $\overline{DO} = \overline{DA} + \overline{AO}$; $-\overline{AD} - \overline{OA} \rightarrow -\frac{1}{4}\bar{c} - \bar{a}$

$\overline{OE} = \overline{OB} + \overline{BE} \rightarrow \bar{b} + \frac{5}{6}\bar{a}$

$\overline{DE} = -\frac{1}{4}c - \bar{a} + \bar{b} + \frac{5}{6}\bar{a} \rightarrow \frac{5}{6}\bar{a} - \bar{a} + \bar{b} - \frac{1}{4}\bar{c}$

$\overline{DE} = 1 - \frac{5}{6}\bar{a} + \bar{b} - \frac{1}{4}\bar{c}$

$\overline{DE} = \frac{1}{6}\bar{a} + \bar{b} - \frac{1}{4}\bar{c}$

7) $\overline{OA} = \vec{a} + 2\vec{b}$, $\overline{OB} = 3\bar{a} + 2\bar{b}$, $\overline{OC} = 5\bar{a} + x\vec{b}$

$\overline{AC} = \overline{AO} + \overline{OC}$; $\overline{AD} = -\overline{OA} + OC \rightarrow -\bar{a} - 2\bar{b} + 5\bar{a} + x\bar{b}$

$\overline{AC} = 4\vec{a} + x\vec{b} - 2\vec{b}$

$\overline{BC} = \overline{BO} + \overline{OC}$; $= \overline{OB} + \overline{OC}$; $= -(3\bar{a} + 2\bar{b}) + 5\vec{a} + x\vec{b}$

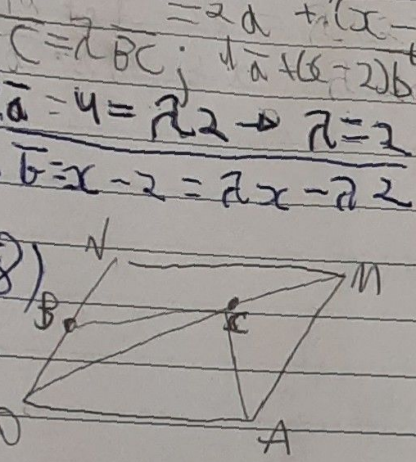
$= 2a + (x - 2)b$; $\{\overline{AC}, \overline{BC}\}$ L D $\Leftrightarrow \exists \in \mathbb{R}$ tal que

$\overline{AC} = \lambda\overline{BC}$; $4\bar{a} + (x-2)\bar{b} = \lambda(2\bar{a} + (x-2)\bar{b})$

R: $\bar{a}$: $4 = \lambda 2 \rightarrow \lambda = 2$; $4 - 2\lambda = 0 \rightarrow \lambda = 2$

R: $\bar{b}$: $x - 2 = \lambda x - \lambda 2 \rightarrow x - 2 = 2x - 4 \rightarrow 2x - x = -2 + 4$ $\boxed{x = 2}$

8)



$\overline{OB} = \frac{1}{n} ON$; A, B, C são colineares $\Leftrightarrow \overline{AB} \parallel \overline{OC}$

$OC = \frac{1}{1+m} OM$

• $\overline{BC} = \overline{BO} + \overline{OC}$

$= -OB + OC$

$= -\frac{1}{n} ON + \frac{1}{1+m}$

• $\overline{CA} = \overline{CO} + \overline{OA}$

$-\overline{OC} + NM - \frac{1}{1+m} OM + NO + OM$

$= -\frac{1}{1+m} \overline{OM} - ON + OM$

$= \frac{-1 + (1+m)OM - ON}{1+m}$

$\rightarrow CA = -\vec{ON} + \frac{1}{1+m} \overline{ON}$

$\therefore BC$

9) $\bar{a} = 2\bar{u} + \bar{v}$, $\bar{b} = \bar{u} - 2\bar{u}$

$\alpha\vec{a} + \beta\bar{b} = \bar{0}$ | $\alpha(2\vec{u} + \vec{v}) + \beta(\vec{u} - 2\vec{v}) = \vec{0}$

$\alpha(2\vec{u}) + \alpha(\vec{v}) + \beta(\vec{u}) + \beta(-2\vec{v}) = \vec{0}$ | $(2\alpha + \beta)\vec{u} + (\alpha - 2\beta)\vec{v} = \vec{0}$

$\begin{cases} 2\alpha + \beta = 0 \\ \alpha - 2\beta = 0 \end{cases} \Rightarrow$ $2(2\beta) + \beta = 4\beta + \beta = 5\beta = 0 \Rightarrow \beta = 0 \Rightarrow \alpha = 0$

$\alpha = \beta = 0$ é L.I.

10) $\{\vec{u}, \vec{v}, \vec{w}\}$ é L.I

1 → a) $\{\bar{u} + \bar{v}, \bar{u}\ \bar{v} + w, \bar{u} + \bar{v} + \bar{w}\}$ é L.I

$\alpha(\bar{u} + \bar{v}) + \beta(\bar{u} - \bar{v}) + \gamma(\bar{u} + \bar{v} + w) = 0$

2 — base $\{\bar{u}, \bar{v}, \bar{w}\}$ (L.I.)

