[illegible]

2025.1.08.035

1.

$\mathcal{E} = (\bar{i}, \bar{j}, \bar{k})$ é ortonormada e base

a)

$\|u\| = \sqrt{x^2 + y^2 + z^2}$ $\quad \begin{matrix} i & j & k \\ x & y & z \end{matrix}$

a) $\vec{v} = (1, 1, 1)_{\mathcal{E}}$

$\|\vec{u}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

b) $\vec{u} = 3\vec{i} + 4\vec{k}$

$x = 3, \ y = 0, z = 4$

$\|\vec{u}\| = \sqrt{3^2 + 0^2 + 4^2} = \sqrt{9 + 0 + 16} = \sqrt{25} = 5 = \|\vec{u}\|$

c) $\vec{v} = -\vec{i} + \vec{j}$

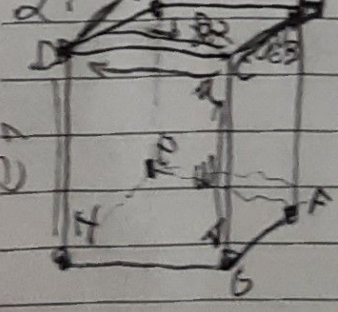
$x = -1; y = 1, z = 0$

$\|u\| = \sqrt{(-1)^2 + 1^2 + 0^2} = \sqrt{1+1} = \sqrt{2}$

d) $\vec{u} = 4\vec{i} + 3\bar{j} - \bar{k}$

$\|\vec{u}\| = \sqrt{4^2 + 3^2 + (-1)^2} = \sqrt{16 + 9 + 1} = \sqrt{26}$

2.



$\vec{e_1} = \overline{DH}$
$\vec{e_2} = \overline{DC}$
$\vec{e_3} = \overline{DA}$
$\bar{u} = \overline{CD} + \overline{CB}$
$\bar{v} = \bar{e}_2 + \overline{CB}$
$\bar{w} = GC$

a) E é ortonormal tem arestas que ... os vectores e_i são paralelos e são perpendiculares entre si
$\therefore$ E é ortonormal

2 3 1 2
1 3

$f_1 \cdot f_2 = \left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \cdot \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = \left(\frac{-1}{2}, 0, \frac{1}{2}\right) = \left(\frac{-1}{2} + 0 + \frac{1}{2}\right)$

b) $\vec{u} = \overline{CD} + \overline{CB}$

$\vec{u} = (D - C) + (B - C)$

$\vec{u} = ((0,1,0) - (1,1,0)) + ((1,1,1) - (1,1,0))$

$\vec{u} = (-1,0,0) + (0,0,1)$

R: $\vec{u} = (-1,0,1)$

$\vec{v} = \overline{DC} + \overline{CB}$

$\vec{v} = (C - D) + (B - C)$

$\vec{v} = ((1,1,0) - (0,1,0)) + ((1,1,1) - (1,1,0))$

$\vec{v} = (1,0,0) + (0,0,1)$

$\vec{v} = (1,0,1)$

$\vec{w} = GC$

$\vec{w} = C - G \Rightarrow \vec{w} = (1,1,0) - (1,0,0)$

$\vec{w} = (0,1,0)$

∴ E é a base canónica, as coordenadas de um vector na base E são as $\vec{w}$

$V = f_3 = (0,1,0)$

c) $f_1 = \frac{\vec{u}}{\|\vec{u}\|}$ $\qquad$ $f_2 = \frac{\vec{v}}{\|\vec{v}\|}$

$f_1 = \frac{(-1,0,1)}{\sqrt{-1^2+0^2+1^2}}$ $\qquad$ $f_2 = \frac{(1,0,1)}{\sqrt{1^2+0^2+1^2}}$

$f_1 = \frac{(-1,0,1)}{\sqrt{2}}$ $\qquad$ $f_2 = \frac{(1,0,1)}{\sqrt{2}}$

$f_1 = \left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ $\qquad$ $f_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

verificar ortogonal

$\Rightarrow f_2 \cdot f_3 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \cdot (0,1,0)$

$f_2 \cdot f_3 = (0,0,0) \Rightarrow (0+0+0) \Rightarrow 0$

$f_1 \cdot f_3 = \left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \cdot (0,1,0)$

$f_2 \cdot f_3 = (0,0,0) \Rightarrow (0+0+0) = 0$

Credeal

2 3   1 2
1 3

$f_1 \cdot f_2 = \left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \cdot \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = \left(\frac{-1}{\sqrt{4}}, 0, \frac{1}{\sqrt{4}}\right) = \frac{-1}{2} + 0 + \frac{1}{2} = 0$

$\left\|\left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\right\| = \sqrt{\left(\frac{-1}{\sqrt{2}}\right)^2 + 0^2 + \left(\frac{1}{\sqrt{2}}\right)^2} \to \sqrt{\left(\frac{1}{4}\right) + 0 + \left(\frac{1}{4}\right)} \to \sqrt{\frac{1}{2} + \frac{1}{2}} \to \sqrt{1} \to 1$

$\left\|\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\right\| = \sqrt{\frac{1}{2} + 0 + \frac{1}{2}} = \sqrt{1} = 1 \qquad \|(0,1,0)\| = \sqrt{0^2+1^2+0^2} = \sqrt{1} = 1$

∴ Cada vector tem norma $= 1$ e o produto escalar entre dois vetores distintos é $= 0$, logo, $F_1 = (f_1, f_2, f_3)$ é uma base ortonormal.

d) $E = (\vec{e_1}, \vec{e_2}, \vec{e_3}) \Rightarrow E = (\vec{DH}, \vec{DC}, \vec{DA}) \to E = ((H-D), (C-D), (A-D)) \to$
$\to E = ((0,-1,0), (1,0,0), (0,0,1))$

$F = (\vec{f_1}, \vec{f_2}, \vec{f_3}) = \left(\left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), (0,1,0)\right)$

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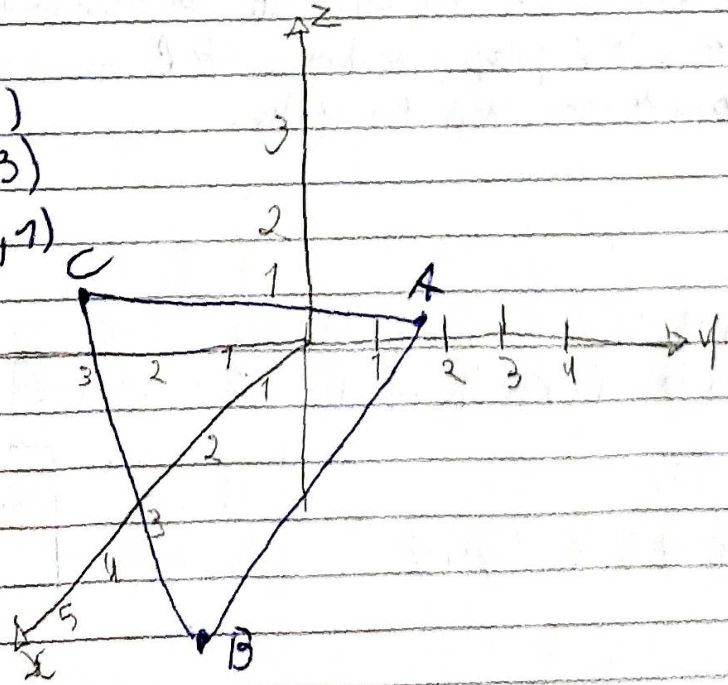
e) $\vec{HB} \Rightarrow B - H \Rightarrow (1,1,1) - (0,0,0) \Rightarrow HB = (1,1,1)$

$$\begin{bmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ -1 \end{bmatrix} \Rightarrow HB = \begin{bmatrix} 0 \\ \frac{2}{\sqrt{2}} \\ -1 \end{bmatrix}$$

$$\boxed{\begin{array}{l} HB = (1,1,1)_E \text{ e} \\ HB = \left(0, \frac{2}{\sqrt{2}}, -1\right)_E \end{array}}$$

3)
$A = (2,4,3)$
$B = (5,1,-3)$
$C = (0,-3,1)$



Credeal

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$$\begin{array}{r} 16 \\ \times 2 \\ \hline 32 \end{array}$$

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a) $\vec{AB}$ $\quad$ $\vec{BC}$ $\quad$ $\vec{CA}$

$B-A$ $\quad$ $C-B$ $\quad$ $A-C$

$(5,1,-3)-(2,4,3)$ $\quad$ $(0,-3,1)-5,1,-3)$ $\quad$ $(2,4,3)-(0,-3,1)$

$\therefore R: \vec{AB}=(3,-3,-6)$ $\quad$ $\vec{BC}=(-5,-4,4)$ $\quad$ $\vec{CA}=(2,7,2)$

b) $\|\vec{AB}\|$ $\quad$ $\|\vec{BC}\|$ $\quad$ $\|\vec{CA}\|$

$\sqrt{3^2+(-3)^2+(-6)^2}$ $\quad$ $\sqrt{(-5)^2+(-4)^2+4^2}$ $\quad$ $\sqrt{2^2+7^2+2^2}$

$9+9+36=\sqrt{54}$ $\quad$ $25+16+16=\sqrt{57}$ $\quad$ $\sqrt{4+49+4}=\sqrt{54}$

Sim, é um triangulo isósceles pois há 2 Lados da mesma medida.

c) $M_{AB}=\left(\frac{5+2}{2},\frac{1+4}{2},\frac{-3+3}{2}\right)=\left(\frac{7}{2},\frac{5}{2},0\right)$

$M_{BC}=\left(\frac{0+5}{2},\frac{-3+1}{2},\frac{1-3}{2}\right)=\left(\frac{5}{2},-1,-1\right)$

$M_{CA}=\left(\frac{2+0}{2},\frac{4-3}{2},\frac{3+1}{2}\right)=M_{CA}=\left(1,\frac{1}{2},2\right)$

$CM_{AB}=M_{AB}-C=\left(\frac{7}{2}-0,\frac{5}{2}+3,0-1\right)=\left(\frac{7}{2},\frac{11}{2},-1\right)$

$CM_{AB}\cdot AB=\left(\frac{7}{2},\frac{11}{2},-1\right)\cdot(3,-3,-6)=\left(\frac{21}{2}-\frac{33}{2}+\frac{6}{1}\right)=-\frac{12}{2}+\frac{6}{1}=\frac{-12+12}{2}=0$