$\bar{u} + \bar{v} = (1, 1, 0$
$\bar{u} - \bar{v} + \bar{w} = (1, -1, 1)$
$\bar{u} + \bar{v} + w = (1, 1, 1)$

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = (\ldots)$$

$t = a\vec{u} + b\vec{v} + cw$; L.I
$\{\bar{u} + \bar{t}, \vec{v} + \bar{t}, \bar{w} + \bar{t}\}$

$a + b + c \neq -1$
Combinação linear.
$x(\bar{u} + \bar{t}) + y(\bar{v} + \bar{t}) + ?$

Credea

11.

a) $A=(1,3,2)$
$B=(1,0,-1)$
$C=(1,1,0)$

Se $P=(x_1,y_1,z_1)$ e $Q=(x_2,y_2,z_2)$, $\vec{PQ}=Q-P=(x_2-x_1,y_2-y_1,z_2-z_1)$

- $\vec{AB}=B-A=(1-1,0-3,-1-2)=(0,-3,-3)$
- $\vec{BC}=C-B=(1-1,1-0,0-(-1))=(0,1,1)$
- $\vec{CA}=A-C=(1-1,3-1,2-0)=(0,2,2)$

b) $\cdot\vec{AB}=(0,-3,-3)$, $\cdot\vec{BC}=(0,1,1)$

$\frac{2}{3}\vec{BC}=\left(0,\frac{2}{3},\frac{2}{3}\right)$ $\quad \vec{AB}+\frac{2}{3}\vec{BC}=\left(0,-3+\frac{2}{3},-3+\frac{2}{3}\right)=\left(0,-\frac{7}{3},-\frac{7}{3}\right)$

c) $C=(1,1,0)$, $\vec{AB}=(0,-3,-3)$ $\frac{1}{2}\vec{AB}=\left(0,-\frac{3}{2},-\frac{3}{2}\right)$

$C+\frac{1}{2}\vec{AB}=\left(1+0,1-\frac{3}{2},0-\frac{3}{2}\right)=\left(1,-\frac{1}{2},-\frac{3}{2}\right)$

d) $\cdot A=(1,3,2)$, $\cdot BC=(0,1,1)$, $\cdot 2\vec{BC}=(0,2,2)$

$A-2\vec{BC}=(1-0,3-2,2-2)=(1,1,0)=C$ ☺

12

a) $(0,2)$ não é multiplo de $(2,3)$
LI

b) $(-2,0)=-\frac{2}{3}(3,0)$ é L.D.

c) $\{(2,3,4),(0,3,3)\}$
dois vetores em $\mathbb{R}^3$. $(0,3,3=\alpha(2,3,4))$ L.I

d) $\{(1,-1,2),(1,1,0),(1,-1,1)\}$ $\begin{bmatrix}1 & -1 & 2\\ 1 & 1 & 0\\ 1 & -1 & 1\end{bmatrix}$

$\mathcal{B}$

/ /

d) $\rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{matrix} L_2 \leftarrow L_2 - L_1 \\ \sim \\ L_3 \leftarrow L_3 - L_1 \end{matrix} \begin{bmatrix} 1 & -1 & 2 \\ 0, & 2, & -2 \\ 0, & 0, & -1 \end{bmatrix}$ 3 ~~pontos~~ vetores são LI

e) $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix} \begin{matrix} L_2 \leftarrow L_2 + L_1 = \\ \sim \\ L_3 \leftarrow L_3 + L_1 = \end{matrix} \begin{bmatrix} 1, & -1, & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} \sim_{L_3 \leftarrow L_3 - L_2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ posto 3 LI

f) $\{(1,0,1), (0,0,1), (2,0,5)\}$

$(2,0,5) = 2(1,0,1) + 3(0,0,1) = (2,0,2) + (0,0,3) = 2,0,5$

é LD

13 -

a) $\vec{w} = a\vec{u} + b\vec{v} \Rightarrow (1,1) = a(2,-1) + b(1,-1)$

$M = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \det(M) = -2 + 1 = -1$

$M_a = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow \det(M_a) = -1 - 1 = -2 \quad a = \frac{-2}{-1} = 2$

$M_b = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \quad 2 + 1 = \frac{3}{-1} = -3 \quad w(1,1) = \underset{a}{2}\underset{u}{(2,-1)} + \underset{b}{3}\underset{v}{(1,-1)}$

b) $\vec{z} = (1,2,3) \quad \vec{a} = (1,1,1), \vec{b} = (0,1,1), \vec{c} = (1,1,0)$ Esc. α, β, γ

$(1,2,3) = \alpha(1,1,1) + \beta(0,1,1) + \gamma(1,1,0)$

$(1,2,3) = (\alpha + \gamma, \alpha + \beta + \gamma, \alpha + \beta)$

$\begin{cases} \alpha + \gamma = 1 \\ \alpha + \beta + \gamma = 2 \\ \alpha + \beta = 3 \end{cases} \quad \alpha + \beta = 3 \Rightarrow \beta = 3 - \alpha$