O produto escalar é $=0$ logo CM e AB são ortogonais, portanto, a inclinada relativa à AB é perpendicular a AB e passa pelo ponto médio de AB, logo, ela coincide com sua ~~a~~ mediatriz.

d) θBCA:

$B-C$ $\quad$ $CA=(2,7,2)$

$(5,1,-3)-(0,-3,1)=CB=(5,4,-4)$ $\quad$ $CB\cdot CA=(10+28-8)=30$

$\|CB\|=\sqrt{5^2+4^2+(-4)^2}=\sqrt{57}$. $\|\vec{CA}\|=\sqrt{5^2+4^2+(-4)^2}=\sqrt{25+16+16}=\sqrt{57}$

$\cos\theta B\hat{C}A=\frac{\vec{CB}\cdot\vec{CA}}{\|\vec{CB}\|\cdot\|\vec{CA}\|}=\frac{30}{\sqrt{57}\cdot\sqrt{57}}=\frac{30}{57}\overset{3\div}{=}\frac{10}{19}\rightarrow$ $\boxed{\cos\theta B\hat{C}A=\operatorname{arc}\cos\left(\frac{10}{19}\right)}$

15. 16 10+8 5+ 3+-5

e) $\vec{AB}+\vec{BC}+\vec{CA}$ $(3,-3,-6)+(-5,-4,4)+(2,7,2)$
$\Rightarrow (0,0,0)$

Em soma desse resultado em 0 porque se trata de um deslocamento total percorrido pelos lados do triângulo a partir de um dos vértices e retornando a ele

11) a))

$|\vec{u}\cdot\vec{v}| \leq \|\vec{u}\|\cdot\|\vec{v}\|$ igualdade ocorre se e somente se $\vec{u}$ e $\vec{v}$ são paralelos

demonstração $\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cdot\cos\theta$ Logo

porque $|\cos\theta| \leq 1$. A igualdade vale se e somente se, $|\cos\theta| = 1$, ou seja, $\theta = 0°$ ou $\theta = 180°$, o que significa que $\vec{u}$ e $\vec{v}$ são paralelos | $|\vec{u}\cdot\vec{v}| = \|\vec{u}\|\cdot\|\vec{v}\|\cdot|\cos\theta| \leq \|\vec{u}\|\cdot\|\vec{v}\|$

b) $\|\vec{u}+\vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$

Prova:

$$\|\vec{u}+\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = \|\vec{u}\|^2 + 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$$

temos que

$$\vec{u}\cdot\vec{v} \leq \|\vec{u}\|\cdot\|\vec{v}\| \rightarrow$$

$$\Rightarrow \|\vec{u}+\vec{v}\|^2 \leq \|\vec{u}\|^2 + 2\|\vec{u}\|\cdot\|\vec{v}\| + \|\vec{v}\|^2 = (\|\vec{u}\| + \|\vec{v}\|)^2$$

Como os dois lados são ≥ 0, podemos fazer a raiz quadrada

c)

$$\|\vec{u}+\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = \|\vec{u}\|^2 + 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$$

$$\|\vec{u}-\vec{v}\|^2 = (\vec{u}-\vec{v})\cdot(\vec{u}+\vec{v}) = \|\vec{u}\|^2 - 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$$

$$\|\vec{u}+\vec{v}\|^2 - \|\vec{u}-\vec{v}\|^2 = (\|\vec{u}\|^2 + 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2) - (\|\vec{u}\|^2 - 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2)$$

$$= 2\vec{u}\cdot\vec{v} + 2\vec{u}\cdot\vec{v} = 4\vec{u}\cdot\vec{v}$$

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5-

$(1^2+0^2+1^2)=2$

a) $\vec{u}=(1,0,1), \vec{v}=(-2,10,2$ $\quad \|\vec{u}\|=\sqrt{2}$

$\vec{u}\cdot\vec{v}=(1,0,1)\cdot(-2,10,2)=0$

$\|\vec{v}\|=\sqrt{(-2)^2+10^2+2^2}$

$\sqrt{4+100+4}=\sqrt{108}$

$=\dfrac{0}{\sqrt{2}\cdot\sqrt{108}}=0$

$\theta=\arccos(0)$ rad

b) $\vec{u}=(-1,1,1), \vec{v}=(1,1,1)$ $\quad \|\vec{u}\|=\sqrt{1^2+1^2+1^2}$ $\quad \|\vec{v}\|=\sqrt{1^2+1^2+1^2}$

$\vec{u}\cdot\vec{v}=\cancel{-1}+\cancel{1}+1=1$ $\quad \sqrt{1+1+1}=\sqrt{3}$ $\quad \sqrt{1+1+1}=\sqrt{3}$

$\dfrac{1}{\sqrt{3}\cdot\sqrt{3}}=\dfrac{1}{\sqrt{9}}=\dfrac{1}{3}=\theta=\arccos\left(\frac{1}{3}\right)$ rad

c) $\vec{u}=(3,3,0), \vec{v}=(2,1,-2)$

$\vec{u}\cdot\vec{v}=6+3+0=9$ $\quad \|\vec{u}\|=\sqrt{3^2+3^2+0}=\sqrt{18}$

$\|\vec{v}\|=\sqrt{2^2+1^2+(-2)^2}=\sqrt{9}=3$

$\cos\theta=\dfrac{9}{\sqrt{18}\cdot 3}=\dfrac{9}{3\sqrt{18}}=\dfrac{3}{\sqrt{18}}=\dfrac{3}{3\sqrt{2}}=\dfrac{1}{\sqrt{2}}\Rightarrow\theta=\cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

$\theta=\arccos\left(\dfrac{1}{\sqrt{2}}\right)$ rad

d) $\vec{u}=(\sqrt{3},1,0), \vec{v}=(\sqrt{3},1,2\sqrt{3})$

$\vec{u}\cdot\vec{v}=\sqrt{3}\cdot\sqrt{3}+1\cdot 1+0\cdot 2\sqrt{3}=3+1+0=4$

$\|\vec{u}\|=\sqrt{3+1+0}=\sqrt{4}=2$ $\quad \cos\theta=\dfrac{4}{2\cdot 4}=\dfrac{1}{2}\Rightarrow$

$\|\vec{v}\|=\sqrt{3+1+12}=\sqrt{16}=4$

$\theta=\arccos\left(\dfrac{1}{2}\right)$ rad

Credeal

6-

Ortogonal para que $\vec{u}$ e $\vec{v}$ sejam ortogonal $\vec{u} \cdot \vec{v} = 0$

a) $\vec{u} = (x+1, 1, 2)$, $\vec{v} = (x-1, -1, -2)$

P.E.

$(x+1)(x-1) + 1 \cdot (-1) + 2 \cdot (-2) = 0$

$(x+1) \cdot (x-1) = x^2 - 1$

$x^2 - 1 - 1 - 4 = 0 \Rightarrow x^2 - 6 = 0 \to x = \pm\sqrt{6}$

b) $\vec{u} = (x, x, 4)$, $\vec{v} = (4, x, 1$

$x \cdot 4 + x \cdot x + 4 \cdot 1 = 0$

$4x + x^2 + 4 = 0$

$x^2 + 4x + 4 = 0 \to (x+2)^2 = 0 \Rightarrow x = -2$

7-

a) 1. $\vec{u} \cdot \vec{v} = 0$:

$\vec{}$ $\vec{}$ $4x - y + 5z = 0$

2. $\vec{u} \cdot \vec{w} = 0$:

$x - 2y + 3z = 0$

3. $\vec{u} \cdot (1, 1, 1) = -1$

$x + y + z = -1$

$\begin{cases} 4x - y + 5z = 0 \\ x - 2y + 3z = 0 \end{cases}$ $L_2 \cdot 2 = 4x - 8y + 12z = 0$

$(4x - y + 5z - (4x - 8y + 12z) = 0$

$-1y + 5z - (-8y + 12)z = 0$

$7y - 7z = 0 \Rightarrow y = z$

$x = -7$ $\quad$ $x - 2z + 3z = 0 \Rightarrow x + z = 0 \Rightarrow x = -z$

$y = z$

$z = z$

$\vec{u} = (-z, z, z)$

$x + y + z = -1 \Rightarrow -z + z + z = -1 \Rightarrow z = -1$

$x = 1$, $y = -1$, $z = -1$ $\vec{u} = (1, -1, -1$

b) $\vec{v} \cdot \vec{w} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{vmatrix} \begin{matrix} i & j \\ 2 & 3 \\ 2 & -4 \end{matrix}$ $18i - 2j - 8k - 12j - 4i - 6k = 0$

$14i - 14j - 14k = 0$

$(14a, -14a, -14a) = 3\sqrt{3}$

$\sqrt{(14a)^2 + (-14a)^2 + (-14a)^2} = \sqrt{(196a^2) \cdot 3} = \sqrt{588} = 3\sqrt{3}$

$\sqrt{3} \cdot \sqrt{196} \cdot |u| = 3\sqrt{3}$

$\cancel{\sqrt{3}} \cdot 14 \cdot |u| = 3\cancel{\sqrt{3}}$

$14|u| = 3$

$|u| = \pm\frac{3}{14}$

para $u = \frac{3}{14}$; $\frac{3}{14}(14, -14, -14)$

$\vec{u} = (3, -3, -3)$

para $a = -\frac{13}{4}$; $\vec{u} = (-3, 3, 3)$

$\vec{u_1} \cdot \vec{i} = (3, -3, -3) \cdot (1, 0, 0)$

$\vec{u_1} \cdot \vec{i} = 3 \Rightarrow 3 > 0 \Rightarrow$ forma ângulo agudo

ângulo agudo = produto escalar positivo

$\vec{u}_2 \cdot \vec{i} = (-3, 3, 3) \cdot (1, 0, 0)$

$\vec{u}_2 \cdot \vec{i} = -3 \Rightarrow -3 < 0 \Rightarrow$ não forma agudo

R: $\therefore \vec{u_1}$ forma ângulo agudo com $\vec{i}$, mas $\vec{u}_2$ não.