$\alpha + (3 - \alpha) + \gamma = 2 \Rightarrow 3 + \gamma = 2 \Rightarrow \gamma = -1$

Da 1ª: $\alpha + \gamma = 1 \Rightarrow \alpha - 1 = 1 \quad \alpha = 2$

$\therefore \beta = 3 - \alpha = 1$

R: $(1,2,3) = 2(1,1,1) + 1(0,1,1) - 1(1,1,0)$

$\therefore \vec{z}$ na base $\{\vec{a}, \vec{b}, \vec{c}\}$ é $(2,1,-1)$

14-

a) afirmo que: $\vec{v} = \lambda \vec{u}$ ou $\vec{u} = \lambda \vec{v}$

foi afirmado que: $\vec{u} = (1, m-1, m)$, $\vec{v} = (m, 2n, 4)$

se $\vec{u}$ e $\vec{v}$ é LD, $\lambda \in \mathbb{R}$ tal que: $(m, 2n, 4) = \lambda(1, m, m+1)$

$$\begin{cases} m = \lambda \cdot 1 \Rightarrow \lambda = m \\ 2n = \lambda(m-1) = m(m-1) \\ 4 = \lambda m = m^2 \end{cases}$$

$m^2 = 4 \Rightarrow m = \pm 2$

$m_1 = 2 \therefore \lambda = 2, 2n = 2(2-1) = 2 \quad n = 1$

$m_2 = -2 \therefore \lambda = -2, 2n = -2(-2-1) = -2(-3) = 6 \Rightarrow n = 3$

$\therefore$ • (m, n) que tornam os vetores LD são: $(2,1)$ ou $(-2,3)$

b) $\vec{u} = (1, m, n+1)$ $\vec{v} = (m, n+1, 8)$

$(m, n+1, 8) = \lambda(1, m, n+1)$	$n+1 = m^2 \Rightarrow n = m^2 - 1$
$\begin{cases} m = \lambda \cdot 1 \Rightarrow \lambda = m \\ n+1 = \lambda m = m^2 \\ 8 = \lambda(n+1) = m(n+1) \end{cases}$	$8 = m(n+1) = m(m^2 \cancel{-1+1}) = m \cdot m^2$ $\Rightarrow m^3 \Rightarrow m^3 = 8 \Rightarrow 2 \Rightarrow n = 2^2 - 1 \Rightarrow$ $4 - 1 \Rightarrow 3$

$\therefore (m, n) = 2, 3$ para ser LD

16-

$B = \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$

$f_1 = (1, 1, 0)_B$
$f_2 = (1, 0, 1)_B$
$f_3 = (1, 1, -1)_B$

a) $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $\left|\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{matrix}\right|\begin{matrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{matrix}$ $0+0+1+1-1+0=0$

$\therefore$ C é a base de $\mathbb{R}^3$ porque det (M) $\neq 0$

b) $\vec{u} = (2, 3, 7)_C \therefore \vec{u} = 2f_1 + 3f_2 + 7f_3$

$\vec{u} = (2(1,1,0)) + (3(1,0,1)) + 7(1,1,-1)$

$\vec{u} = (2,2,0) + (3,0,3) + (7,7,-7) = (12, 9, -4)$

$\therefore \vec{u} = (12, 9, -4)_B$

c)

$\left[\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{matrix}\middle|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right] \underset{\sim}{L_2 \leftarrow L_2 - L_1} \left[\begin{matrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{matrix}\middle|\begin{matrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right] \underset{\sim}{L_2 \leftarrow (-1) \cdot L_2}$

$\left[\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{matrix}\middle|\begin{matrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{matrix}\right] \underset{\sim}{L_3 \leftarrow L_3 - L_2} \left[\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{matrix}\middle|\begin{matrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{matrix}\right] \underset{\sim}{L_3 \cdot -1 \leftarrow L_3} \left[\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\middle|\begin{matrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{matrix}\right]$

$\underset{\sim}{L_1 \leftarrow L_1 - L_3}$ $\left[\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\middle|\begin{matrix} 0 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{matrix}\right] \underset{\sim}{L_1 \leftarrow L_1 - 1 \cdot L_3} \left[\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\middle|\begin{matrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{matrix}\right]$

$\underset{\sim}{L_1 \leftarrow L_1 - L_2} \left[\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\middle|\begin{matrix} -1 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{matrix}\right]$ $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$

/ /

$$[\vec{v}]_B = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} \qquad P^{-1} = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$

$$[\vec{v}]_C = P^{-1} \cdot [\vec{v}]_B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} =$$

$$\begin{bmatrix} (-1\cdot 2)+(2\cdot 3)+(1\cdot 7) \\ (1\cdot 2)+(-1\cdot 3)+(0\cdot 7) \\ (1\cdot 2)+(-1\cdot 3)+(-1\cdot 7) \end{bmatrix} = \begin{bmatrix} 11 \\ -1 \\ -8 \end{bmatrix} \therefore [\vec{v}]_C = \begin{bmatrix} 11 \\ -1 \\ -8 \end{bmatrix}$$