Credeal

c)

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θ entre dois vetores $\vec{a}$ e $\vec{b}$ é dado por

$$\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{u}\| \|\vec{v}\|}$$

calculamos cos do ângulo entre $\vec{u} + \vec{v}$ e $\vec{u} - \vec{v}$

PE: $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = \|\vec{u}\|^2 - \|\vec{v}\|^2 = 5 - 1 = 4$

Norma

$$\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\vec{u} \cdot \vec{v} = 5 + 1 + 2(\sqrt{5} \cdot 1 \cdot \cos(\pi/4)) =$$

$$\to 6 + 2\sqrt{5} \cdot \frac{\sqrt{2}}{2} = 6\sqrt{10}$$

$$\|\vec{u} - \vec{v}\|^2 = 5 + 1 - 2\sqrt{5} \cdot \frac{\sqrt{2}}{2} = 6 - \sqrt{10}$$

$$\|\vec{u} + \vec{v}\| = \sqrt{6 + \sqrt{10}}, \quad \|\vec{u} - \vec{v}\| = \sqrt{6 - \sqrt{10}}$$

PM:

$$\|\vec{u} + \vec{v}\| \cdot \|\vec{u} - \vec{v}\| = \sqrt{(6 + \sqrt{10}) \cdot (6 - \sqrt{10})} = \sqrt{36 - 10} = \sqrt{26}$$

cos do ângulo:

$$\cos(\theta) = \frac{4}{\sqrt{26}} \to \theta = \arccos\left(\frac{4}{\sqrt{26}}\right)$$

$$\therefore \theta = \arccos\left(\frac{4}{\sqrt{26}}\right) \approx 0{,}566$$

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8) $proj_{\vec{u}}\vec{v} = (\vec{v}\cdot\vec{u})\cdot\vec{u}$

a)

$\vec{v} = (1,-1,2),\ \vec{u} = (3,-1,1)$

produto escalar $\vec{v}\cdot\vec{u} = 1\cdot3 + (-1)\cdot(-1) + 2\cdot1 = 3+1+2 = 6$

Projeção: $proj_{\vec{u}}\vec{v} = 6\cdot(3,-1,1) = (18,-6,6)$

b)

$\vec{v} = (1,3,5),\ \vec{u} = (-3,1,0)$

$\vec{v}\cdot\vec{u} = 1\cdot(-3) + 3\cdot1 + 5\cdot0 = -3+3+0 = 0$

$proj_{\vec{u}}\vec{v} = 0\cdot(-3,1,0) = (0,0,0)$

c) $\vec{v} = (-1,1,1)\ \vec{u} = (-2,1,2)$

$\vec{v}\cdot\vec{u} = (-1)\cdot(-2) + 1\cdot1 + 1\cdot2 = 2+1+2 = 5$

$proj_{\vec{u}}\vec{v} = 5(-2,1,2) = (-10,5,10)$

d) $\vec{v} = (1,2,4),\ \vec{u} = (-2,-4,-8)$

$\vec{v}\cdot\vec{u} = -2 + (-8) + (-32) = -42$

$proj_{\vec{u}}\vec{v} = -42\cdot(-2,-4,-8) = (84,168,336)$

9)

$\vec{u} = 2\vec{i} - 2\vec{j} + \vec{k} = (2,-3,1)$

$\vec{v} = 3\vec{i} - 6\vec{j} = (3,-6,0)$

proj ortogonal

$proj_{\vec{u}}\vec{v} = \frac{\vec{v}\cdot\vec{u}}{\|\vec{u}\|^2}\cdot\vec{u}$

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a)

$\vec{v} \cdot \vec{u} = 3 \cdot 2 + (-6) \cdot (-2) + 0 \cdot 1 = 6 + 12 + 0 = 18$

Norma ao quadrado de $\vec{u}$:

$\|\vec{u}\|^2 = 2^2 + (-2)^2 + 1^2 = 4 + 4 + 1 = 9$

projeção: $proj_{\vec{u}} \vec{v} = \frac{18}{9} \cdot \vec{u} = 2 \cdot (2, -2, 1) = (4, -4, 2)$

$\therefore \vec{u} \cdot \vec{v} = 2 \cdot 3 + (-2)(-6) + 1 \cdot 0 = 6 + 12 + 0 = 18$

$\|\vec{v}\|^2 = 3^2 + (-6)^2 + 0^2 = 9 + 36 = 45$

$proj_{\vec{v}} \vec{u} = \frac{18}{45} \cdot \vec{v} = \frac{2}{5} \cdot (3, -6, 0) = \left(\frac{6}{5}, -\frac{12}{5}, 0\right)$

b) $\vec{p}$ é paralelo a $\vec{u}$ $\rightarrow$ é a projeção de $\vec{v}$ sobre $\vec{u}$

$\vec{q}$ é ortogonal a $\vec{u}$ $\rightarrow$ $\vec{q} = \vec{v} - \vec{p}$

$\vec{p} = (4, -4, 2)$

$\vec{q} = \vec{v} - \vec{p} = (3, -6, 0) - (4, -4, 2) = (-1, -2, -2)$

c)

$A = \|\vec{u} \times \vec{v}\|$

$\vec{u} = (2, -2, 1), \ \vec{v} = (3, -6, 0)$

$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 1 \\ 3 & -6 & 0 \end{vmatrix} = +\vec{i} \cdot (-2 \cdot 0 - 1 \cdot (-6)) - \vec{j} \cdot (2 \cdot 0 - 1 \cdot 3) + \vec{k} \cdot (2 \cdot (-6) - (-2) \cdot 3)$

$= +\vec{i}(6) - \vec{j}(-3) + \vec{k}(-12 + 6) = (6, 3, -6)$

$\|\vec{u} \times \vec{v}\| = \sqrt{6^2 + 3^2 + (-6)^2} = \sqrt{36 + 9 + 36} = \sqrt{81} = 9$

Credeal

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10)

a)

$\vec{u} = 3i + 3j = (3,3,0),\ \vec{v} = 5\vec{i} + 4j = (5,4,0)$

P.V.:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{vmatrix} = \vec{k}(3 \cdot 4 - 3 \cdot 5) = \vec{k}(12 - 15) = -3\vec{k}$$

norma: $\|\vec{u} \times \vec{v}\| = \|(0,0,-3)\| = 3$

b)

$\vec{u} = (7,0,-5),\ \vec{v} = (1,2,-1)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 0 & -5 \\ 1 & 2 & -1 \end{vmatrix} = i(0 \cdot (-1) - (-5) \cdot (2)) - \vec{j}(7 \cdot (-1) - (-5)(1)) + \vec{k}(7 \cdot 2 - 0 \cdot 1)$$

$= \vec{i}(10) - \vec{j}(-7+5) + \vec{k}(14) = (10,2,14)$

norma: $\|\vec{u} \times \vec{v}\| = \sqrt{10^2 + 2^2 + 14^2} = \sqrt{100+4+196} = \sqrt{300} = 10\sqrt{3}$

c)

$\vec{u} = (1,-3,1),\ \vec{v} = (1,1,4)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix} = \vec{i}((-3) \cdot (4) - 1 \cdot 1) - \vec{j}(1 \cdot 4 - 1 \cdot 1) + \vec{k}(1 \cdot 1 - (-3) \cdot (1))$$

$= \vec{i}(-12-1) - \vec{j}(4-1) + \vec{k}(1+3) = (-13,-3,4)$

$\|\vec{u} \times \vec{v}\| = \sqrt{(-13)^2 + (-3)^2 + 4^2} = \sqrt{169+9+16} = \sqrt{194}$

d) $\vec{u} = (2,1,2),\ \vec{v} = (4,2,4)$ $\vec{v} = 2\vec{u}$ + são vetores paralelos, então $\vec{u} \times \vec{v} = \vec{0}$, $\|\vec{u} \times \vec{v}\| = 0$

d) $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{vmatrix} = \vec{i}\,(1 \cdot 4 - 2 \cdot 2) - \vec{j}\,(2 \cdot 4 - 2 \cdot 4) + \vec{k}\,(2 \cdot 2 - 1 \cdot 4)$

$= \vec{i}\,(4-4) - \vec{j}\,(8-8) + \vec{k}\,(4-4)$

$= \vec{i}\,(0) - \vec{j}\,(0) + \vec{k}\,(0)$

$= 0\vec{i} + 0\vec{j} + 0\vec{k}$

11

a) $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \cdot \|\vec{v}\| \sin(\theta)$

$\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2 \|\vec{v}\|^2 \sin^2(\theta)$

E:

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta \Rightarrow (\vec{u} \cdot \vec{v})^2 = \|\vec{u}\|^2 \|\vec{v}\|^2 \cos^2(\theta)$

somando:

$\|\vec{u} \times \vec{v}\|^2 + (\vec{u} \cdot \vec{v})^2 = \|\vec{u}\|^2 \|\vec{v}\|^2 (\sin^2(\theta) + \cos^2(\theta)) = \|\vec{u}\|^2 \|\vec{v}\|^2$

Logo

$\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2 \|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$

demonstrado!

b)

dados: $\vec{u} \cdot \vec{v} = 3$, $\|\vec{u}\| = 1$, $\|\vec{v}\| = 5$

$\|\vec{u} \times \vec{v}\|^2 = (1)^2 \cdot (5)^2 - 3^2 = 25 - 9 = 16 = \|\vec{u} \times \vec{v}\| = \sqrt{16} = 4$

c)

Lado do triângulo l $\|\vec{AB} \times \vec{AC}\|$

Área $= \frac{1}{2} \|\vec{AB} \times AC$

Área do triângulo equilátero é:

Área $= \frac{\sqrt{3}\, l^2}{4}$

Logo:

$\frac{1}{2} \|\vec{AB} \times \vec{AC}\| = \frac{\sqrt{3}}{4} l^2 \rightarrow \|\vec{AB} \times \vec{AC}\| =$

$\hookrightarrow \frac{\sqrt{3}}{2} l^2$

/ /

12.

a)

$$\begin{cases} \vec{x} \cdot (2\vec{i} + 3\vec{j} + 4\vec{k}) = 9 \\ \vec{x} \times (-\vec{i} + \vec{j} - \vec{k}) = -2\vec{i} + 2k \end{cases} \qquad x = (a, b, c)$$

eq 1: $2a + 3b + 4c = 9$ I

II $= -b - c = -2$

eq 2: $\begin{vmatrix} i & j & k \\ a & b & c \\ -1 & 1 & -1 \end{vmatrix} \begin{matrix} i & j \\ a & b \\ -1 & 1 \end{matrix}$

$-b_i - cj + ak + aj - ci + bk$

$i(-b-c) - j(c-a) + k(a+b)$

$-2i + 0j + 2k$

II $= -b - c = -2$

III $= a = c$

IV $= a + b = 2$

$$\begin{cases} \text{I} = 2a + 3b + 4c = 9 \\ \text{III} = a = c \\ \text{IV} = a + b = 2 \end{cases}$$

$\therefore \vec{x} = \vec{i} + \vec{j} + \vec{k}$ $\Rightarrow b = 2 - a$

I $= 2a + 6 - 3a + 4a = 9$

$3a = 3 \quad c = a = 1$

$a = 1 \quad b = 2 - 1 = 1$

b)

$$\begin{cases} \vec{x} \times (1, 0, 1) = 2(1, 1, -1) = (2, 2, -2) \\ \|\vec{x}\| = \sqrt{6} \end{cases}$$

$\vec{x} = (a, b, c)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ 1 & 0 & 1 \end{vmatrix} = \vec{i}(b \cdot 1 - c \cdot 0) - \vec{j}(a \cdot 1 - c \cdot 1) + \vec{k}(a \cdot 0 - b \cdot 1) \Rightarrow (b, c-a, -b)$

$b, c - a, -b = 2, 2, -2 \Rightarrow b = 2, \ c - a = 2, \ b = -2 \Rightarrow c = a + 2$

Credeal

1) b)

a)

Nova: $\vec{x} = (a, 2, a+2)$

$$\|x\|^2 = a^2 + 4 + (a+2)^2 = a^2 + 4 + a^2 + 4a + 4 = 2a^2 + 4a + 8$$

$2a^2 + 4a + 8 = 6 \Rightarrow 2a^2 + 4a + 2 = 0 \Rightarrow a^2 + 2a + 1 = 0 \Rightarrow (a+1)^2 = 0 \Rightarrow$

$a = -1 \Rightarrow b = 2, c = a + 2 = 1$

$$\therefore \vec{x} = -\vec{i} + 2\vec{j} + \vec{k}$$

c)

- $\|\vec{x}\| = \sqrt{3} \Rightarrow a^2 + b^2 + c^2 = 3$
- $\vec{x} \cdot \vec{u} = 0$, com $\vec{u} = (-3, 0, 3)$
- $\vec{x} \cdot \vec{v} < 0$, com $\vec{v} = (2, -2, 1)$ (ângulo obtuso

$$\vec{x} \cdot \vec{u} = -3a + 0 + 3c = 0 \Rightarrow a = c$$

Nova:

$$a^2 + b^2 + c^2 = 3 \Rightarrow 2a^2 + b^2 = 3 \text{ (já que } a = c)$$

Ângulo obtuso com $\vec{j} \rightarrow \vec{x} \cdot \vec{v} < 0$:

$$\vec{x} \cdot \vec{v} = 2a - 2b < 0 \Rightarrow a < b$$

? $a = 1$

$c = 1, \therefore 2(1)^2 + b^2 = 3 \Rightarrow 2 + b^2 = 3 \Rightarrow b^2 = 1 \Rightarrow b = \pm 1$

$b = 1 \Rightarrow a = 1 < b$ × não satisfaz

$b = -1 \Rightarrow a = 1 \Rightarrow$ ✓ (ângulo obtuso)

$$\therefore \vec{x} = \vec{i} - \vec{j} + \vec{k}$$

Credeal

/ /

13.

1)

Dado

paralelogramo ABCD

$\vec{AB} = (1,1,-1)$

$A = (3,2,-1)$

$D = (5,3,3)$

$\rightarrow AD$

$\vec{AD} = D - A = (5,3,3) - (3,2,-1) = (2,1,4)$

Produto vetorial $AB \times AD$

$\vec{AB} \times AD = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & 1 & 4 \end{vmatrix} = i(1.4 - (-1).1) - j(1.4 - (-1).2)$
$k(1.1 - 1.2) = (4+1)i - (4+2)j + 1k =$

$||\vec{AB} \times \vec{AD}|| = \sqrt{5^2 + (-6)^2 + (-1)^2} \rightarrow$ $= 5i - 6j - 1k$

$\sqrt{25 + 36 + 1} = \sqrt{62}$ Δ

2) $AB = (-1,1,0)$

$AC = (0,1,3)$

~~Produto~~

$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} = i(1.3 - 0.1) - j(-1.3 - 0.0) + k$
$+ k(-1 \cdot 1 - 1.0) = 3i + 3j - 1k$

$|\vec{AB} \times \vec{AC}| = \sqrt{3^2 + 3^2 + (-1)^2} = \sqrt{9+9+1} = \sqrt{19}$

$\frac{1}{2} |\vec{AB} \times \vec{AC}|$ $\quad A = \frac{1}{2} \cdot \sqrt{19} = A_{ABC}\ \frac{\sqrt{19}}{2}$

Credeal

1V.

a) Ambos os lados representam o produto misto entre os vetores $\vec{u}, \vec{v}, \vec{w}$, e o valor numérico é:

Demonstração: $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$

$$[\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Esse determinante é invariante

Por ciclicidade, ou seja: $[\vec{u}, \vec{v}, \vec{w}] = [\vec{v}, \vec{w}, \vec{u}] = [\vec{w}, \vec{u}, \vec{v}]$

Portanto, a igualdade: $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$

é verdadeira pois ambos representam o mesmo valor escalar do produto misto

b)

Dados:

$\vec{u} = (1, 3, 2)$

$\vec{v} = (0, 1, -2)$

$\vec{w} = (-1, 2, 0)$

Calcular o produto misto $[\vec{u}, \vec{v}, \vec{w}]$:

$$C_1 \Rightarrow \begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{vmatrix} = 1 \cdot (1 \cdot 0 - (-2 \cdot 2)) - 3 \cdot (0 \cdot 0 - (-2) \cdot (-1)) + 2 \cdot (0 \cdot 2 - 1 \cdot (-1))$$
$$= 1(0+4) - 3(0-2) + 2(0+1) = 4 - 6 + 2 = \boxed{0}$$

Misto de $[\vec{v}, 2\vec{w}, \vec{u}]$

$$C_2 \Rightarrow \begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{vmatrix} = 0(4 \cdot 2 - 0 \cdot 3) - 1(-2 \cdot 2 - 0 \cdot 1) + (-2)(-2 \cdot 3 - 4 \cdot 1)$$
$$= 0 - 1(-4) + (-2)(-6 - 4) = 4 + 20 = \boxed{24}$$

$\boxed{0}$

$$\rightarrow [\vec{u}, 3\vec{v} - 2\vec{u}, \vec{w} + 3\vec{u}]$$

Vetores intermediários:

$3\vec{v} = (0, 3, -6)$
$2\vec{u} = (2, 6, 4)$
$3\vec{u} = (3, 9, 6)$

$3\vec{v} - 2\vec{u} = (0-2, 3-6, -6-4) = (-2, -3, -10)$
$\vec{w} + 3\vec{u} = (-1+3, 2+9, 0+6) = (2, 11, 6)$

$$\begin{vmatrix} 1 & 3 & 2 \\ -2 & -3 & -10 \\ 2 & 11 & 6 \end{vmatrix} = 1((-3)(6) - (-10)(11)) - 3((-2)(6) - (-10)(2)) + 2((-2)(11) - (-3)(2)) = 1(-18+110) - 3(-12+20) + 2(-22+6) =$$
$$= 1(92) - 3(8) + 2(-16) = 92 - 24 - 32 = \boxed{36}$$

$\therefore [\vec{u}, 3\vec{v} - 2\vec{u}, \vec{w} + 3\vec{u}] = 36$

15-

Dados:

- $\vec{AB} = (1, 0, 1)$
- $\vec{BE} = (2, 2, 2)$
- $\vec{AF} = (3, 5, 6)$

$\vec{AD} = \vec{AF} - \vec{BE} = (3, 5, 6) - (2, 2, 2) = (1, 3, 4)$
$\vec{AE} = \vec{AB} + \vec{BE} = (1, 0, 1) + (2, 2, 2) = (3, 2, 3)$

a)

$A_{ret} = \|\vec{AB} \times \vec{AD}$

$AB \times AD = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 1 & 3 & 4 \end{vmatrix} = -i(0\cdot 4 - 1\cdot 3) + j(1\cdot 4 - 1\cdot 1) + K(1\cdot 3 - 0\cdot 1) =$
$= (-3, -3, 3)$

$\|\vec{AB} \times AD\| = \sqrt{(-3)^2 + (-3)^2 + 3^2} = \sqrt{27} = 3\sqrt{3}$

b)

$V_{altura} = |\vec{AB} \cdot (\vec{AD} \times \vec{AE})|$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 3 \\ 3 & 2 & 3 \end{vmatrix} = \vec{i}(3\cdot 3 - 3\cdot 2) + j(0\cdot 3 - 3\cdot 3) - K(0\cdot 2 - 3\cdot 3) = (9-6, 0-9, 0-9$
$= (3, 9, -9$

$(1,0,1)\cdot(3,9,-9) = 1\cdot 3 + 0 + 1\cdot(-9) = 3 - 9 = -6 \Rightarrow |-6| = 6$

c)

$h_{altura} = \frac{V}{Area\,base} = \frac{6}{3\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

d)

~~$V = \frac{1}{6}|AB \cdot (AD \times AE$~~

d) $\frac{1}{6}|(AB \cdot AD) AE|$ volume do tetraedo

$\frac{1}{6}\cdot$

$V = \frac{1}{6}|\vec{u} \cdot (\vec{v} \times \vec{w})|$

$V = \frac{1}{6}\cdot 6 = 1$

e)

/ /

1)

$\frac{1}{3}$ Área base · h = altura do tetraedro

$$\frac{1}{3} \cdot \frac{3\sqrt{3}}{1} \cdot \frac{2\sqrt{3}}{3} \rightarrow \frac{6\sqrt{9}}{9} \rightarrow \frac{6 \cdot 3}{9} \rightarrow \frac{18}{9} = 2$$

Volume = 2 u.v³

Credeal